

VELOCITY AND THE VARIABILITY OF MONEY GROWTH: EVIDENCE FROM A VARMA, GARCH-M MODEL

APOSTOLOS SERLETIS
University of Calgary

AKBAR SHAHMORADI
University of Tehran

This paper uses recent advances in financial econometrics to test the Friedman hypothesis that money supply volatility Granger-causes velocity. Comparisons are made among simple-sum and Divisia velocity series at the M1 and M2 levels of monetary aggregation, using quarterly data from 1959:1 to 2004:3. The conclusion is that the Friedman hypothesis cannot be rejected if money supply volatility is modeled explicitly, using models that capture important volatility effects that previous work has ignored.

Keywords: Multivariate GARCH, Variability of Money Growth, Simple-Sum, Divisia

1. INTRODUCTION

A substantial amount of attention has been focused on the behavior of the velocity of money in the United States, primarily as the result of a relatively sudden and unanticipated decline in the velocity series in 1981. The most powerful element of this statistically unusual decline in velocity is the collapse of the longer-run relationship connecting money to both income and prices. In fact, whether velocity is stable or at least predictable is essential to any empirical interpretation of the monetarist position and especially relevant for some problems of potential importance in the practical conduct of monetary policy.

The debate that has arisen mostly concerns the rather abrupt decline in velocity, and quite a few specific hypotheses regarding the determinants of velocity have evolved from this discussion. Among the propositions advanced, those most often cited involve the influence of structural changes in the financial sector, tax cuts, inflation (or expected inflation), changes in government expenditures, changes in energy prices, and money growth along with its variability—see especially Hall and Noble (1987), Judd and Motley (1984), Tatom (1983), Santoni (1987), and

We would like to thank an anonymous referee for very useful comments that improved the paper considerably. Serletis also gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada (SSHRC). Address correspondence to: Apostolos Serletis, Department of Economics, University of Calgary, Calgary, Alberta, T2N 1N4, Canada; e-mail: Serletis@ucalgary.ca; Web: <http://econ.ucalgary.ca/serletis.htm>.

Fisher and Serletis (1989), for more on these and other suggested (and sometimes empirically supported) influences.

It is the variability of money growth that we wish to explore as an influence on velocity in this paper. There are essentially two avenues of influence, proposed by Friedman (1983, 1984). The first involves the behavior of real income and the second concerns the behavior of the demand for money (accommodated by money supply). Although for the *real income effect* Friedman's position is not precisely stated in this particular paper, for the *money demand effect* he argues that increased uncertainty in financial markets, because of the increased volatility of money after the 1979 change of policy-operating techniques in the United States, has produced both an increased demand for money for (essentially) precautionary purposes and, assuming that the money supply process accommodates the pressure, a rise in money holdings relative to nominal income. Thus, the authorities willing, the equilibrium stock of money would increase and velocity would decrease, with increases in the volatility of money growth.

Papers by Hall and Noble (1987), Brocato and Smith (1989), Fisher and Serletis (1989), and Thornton (1995) have offered mixed results regarding Friedman's money supply volatility hypothesis. They all used moving (sample) standard deviations of money growth rates to measure the variability of money growth and the Granger-causality method to test the general hypothesis that the variability of money growth causes velocity to change. Some of these papers also paid attention to misspecification issues as well as to issues of deterministic versus stochastic trend, on which the appropriate distribution theory depends crucially.

In extending the works of these authors, we have two objectives in this paper. Our first objective is to test the Friedman hypothesis using recent advances in the financial econometrics literature. In doing so, we specify and estimate a multivariate GARCH-M model of money growth and velocity, allowing for the effects of other determinants on velocity as well for the possibilities of spillovers and asymmetries in the variance-covariance structure for money growth and velocity growth. Our second objective is to investigate four measures of velocity to deal with anomalies that arise because of different definitions of money. The money measures employed are quarterly simple-sum and Divisia indices (from 1959:1 to 2004:3) for the United States.

The paper is organized as follows. Section 2 briefly describes the traditional approach to testing the Friedman hypothesis and reviews the relevant literature. Section 3 provides a description of the multivariate GARCH-M model of money growth and velocity growth that we use to test for Granger causality from money growth to velocity. Section 4 discusses the data and Section 5 presents and discusses the empirical results. The final section concludes the paper.

2. THE CONVENTIONAL APPROACH AND RESULTS

Hall and Noble (1987), Brocato and Smith (1989), Mehra (1989), Fisher and Serletis (1989), and Thornton (1995) all use the Granger-causality method to test

Friedman's hypothesis that money growth volatility is causal factor in changes in velocity. In doing so, they assume that the relevant information is contained in the present and past values of these variables and use the following bivariate autoregressive representation

$$v_t = \alpha_0 + \sum_{j=1}^r \alpha_j v_{t-j} + \sum_{j=1}^s \beta_j \text{VOL}_{t-j} + \varepsilon_t, \quad (1)$$

where $v_t = \Delta \log V_t$, V_t is the level of money velocity, VOL_t is the level of money growth volatility (calculated as a moving standard deviation of money growth), Δ is the difference operator, and ε_t is a white noise disturbance.

In this framework, the Granger procedure requires that we first estimate equation (1) by ordinary least squares to obtain the unrestricted sum of squared residuals, SSR_u . Then, by running another regression under the restriction that all β_j 's are zero, the restricted sum of squared residuals, SSR_r , is obtained. If ε_t is white noise, then the statistic,

$$\frac{(SSR_r - SSR_u)/s}{SSR_u/(T - r - s - 1)},$$

has an asymptotic F -distribution with the numerator having degrees of freedom of s and the denominator of $T - r - s - 1$, where T is the number of observations and 1 is subtracted out to account for the constant term in equation (1). If the null hypothesis of $\beta_j = 0$ for $j = 1, \dots, s$ cannot be rejected, then the conclusion is that the data do not show causality. If the null hypothesis is rejected, then the conclusion is that the data do show causality.

For monetary variability, the literature has produced mixed results. In particular, Hall and Noble (1987) using quarterly data (from 1963:1 to 1984:2), define monetary variability as an eight-quarter standard deviation of M1 growth, and find a causal relation from money growth variability to velocity. Fisher and Serletis (1989) test the same hypothesis that Hall and Noble (1987) did, using monthly U.S. data (covering the period 1970 through mid-1985), and search for the relationship over nine simple-sum and Divisia measures of the money stock, in an attempt to deal with anomalies that arise because of different definitions of money. Although they find some slight differences across the different monetary aggregates of equal breadth, they claim that the influence of money growth variability shows up on velocity in a (Granger) causal sense.

However, Brocato and Smith (1989), using monthly U.S. data from 1962:2 through 1985:9, reestimate the Hall and Noble (1987) equation over the full period, as well as over pre- and post-October 1979 periods. They find evidence to support the Friedman hypothesis in the pre-October 1979 period but find no evidence that the variability of money growth influenced velocity thereafter. They conclude that the change in Federal Reserve operating procedures in October 1979 contributed to a breakdown in the money growth/velocity growth relationship and that this finding runs counter to the Friedman hypothesis, which would suggest

stronger causality results after 1979, given the increase in money growth variability that took place.

The Brocato and Smith (1989) finding has also been confirmed by Mehra (1989), who shows that the Granger-causality result reported by Hall and Noble (1987) is not robust to some changes of specification and the sample period. Moreover, Thornton (1995) extends this work by testing Friedman’s hypothesis using quarterly data (over the period from 1961 to 1990) for nine countries—Australia, Austria, Canada, France, Germany, Italy, Japan, Switzerland, and the United Kingdom. He finds no evidence of the hypothesis that money supply volatility causes income velocity to change and concludes that “the Friedman hypothesis would appear to have little general applicability.”

3. AN ALTERNATIVE APPROACH

The conventional approach to testing the Friedman hypothesis that money supply volatility Granger-causes velocity uses moving standard deviations of money growth rates as measures of monetary variability. Such measures, however, are inappropriate because they are ad hoc, nonparametric estimates. In this paper, we use an extremely general asymmetric GARCH in Mean model of velocity and money growth to test the Friedman hypothesis. The model allows for the effects of other determinants on velocity, for the possibilities of spillovers and asymmetries in the variance-covariance structure, and also for the separate examination of the effects of the volatility of anticipated and unanticipated changes in money growth.

We use an extended version of a VARMA (vector autoregressive moving average) GARCH in mean model, in velocity growth (v_t) and money growth (μ_t), as follows

$$\begin{aligned}
 \mathbf{y}_t = & \mathbf{a} + \sum_{i=1}^p \Gamma_i \mathbf{y}_{t-i} + \sum_{j=1}^q \Psi_j \mathbf{h}_{t-j} + \sum_{k=1}^r \Phi_k \mathbf{z}_{t-k} + \sum_{l=1}^s \Theta_l \mathbf{e}_{t-l} \\
 & + \sum_{m=0}^t \Lambda_m \mathbf{x}_{t-m} + \mathbf{e}_t \tag{2}
 \end{aligned}$$

$$\mathbf{e}_t \mid \Omega_{t-1} \sim (\mathbf{0}, \mathbf{H}_t), \quad \mathbf{H}_t = \begin{bmatrix} h_{v_t} & h_{v_t \mu_t} \\ h_{v_t \mu_t} & h_{\mu_t} \end{bmatrix},$$

where Ω_{t-1} denotes the available information set in period $t - 1$ and

$$\begin{aligned}
 \mathbf{y}_t = & \begin{bmatrix} v_t \\ \mu_t \end{bmatrix}; \mathbf{e}_t = \begin{bmatrix} e_{v_t} \\ e_{\mu_t} \end{bmatrix}; \mathbf{h}_t = \begin{bmatrix} h_{v_t} \\ h_{\mu_t} \end{bmatrix}; \mathbf{a} = \begin{bmatrix} a_v \\ a_\mu \end{bmatrix}; \\
 \mathbf{x}_t = & \begin{bmatrix} g_t \\ \Delta R_t \end{bmatrix}; \Gamma_i = \begin{bmatrix} \gamma_{11}^{(i)} & \gamma_{12}^{(i)} \\ \gamma_{21}^{(i)} & \gamma_{22}^{(i)} \end{bmatrix}; \Psi_j = \begin{bmatrix} \psi_{11}^{(j)} & \psi_{12}^{(j)} \\ \psi_{21}^{(j)} & \psi_{22}^{(j)} \end{bmatrix};
 \end{aligned}$$

$$\Phi_k = \begin{bmatrix} \phi_{11}^{(k)} & \phi_{12}^{(k)} \\ \phi_{21}^{(k)} & \phi_{22}^{(k)} \end{bmatrix}; \Theta_l = \begin{bmatrix} \theta_{11}^{(l)} & \theta_{12}^{(l)} \\ \theta_{21}^{(l)} & \theta_{22}^{(l)} \end{bmatrix}; \Lambda_m = \begin{bmatrix} \lambda_{11}^{(m)} & \lambda_{12}^{(m)} \\ \lambda_{21}^{(m)} & \lambda_{22}^{(m)} \end{bmatrix};$$

$$z_{t-k} = \begin{bmatrix} z_{v_{t-k}} \\ z_{\mu_{t-k}} \end{bmatrix}; z_{i_{t-k}} = \frac{e_{i_{t-k}}}{\sqrt{h_{i_{t-k}}}}, \text{ for } i, j = v, \mu .$$

Notice that h_{t-j} and z_{t-k} have been introduced to take anticipated and unanticipated volatilities into account. Moving average terms are added to get rid of potential serial correlation; we will return to this issue in Section 5. Finally, regarding underlying cointegration concerns and the possible existence of error-correction terms that should be added to the model, we note that most of the previous literature cannot reject the null of no cointegration between velocity and money supply.

It is to be noted that previous research, in the framework of equation (1), ignores influences of other determinants of velocity such as income and interest rates. Friedman’s hypothesis, however, is about the effects of changes in the volatility of the supply of money. Hence, in the context of our model, we control for the effects that are a result of movements in money demand. In particular, because demand functions for M1 and M2 have been affected by the continuing wave of financial innovations and the interest rate deregulation, we capture their influences on velocity by including the change in the interest rate (ΔR_t) and the growth rate of real GDP (g_t) in equation (2). Hence, we investigate whether money growth volatility affects velocity, given its other determinants.

Multivariate GARCH models require that we specify volatilities of v and μ , measured by conditional variances. Several different specifications have been proposed in the literature, including the VECH model of Bollerslev, Engle, and Wooldridge (1988), the CCORR model of Bollerslev (1990), the FARCH specification of Engle, Ng, and Rothschild (1990), the BEKK model proposed by Engle and Kroner (1995), and the DCC model of Engle (2002). However, none of these specifications is basically capable of capturing the asymmetric volatility effects of v and μ in order to deal with good and bad news about money growth—in the sense that a negative money supply shock may generate a different response than a positive shock of the same magnitude.

In this regard, we use an asymmetric version of the BEKK model, introduced by Grier et al. (2004), as follows

$$H_t = C'C + \sum_{j=1}^f B'_j H_{t-j} B_j + \sum_{k=1}^g A'_k e_{t-k} e'_{t-k} A_k + D'u_{t-1} u'_{t-1} D \quad (3)$$

where C , B_j , A_k , and D are $n \times n$ matrices (for all values of j and k), with C being a triangular matrix to ensure positive definiteness of H . This specification allows past volatilities, H_{t-j} , as well as lagged values of ee' and uu' , to show up in estimating current volatilities of velocity and money supply, where $u_t = (u_v, u_\mu)'$ captures potential asymmetric responses. In particular, if money

growth is higher than expected, we take that to be bad news. We therefore capture bad news about money growth by a positive money growth residual, by defining $u_{\mu_t} = \max\{e_{\mu_t}, 0\}$. We also capture news about velocity growth by defining $u_{v_t} = \max\{e_{v_t}, 0\}$ and $u_{v_t} = \min\{e_{v_t}, 0\}$. In what follows we report the results for the case when $u_{v_t} = \max\{e_{v_t}, 0\}$. The results for the case when $u_{v_t} = \min\{e_{v_t}, 0\}$ are available on request and are qualitatively similar to the ones reported in the paper.

There are $n + n^2(p + q + r + s + t) + n(n + 1)/2 + n^2(f + g + 1)$ parameters in (2)–(3) and in order to deal with estimation problems in the large parameter space we assume that $f = g = 1$ in equation (3), consistent with recent empirical evidence regarding the superiority of GARCH(1,1) models—see, for example, Hansen and Lunde (2005).

It is also to be noted that we have not included an inflation measure in the model (in either y_t or z_{t-k}), although it would seem to be important to distinguish between the effect of money supply volatility and inflation volatility. We have kept the dimension of the model low because of computational and degree of freedom problems in the large parameter space. For example, with $n = 2$, $p = q = r = s = t = 2$ in equation (2) and $f = g = 1$ in equation (3), the model has 61 parameters to be estimated. If we introduce one more variable in the system (like the inflation rate), then we would have to estimate 135 parameters. Moreover, the tests that we conduct in Section 5 indicate that the exclusion of such a measure is not expected to result in significant misspecification error.

4. THE DATA

As it was mentioned in the introduction, the money measures employed are official simple-sum aggregates and Barnett's (1980)—see also Barnett, Fisher, and Serletis (1992)—monetary services indices (also known as Divisia aggregates) at the M1 and M2 levels of monetary aggregation. The broad measures of money (M3 and MZM) are not used in this paper because stable demand functions for these measures are unlikely to exist. The data were obtained from the St. Louis MSI database, maintained by the Federal Reserve Bank of St. Louis as a part of the Bank's Federal Reserve Economic Database (FRED)—see Anderson, Jones, and Nesmith (1997a,b) for details regarding the construction of the Divisia aggregates and related data. We use quarterly data from 1959:1 to 2004:3.

A battery of unit root and stationarity tests are conducted in Table 1 in the growth rates of the velocity and money series. In particular, in the first three columns of Table 1 we report p -values [based on the response surface estimates given by MacKinnon (1994)] for the augmented Weighted Symmetric (WS) unit root test (see Pantula et al. [1994]), the augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)], and the nonparametric $Z(t_{\hat{\alpha}})$ test of Phillips (1987) and Phillips and Perron (1988). Moreover, given that unit root tests have low power against relevant alternatives, in the last two columns of Table 1 we present Kwiatkowski et al. (1992) tests, known as KPSS tests, for level and trend stationarity. As can be seen, the null hypothesis of a unit root can be rejected at conventional significance

TABLE 1. Unit root and stationarity tests

Monetary aggregate	Series	Unit root tests			KPSS	
		WS	ADF	$Z(t_{\hat{\alpha}})$	$\hat{\eta}_{\mu}$	$\hat{\eta}_{\tau}$
Sum M1	v	0.006	0.019	0.000	0.162	0.077
	μ	0.026	0.046	0.000	0.219	0.211
Sum M2	v	0.000	0.000	0.000	0.085	0.082
	μ	0.128	0.138	0.000	0.465	0.174
Divisia M1	v	0.002	0.014	0.000	0.210	0.068
	μ	0.026	0.043	0.000	0.218	0.217
Divisia M2	v	0.000	0.000	0.000	0.199	0.112
	μ	0.019	0.030	0.000	0.188	0.127

Notes: Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of unit root tests. The 5% critical values for the KPSS $\hat{\eta}_{\mu}$ and $\hat{\eta}_{\tau}$ test statistics [given in Kwiatkowski et al. (1992)] are 0.463 and 0.146.

levels. Moreover, the t -statistics $\hat{\eta}_{\mu}$ and $\hat{\eta}_{\tau}$ that test the null hypotheses of level and trend stationarity are small relative to their 5% critical values of 0.463 and 0.146 (respectively), given in Kwiatkowski et al. (1992).

In Table 2, we conduct a series of Ljung-Box (1979) tests for serial correlation—the Q -statistics are asymptotically distributed as $\chi^2(36)$ on the null of no autocorrelation. Clearly, there is significant serial dependence in the data. Moreover, a Ljung-Box test for serial correlation in the squared data provides strong evidence of conditional heteroscedasticity, which is also confirmed by an ARCH test (in the second last column of Table 2), distributed as a $\chi^2(1)$ on the null of no ARCH. Finally, a Jarque-Bera (1980) test for normality, distributed as a $\chi^2(2)$ under the null hypothesis of normality, suggests that Sum M1, Divisia M1, and Divisia M2 velocity growth, as well Sum M2 money growth, fail to satisfy the null hypothesis of the test.

TABLE 2. Tests for serial correlation, arch, and normality

Monetary aggregate	Series	$Q(4)$	$Q(12)$	$Q^2(4)$	$Q^2(12)$	ARCH(4)	J-B
Sum M1	v	0.000	0.112	0.000	0.148	0.041	0.003
	μ	0.000	0.000	0.000	0.000	0.088	0.261
Sum M2	v	0.214	0.000	0.000	0.003	0.017	0.317
	μ	0.000	0.000	0.000	0.000	0.376	0.001
Divisia M1	v	0.000	0.000	0.096	0.079	0.123	0.025
	μ	0.000	0.000	0.000	0.000	0.028	0.384
Divisia M2	v	0.000	0.000	0.025	0.001	0.001	0.046
	μ	0.000	0.000	0.000	0.000	0.371	0.848

Note: Numbers are marginal significance levels.

5. EMPIRICAL EVIDENCE

Initially, we used the AIC and SIC criteria to select the optimal values of $p, q, r, s,$ and t in (2). However, because of computational difficulties in the large parameter space and remaining serial correlation and ARCH effects in the standardized residuals, we set $p = q = r = s = t = 2$ in equation (2). It is to be noted that by setting $s = 2$ we use scarce degrees of freedom to estimate the parameters of the MA part of the VARMA GARCH-M model, but we do so because the null hypothesis of no autocorrelation is primarily rejected for $s < 2$. Hence, with $n = 2, p = q = r = s = t = 2$ in equation (2), and $f = g = 1$ in equation (3), we estimate a total of 61 parameters. Quasi-maximum likelihood (QML) estimates of the parameters and diagnostic test statistics are presented in Tables 3–6, for the Sum M1, Sum M2, Divisia M1, and Divisia M2 monetary aggregates, respectively.

We conduct a battery of misspecification tests, using robustified versions of the standard test statistics based on the standardized residuals,

$$z_{j_t} = \frac{e_{j_t}}{\sqrt{\hat{h}_{j_t}}}, \quad \text{for } j = v, \mu.$$

As shown in Tables 3–6, the Ljung-Box Q -statistic for testing serial correlation cannot reject the null of no autocorrelation (at conventional significance levels) for the values and the squared values of the standardized residuals, suggesting that there is no evidence of conditional heteroscedasticity.

In addition, the failure of the data to reject the null hypotheses of $E(z) = 0$ and $E(z^2) = 1$, implicitly indicates that the multivariate asymmetric GARCH-M model does not bear significant misspecification error—see, for example, Kroner and Ng (1998).

In Table 7, we also present diagnostic tests suggested by Engle and Ng (1993) and Kroner and Ng (1998), based on the “generalized residuals,” defined as $e_{i_t}e_{j_t} - h_{i_tj_t}$ for $i, j = v, \mu$. For all symmetric GARCH models, the news impact curve—see Engle and Ng (1993)—is symmetric and centered at $e_{i_{t-1}} = 0$. A generalized residual can be thought of as the distance between a point on the scatterplot of $e_{i_t}e_{j_t}$ from a corresponding point on the news impact curve. If the conditional heteroscedasticity part of the model is correct, $E_{t-1}(e_{i_t}e_{j_t} - h_{i_tj_t}) = 0$ for all values of i and j , generalized residuals should be uncorrected with all information known at time $t - 1$. In other words, the unconditional expectation of $e_{i_t}e_{j_t}$ should be equal to its conditional one, $h_{i_tj_t}$. For example, if the model (2)–(3) gives a covariance news impact surface—a three-dimensional graph of $h_{v\mu_t}$, against e_{v_t} and e_{μ_t} —which is too low whenever the shock to the money growth rate is negative ($e_{\mu_t} < 0$), then the vertical distance between $h_{v\mu_t}$ and $e_{v_t}e_{\mu_t}$ will tend to be positive.

The Engle and Ng (1993) and Kroner and Ng (1998) misspecification indicators test whether we can predict the generalized residuals by some variables observed in the past, but which are not included in the model—this is exactly the intuition behind $E_{t-1}(e_{i_t}e_{j_t} - h_{i_tj_t}) = 0$. In this regard, we follow Kroner and Ng (1998)

TABLE 3. The Multivariate Asymmetric GARCH-M Model: Sum M1

Model: Equations (2) and (3) with $p = q = r = s = t = 2$ and $f = g = 1$						
Conditional mean equation						
$\mathbf{a} = \begin{bmatrix} 0.413 \\ (0.016) \\ 0.498 \\ (0.015) \end{bmatrix}; \Gamma_1 = \begin{bmatrix} 0.693 & 0.374 \\ (0.003) & (0.003) \\ -0.100 & 0.297 \\ (0.003) & (0.003) \end{bmatrix}; \Gamma_2 = \begin{bmatrix} 0.041 & -0.330 \\ (0.003) & (0.003) \\ 0.309 & 0.645 \\ (0.003) & (0.003) \end{bmatrix};$						
$\Psi_1 = \begin{bmatrix} -0.066 & -0.079 \\ (0.001) & (0.002) \\ -0.076 & 0.049 \\ (0.001) & (0.002) \end{bmatrix}; \Psi_2 = \begin{bmatrix} -0.017 & 0.147 \\ (0.001) & (0.002) \\ -0.046 & -0.009 \\ (0.001) & (0.002) \end{bmatrix}; \Phi_1 = \begin{bmatrix} -0.716 & -0.326 \\ (0.032) & (0.031) \\ -1.896 & -0.210 \\ (0.032) & (0.033) \end{bmatrix};$						
$\Phi_2 = \begin{bmatrix} 0.778 & 0.249 \\ (0.033) & (0.032) \\ 1.648 & 0.405 \\ (0.033) & (0.034) \end{bmatrix}; \Theta_1 = \begin{bmatrix} -0.357 & -0.456 \\ (0.011) & (0.011) \\ 0.108 & 0.045 \\ (0.011) & (0.012) \end{bmatrix}; \Theta_2 = \begin{bmatrix} -0.148 & -0.037 \\ (0.012) & (0.013) \\ -0.148 & -0.297 \\ (0.012) & (0.014) \end{bmatrix};$						
$\Lambda_1 = \begin{bmatrix} 0.847 & 0.357 \\ (0.004) & (0.028) \\ 0.091 & -0.032 \\ (0.004) & (0.026) \end{bmatrix}; \Lambda_2 = \begin{bmatrix} -0.765 & 0.784 \\ (0.004) & (0.031) \\ 0.173 & -0.714 \\ (0.004) & (0.033) \end{bmatrix}; \Lambda_3 = \begin{bmatrix} -0.076 & 3.261 \\ (0.004) & (0.028) \\ -0.228 & -2.880 \\ (0.004) & (0.038) \end{bmatrix}$						
Residual diagnostics						
	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{v_t}	0.091 [0.214]	0.966 [0.982]	4.086 [0.394]	3.619 [0.460]	13.611 [0.326]	8.342 [0.757]
z_{μ_t}	-0.080 [0.265]	0.937 [0.968]	2.633 [0.620]	5.716 [0.221]	16.154 [0.184]	9.156 [0.689]
Conditional variance-covariance structure						
$\mathbf{C} = \begin{bmatrix} 2.567 & -2.317 \\ (0.014) & (0.027) \\ & 0.334 \\ & (0.072) \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0.696 & 0.192 \\ (0.009) & (0.023) \\ 0.407 & 0.474 \\ (0.011) & (0.020) \end{bmatrix};$						
$\mathbf{A} = \begin{bmatrix} 0.039 & -0.189 \\ (0.015) & (0.014) \\ 0.274 & -0.581 \\ (0.014) & (0.013) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} -0.004 & 0.003 \\ (0.791) & (1.015) \\ -0.259 & 0.270 \\ (0.026) & (0.036) \end{bmatrix}$						
Hypothesis testing						
Diagonal VARMA	$H_0: \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(i)} = \theta_{21}^{(i)} = 0$, for $i, l = 1, 2$					[0.000]
No GARCH	$H_0: \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j					[0.000]
No GARCH-M	$H_0: \psi_{ij}^k = \phi_{ij}^k = 0$, for all i, j, k					[0.000]
No asymmetry	$H_0: \delta_{ij} = 0$, for $i, j = 1, 2$					[0.000]
Diagonal GARCH	$H_0: \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$					[0.000]
No anticipated causality	$H_0: \psi_{12}^{(1)} = \psi_{12}^{(2)} = 0$					[0.000]
No unanticipated causality	$H_0: \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$					[0.000]
No causality	$H_0: \psi_{12}^{(1)} = \psi_{12}^{(2)} = \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$					[0.000]

and Shields et al. (2005) and define two sets of misspecification indicators. In a two-dimensional space, we first partition $(e_{v_{t-1}}, e_{\mu_{t-1}})$ into four quadrants in terms of the possible sign of the two residuals. Then, to shed light on any possible sign

TABLE 4. The Multivariate Asymmetric GARCH-M Model: Sum M2

Model: Equations (2) and (3) with $p = q = r = s = t = 2$ and $f = g = 1$						
Conditional mean equation						
$\mathbf{a} = \begin{bmatrix} -0.317 \\ (0.000) \\ 0.738 \\ (0.000) \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} 0.627 & -0.024 \\ (0.000) & (0.000) \\ 0.014 & 0.672 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} 0.021 & 0.013 \\ (0.005) & (0.000) \\ 0.279 & 0.349 \\ (0.003) & (0.000) \end{bmatrix};$						
$\mathbf{\Psi}_1 = \begin{bmatrix} -0.010 & -0.005 \\ (0.000) & (0.000) \\ -0.007 & 0.012 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Psi}_2 = \begin{bmatrix} -0.076 & 0.193 \\ (0.000) & (0.000) \\ -0.162 & 0.030 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Phi}_1 = \begin{bmatrix} -0.016 & -0.150 \\ (0.001) & (0.001) \\ -0.842 & 0.039 \\ (0.000) & (0.000) \end{bmatrix};$						
$\mathbf{\Phi}_2 = \begin{bmatrix} 0.029 & -0.100 \\ (0.001) & (0.001) \\ -0.342 & 0.011 \\ (0.001) & (0.001) \end{bmatrix}; \mathbf{\Theta}_1 = \begin{bmatrix} -0.057 & -0.054 \\ (0.000) & (0.000) \\ 0.134 & -0.278 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} -0.518 & -0.221 \\ (0.001) & (0.000) \\ -0.004 & -0.409 \\ (0.000) & (0.000) \end{bmatrix};$						
$\mathbf{\Lambda}_1 = \begin{bmatrix} 0.755 & 0.745 \\ (0.000) & (0.013) \\ 0.163 & -0.309 \\ (0.000) & (0.003) \end{bmatrix}; \mathbf{\Lambda}_2 = \begin{bmatrix} -0.748 & 0.809 \\ (0.000) & (0.003) \\ 0.089 & -0.686 \\ (0.000) & (0.002) \end{bmatrix}; \mathbf{\Lambda}_3 = \begin{bmatrix} 0.013 & 1.771 \\ (0.000) & (0.009) \\ -0.273 & -1.455 \\ (0.0000) & (0.0033) \end{bmatrix}$						
Residual diagnostics						
	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{v_t}	0.049 [0.505]	1.006 [0.997]	5.510 [0.238]	6.283 [0.178]	17.558 [0.129]	11.870 [0.456]
z_{μ_t}	-0.037 [0.615]	0.998 [0.999]	3.998 [0.406]	3.584 [0.465]	13.609 [0.326]	12.940 [0.373]
Conditional variance-covariance structure						
$\mathbf{C} = \begin{bmatrix} 2.114 & -1.803 \\ (0.000) & (0.000) \\ & 0.591 \\ & (0.000) \end{bmatrix}; \mathbf{B} = \begin{bmatrix} -0.078 & 0.136 \\ (0.000) & (0.000) \\ 0.115 & -0.043 \\ (0.000) & (0.000) \end{bmatrix};$						
$\mathbf{A} = \begin{bmatrix} 0.465 & 0.040 \\ (0.000) & (0.000) \\ 0.028 & 0.420 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} -0.048 & 0.117 \\ (0.002) & (0.001) \\ -0.297 & -0.024 \\ (0.000) & (0.006) \end{bmatrix}$						
Hypothesis testing						
Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(i)} = \theta_{21}^{(i)} = 0, \text{ for } i, l = 1, 2$					[0.000]
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0, \text{ for all } i, j$					[0.000]
No GARCH-M	$H_0 : \psi_{ij}^k = \phi_{ij}^k = 0, \text{ for all } i, j, k$					[0.000]
No asymmetry	$H_0 : \delta_{ij} = 0, \text{ for } i, j = 1, 2$					[0.000]
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$					[0.000]
No anticipated causality	$H_0 : \psi_{12}^{(1)} = \psi_{12}^{(2)} = 0$					[0.000]
No unanticipated causality	$H_0 : \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$					[0.000]
No causality	$H_0 : \psi_{12}^{(1)} = \psi_{12}^{(2)} = \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$					[0.000]

bias of the model, we define the first set of indicator functions as $I(e_{v_{t-1}} < 0)$, $I(e_{\mu_{t-1}} < 0)$, $I(e_{v_{t-1}} < 0; e_{\mu_{t-1}} < 0)$, $I(e_{v_{t-1}} > 0; e_{\mu_{t-1}} < 0)$, $I(e_{v_{t-1}} < 0; e_{\mu_{t-1}} > 0)$ and $I(e_{v_{t-1}} > 0; e_{\mu_{t-1}} > 0)$, where $I(\cdot)$ equals one if the argument is true and zero

TABLE 5. The Multivariate Asymmetric GARCH-M Model: Divisia M1

Model: Equations (2) and (3) with $p = q = r = s = t = 2$ and $f = g = 1$												
Conditional mean equation												
$\mathbf{a} = \begin{bmatrix} 0.200 \\ (0.000) \\ 0.063 \\ (0.000) \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} 0.562 & 0.332 \\ (0.000) & (0.000) \\ -0.014 & 0.276 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} 0.032 & -0.319 \\ (0.000) & (0.000) \\ 0.335 & 0.698 \\ (0.000) & (0.000) \end{bmatrix};$												
$\mathbf{\Psi}_1 = \begin{bmatrix} -0.036 & -0.044 \\ (0.000) & (0.000) \\ -0.106 & 0.035 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Psi}_2 = \begin{bmatrix} -0.136 & 0.174 \\ (0.000) & (0.000) \\ -0.051 & 0.046 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Phi}_1 = \begin{bmatrix} -0.589 & -0.855 \\ (0.000) & (0.000) \\ -1.626 & -0.563 \\ (0.000) & (0.000) \end{bmatrix};$												
$\mathbf{\Phi}_2 = \begin{bmatrix} 1.890 & 0.811 \\ (0.000) & (0.000) \\ 1.368 & 1.747 \\ (0.002) & (0.000) \end{bmatrix}; \mathbf{\Theta}_1 = \begin{bmatrix} -0.626 & -0.254 \\ (0.000) & (0.000) \\ -0.177 & -0.644 \\ (0.001) & (0.000) \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} 0.062 & 0.264 \\ (0.000) & (0.001) \\ -0.499 & -0.754 \\ (0.000) & (0.002) \end{bmatrix};$												
$\mathbf{\Lambda}_1 = \begin{bmatrix} 0.800 & 0.871 \\ (0.000) & (0.001) \\ 0.148 & -0.679 \\ (0.000) & (0.001) \end{bmatrix}; \mathbf{\Lambda}_2 = \begin{bmatrix} -0.730 & 0.453 \\ (0.000) & (0.001) \\ 0.176 & -0.373 \\ (0.000) & (0.002) \end{bmatrix}; \mathbf{\Lambda}_3 = \begin{bmatrix} -0.046 & 2.482 \\ (0.000) & (0.002) \\ -0.271 & -2.209 \\ (0.000) & (0.003) \end{bmatrix}$												
Residual diagnostics												
	Mean		Variance		$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$				
z_{v_t}	0.043	[0.563]	0.993	[0.997]	5.464	[0.242]	4.261	[0.371]	20.283	[0.061]	5.473	[0.940]
z_{μ_t}	-0.013	[0.855]	0.962	[0.983]	4.834	[0.403]	4.555	[0.336]	25.758	[0.011]	6.795	[0.870]
Conditional variance-covariance structure												
$\mathbf{C} = \begin{bmatrix} 1.647 & -1.512 \\ (0.000) & (0.000) \\ & 0.411 \\ & (0.000) \end{bmatrix}; \mathbf{B} = \begin{bmatrix} -0.013 & 0.028 \\ (0.000) & (0.000) \\ 0.561 & -0.525 \\ (0.000) & (0.000) \end{bmatrix};$												
$\mathbf{A} = \begin{bmatrix} 0.549 & 0.458 \\ (0.000) & (0.000) \\ 0.259 & 0.795 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} 0.413 & -0.449 \\ (0.000) & (0.000) \\ -0.555 & 0.521 \\ (0.000) & (0.000) \end{bmatrix}$												
Hypothesis testing												
Diagonal VARMA	$H_0: \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(i)} = \theta_{21}^{(i)} = 0$, for $i, l = 1, 2$ [0.000]											
No GARCH	$H_0: \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j [0.000]											
No GARCH-M	$H_0: \psi_{ij}^k = \phi_{ij}^k = 0$, for all i, j, k [0.000]											
No asymmetry	$H_0: \delta_{ij} = 0$, for $i, j = 1, 2$ [0.000]											
Diagonal GARCH	$H_0: \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$ [0.000]											
No anticipated causality	$H_0: \psi_{12}^{(1)} = \psi_{12}^{(2)} = 0$ [0.000]											
No unanticipated causality	$H_0: \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$ [0.000]											
No causality	$H_0: \psi_{12}^{(1)} = \psi_{12}^{(2)} = \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$ [0.000]											

otherwise. Significance of any of these indicator functions indicates that the model (2)–(3) is incapable of predicting the effects of some shocks to either v_t or μ_t . Moreover, because the possible effect of a shock could be a function of both the

TABLE 6. The Multivariate Asymmetric GARCH-M Model: Divisia M2

Model: Equations (2) and (3) with $p = q = r = s = t = 2$ and $f = g = 1$						
Conditional mean equation						
$\mathbf{a} = \begin{bmatrix} -0.346 \\ (0.000) \\ 1.158 \\ (0.000) \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} 0.614 & 0.061 \\ (0.000) & (0.000) \\ 0.067 & 0.682 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} 0.098 & -0.010 \\ (0.000) & (0.000) \\ 0.197 & 0.306 \\ (0.000) & (0.000) \end{bmatrix};$						
$\mathbf{\Psi}_1 = \begin{bmatrix} 0.116 & -0.085 \\ (0.000) & (0.000) \\ 0.018 & -0.027 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Psi}_2 = \begin{bmatrix} 0.086 & -0.062 \\ (0.000) & (0.000) \\ -0.122 & -0.165 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Phi}_1 = \begin{bmatrix} -0.669 & -1.331 \\ (0.003) & (0.003) \\ -0.565 & 0.049 \\ (0.003) & (0.002) \end{bmatrix};$						
$\mathbf{\Phi}_2 = \begin{bmatrix} 0.231 & -0.557 \\ (0.004) & (0.003) \\ 0.333 & 0.125 \\ (0.003) & (0.002) \end{bmatrix}; \mathbf{\Theta}_1 = \begin{bmatrix} -0.223 & 0.302 \\ (0.001) & (0.001) \\ -0.031 & 0.020 \\ (0.001) & (0.001) \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} -0.357 & -0.006 \\ (0.001) & (0.002) \\ -0.096 & -0.484 \\ (0.001) & (0.002) \end{bmatrix};$						
$\mathbf{\Lambda}_1 = \begin{bmatrix} 0.839 & 0.355 \\ (0.000) & (0.005) \\ 0.073 & 0.099 \\ (0.000) & (0.007) \end{bmatrix}; \mathbf{\Lambda}_2 = \begin{bmatrix} -0.724 & 0.768 \\ (0.000) & (0.004) \\ 0.034 & -0.735 \\ (0.000) & (0.004) \end{bmatrix}; \mathbf{\Lambda}_3 = \begin{bmatrix} -0.150 & 2.249 \\ (0.000) & (0.007) \\ -0.114 & -1.850 \\ (0.000) & (0.008) \end{bmatrix}$						
Residual diagnostics						
Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$	
z_{v_t} 0.092 [0.194]	0.910 [0.954]	2.653 [0.617]	1.975 [0.740]	4.964 [0.326]	4.533 [0.971]	
z_{μ_t} -0.105 [0.139]	0.912 [0.960]	0.826 [0.934]	13.670 [0.934]	13.670 [0.322]	10.226 [0.596]	
Conditional variance-covariance structure						
$\mathbf{C} = \begin{bmatrix} 1.950 & -1.701 \\ (0.000) & (0.000) \\ & 0.443 \\ & (0.000) \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0.292 & -0.307 \\ (0.028) & (0.001) \\ & 0.302 \\ & (0.005) & (0.000) \end{bmatrix};$						
$\mathbf{A} = \begin{bmatrix} 0.318 & 0.511 \\ (0.001) & (0.001) \\ & 0.034 & 0.780 \\ & (0.001) & (0.000) \end{bmatrix}; \mathbf{D} = \begin{bmatrix} 0.350 & -0.1661 \\ (0.001) & (0.0011) \\ & 0.358 & -0.362 \\ & (0.003) & (0.001) \end{bmatrix}$						
Hypothesis testing						
Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(i)} = \theta_{21}^{(i)} = 0, \text{ for } i, l = 1, 2$					[0.000]
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0, \text{ for all } i, j$					[0.000]
No GARCH-M	$H_0 : \psi_{ij}^k = \phi_{ij}^k = 0, \text{ for all } i, j, k$					[0.000]
No asymmetry	$H_0 : \delta_{ij} = 0, \text{ for } i, j = 1, 2$					[0.000]
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$					[0.000]
No anticipated causality	$H_0 : \psi_{12}^{(1)} = \psi_{12}^{(2)} = 0$					[0.000]
No unanticipated causality	$H_0 : \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$					[0.000]
No causality	$H_0 : \psi_{12}^{(1)} = \psi_{12}^{(2)} = \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$					[0.000]

size and the sign of the shock, we define a second set of indicator functions, $e_{v_{t-1}}^2 I(e_{v_{t-1}} < 0)$, $e_{\mu_{t-1}}^2 I(e_{\mu_{t-1}} < 0)$, $e_{v_{t-1}}^2 I(e_{v_{t-1}} < 0)$, and $e_{\mu_{t-1}}^2 I(e_{\mu_{t-1}} < 0)$. These indicators are technically scaled versions of the former ones, with the magnitude of the shocks as a scale measure.

TABLE 7. Diagnostic tests based on the News Impact Curve

	Sum M1	Sum M2	Divisia M1	Divisia M2
$I(e_{v_{t-1}} < 0)$	0.245	0.255	0.751	0.147
$I(e_{\mu_{t-1}} < 0)$	0.236	0.129	0.788	0.094
$I(e_{v_{t-1}} < 0, e_{\mu_{t-1}} < 0)$	0.528	0.291	0.207	0.469
$I(e_{v_{t-1}} > 0, e_{\mu_{t-1}} < 0)$	0.885	0.633	0.573	0.358
$I(e_{v_{t-1}} < 0, e_{\mu_{t-1}} > 0)$	0.542	0.360	0.651	0.476
$I(e_{v_{t-1}} > 0, e_{\mu_{t-1}} > 0)$	0.646	0.906	0.872	0.079
$e^2_{v_{t-1}} I(e_{v_{t-1}} < 0)$	0.753	0.787	0.670	0.454
$e^2_{v_{t-1}} I(e_{\mu_{t-1}} < 0)$	0.180	0.097	0.961	0.044
$e^2_{\mu_{t-1}} I(e_{v_{t-1}} < 0)$	0.306	0.306	0.922	0.258
$e^2_{\mu_{t-1}} I(e_{\mu_{t-1}} < 0)$	0.712	0.389	0.498	0.269

Note: Numbers are tail areas of tests.

We conducted indicator tests for h_{vv} , $h_{\mu v}$, and $h_{\mu\mu}$ (that is 120 tests), but in Table 7 we only report the results for $h_{\mu v}$ which is the focus of attention in this paper. As can be seen in Table 7, all indicators (except the one for Divisia M2) fail to reject the null of no misspecification—all test statistics in Table 7 are distributed as $\chi^2(1)$. Hence, our model (2)–(3) captures the effects of all sign bias and sign-size scale depended shocks in predicting volatility and there is no significant misspecification error. This means that the exclusion of an inflation measure (in either y_t or z_{t-k}) is not expected to lead to significant misspecification problems.

Turning now back to Tables 3–6, the diagonality restriction, $\gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$ for $i, l = 1, 2$, is rejected, meaning that the data provide strong evidence of the existence of dynamic interactions between v_t and μ_t . The null hypothesis of homoscedastic disturbances requires the **A**, **B**, and **D** matrices to be jointly insignificant (that is, $\alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$ for all i, j) and is rejected at the 0.01% level or better, suggesting that there is significant conditional heteroscedasticity in the data. The null hypothesis of symmetric conditional variance-covariances, which requires all elements of the **D** matrix to be jointly insignificant (that is, $\delta_{ij} = 0$ for all i, j), is rejected at the 0.01% level or better, implying the existence of some asymmetries in the data which the model is capable of capturing. Also, the null hypothesis of a diagonal covariance process requires the off-diagonal elements of the **A**, **B**, and **D** matrices to be jointly insignificant (that is, $\alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$), but these estimated coefficients are jointly significant at the 0.01% level or better.

Thus the inflation-velocity growth process is strongly conditionally heteroscedastic, with innovations to inflation significantly influencing the conditional variance of velocity growth in an asymmetric way. Moreover, the sign as well as the size of inflation innovations are important. To establish the relationship between the volatility of money growth and velocity, in the last three rows of Tables 3–6, we test the following three null hypotheses: the null that the volatility of anticipated

money growth does not (Granger) cause velocity, $\psi_{12}^{(1)} = \psi_{12}^{(2)} = 0$, the null that the volatility of unanticipated money growth does not (Granger) cause velocity, $\phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$, and the null that the volatility of anticipated and unanticipated money growth does not (Granger) cause velocity, $\psi_{12}^{(1)} = \psi_{12}^{(2)} = \phi_{12}^{(1)} = \phi_{12}^{(2)} = 0$. It is clear that all these null hypotheses are rejected—the causality tests are carried out in terms of both Wald and LR statistics, producing the same results. Hence, we find strong evidence in support of Friedman's money growth volatility hypothesis.

We also have investigated the robustness of our results by reestimating our asymmetric GARCH in Mean model over the pre- and post-October 1979 periods (because of the announced change in Federal Reserve operating procedures in October 1979) with each of the four money measures. The results (available on request) are consistent with those in Tables 3–6, suggesting that the money growth variability/velocity relationship is robust to monetary policy regimes.

Finally, it should be noted that we have relied on the asymptotic distributions of the underlying test statistics. Although this issue could be explored using properly designed simulations, we have investigated the robustness of our results to the use of monthly data over the same sample period (a total of 549 observations). The results (available upon request) are qualitatively similar to the ones reported here.

6. CONCLUSIONS

This paper provides a study of the relationship between velocity and the variability of money growth, using recent advances in the financial econometrics literature. In particular, the central hypothesis of the paper is that VARMA GARCH-in-mean volatility of unanticipated money growth has a more systematic causal relation to the velocity of money than other measures of volatility. In contrast to Brocato and Smith (1989), Mehra (1989), and Thornton (1995), we find evidence of Friedman's hypothesis that the variability of money growth helps predict velocity and that the money/velocity relationship is robust to monetary policy procedures. We also find that the money variability/velocity relationship is robust to alternative methods of aggregating monetary assets.

REFERENCES

- Anderson, R.G., B.E. Jones, and T.D. Nesmith (1997a) Monetary aggregation theory and statistical index numbers. *The Federal Reserve Bank of St. Louis Review* 79, 31–51.
- Anderson, R.G., B.E. Jones, and T.D. Nesmith (1997b) Building new monetary services indexes: Concepts, data and methods. *The Federal Reserve Bank of St. Louis Review* 79, 53–82.
- Barnett, William A. (1980) Economic monetary aggregates: An application of aggregation and index number theory. *Journal of Econometrics* 14, 11–48.
- Barnett, William A., Douglas Fisher, and Apostolos Serletis (1992) Consumer theory and the demand for money. *Journal of Economic Literature* 30, 2086–2119.
- Bollerslev, T. (1990) Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics* 72, 498–505.
- Bollerslev, T., Robert F. Engle, and J. Wooldridge (1988) A capital asset pricing model with time varying covariances. *Journal of Political Economy* 96, 143–172.

- Brocato, Joe and Kenneth L. Smith (1989) Velocity and the variability of monetary growth: Evidence from Granger-causality tests. *Journal of Money, Credit, and Banking* 21, 258–261.
- Dickey, David A., and Wayne A. Fuller (1981) Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica* 49, 1057–1072.
- Engle Robert F. (2002) Dynamic conditional correlation: A simple class of multivariate GARCH models. *Journal of Business and Economic Statistics* 20, 339–350.
- Engle, Robert F. and Kenneth F. Kroner (1995) Multivariate simultaneous generalized ARCH. *Econometric Theory* 11, 122–150.
- Engle, Robert F. and Victor K. Ng (1993) Measuring and testing the impact of news on volatility. *The Journal of Finance* 5, 1749–1778.
- Engle, Robert F., Victor K. Ng, and M. Rothschild (1990) Asset pricing with a factor-arch covariance structure: Empirical estimates for treasury bills. *Journal of Econometrics* 45, 213–237.
- Fisher, Douglas and Apostolos Serletis (1989) Velocity and the growth of money in the United States, 1970–1985. *Journal of Macroeconomics* 3, 323–332.
- Friedman, Milton (1983) Monetary variability: The United States and Japan. *Journal of Money, Credit, and Banking* 15, 339–343.
- Friedman, Milton (1984) Lessons from the 1979–1982 monetary policy experiment. *The American Economic Review Papers and Proceedings* 74, 397–400.
- Grier, Kevin B., Ólan T. Henry, Nilss Olekalns, and Kalvinder Shields (2004) The asymmetric effects of uncertainty on inflation and output growth. *Journal of Applied Econometrics* 19, 551–565.
- Hall, Thomas E., and Nicholas R. Noble (1987) Velocity and the variability of money growth: Evidence from Granger causality tests. *Journal of Money, Credit, and Banking* 19, 112–116.
- Hansen, Peter R. and Asger Lunde (2005) A forecast comparison of volatility models: Does anything beat a GARCH(1,1)? *Journal of Applied Econometrics* 20, 873–889.
- Jarque, C.M. and A.K. Bera (1980) Efficient tests for normality, homoscedasticity, and serial independence of regression residuals. *Economics Letters* 6, 255–259.
- Judd, John P. and Brian Motley (1984) The great velocity decline of 1982–83: A comparative analysis of M1 and M2. *Federal Reserve Bank of San Francisco Economic Review*, 56–74.
- Kroner, Kenneth F. and Victor K. Ng (1998) Modeling asymmetric comovements of asset returns. *The Review of Financial Studies* 11, 817–844.
- Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., and Shin, Y. (1992) Testing the null hypothesis of stationarity against the alternative of a unit root. *Journal of Econometrics* 54, 159–178.
- Ljung, T. and G. Box (1979) On a measure of lack of fit in time series models. *Biometrika* 66, 66–72.
- MacKinnon, James G. (1994) Approximate asymptotic distribution functions for unit-root and cointegration tests. *Journal of Business and Economic Statistics* 12, 167–176.
- Mehra, Yash P. (1989) Velocity and the variability of money growth: Evidence from Granger causality tests. *Journal of Money, Credit, and Banking* 21, 262–266.
- Pantula, Sastry G., Graciela Gonzalez-Farias, and Wayne Fuller (1994) A comparison of unit-root test criteria. *Journal of Business and Economic Statistics* 12, 449–459.
- Phillips, P.C.B. (1987) Time series regression with a unit root. *Econometrica*, 277–301.
- Phillips, P.C.B., and Pierre Perron (1988) Testing for a unit root in time series regression. *Biometrika* 75, 335–346.
- Santoni, G.J. (1987) Changes in wealth and the velocity of money. *Federal Reserve Bank of St. Louis Review* 67, 16–26.
- Shields, Kalvinder, Nilss Olekalns, Ólan T. Henry, and Chris Brooks (2005) Measuring the response of macroeconomic uncertainty to shocks. *Review of Economics and Statistics* 87, 362–370.
- Tatom, John A. (1983) Was the 1982 velocity decline unusual? *Federal Reserve Bank of St. Louis Review* 67, 5–15.
- Thornton, John (1995) Friedman's money supply volatility hypothesis: Some international evidence. *Journal of Money, Credit, and Banking* 27, 288–292.