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ON A PAPER OF G. MASON

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Mason, [1, Theorem 2.6], proved that for any near-ring R, there are no non-trivial injective R-modules. In his proof he embedded R into a simple R-module G of arbitrarily high cardinality. Assuming the existence of an injective R-module $I \neq 0$, he chose an arbitrary element $x \in I$, $x \neq 0$, mapped $R \to I$ via the R-homomorphism $r \mapsto rx$, and extended this map to a homomorphism $G \to I$. Since G is simple this would mean that I contains a submodule of arbitrarily high cardinality, a contradiction. The above argument falls apart if $rx = 0$ for all $r \in R$. In this case the extended homomorphism from G to I might be the map $g \mapsto 0$ for all $g \in G$. If there exist $r \in R$, $x \in I$ with $rx \neq 0$, then Mason's argument remains valid, except for the fact that the choice of $x \in I$, $x \neq 0$ is not arbitrary. Suppose that $rx = 0$ for all $r \in R$, $x \in I$. Employing the remarks preceding [1, Theorem 2.6] embed $(I, +)$ into a simple group H with $\#H > \#I$, ($\#$ signifying the cardinality of the underlying set). The composition $rh = 0$ for all $r \in R$, $h \in H$ defines an R-module structure on H with I a submodule of H. Extend the identity map on I to an R-homomorphism $f: H \to I$. Then $f(H) = I$, and so $\#H = \#f(H) = \#I$, a contradiction.

The above argument may be used to prove that non-trivial injectives do not exist in other categories, e.g.:

THEOREM. *There are no non-trivial injectives in the category of rings (with unity).*

Proof. Let I be an injective ring, $I \neq 0$. Embed I into a ring A with unity. Extend the identity map on I to a homomorphism $f: A \to I$. Clearly $f(A) = I$, and so I is a ring with unity e. Let Z be the ring of integers. Embed Z into a field F with $\#F > \#I$. There exists a homomorphism $g: Z \to I$ satisfying $g(1) = e$. Extend g to a homomorphism $h: F \to I$. Then $\#F = h(F) \leq \#I$, a contradiction.

The above result (proved differently) as well as the proof of the non-existence of injectives in other categories may be found in [2].

Using the above argument it can be shown that there are no non-trivial injectives in the category of commutative rings (with unity).

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References

1. G. Mason, *Injective and projective near-ring modules*, Compositio Math. **33** (1976), 43–54.
2. A. Klein, *Injectives and simple objects*, J. of Pure and Applied Algebra, **15** (1979), 243–245.

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