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Nonlinear analytic growth rate of a single-mode Richtmyer–Meshkov instability

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Abstract

A perturbation method in which only the most secular terms are retained gives simple results for the weakly nonlinear growth of a single-mode shock-accelerated interface (Vandenboomgaerde *et al.*, 2002). This result can be written as a series in integer powers of time. It can be considered as the Taylor expansion of an analytic function. We believe that an approximation of such a function has been identified; it described the evolution of the instability from linear to intermediate nonlinear regime. Furthermore, this function has no singularity. The relevance of this analytic formula is checked against two-dimensional simulations and experimental data.

Keywords: Instability; Nonlinear; Richtmyer–Meshkov; Theory

1. INTRODUCTION

The Richtmyer–Meshkov (RM) instability occurs at the perturbed interface between two fluids of different densities after its interaction with a shock wave. The perturbation first grows linearly and then goes into an intermediate nonlinear regime. When harmonics appear, the interface can even cease being single valued as it evolves into mushroomlike structures. To study the weakly nonlinear regime of interfacial instability, perturbation methods are commonly used (Holyer, 1979). It has been applied to the RM instabilities by Zhang and Sohn (1997) and Vandenboomgaerde *et al.* (2002). As Cartesian (x, z) coordinates are used to describe the interface $z = \eta(x, t)$, only a single-valued shape can be computed. The perturbation expansions have a finite range of validity in time t_v; numerical simulations show that it can be estimated by $a_0 k\sigma t_v = 1$, where $a_0 k$ is the initial wave steepness and σ the initial growth rate.

In Section 2, we present the principle of analytic continuation of the perturbation expansion for single mode RM instability. In Setion 3, some hypothesis are made to build an analytic function for the amplitude of the perturbation. In Setion 4, this function is compared with numerical results and experimental data. We conclude with some remarks.

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2. PERTURBATION EXPANSION FOR THE GROWTH RATE OF A SINGLE MODE RM INSTABILITY

The full perturbation method derived by Zhang and Sohn (1997) has been drastically simplified by Vandenboomgaerde *et al*. (2002). Keeping at each order only the terms with the highest power in time transforms the solution from an n^2 to an n algorithm at the nth order. Approximate perturbation expansions are obtained. The validity of such approximations has been checked by comparisons with simulations for single mode and multimode configurations. For single-mode ones, the shape of the interface can be written as

$$k\eta(x, t) = \sum_{n=1}^{\infty} (a_0 k\sigma t)^n \sum_{j=1}^{n} a_j^{(n)} \cos jkx, \tag{1}$$

where $a_j^{(n)}$ are real functions that are functions of the Atwood number. This number is defined as $A = (\rho' - \rho)/(\rho' + \rho)$, where ρ is the density of the first shocked fluid. From expression (1) the half crest-to-crest amplitude, $a(t)$, of the interface can be derived:

$$ka(T) = \sum_{p=0}^{\infty} P_{2p+1}[A] T^{2p+1}, \tag{2}$$

where $T = a_0 k\sigma t$, and P_{2p+1} are polynomials of the $2p$th degree. Numerical studies have shown that the truncated series (2) diverge at $T \approx 1$. On the other hand, the full series (2) can be considered as the Taylor expansion of an unknown analytic function $F(A, T)$. This function allows analytic continuation of the series (2) beyond $T = 1$.

3. HYPOTHESIS FOR THE ANALYTIC CONTINUATION

The P_{2p+1} polynomials have peculiar shapes. By inspection among the orthogonal polynomial classes, we have found that they fit quite well modified Jacobi's polynomials (the Jacobi's polynomials will be hereafter noted JP). In Figure 1 are plotted the first six odd P_{2p+1} polynomials and the corresponding modified Jacobi's polynomials (noted Q_{2p+1}) defined as

$$Q_{2p+1} = \frac{1}{p+1}\,\mathrm{JP}[2p, \alpha, \alpha, \beta A] \quad \text{with } \alpha = 0.1 \text{ and } \beta = 0.79.$$

As can be seen, the Q_{2p+1} polynomial seems to be a good approximation of the P_{2p+1} polynomial. They have the same parity, same number of roots, and very close values for all Atwood numbers. The main hypothesis of this study is to assume that the discrepancies between P_{2p+1} and Q_{2p+1} are negligible in the computation of series (2). We shall now write

$$ka(T) \approx \sum_{p=0}^{\infty} Q_{2p+1}[A]\,T^{2p+1}. \tag{3}$$

The generating function G for the Jacobi's polynomials Q is

$$G[\beta A, T] = 2^{2\alpha} u^{-1} (1 - T + u)^{-\alpha} (1 + T + u)^{-\alpha} \quad \text{with } u = \sqrt{1 - 2 \times \beta A T + T^2}. \tag{4}$$

Combining Eqs. (3) and (4), an approximation for the nonlinear growth rate of a single-mode RM instability can be built:

$$k(a(T) - a_0) \approx \frac{1}{2}T + \frac{1}{T^3}\int_0^T \frac{\tau^3}{2}\{G(A,\tau) + G(A,-\tau)\}\,d\tau. \tag{5}$$

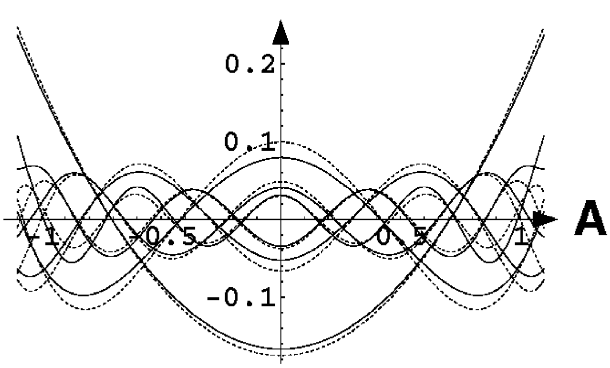

Fig. 1. *P* and *Q* polynomials as defined by Eqs. (2) and (3), respectively. First ones are in full lines and the others are in dashed lines.

Eq. (5) has no singularity and is valid beyond $T = 1$. In Figure 2, the 9th- and 11th-order Taylor expansions (2) are plotted for three different values of the Atwood number (full line). These curves are compared with the approximate analytic continuation (5; dashed lines). For $T < 1$, analytic function and Taylor series cannot be distinguished. After that time, whereas expansions diverge, Eq. (5) still describes a nonlinear growth.

4. COMPARISONS WITH EXPERIMENTS

The code CADMÉE (Mügler *et al.*, 1996; Mügler & Gauthier, 2000) has been used to estimate the growth of a perturbed He/air interface subjected to a 1.09 Mach number shock wave. The wave number and the amplitude are $k = 224.855\ \mathrm{m}^{-1}$ and $a_0 = 0.35 \times 10^{-3}$ m, respectively. The low

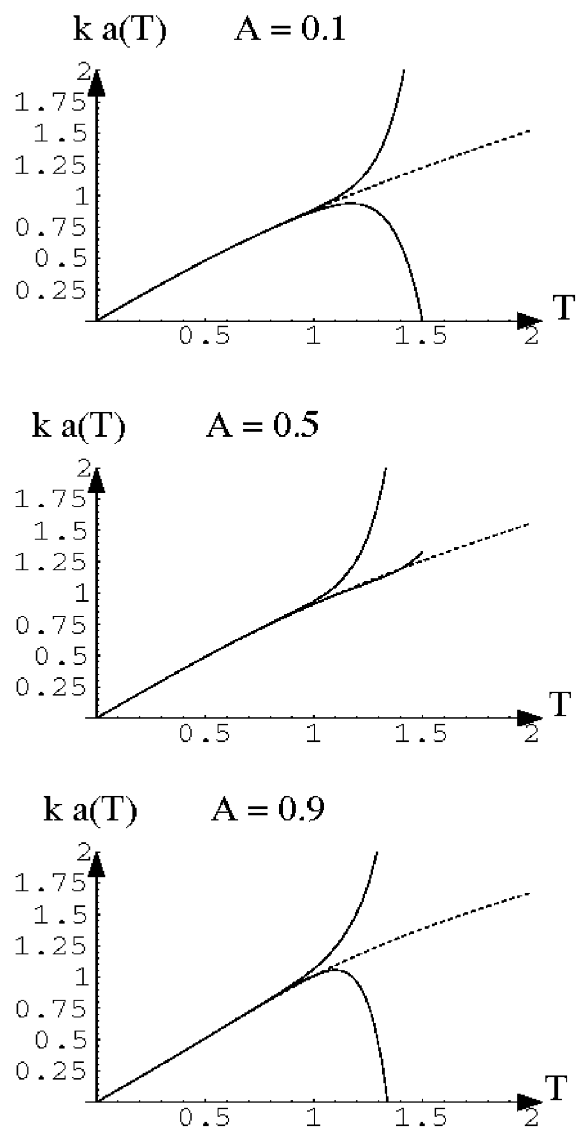

Fig. 2. Growth rates obtained by the perturbation series and the analytic continuation for three values of the Atwood number, in full and dashed lines, respectively.

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value of the Mach number was chosen to keep the flow in a quasi-incompressible and irrotational regime. The values of k and a_0 were chosen to start the instability in the linear regime ($a_0 k = 0.08$) and reach easily the weakly nonlinear regime. The choice of such parameters has been motivated by the framework of the theoretical model. In Figure 3a, the results of the simulations, the 9th and 11th Taylor expansions and the analytic continuation are plotted in full, dashed, and dash-dotted lines, respectively.

Equation (5) gives a good estimate even for times beyond the divergence of the perturbation series. This good agreement still holds when the interface becomes multivalued (see Fig. 3b).

Another comparison has been made with an experiment done by Jacobs and Krivets (2001). Figure 4 shows experimental results (circles) and the theoretical result (dash-dotted line). For this experiment, the wave steepness is $a_0 k = 0.23$, the Mach number is equal to 1.3 and the two gases are air and SF_6.

Once again, analytic continuation of the nonlinear growth rates obtained by the approximate perturbation method gives a very good estimate of the experimental results. This good agreement still holds in the intermediate nonlinear regime for values of $a(t)k$ larger than one.

5. CONCLUDING REMARKS

We have derived an approximate analytic continuation of the perturbation series for the single-mode RM instability. We emphasize that no parameter is needed to build this model. For all values of the Atwood number, the results of this model cannot be distinguished from those of the full perturbation method before their time of divergence. After that time, the nonlinear growth rate obtained by this model has been compared successfully with both numerical and experimental data up to $a(t)k = 5.5$.

However, it should be noticed that the limit of this model as time t tends to infinity gives a linear growth:

$$\lim_{T\to\infty} k(a(T) - a_0) = T/2$$

This behavior at large time disagrees with the T^α or $\ln T$ time dependence often found for highly nonlinear RM instabilities (see Prasad *et al.*, 2000, and references within). This discrepancy comes either from the approximation and hypothesis leading to the model or from the fact that we study only a single-mode perturbation.

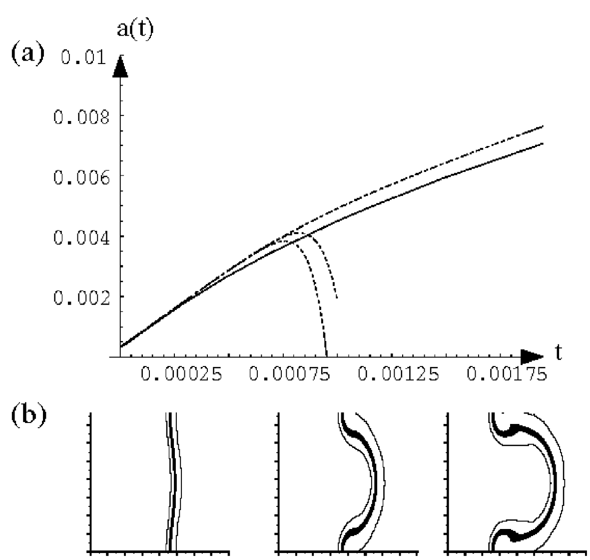

Fig. 3. a: Growth rates obtained by the CADMÉE simulation, the Taylor expansions, and the analytic continuation, in full, dashed, and dash-dotted lines, respectively. b: Shapes of the interface at different times ($t = 0.25$ ms, 1 ms and 1.75 ms).

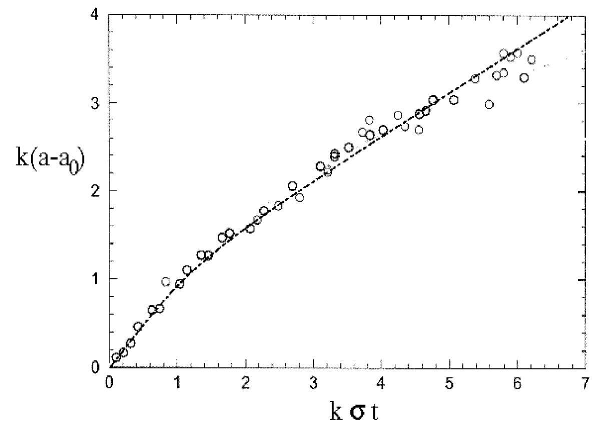

Fig. 4. Growth of the Jacobs and Krivets (2001) experiment (circles) versus scaled time. Growth from the analytic continuation (dash-dotted line).

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