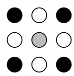
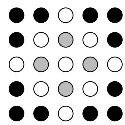
## 103.02 Proofs without words: identities in triangular numbers

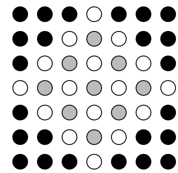
*A triangular number relation*



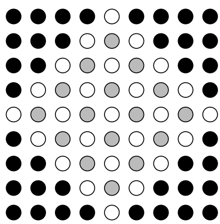
$4T_1 + 1^2 + 2^2 = 3^2$



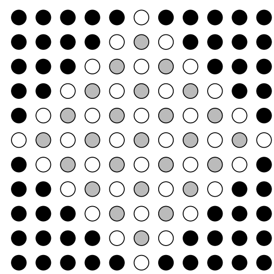
$4T_2 + 2^2 + 3^2 = 5^2$



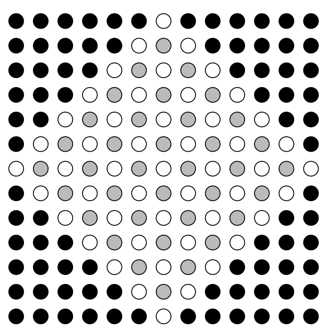
$4T_3 + 3^2 + 4^2 = 7^2$



$4T_4 + 4^2 + 5^2 = 9^2$

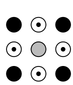


$4T_5 + 5^2 + 6^2 = 11^2$

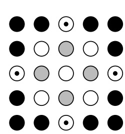


$4T_n + n^2 + (n + 1)^2 = (2n + 1)^2$

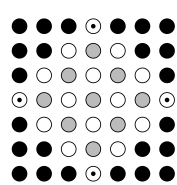
*A companion result*



$4T_2 - 4 + 0^2 + 1^2 = 3^2$



$4T_3 - 4 + 1^2 + 2^2 = 5^2$



$4T_4 - 4 + 2^2 + 3^2 = 7^2$

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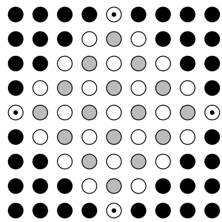
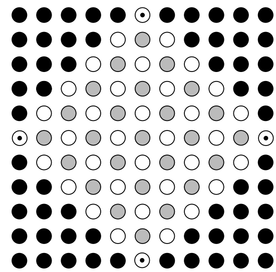
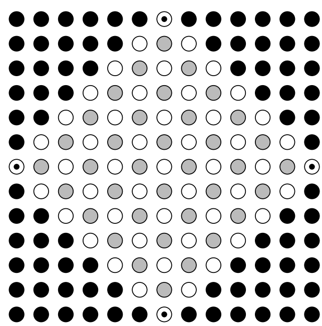


$4T_5 - 4 + 3^2 + 4^2 = 9^2$



$4T_6 - 4 + 4^2 + 5^2 = 11^2$



$$4T_n - 4 + (n - 2)^2 + (n - 1)^2 = (2n - 1)^2$$

*References*

1. Claudi Alsina and Roger B. Nelsen, *A mathematical space odyssey: solid geometry in the 21st Century* (Chapter 2) MAA (2015).
2. Roger B. Nelsen, *Proofs Without Words II: more exercises in visual thinking* (Volume 2) p. 98. MAA (2000).

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## 103.03 Quotients of hypotenuses of Pythagorean triples $(a, b, b + 1)$ and finite differences

Pythagorean triples $(a, b, b + 1)$, for which the length of the hypotenuse is one more than the length of the longer leg, form the sequence

$$(3, 4, 5), (5, 12, 13), (7, 24, 25), (9, 40, 41), (11, 60, 61), (13, 84, 85), \ldots,$$

that satisfies $\left(2n + 1, n\left(2n + 2\right), n\left(2n + 2\right) + 1\right)$, for $n$ a positive integer. O'Shea [1, p. 242] noticed that the following fractions relating successive hypotenuses are all equal to 2 with a remainder of 4:

$$\frac{5 + 25}{13}, \frac{13 + 41}{25}, \frac{25 + 61}{41}, \frac{41 + 85}{61}, \ldots .$$