Forum

The *Nautical Almanac*'s Faulty Calculator Instructions

Mike Pepperday

(Merlin Navigation and Marine Pty Ltd, Sydney, Australia)

The calculator instructions published in the *Nautical Almanac* since 1989 are impractical. The sextant corrections should include a passage correction and exclude semi-diameter; the celestial triangle solution is clumsy and not sufficiently comprehensive; the specified computation of a position line is a radical departure from standard method and will not work; the computation of the fix omits error assessment, and the directions for 'iterating' the fix are eccentric and superfluous. The instructions should be re-written to conform with the practice of celestial navigation or deleted from the *Almanac*.

In 1989 the *Nautical Almanac*, published jointly by the UK and US Governments, appeared with two new inclusions: a discussion of calculator sight reduction mathematics and a set of sight reduction tables. The inclusions have reappeared every year since. The calculator instructions, presented on *Nautical Almanac* pages 277–283 and written by the Royal Greenwich Observatory (RGO), cover altitude corrections, position line (azimuth and intercept), and the least squares fix.

Twenty years ago, the electronic calculator swept trigonometric tables from the shelves of all the professions except one: navigation. Even today, standard navigation text books barely recognize the existence of calculators and, whereas in other professions the educational institutions are in the vanguard of innovation, celestial navigation is being taught much as it was fifty years ago – with tables, not calculators. Apparently, the popular demand for calculator mathematics to be presented in the *Almanac* was nil.

This is not only a contrast with other professions but is something of a paradox as the past decade has seen a boom in sales of 'navigation computers'. These are ready-made pocket computers for navigation – essentially celestial navigation – at sea. Total sales by now must be several hundred thousand. At one stage in the mid eighties, there were fourteen brands vying for the English-speaking navigator's dollar, but the market has settled down and there are now five. They are the Celesticomp V, the CN2000, the Merlin II, the NC99, and the Petrel. They are available in the chandleries of the USA, Australia and New Zealand but generally not offered in Britain. All are pocket computers programmed in BASIC. All five contain Sun, Moon and Aries almanacs; four contain stars, and three contain planet almanacs. All compute the 'least squares' fix.

Each computer is the outcome of a lineage of previous models reflecting years of customer feedback and competitive pressure. Though they vary in many details, they long since converged to a common operating procedure: you enter clock time and sextant altitude, and the computer displays azimuth and intercept. After entering two or more sights, if you press the fix button you will see the fix so formed. You can delete sights, enter more sights, and look at the fix at any stage. One brand limits a fix to forty sights; the others set no limit to the number of sights per fix. Position lines may, of course, be plotted in the usual manner.

Where did the computer makers find the mathematics to program their products? Since the late seventies (at least), the RGO has privately published a series of papers, called 'Technical Notes', which set out some of the astronomical mathematics. These 'Notes' are available from the Observatory and some of them have been used as source material by some of the commercial computer makers. Effectively the navigation and marketing skills of the computer makers enabled the astronomical and mathematical skills of the RGO (among others) to reach and assist thousands of sailors. It was, and is, an effective process.

The American and British Almanac offices have never said why they decided to publish 'customer direct' in the *Nautical Almanac*. Nor have they said why they decided to publish navigation mathematics (which is available in books off the chandlery shelf) rather than ephemeris mathematics (not so readily available) which would reflect their expertise. Whatever the intent, the publication has not been a success. They seem to have prompted no review, no public comment, no apparent acknowledgement at all. In short, despite the prestige of the *Nautical Almanac*, the instructions have achieved no acceptance.

There is a simple reason: they are impractical. In every sphere of activity, the introduction of electronic computation compels changes to traditional procedures. However, the departures from both traditional and modern practices which are set out in the *Almanac* are idiosyncratic and inappropriate. As far as computers in navigation are concerned, it is no longer early days. Although the college navigation classes tend to ignore the commercially available computers, for years virtually every recreational sailor who navigates with a sextant has been using one. They are a standard against which anyone presuming to tell navigators how to do it, may be judged.

The major faults occur in Section 8 to Section 11 but there are problems of one sort or another throughout.

Section 1, the Introduction, states that the calculator, or 'microcomputer' (the usual term is pocket computer or hand-held computer), should preferably be programmable. Quite right. In practice, the computing of position lines with an ordinary, non-programmable, scientific calculator is unworkable. Any number of people have promoted the idea but it has never got beyond the lounge room. Even if such calculation is restricted to solving the bare astronomical triangle, it is just too lengthy and too strange for most sailors.

The potential user of these *Almanac* instructions would thus be someone who owns a programmable calculator or pocket computer. Perhaps a civil engineer planning to go cruising, or a ship's officer with a taste for trigonometry. Who the intended reader is, is not stated. Eight pages covering sight reduction could never be for novices and the academic style – passive voice, mathematical terminology – also presumes a certain background. Though the prospective readership is limited, knowledge is a good thing and for those interested, it would be handy to have a reference. To function as a reference such instructions would need to be reasonably comprehensive and to reflect the practice of (calculator) navigation.

The Introduction says the astronomical data are assumed to be taken from the *Nautical Almanac*. This is probably unrealistic. While it would depend on individual enthusiasm, chances are that those who went as far as programming the fix would also program almanacs, at least for the Sun and Aries. But in that case, the textbook which provides the almanac algorithms would also provide the other mathematics of sight reduction.

That in turn raises the question: what is the point of including these mathematics in the *Nautical Almanac*?

Section 2 sets out the notation with sign conventions and numerical limits. It is clear and to the point. Abbreviations rather than symbols are used, which is a help to readability. There are two quibbles. The concept of 'apparent altitude' is superfluous when a calculator is being applied since only a sextant altitude and a final corrected altitude are needed. Also, the quantity defined as 'index error' would be better termed 'index correction', a correction being the amount to be added to an erroneous value to correct it. It is always clearer to speak of corrections rather than errors.

Section 3 is a discussion of how to convert minutes to degrees by dividing by 60. It is an odd thing to be explaining to navigators.

Sections 4 and 5 comprise a lengthy explanation of how to interpolate for GHA or Dec from two values taken from the *Almanac*. To interpolate means to find, by proportion, a value in between the ones listed. Rather than explain it, however, it should be omitted altogether. One of the computers in the Eighties took the interpolation approach and quickly vanished. It is unnecessary bother. It is much easier to enter the v and d corrections. It might have been better to set out the arithmetic of these v and d corrections.

It would also have been instructive to show how Aries can be computed from its value at the beginning of the month, for example:

$$\text{GHA Aries} = 0.98565\ (D-1) + 15.0411 \times T + A$$

where D is the day of the month, T is Universal Time in hours, and A is GHA Aries at O hours on the first of the month. It is also possible to write a complete, *Almanac*-independent, prediction for GHA Aries as an even simpler expression providing it is accompanied by a short list of month and year codes.

Corresponding formulae to compute the position of the Sun are lengthier but would have greatly enhanced the usefulness of these mathematical instructions.

Sections 6 and 7 present a celestial triangle solution. That is, the computation of azimuth and altitude given lat, long, Dec and GHA.

It says "LHA = GHA + long" and then "add or subtract multiples of 360° to set LHA in the range 0° to 360°." This is poor advice. LHA of itself is not of interest to anyone and may be left as it is.

Only where a result is needed for human consumption must it be brought within 0° and 360°. In such a case it is not necessary to go testing whether it is within this range and then adding or subtracting 360s till it is. Instead test nothing but simply subtract 360 × INT(N/360) where N is the value to be brought into range and INT has the BASIC language meaning of 'integer value less than'. This situation does not actually arise during sight reduction.

Directions are given to find azimuth and altitude by the spherical cosine rule and the five parts formula. The formulae are divided in an unconventional way with some intermediate parameters introduced. The reason for this is not stated. Then it says:

If $X > +1$ set $X = +1$
If $X < -1$ set $X = -1$
$A = \cos^{-1} X$
where $\cos^{-1}$ is the inverse function of cosine.

X has been computed from spherical formulae so how could X possibly lie outside the range ± 1? Normally it cannot. Perhaps it has something to do with the way the formulae have been divided. The Royal Greenwich Observatory has been in the business of

https://doi.org/10.1017/S0373463300011152 Published online by Cambridge University Press

manipulating spherical triangles for centuries so there must be some explanation. It ought to be given, if only because those 'IFS' can be a nuisance to program.

If you didn't know what $\cos^{-1}$ was, would the statement about it make you wiser? What, actually, is an 'inverse function'?

The choice of formulae to use to solve the celestial triangle seems to be to some extent a matter of taste but 'IF' is always bad taste. IF-testing is intellectually sloppy, introduces the risk of overlooking some contingency, eats computer space, makes a program hard to read and is practically always unnecessary. For example, the instructions:

$$A = \cos^{-1}X.$$
If LHA > 180° then Z = A
Otherwise Z = 360 − A

could have been avoided with the single expression:

$$Z = (\cos^{-1}X - 180°) \times \text{SGN} \sin \text{LHA} + 180°$$

where SGN means the sign of sin LHA.

That is still clumsy and the first 180° would vanish if the previous five parts formula had had its signs reversed so X was its reciprocal.

There is often a better way. Cosine rule and five parts formula are suited to some BASIC computers, but calculators should apply the tangent formula. Where a result lies in the range 0°–360°, a formula using tangent will give the unique result since arctan, unlike arcsin or arccos, is defined through 360°. It is not necessarily briefer but some calculators do not provide IF, SGN, or INT, whereas such machines always provide the two-argument arctan; namely ,the rectangular-polar key. Final azimuth, Z, is given in a single expression (without the X or A above) by:

$$Z = \tan^{-1} [\sin \text{LHA}/(\cos \text{LHA} \sin \text{lat} - \tan \text{Dec} \cos \text{lat})] + 180°$$

assuming the ratio is not divided but directly evaluated with the two-argument arctan. The computed altitude may be found by the cosine rule or, if the calculator space is tight, via the sine rule by multiplying the other value which the rec-pol function yielded by cos Dec to give the cosine of the altitude. (This altitude will be unsigned.)

On computers with a BASIC which lacks the rec–pol function, the tangent quotient could be multiplied by the sign of the numerator; that is, SGN sin LHA. There would be no IFS, and certainly no impossible values exceeding 1, but there would be a disadvantage since a division requires the program (the programmer) to ensure that the denominator is never zero. It cannot be zero when computing azimuth, but on a computer, the formulae will usually be written as a general purpose subroutine for spherical triangle solution which means that the other applications (such as great circle, star identification, ephemeris computation) must take a possible zero denominator into account. The five parts formula also involves division but, since the denominator is a value output from the previous cosine rule, it would be practically impossible for it to be zero.

In sum, a reference to solving the celestial triangle might set out three possibilities. For most computers or calculators: tan formula followed by cos rule; for small calculators: tan followed by sine rule by-product; for BASIC computers without rec-pol: cosine rule and five parts formula. The azimuth formula should always be structured to come out in reverse so that with 180° added it falls automatically within the 0°–360° range.

Solving the celestial triangle does not finish finding the computed altitude. When Dec and GHA were found, the semi-diameter correction was also determined. This should now be applied to computed altitude and not, as the *Almanac* directs, to the sextant reading (see below).

Sections 8 and 9 set out the sextant altitude corrections. There are two bouquets: the refraction correction given is the sensible formula – it works on altitudes from zero to 90° and few knew of it before this publication – and the relationship between the Moon's semi-diameter and horizontal parallax is presented, which saves an entry if looking up the Moon in the *Almanac.* There are three brickbats.

The first is the minor one that the concept 'apparent altitude' is superfluous: the sextant reading has to be turned into the corrected altitude and this intermediate quantity is not needed. Mention made of "the height of eye above the horizon" should read "above sea level".

The second is more important: the correction for semi-diameter should not be applied to observed altitude but to computed altitude. The reason for this is so that a lower limb becomes effectively (administratively) a different body from the centre or the upper limb. It saves having the computer pester the navigator about which limb was observed; instead, the user simply nominates the body – which includes the limb. All the commercial machines do this. It is actually more logical than the traditional way. Computers are logical sometimes.

The third deficiency is vital. The *Almanac* instructions have omitted what might be termed 'passage correction'. This is a correction to the observed altitude for the passage of the vessel during the period of the observations; that is, to advance or retard the position line over the time between the sight and the required time of fix. Passage correction in degrees is:

$$\cos\,(\text{course}-\text{azimuth})\times(\text{fix time}-\text{sight time})\times\text{Speed}/60$$

where time is in hours and speed is in knots. Advancing a position line is not traditionally thought of as an altitude correction but in fact this formula simply 'runs up' or 'runs back' the position line in the same way as the navigator has always done – by moving it in the direction of the course. It can be a large correction and all the commercial computers apply it. Other RGO publications set it out but these instructions in the *Almanac* omit it and, instead, account for the passage of the vessel by moving the DR position – a method which is quite inappropriate (see below).

In sum, the corrections to the sextant reading should be: index, dip, refraction, passage and parallax. The corrected altitude boils down to one long but fairly simple expression:

$$\begin{aligned}\text{Ho} = \text{Hs}+\text{I}/60-.03\,\sqrt{\text{h}}-.0167/\tan\,(\text{Hs}+7.31/(\text{Hs}+4.4))\\ +\text{passage correction as above}+\text{HP}\times\cos\,\text{Hs}\end{aligned}$$

The *Almanac* also gives the pressure, temperature and oblateness corrections. Though usually ignored, it is reasonable that they be set out since a reference should be complete. Presenting them lets the reader decide and shows, if nothing else, why they may be neglected.

Section 10, headed 'position from intercept and azimuth using a chart', is mistitled – in two senses. The section applies to the computed fix as well as a chart fix – it is so applied on the very next page – and the section does not actually discuss position. It discusses the position line. The only mention of finding position is the uninformative last sentence: "Two or more position lines are required to determine a fix".

The first thing the section says to do is estimate a fix position and then compute a DR lat and long at the time of each sight. This is incorrect.

It is the great virtue of the intercept method that there is no need to be fussy about DR position. DR uncertainty is of no consequence. Not even a hundred-mile error would matter. Yet, in the case of the example given in the *Almanac* the three different DRs for the three stars are only two to five miles apart!

Such precision not only involves much unnecessary computing but, to find the fix on the chart – which is what this section is allegedly about – you'd have to plot the three DRs in order to plot the three intercepts. If you took seven shots you'd have to plot seven different DRs each with its own associated intercept. If you took twenty...

On the commercially available computers all intercepts are, naturally, from a single DR latitude/DR longitude. Apart from ease of plotting, the single DR position also means that, where several sights of a body are taken, the intercepts are directly comparable and any erroneous reading is obvious. These machines accept any number of observations except for one model which limits the number of sights per fix to forty.

The *Almanac* instructions are an astonishing departure from standard procedure. No explanation or justification is offered. On the contrary it says: "The position is calculated..." as though it were the normal thing to do. It isn't and it won't work. The whole of section 10 should be deleted.

Before leaving it, however, we should note that the formulae it gives to compute the DR position are not suitable for that purpose. To compute dead reckoning find the latitude first *then* the longitude whereby the longitude formula uses, not the old latitude (as advised in the *Almanac*), and not the newly-computed latitude, but the average of the two. This is not for an exact result but for a consistent result. The navigator who updates DR will occasionally make a mis-entry and will require to undo it so the formulae should work the same for a negative time or distance as for a positive.

Section 11 sets out the mathematics for computing the fix by the 'least squares' method. This yields the 'best fit' fix when there are more than two position lines. As presented it is fairly concise but (a) it can bear some simplifying, (b) an extra calculation for accuracy should be added, and (c) the concoction about 'iteration' should be excised.

Instead of finding A, B, C, find n, B′, C′ where n is the count of the number of sights (so far – that is, at any stage of entering sights) B′ is the sum of sin 2Z, and C′ is the sum of cos 2Z. Find D and E as shown. Also find F where F is the sum of p^2. F is for error estimation and is set out in other RGO publications but has been omitted from these *Almanac* instructions. Every time a sight – that is, an azimuth-intercept pair – is computed, this set of summations n, B′, C′, D, E, F is added to. For deletion of a sight from the fix, subtract the values.

To view the fix formed from the lines entered at any stage, compute $B = B'/2$, $C = (n-C')/2$, $A = n-C$, and compute G as shown in the *Almanac* then:

$$\text{dlat} = (DC-EB)/G$$
$$dL = (AE-BD)/G$$
$$S = 60 \times \sqrt{[(F-D\times \text{dlat}-E\times dL)/(n-2)]}$$

where S is the 'standard error', in miles, of the fix which is given by:

$$\text{lat} = \text{DR lat}+\text{dlat}$$
$$\text{long} = \text{DR long}+dL/\cos \text{lat}.$$

And that is the end of the calculation. Essentially, it is a mathematical way of drawing the lines and marking the fix in the middle of them, much as navigators have always done. Sights can be deleted or more added. The definition of best fit fix – that is, the point where the sum of the squares of the observational discrepancies is the minimum – should be stated and illuminated with some discussion of the problem of systematic error: the example given has three bodies well distributed around the horizon but there is no explanation of why. Error S, without which the navigator has no measure of the quality of the fix, would also bear some discussing. Essentially it may be viewed as a sort of average error: if it is larger than a mile or two, look for an explanation.

But this is not the end of the calculation for the *Almanac*. According to the Almanac offices of the United States and Great Britain, the above computation does not give the fix but only a new estimate of the fix. After a century of use at sea one would have thought that, if the intercept method were deficient, someone might have noticed. Yet the *Almanac* instructions simply presume it is deficient. The whole calculation is, allegedly, now to be repeated with this new 'estimate' instead of the previous one. As well as repeating the above calculations for the best fit fix, this includes computing yet another set of DR positions for each and every sight!

This eccentric 'iterative' procedure is also presented in other RGO publications – there also as though it were normal. Even land surveyors, who are at home with iterative procedures and who look for an accuracy of metres rather than miles, do not iterate a celestial fix. It is an extraordinary complication and, if the Royal Greenwich Observatory and the United States Naval Observatory were not promoting it, year in, year out, in their official *Nautical Almanac*, it would not warrant serious discussion.

The *Almanac*'s fix example is defective in two other respects. Firstly, real navigators who compute least squares fixes enter sextant altitudes in degrees and minutes. The *Almanac* example (revised every year) gives for entry corrected altitudes in decimal degrees, perhaps with the idea that this would help the programmer. It probably will not because the programmer using this example as a test of a least squares routine will already have the altitude corrections programmed and will find it necessary to insert some corrections backwards to reconstruct some sextant readings.

Secondly, the example consists of three bodies with one sight each whereas several sights of each body should be taken. Back in section 1, the Introduction, the possibility of removing doubtful sights from the fix is mentioned. How can a doubtful sight be noticed if there is only one? Single sights are often presented in examples and it tends to leave the wrong impression. If the example is only to illustrate a principle then that should be stated.

The *Nautical Almanac* is printed annually and distributed in tens of thousands of copies. It has been around for a couple of centuries and though it changed a few times to reflect evolving technology and new ideas, it has probably always confined itself to astronomical positions. The inclusion of sight reduction mathematics and of sight reduction tables would therefore appear to be significant innovations.

As far as this author is aware there was no prior discussion of, or public notification of, either innovation and in the years since there has appeared no explanation for them and no review of them. Except for a couple of passing mentions of the sight reduction tables in the American press, they seem to have been ignored.

The three-century history of the Royal Greenwich Observatory has been a colourful one with seasons of flowerings and witherings. Through it all the problem of – the necessity of – navigation by astronomical bodies remained. Perhaps the RGO is now in terminal decline. It was created to help navigators: the *Nautical Almanac* was its triumph and its ongoing *raison d'etre*. But sextant navigation has been irrelevant for ships for a decade and for aircraft for much longer. These users carry the printed *Almanac* only to satisfy obsolescent regulations. As long as recreational sailors needed celestial, the RGO's publication of their astronomical 'Technical Notes' provided a service (albeit effectively a public subsidy to private yachtsmen) which kept up with the times and was in line with the RGO's purpose.

The inclusion of calculator instructions in the *Almanac* could be criticised for

https://doi.org/10.1017/S0373463300011152 Published online by Cambridge University Press

competing directly with private enterprise texts on calculator navigation. As it stands, private publishers have nothing to fear and, owing to its brevity and limited scope, this would probably never be a serious objection. Now that GPS has brought the History of Navigation to an end, sextant navigation is becoming irrelevant to yachts and soon there will remain only the hobby (lounge room) market. Perhaps consideration should be given to privatizing the RGO's navigation publishing functions.

Private or public, as long as the *Nautical Almanac* is being printed and being bought, a reference setting out appropriate, practical mathematics could be a convenience and could influence celestial navigation classes (where there is probably an enormous potential readership). Such instructions would take into account how navigation is thought of and how it is performed. Any deviation should be regarded as extraordinary and would need to be explicitly justified. In its present form the *Nautical Almanac*'s calculator segment is unsatisfactory and should either be set to rights or deleted.

KEY WORDS

1. Astro. 2. Computers. 3. Reduction and plotting.

Visual Approach Guidance Indicator Systems

J. R. C. Young

In their paper[1] on 'A history of visual approach guidance indicator systems in Australia', Clark and Antonenko raise a number of points which invite response, although to a certain extent, comment seems to be pre-empted by one of their sentences: 'Haphazard trials of visual approach aids were shown to be readily subject to bias, and were consequently depreciated by Millar',[2] no less.

As one of those deputed to assess them, I should say at once that I hold that the Australian T-VASIS were admirable, despite one major reservation which was subsequently put right and another which I did not fully appreciate at the time.

The paper diverges quite widely from what is purported to be its basic themes: that T-VASIS are better than VASIS or PAPIS because their many gradations allow an assessment of a change in rate of descent; further, that they incorporate a positioning of lights that is ergonomically correct for a descent aid, and that they do not depend upon the colour vision of the pilot using them. It also notes that, like its rivals, it includes supplementary information regarding the horizon line at or near the point of touch-down as an aid to final corrective alignment.

Such are the divergences that my first reaction was to detail my experience with night landing and approach aids dating from those in use in the RAF pre-war, then the old International Lighting System with coloured runway lights, the German system of surface floodlighting, and the completely new system developed as part of the wide-ranging concept of fortress Britain – depending, as the other systems did not, upon the night vision aspects of human capability – the Glim Lamp Flare Path and its developments Drem MkI and MkII. The Glim Lamp Flare Path and Drem systems were abandoned for a variety of reasons, including (*a*) the fact that, in front-line situations, night vision was disabled, and (*b*) the eventual return of peace-time lighting. Other supplementary reasons were the maintenance problem, the amount of real estate involved around so many airports and the very narrow approach path clearly designated.

Another aspect of the paper is its attempt to link the superiority of the T-VASIS to an