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Mixing input-output pseudolinearization and gain scheduling techniques for stabilization of mobile robots with two independently driven wheels

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SUMMARY

In this paper, we propose a two-loop structure to transform and stabilize the kinematic model of a nonholonomic mobile robot with two independently driven wheels. This two-loop structure consists of input-output pseudolinearization and gain scheduling techniques. A comparison with previous methods is included. The main contribution of this paper is to apply a input-output pseudolinearization transformation method and to use an effective pole-assignment strategy for stabilizing a mobile robot with two independently driven wheels. The proposed method has demonstrated superiority over previous methods.

KEYWORDS: Mobile robot; Input-output pseudolinearization; Gain scheduling; Linear parameter-varying system.

1. INTRODUCTION

The guidance of a nonholonomic system to an arbitrary point and having it follow a desired trajectory in a state space are, in general, quite difficult.[1–14] An example of a nonholonomic system is a mobile robot. Mobile robots either have car-like wheels,[2] with two that are independently driven,[3] or have omni-directional wheels.[4] For nonholonomic control systems it is very important to define a model representation before designing the stabilizing controller. The model of nonholonomic systems can be divided into kinematic models, which are affine nonlinear driftless systems, and dynamic models, which are affine nonlinear system.[1] The stabilized methods for nonholonomic control systems are largely dependent on the modeling techniques. For two different model representations the same control method cannot be applied to both of them. The stabilization problems are concerned with designing the feedback laws which guarantee that an equilibrium of the closed-loop system is asymptotically stable. Previous works on stabilization of mobile robots use discontinuous time-invariant stabilization,[5] time-varying stabilization,[6] and hybrid feedback laws.[7] Most of the previous works in stabilization of mobile robots are based on the so-called chain form or power form. To transform into these forms, one is required to apply techniques of Lie algebra and exterior differential systems, because these systems are still nonlinear driftless systems. Therefore, complicated controllers are needed to stabilize these systems. That is, these methods must use an unsmooth and complicated trajectory for the feedback stabilization of nonholonomic systems. Moreover, these stabilizing structures are all single feedback loop systems for a new nonlinear driftless system. In this paper, a new controller structure and improved trajectory are proposed to stabilize mobile robots with independently driven wheels.

The notion of pseudolinearization was introduced by Champetier et al.[15–17] as a method to approximately linearize the input-state behavior of a general nonlinear system. That is, pseudolinearization involves the computation of a state feedback and state coordinate transformation so that the resulting closed-loop state equation in the new coordinates has a family of linearizations that is independent of the closed-loop operating point. Moreover, the necessary and sufficient condition for existence of a state feedback and state coordinate change that transforms a given nonlinear system into a pseudo-normal form is input-output pseudolinearization.[18] This method is better for application in a mobile robot than the state space exact linearization.[19] The gain scheduling technique is commonly used in designing controllers for linear time-varying and nonlinear systems.[20,21] Roughly speaking, design of controllers by the gain scheduling technique is as follows: (1) linear time-invariant approximations are obtained; (2) linear time-invariant controllers are designed for each linearized representation of the system at the selected operating points, so that stability and certain performance objectives are achieved; and (3) these controllers are then linked together in order to obtain a single controller for the entire range of the system operation. Shahruz and Behtash[20] first proposed a new algorithm to design a controller for linear MIMO systems whose dynamics depend on a time-varying parameter. In this paper we first apply the gain scheduling technique to design the controller for stabilizing the control of a mobile robot after the pseudo-normal form has been obtained by utilizing the input-output pseudolinearization.

In this paper, a mobile robot with two independently driven wheels is considered, which can be steered to any position in the free space. Position means the location and the orientation of the robot. However, the robot's

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freedom of motion is limited; it cannot move sideways. Thus, complicated maneuvering is needed to bring the robot to an arbitrary position and to follow an arbitrary trajectory. The problems of stabilizing the robot at a specific position are solved by a two feedback-loop control structure by the input-output pseudolinearization and the gain scheduling technique. In particular, the input-output pseudolinearization is used in the inner loop and gain scheduling techniques (pole-assignment type algorithm) for stabilization of mobile robots is used in the outer loop. Input-output pseudolinearization is first used to transform the MIMO nonlinear system to a pseudo-normal form. This procedure is called an input-output pseudolinearization loop. We then show that the pseudo-normal form can be transformed into a linear parameter-varying system, which can be easily be stabilized by the common gain scheduling technique. This procedure is called a pole-placement loop. That is, a two-loop control structure is proposed to transform and stabilize the kinematic model of a nonholonomic mobile robot. Moreover, the proposed method first transforms the nonlinear driftless system to a linear parameter-varying system. It turns out that the proposed structure has a much simpler mathematical method in the process of transformation and has a smoother and simpler trajectory than previous methods. In summary, the proposed method is much easier than previous works for stabilization of mobile robots, moreover the proposed method has demonstrated superiority over previous methods.

2. INPUT-OUTPUT PSEUDOLINEARIZATION

In this section, we summarize some concepts and results about the input-output pseudolinearization. Generally speaking, pseudolinearization involves the computation of a state feedback and state coordinate change such that the resulting closed-loop state equation in the new coordinates has a family of linearizations that is independent of the closed-loop operating point. By the same token, a nonlinear system is in pseudo-normal form if its family of linearizations has an input-output behavior that is independent of the operating point. Consider an m-input, m-output, and n-dimensional nonlinear system

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= h(x(t)), \end{aligned} \tag{1}$$

where $f(\cdot, \cdot)$ and $h(\cdot)$ are smooth functions and satisfy $f(0, 0) = 0$, $h(0) = 0$. We also assume that equation (1) has a parameterized constant operating point family described by the smooth function $[x(\alpha), u(\alpha), y(\alpha)]$, $\alpha \in \Gamma$, where Γ is an open set containing the origin in R^n. That is,

$$\begin{aligned} f(x(\alpha), u(\alpha)) &= 0, \\ h(x(\alpha)) &= y(\alpha), \qquad \alpha \in \Gamma. \end{aligned} \tag{2}$$

The problem of transformation to Pseudo-Normal form is stated below: Given system equation (1) with a constant operating point family $[x(\alpha), u(\alpha), y(\alpha)]$, $\alpha \in \Gamma$, and a linearization family satisfying $rank(\partial x/\partial \alpha)(\alpha) = m$, find (if possible) positive integers $\rho_1, \ldots, \rho_m$, and an admissible state coordinate change and state feedback, such that the resulting closed-loop system in the new coordinates with a constant operating point family $[z(\alpha), v(\alpha), y(\alpha)]$, $\alpha \in \Gamma$ has a corresponding linearization family described by

$$\bar{A}(\alpha) = \begin{bmatrix} A_{00}(\alpha) & A_{01}(\alpha) \\ 0 & A_{11} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}, \quad \bar{C} = [0 \quad C_1], \tag{3}$$

where

$$A_{11} = blockdiag\left(\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \rho_i \times \rho_i, i = 1, \ldots, m \right),$$

$$B_1 = blockdiag\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \rho_i \times 1, i = 1, \ldots, m \right),$$

and

$$C_1 = blockdiag([1 \quad 0 \quad \cdots \quad 0], 1 \times \rho_i, i = 1, \ldots, m).$$

It is easy to see that for each $\alpha \in \Gamma$, equation (3) specifies a linear system in normal form. Furthermore, for all $\alpha \in \Gamma$, from the transfer matrix corresponding to equation (4)

$$G(s) = diag\left\{ \frac{1}{s^{\rho_1}}, \ldots, \frac{1}{s^{\rho_m}} \right\}, \tag{4}$$

it is clear that a nonlinear system in pseudo-normal form is input-output pseudolinearized. This means, one can obtain the pseudo-normal form when using the input-output pseudolinearization method. The procedure of input-output pseudolinearization is based on Definition 1 and Theorem 1 below.

Definition 1.[18] At each $\alpha \in \Gamma$ the pointwise relative degree of the linearization family is the m-tuple of positive integers $[\rho_1(\alpha), \ldots, \rho_m(\alpha)]$ satisfying, for $i = 1, \ldots, m$,

$$c_i(\alpha) A^{j-1}(\alpha) B(\alpha) = 0, \quad j = 1, \ldots, \rho_i(\alpha) - 1 \tag{5}$$

$$c_i(\alpha) A^{\rho_i(\alpha)-1}(\alpha) B(\alpha) \neq 0, \tag{6}$$

where $c_i(\alpha)$ denotes the ith row of $C(\alpha)$.

Theorem 1.[18] Suppose system equation (1) has a linearization family that satisfies

$$rank \frac{\partial x(\alpha)}{\partial \alpha}(\alpha) = m \text{ and } \dim [B(\alpha) \cap x(\alpha)] = d, \tag{7}$$

where m is the number of inputs and d is a positive integer. There then exists a transformation to pseudo-normal form if and only if

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(i) the linearization family,

$$A(\alpha)=\frac{\partial f}{\partial x}(x(\alpha),u(\alpha)),$$

$$B(\alpha)=\frac{\partial f}{\partial u}(x(\alpha),u(\alpha)),$$

$$C(\alpha)=\frac{\partial h}{\partial x}(x(\alpha))\quad\text{and } \alpha\in\Gamma,$$

has constant pointwise relative degree $[\rho_1,\ldots,\rho_m]$ at each $\alpha\in\Gamma$.

(ii) matrix

$$M(\alpha)=\begin{bmatrix} c_1(\alpha)A^{\rho_1-1}(\alpha)B(\alpha)\\ \vdots \\ c_m(\alpha)A^{\rho_m-1}(\alpha)B(\alpha)\end{bmatrix}$$

is inevitable at each $\alpha\in\Gamma$.

(iii) the distribution D is involutive on Γ.

It is also noted that involutivity is not necessary to achieve input-output pseudolinearization.[20]

The pseudo-normal form is not a linear time-invariant system; therefore, a linear time-invariant controller cannot be applied. When the pseudo-normal form is a linear parameter-varying system, the gain scheduling controllers (Appendix) are generally effective for stabilizing the system.

3. MAIN RESULTS

In this section we shall show the proposed method to transform and stabilize the kinematic model of a nonholonomic mobile robot with two independently driven wheels. The mobile robot as shown in Figure 1 is located on a 2-dimensional plane in which a global Catesian coordinate system is defined. The reference point of the robot is located at the center of the driving wheels. The robot processes three degrees of freedom in its positioning by (x_1, x_2, θ), where the heading angle θ is measured counterclockwise from the x_1-axis. The physical parameters of the mobile robot are listed below:

q_1, q_2: angles of the driving wheels
u_1, u_2: speed of the two independently driven wheels
r: radius of the driving wheels
w: half of the distance between the two centers of the driving wheels

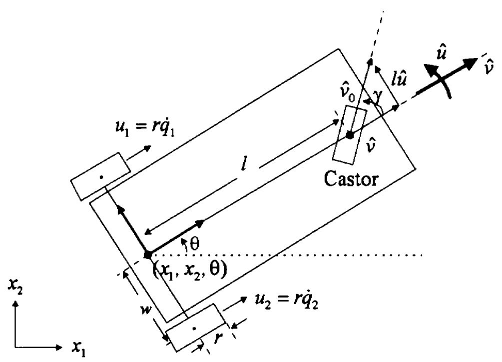

Fig. 1. The kinematic model of a mobile robot with two independently driven wheels and one free front wheel.

$\hat{v}$: linear velocity of the robot
$\hat{u}$: rotational velocity of the robot.

The kinematics model of a mobile robot with two independently driven wheels can be represented by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\cos\theta\\ \frac{1}{2}\sin\theta\\ \frac{1}{2w}\end{bmatrix}u_1+\begin{bmatrix}\frac{1}{2}\cos\theta\\ \frac{1}{2}\sin\theta\\ -\frac{1}{2w}\end{bmatrix}u_2. \tag{8}$$

The stabilization of a mobile robot to a target position is a two-input and three-state problem as represented in (8), which one cannot obtain the normal form by using only state space exact linearization.[18,19] For simplicity, the target position will be set at the origin in this analysis. It is known that involutivity is not necessary for input-output pseudolinearization. Therefore, the pseudo-normal form can be obtained by applying Theorem 1. To attempt transforming equation (8) to pseudo-normal form, two output variables need to be defined,

$$O_1=k_1x_1+k_2x_2,$$
$$O_2=\theta,$$

where k_1 and k_2 are constants.

The system of equation (8) has a family of constant operating points conveniently parameterized by the first and third components of the state vector yielding

$$x(\alpha)=\begin{bmatrix}\alpha_1\\ 0\\ \alpha_3\end{bmatrix},\quad u=\begin{bmatrix}0\\0\end{bmatrix}\quad\text{and}\quad O(\alpha)=\begin{bmatrix}k_1\alpha_1\\ \alpha_3\end{bmatrix}, \tag{9}$$

where $\alpha_1=x_1$ and $\alpha_3=\theta$. The corresponding linearization family is described by the parameterized coefficient matrices

$$A(\alpha)=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},\quad B(\alpha)=\begin{bmatrix}\frac{1}{2}\cos\alpha_3 & \frac{1}{2}\cos\alpha_3\\ \frac{1}{2}\sin\alpha_3 & \frac{1}{2}\sin\alpha_3\\ \frac{1}{2w} & -\frac{1}{2w}\end{bmatrix} \tag{10}$$

and

$$C(\alpha)=\begin{bmatrix}k_1&k_2&0\\0&0&1\end{bmatrix}.$$

From equation (9), we can obtain $rank(\partial x(\alpha)/\partial\alpha)(\alpha)=2$ and $\dim[B(\alpha)\cap x(\alpha)]=1$. Thus, the basic requirement of Theorem 1 is satisfied. Moreover, conditions (i)–(ii) of Theorem 1 can be obtained by the following:

$$c_1A^0(\alpha)B(\alpha)=\left[\frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3\quad \frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3\right], \tag{11}$$

$$c_2A^0(\alpha)B(\alpha)=\left[\frac{1}{2w}\quad -\frac{1}{2w}\right].$$

The pointwise relative degree is then

$$[\rho_1\quad\rho_2]=[1\quad 1], \tag{12}$$

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and M matrix becomes

$$M=\begin{bmatrix} c_1A^0(\alpha)B(\alpha) \\ c_2A^0(\alpha)B(\alpha) \end{bmatrix} = \begin{bmatrix} \frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3 & \frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3 \\ \frac{1}{2w} & -\frac{1}{2w} \end{bmatrix}. \quad (13)$$

Because $rank(\partial x/\partial\alpha)(\alpha)=m$ holds from equation (9), we obtain

$$M^{-1}=\frac{1}{-\frac{1}{2w}(k_1\cos\alpha_3+k_2\sin\alpha_3)} \times \begin{bmatrix} -\frac{1}{2w} & -\left(\frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3\right) \\ -\frac{1}{2w} & \frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3 \end{bmatrix}. \quad (14)$$

The original control input u and the new control input are then related by

$$u=M^{-1}v=\frac{1}{-\frac{1}{2w}(k_1\cos\alpha_3+k_2\sin\alpha_3)} \times \left.\begin{bmatrix} -\frac{v_1}{2w} & -\left(\frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3\right)v_2 \\ -\frac{v_1}{2w} & \left(\frac{k_1}{2}\cos\alpha_3+\frac{k_2}{2}\sin\alpha_3\right)v_2 \end{bmatrix}\right|_{\alpha_3=\theta} \quad (15)$$

where v is the control input for the system in the pseudo-normal form. The state coordinate change relation between x and z is also shown in equation (16):

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_1\sin\theta - x_2\cos\theta \\ k_1x_1+k_2x_2 \\ \theta \end{bmatrix}. \quad (16)$$

Equation (8) can then be transformed to pseudo-linear form by the control input in equation (15) and new state variables in equation (16) as below:

$$\begin{aligned} \dot{z}_1 &= \frac{v_2}{k_1\cos z_3+k_2\sin z_3}(-z_1(k_1\sin z_3-k_2\cos z_3)+z_2), \\ \dot{z}_2 &= v_1, \\ \dot{z}_3 &= v_2, \\ O_1 &= z_2, \\ O_2 &= z_3. \end{aligned} \quad (17)$$

Equation (17) can be rewritten as two subsystems:

$$\text{Subsystem 1} \begin{cases} \dot{z}_3 = v_2, \\ O_2 = z_3. \end{cases}$$

Subsystem 2

$$\begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \frac{-(k_1\sin z_3-k_2\cos z_3)v_2}{k_1\cos z_3+k_2\sin z_3} & \frac{v_2}{k_1\cos z_3+k_2\sin z_3} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1, \\ O_1 = [0 \quad 1]\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}. \end{cases} \quad (18)$$

It is obvious that the pseudo-normal form in equation (18) can be conveniently transformed to a linear parameter-varying system. Subsystem 2 is controllable as long as $v_2 \neq 0$ and $k_1\cos z_3+k_2\sin z_3 \neq 0$, and in this case a continuous-time controller exists that stabilizes Subsystem 2. That is, when Subsystem 1 chooses suitable control v_2 to ensure that $v_2 \neq 0$ in a finite time period, we can replace equation (18) with a linear parameter-varying system. When Subsystem 1 with $z_3(0)=\gamma$ uses static state feedback, then we can obtain

$$z_3=\gamma e^{-k_\alpha t}+\frac{\theta_d}{k_\alpha}(1-e^{-k_\alpha t}), \quad v_2=\theta_d-k_\alpha z_3, \quad (19)$$

where θ_d is the target angle, and k_α is the static feedback gain. Consequently, equation (18) can be simplified as a linear parameter-varying system by substituting equation (19) into equation (18). The linear parameter-varying system is then presented by

$$\begin{aligned} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} \frac{-(k_1\sin z_3-k_2\cos z_3)(\theta_d-k_\alpha z_3)}{k_1\cos z_3+k_2\sin z_3} & \frac{\theta_d-k_\alpha z_3}{k_1\cos z_3+k_2\sin z_3} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1, \\ O_1 &= [0 \quad 1]\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \\ z_3 &= ye^{-k_\alpha t}+\frac{\theta_d}{k_\alpha}(1-e^{-k_\alpha t}). \end{aligned} \quad (20)$$

The solution of equation (20) can be obtanied by a gain scheduling technique.[22,23] The gain scheduling algorithm is given below:

Algorithm 1:[22] Computing the state feedback gain matrices.

Step 1. Choose a matrix $F \in R^{n\times n}$ such that $\sigma(F)=\sigma_d$, and that for all $\beta \in I$, $\sigma(A(\beta)) \cap \sigma(F)=\phi$ (ϕ denotes the empty set), where $\sigma(F)$ denotes all eigenvalues of matrix F.

Step 2. Choose a matrix $\bar{K} \in R^{n_i\times n}$ such that pair $(F, \bar{K})$ is observable.

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Step 3. Obtain at a point $\beta \in I$ the unique solution matrix $T(\beta) \in R^{n\times n}$ of the Lyapunov matrix equation

$$T(\beta)F - A(\beta)T(\beta) = -B(\beta)\bar{K}. \tag{21}$$

Step 4. If $T(\beta)$ at $\beta \in I$ is nonsingular, then the gain matrix is

$$K(\beta) = \bar{K}T^{-1}(\beta). \tag{22}$$

If $T(\beta)$ at $\beta \in I$ is singular, then choose a different $\bar{K}$ in **Step 2** and repeat the process.

Hence input v_1 of pseudo-normal form can be represented as

$$v_1 = -K_1(z_3)z_1 - K_2(z_3)z_2 + R, \tag{23}$$

where R is the reference input. Moreover, from equation (19), we know that input v_2 of pseudo-normal form is

$$v_2 = \theta_d - k_\alpha z_3. \tag{24}$$

Therefore, the input u of a mobile robot can be obtained as

$$u = M^{-1}v. \tag{25}$$

4. COMPARISON WITH PREVIOUS APPROACHES

Most of previous works on stabilizing mobile robots, are based on the chained form or the power form below:

$$\begin{aligned} \dot{z}_1 &= v_1 \\ \dot{z}_2 &= v_2 \\ \dot{z}_3 &= z_1 v_2. \end{aligned} \tag{26}$$

Equation (26) is obtained by some transformable techniques from equation (8).[11,22] It is noted that transformable techniques must use the Lie algebra and exterior differential systems in order to transform equation (8) into equation (26). Hence, complex mathematical methods are applied to equation (8) in the process of transformation. Moreover, previous works[5–7] on stabilization of mobile robots directly use the system with equation (26). These methods are discontinuous time-invariant stabilization,[5] time-varying stabilization,[6] and hybrid feedback laws. From equation (26), one can obtain that the system is still a nonlinear driftless system. Hence, complex methodologies are needed for stabilization of a nonlinear driftless system in the design of a stabilizable controller.

Firstly, consider the problem of stabilizing a system in equation (26) by the discontinuous time-invariant stabilization.[1,5] Define the feedback law

$$\begin{aligned} v_1 &= -z_1 + 2z_2 sign\left(z_3 - \frac{z_1 z_2}{2}\right) \\ v_2 &= -z_2 + 2z_1 sign\left(z_3 - \frac{z_1 z_2}{2}\right), \end{aligned} \tag{27}$$

where *sign* (.) denotes the signum function, the terms v_1 and v_2 are obtained under the suitable Lyapunov equation. Hence, the convergent trajectory must depend on a certain Lyapunov function. Secondly, consider the problem of stabilizing a system in equation (26) by the time-varying stabilization.[1,6] Define the feedback law

$$\begin{aligned} v_1 &= (z_2^2 + z_3^2)\sin t - (z_1 + (z_2^2 + z_3^2)\cos t) \\ v_2 &= -2(z_1 + (z_2^2 + z_3^2)\cos t) \\ &\quad \cdot (z_1 z_3 + z_2)\cos t - (z_1 z_3 + z_2), \end{aligned} \tag{28}$$

the terms v_1 and v_2 are also obtained under a Lyapunov function. Hence, this method also has a complex convergent trajectory. Thirdly, consider the problem of stabilizing a system in equation (26) by the hybrid feedback stabilization.[1,7] Define the hybrid feedback law

$$\begin{aligned} v_1 &= -z_1 + \alpha^k \sin t, \quad 2\pi k \le t \le 2\pi(k+1) \\ v_2 &= -z_2 + |\alpha^k| \cos t \quad 2\pi k \le t \leqq 2\pi(k+1), \end{aligned} \tag{29}$$

where $\{\alpha^k : k = 0, 1, \ldots\}$ is a sequence of scalar parameters. The feedback law construction depends on a parameter-dependent family of a continuous T-periodic control function. The parameter-dependent family of a continuous T-periodic control function is $\alpha^k \sin t$ and $|\alpha^k| \cos t$ in equation (29). Hence, the convergent trajectory is still complex.

The purpose of input-output pseudolinearization is an extension of global linearization which consists of transforming a nonlinear system into a linear system in the whole state space. Thus, a broader class of systems can be linearizable by the input-output pseudolinearization and this technique has much broader applications. In this paper the guidance control problem of a mobile robot as a two feedback loops structure is shown in Figure 2. We first apply this technique to transform the kinematic model of a mobile robot into a linear parameter-varying system. Then the gain scheduling controller is used to effectively stabilize the mobile robot. The gain scheduling controller is easy to implement and practical in many systems. In particular, the gain scheduling controller can perform pole-assignment in the control loop. By placing the closed loop poles in the appropriate left half-plane, simpler convergent trajectory can then be obtained. Therefore, we can get a simple convergence trajectory for stabilization of mobile robots. Finally, the stabilization of a mobile robot can be guaranteed by Theorem 1 as well as assumptions (A)–(A3). Comparison of the proposed approach with previous approaches will be illustrated in the following section.

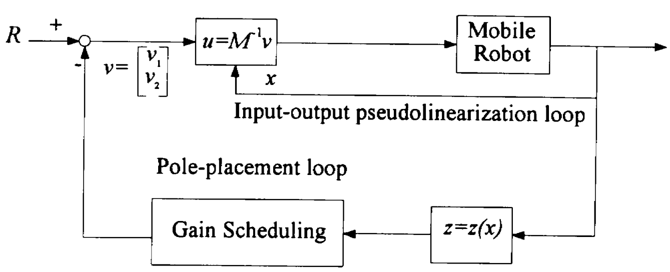

Fig. 2. The structure of the proposed stabilization controller.

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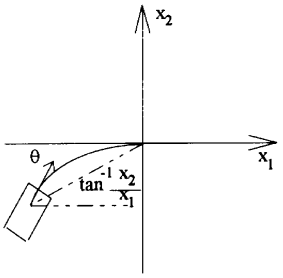

Fig. 3. The condition for one control mode in terms of the robot's initial location and orientation.

5. ILLUSTRATED EXAMPLES

In the first example, we consider controlling the mobile robot to the origin (0.0, 0.0, 0.0) from initial points $\left(-1.0, 0.5, \frac{\pi}{4}\right)$. The output variables are set to

$$O_1 = x_1 + x_2$$

$$O_2 = \theta_d = 0,$$

where $k_1 = 1$ and $k_2 = 1$ in equation (20). The simplified model can then be represented below as a linear

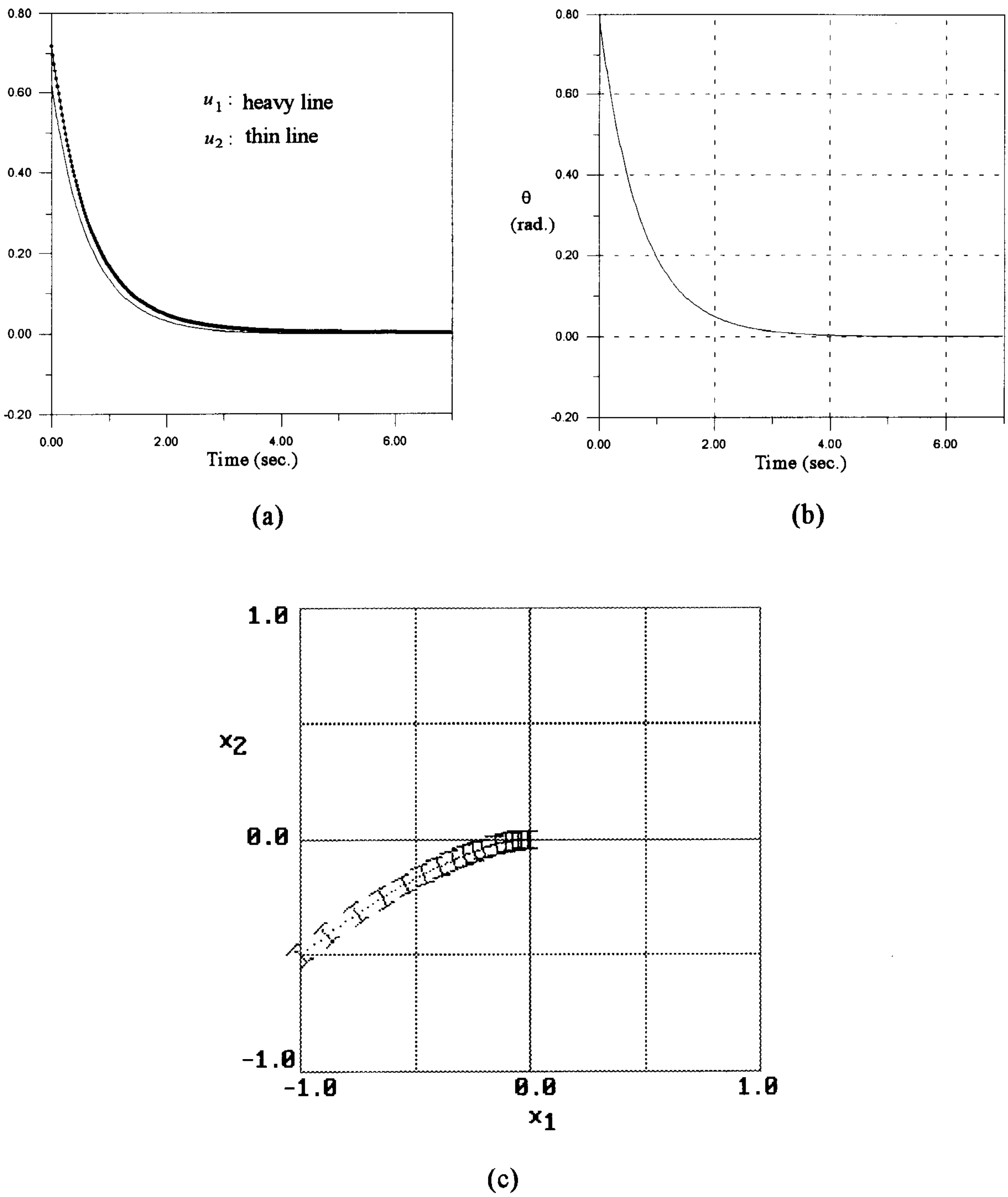

Fig. 4. The simulation results for example 1, where (a) is the speed response u_1 and u_2 of the mobile robots, (b) is the evolution of θ, and (c) is the mobile robot trajectory which converges to the origin from the initial point $\left(-1.0, 0.5, \frac{\pi}{4}\right)$.

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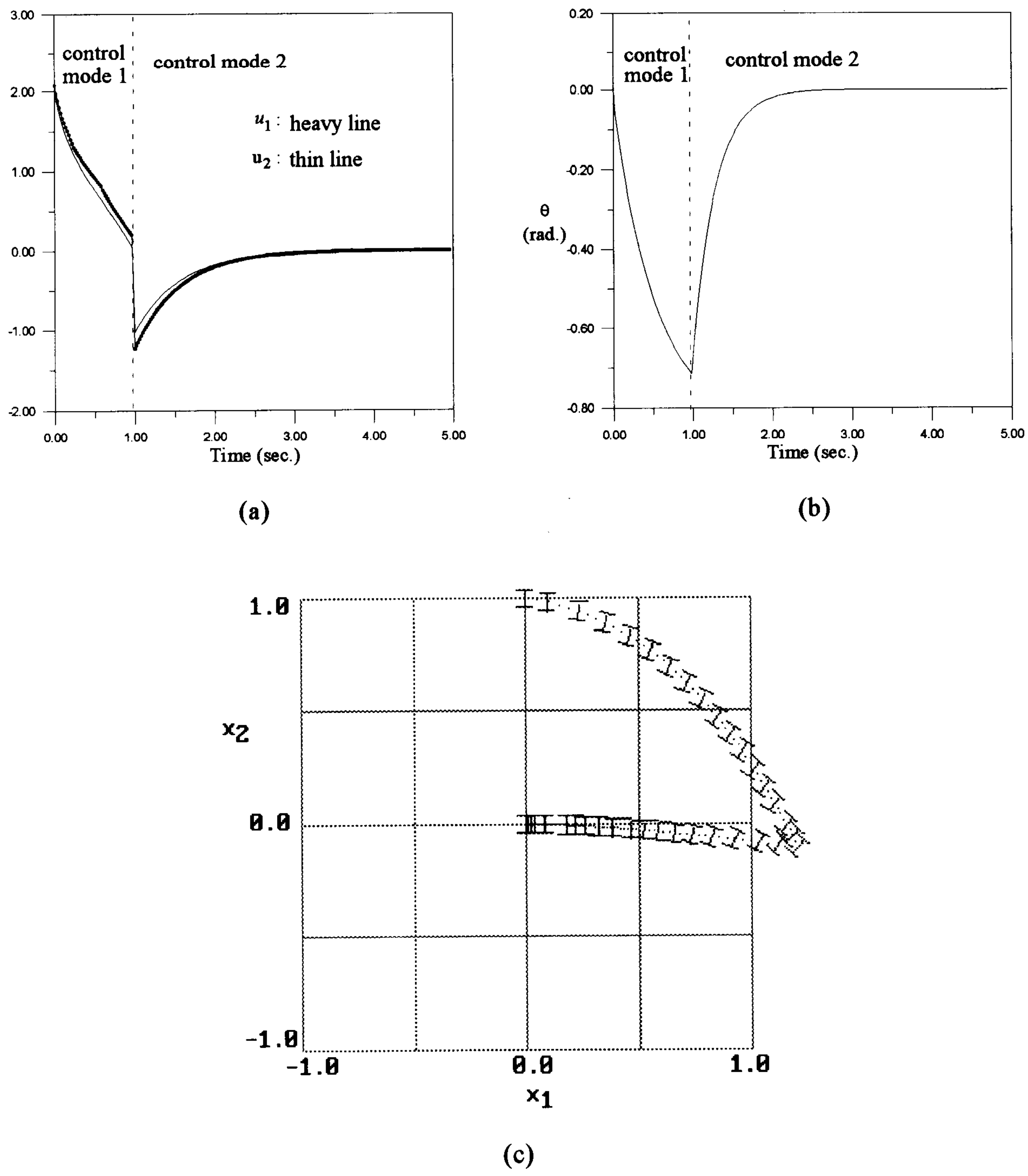

Fig. 5. The simulation results for example 2, where (a) is the speed response u_1 and u_2 of the mobile robots, (b) is the evolution of θ, and (c) is the mobile robot trajectory which converges to the origin from the initial point (0.0, 1.0, 0.0).

parameter-varying system

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}\dfrac{(\sin z_3-\cos z_3)k_\alpha z_3}{\cos z_3+\sin z_3} & \dfrac{-k_\alpha z_3}{\cos z_3+\sin z_3}\\ 0 & 0\end{bmatrix}\times\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}v_1,$$

$$O_1=[0\quad 1]\begin{bmatrix}z_1\\ z_2\end{bmatrix},$$

$$z_3=\gamma e^{-k_\alpha t},\quad 0\le t,$$

where $\gamma=\pi/4$, and $k_\alpha=1.89$. Noted that the above system is uncontrollable when θ is equal to $-45°$ or $135°$. Let $(x_{1initial}, x_{2initial}, \theta_{initial})$ denote the initial location and orientation of a mobile robot. Because θ is planned as an exponential decreasing function, its initial value must satisfy

$$\left|\tan\frac{x_{2initial}}{x_{1initial}}\right|<|\theta_{initial}|, \tag{30}$$

for reaching the origin (see Figure 3); otherwise, the robot needs turn in place to satisfy the above condition. In this example, the initial value of z_3 is not equal to zero and Constraint (30) is satisfied; therefore, one control mode is needed to stabilize the mobile robot at the

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origin. Figure 4(a) shows the speed response u_1 and u_2 of mobile robots, Figure 4(b) show the evolution of θ, and Figure 4(c) displays the resulting motion trajectory of the mobile robot which converges to the origin from the initial point $\left(-1.0, 0.5, \frac{\pi}{4}\right)$.

In the second example, we consider controlling the mobile robot to the origin (0.0, 0.0, 0.0) from the initial point (0.0, 1.0, 0.0). Because the mobile robot is located at x_2-axis and $z_3(0) = 0$, it is required that $\theta_d = 2\pi$ for only one control mode. In that case, the θ will pass through uncontrollable points; therefore, two control modes are needed in this example. The control mode one is to choose a temporary goal orientation $\theta_d \neq 0$, such that $z_3 = (\theta_d/k_\alpha)(1 - e^{-k_\alpha t})$ and Algorithm 1 can then be used. If the initial value of x_2 is positive, then $k_1 = 1$ and $k_2 = -1$ are chosen, and the robot moves to a temporary goal location in the 4th quadrant such that Constraint (30) is satisfied. Similarly, if the initial value of x_2 is negative, then $k_1 = 1$ and $k_2 = 1$ are chosen, and the robot moves to a temporary location in the 1st quadrant. After, we use control mode one to change the location and the orientation of the mobile robot, the control mode two is then used to stabilize the mobile robot to the origin. In the first time interval the output variables are set to

$$O_1 = x_1 - x_2$$
$$O_2 = \theta_d = -\pi/2.$$

Thus, the simplified model can be represented below as a linear parameter-varying system

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{(-\sin z_3 - \cos z_3)\left(-\dfrac{\pi}{2} - k_\alpha z_3\right)}{\cos z_3 - \sin z_3} & \dfrac{-\dfrac{\pi}{2} - k_\alpha z_3}{\cos z_3 - \sin z_3} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1,$$

$$O_1 = [0 \quad 1]\begin{bmatrix} z_1 \\ z_2 \end{bmatrix},$$

$$z_3 = -\frac{\pi}{2k_\alpha}(1 - e^{-k_\alpha t}), \quad 0 \le t \le 1,$$

where the time interval 1.0 seconds and $k_\alpha = 1.85$ are selected such that the Constraint (30) can be satisfied at the beginning of the control mode two. In the second time interval the output variables are set to

$$O_1 = x_1 - x_2$$
$$O_2 = \theta_d = 0.$$

The simplified model can then be represented below as a linear parameter-varying system

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{(\sin z_3 + \cos z_3)k_\alpha z_3}{\cos z_3 - \sin z_3} & \dfrac{-k_\alpha z_3}{\cos z_3 - \sin z_3} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1,$$

$$O_1 = [0 \quad 1]\begin{bmatrix} z_1 \\ z_2 \end{bmatrix},$$

$$z_3 = \gamma e^{-k_\alpha(t-1)}, \quad 1 < t,$$

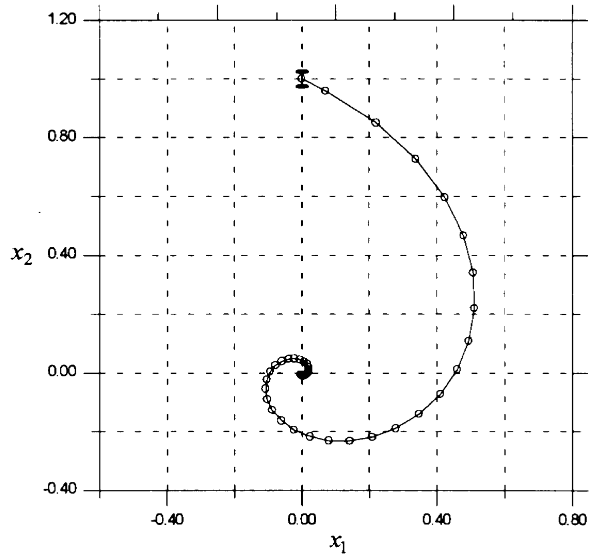

Fig. 6. The mobile robot converges to the origin from the initial point (0.0, 1.0, 0.0) under the discontinuous time-invariant feedback law in equation (27).

where $\gamma = z_3(1)$ and $k_\alpha = 3.5$. Figure 5(a) shows the speed response u_1 and u_2 of mobile robots, Figure 5(b) shows the evolution of θ, and Figure 5(c) displays the resulting motion trajectory of the mobile robots which converges to the origin from initial point (0.0, 1.0, 0.0).

Figure 6 displays the trajectories of the mobile robot converging to the origin from (0.0, 1.0, 0.0) under the discontinuous time-invariant feedback law in equation (27) for system equation (26). Figure 7 displays the trajectories of the mobile robot converging to the origin from (0.0, 1.0, 0.0) under time-varying feedback law in equation (28) for system equation (26). Figure 8 displays the trajectories of the mobile robots converging to the origin from (0.0, 1.0, 0.0) under the hybrid feedback law

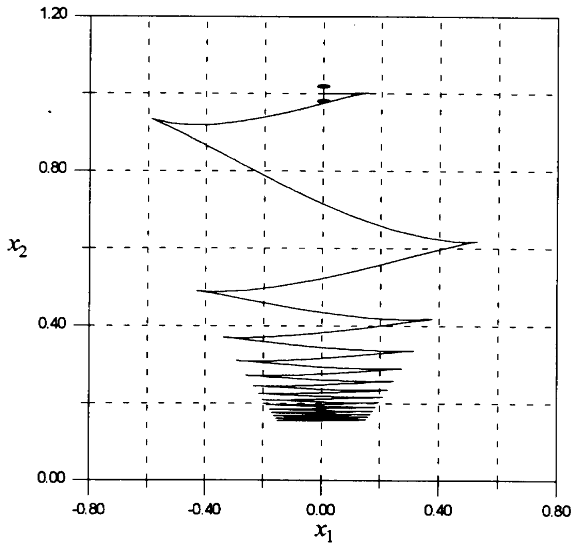

Fig. 7. The mobile robot converges to the origin from the initial point (0.0, 1.0, 0.0) under the time-varying feedback law in equation (28).

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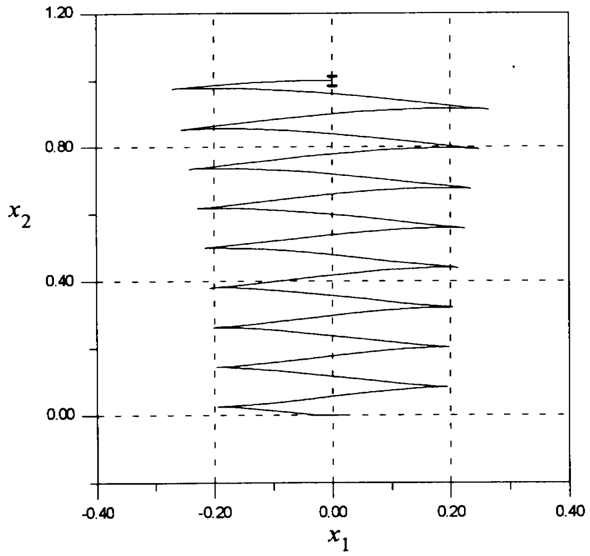

Fig. 8. The mobile robot converges to the origin from the initial point (0.0, 1.0, 0.0) under the hybrid feedback law in equation (29).

in equation (29) for system (26). Compared to these results, the mobile robots in Example 2 did not go through the jagged-like, pendulum motion that robots from other works wen through in order to move to the target position point. In fact, the motion of the mobile robot to the target point by applying the proposed method was much simpler as seen in Figure 5(c). Moreover, the proposed controller is discontinuous and time-varying when using two control modes.

6. CONCLUSIONS

A two feedback control-loop structure for stabilization of the mobile robot with two independently driven wheels has been studied by using the input-output pseudo-linearization and gain scheduling technique. From the simulation results it is shown that the proposed stabilization method is very effective, and a much smoother trajectory of the mobile robot can be obtained. Moreover, based on the characteristic of input-output pseudolinearization,[18] gain scheduling techniques[20] and results of this paper, the proposed method has the potential to control other nonlinear systems than existing linearization methods do.

References

1. I. Kolmanovsky and N.H. McClamroch, "Developments in Nonholonomic Control Problems" *IEEE Control Systems* **15,** No. 6, 20–36 (1995).
2. C. Samson, "Time-varying Feedback Stabilization of Car-like Wheeled Mobile Robots" *Int. J. Robotics Research* **12,** No. 1, 55–64 (1993).
3. D.B. Reister and F.G. Pin, "Time-optimal Trajectories for Mobile Robots with Two Independently Driven Wheels" *Int. J. Robotics Research* **13,** No. 1, p. 38–54 (1994).
4. D.B. Reister, "A New Wheel Control System for the Omnidirectional HERMES-III Robot" *Robotica* **10,** Part 4, 351–360 (1992).
5. A.M. Bloch, M. Reyhanoglu and N.H. McClamroch, "Control and Stabilization of Nonholonomic Dynamic Systems" *IEEE Trans. on Automatic Control* **37,** No. 11, 1746–1757 (1992).
6. J.B. Pomet, "Explicit Design of Time-Varying Stabilizing Control Laws for a Class of Controllable Systems without Drift" *System and Control Letter* **18,** 147–158 (1992).
7. W. Oelen, H. Berghuis, H. Nijmeijer and C. Canudas de Wit, "Hybrid Stabilizing Control on a Real Mobile Robot" *IEEE Robotics and Automation Magazine,* 16–23 (1995).
8. S.S. Lee, J.H. Williams and P.J. Rayment, "An Automatic Guidance System of an Autonomous Vehicle: the Trajectory Generation and the Control Algorithm" *Robotica* **11**, Part 4, 309–314 (1993).
9. A. Hemami, M.G. Mehrabi and R.M.H. Cheng, "Optimal Kinematic Path Tracking Control of Mobile Robots with Front Steering" *Robotica* **12,** Part 1, 563–568 (1994).
10. R.M. DeSantis, "Modelling and Path-tracking Control of a Mobile Wheeled Robot with a Differential Driven" *Robotica* **13,** Part 4, 401–410 (1995).
11. R.M. Murray and S.S. Sastry, "Nonholonomic Motion Planning: Steering Using Sinusoids" *IEEE Trans. on Automatic Control* **38,** No. 5, 700–716 (1993).
12. B. D'Andrea-Novel, G. Campion and G. Basin, "Control of Wheeled Mobile Robots not Satisfying Ideal Velocity Constraints: a Singular Perturbation Approach" *Int. J. Robot and Nonlinear Control* **5,** 243–267 (1995).
13. Y. Kanayama, Y. Kimura, F. Miyazaki and T. Noguchi, "A Stable Tracking Control Method for an Autonomous Mobile Robot" *Proc. IEEE Inter. Conf. on Robotics and Automation* (1990) pp. 384–389.
14. Y. Kanayama and S. Yuta, "Vehicle Path Specification by a Sequence of Straight Lines" *IEEE Journal of Robotics and Automation* **4,** No. 3, 265–276 (1988).
15. C. Champetier, P. Mouyon and C. Reboulet, "Pseudo-linearization of Multi-input Nonlinear System" *Proceedings of 23rd CDC* (1984) pp. 96–97.
16. C. Reboulet and C. Champetier, "A New Method for Linearizing Nonlinear Systems: the Pseudolinearization" *Int. J. Control* **40,** No. 4, 631–638 (1984).
17. C. Champetier, P. Mouyon, and J.F. Magni, "Pseudo-linearization of Nonlinear System by Dynamic Precompensation" *Proceedings of 24th C.D.C.* (1985) pp. 1371–1372.
18. D.A. Lawrence and W.J. Rugh, "Input-output Pseudo-linearization for Nonlinear Systems" *IEEE Trans. on Automatic Control* **39,** No. 11, 2207–2218 (1994).
19. A. Isidori, *Nonlinear Control Systems,* 2nd Edition (Springer-Verlag, Berlin, 1985).
20. S.M. Shahruz and S. Behtash, "Design of Controllers for Linear Parameter-varying System by the Gain Scheduling Technique" *J. Mathematical Analysis and Application* **168,** 195–217 (1992).
21. C.T. Chen, *Linear System Theory and Design* (Holt, Rinehart and Winston, USA, 1984).
22. D. Tilbury, R.M. Murray, and S.S. Sastry, "Trajectory Generation for the N-Trailer Problem Using Goursat Normal Form" *IEEE Trans. on Automatic Control* **40,** No. 5, 802–819 (1995).

APPENDIX: GAIN SCHEDULING TECHNIQUE FOR A LINEAR PARAMETER-VARYING SYSTEM

The linear MIMO parameter-varying system can be represented for all $t \geq 0$ by

$$\dot{x}(t) = A(\beta)x(t) + B(\beta)u(t), \quad x(0) = x_0,$$
$$y(t) = C(\beta)x(t), \tag{31}$$
$$\beta = \beta(t),$$

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where state $x(t) \in R^n$, input $u(t) \in R^{n_i}$, and output $y(t) \in R^{n_o}$; where parameter $\beta = \beta(t) \in [\beta_0 \quad \beta_n] \equiv I \subset R$; coefficient matrices $A(\beta) = [a_{ij}(\beta)] \in R^{n \times n}$, $B(\beta) = [b_{ij}(\beta)] \in R^{m \times n_i}$, and $C(\beta) = [c_{ij}(\beta)] \in R^{n_o \times n}$; and where the number of inputs $n_i \leq n$ and matrix $B(\beta)$ are of full column rank. Here it is assumed that

(A1) The elements of the coefficient matrices *A, B,* and *C* are analytic functions of β.

(A2) The parameter *b* is a continuous and bounded function of *t,* differentiable almost everywhere with a bounded derivative, and is measured for all $t \geq 0$.

(A3) The linear parameter-varying system is completely controllable for all $\beta \in I$.

The gain-scheduling state feedback controller $u(t)$ for the system in (31) is the form

$$u(t) = -K(\beta(t))x(t) + u_r(t), \tag{32}$$

where $K(\beta) \in R^{n_i \times n}$ is the state feedback gain matrix and $u_r(t) \in R^{n_i}$ is the reference input. The state feedback gain matrices $K(\beta(t))$ can be obtained in references 20 and 21.

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