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THE NEW KEYNESIAN WAGE PHILLIPS CURVE: CALVO VS. ROTEMBERG

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We systematically evaluate how to translate a Calvo wage duration into an implied Rotemberg wage adjustment cost parameter in medium-scale New Keynesian DSGE models by making use of the well-known equivalence of the two setups at first order. We consider a wide range of felicity functions and show that the assumed household insurance scheme and the presence of labor taxation greatly matter for this mapping, giving rise to differences of up to one order of magnitude. Our results account for the inclusion of wage indexing, habit formation in consumption, and the presence of fixed costs in production. We also investigate the conditional and unconditional welfare implications of the wage-setting schemes under efficient and distorted steady states.

Keywords: Wage Phillips Curve, Wage Stickiness, Rotemberg, Calvo, Welfare

1. INTRODUCTION

Studying the Great Recession, economists have increasingly come to rely on nonlinear macroeconomic models, be it to study the effects of uncertainty shocks as drivers of business cycles [e.g. Fernández-Villaverde et al. (2011); Born and Pfeifer (2014), and Fernández-Villaverde et al. (2015)] or to model the zero lower bound for the nominal interest rate [e.g. Johannsen (2014) and Plante et al. (2018)].[1] However, the use of nonlinear solution techniques often makes it impractical to use Calvo (1983)–Yun (1996)-type nominal rigidities. First, Calvo rigidities introduce an additional state variable in the form of price/wage dispersion. Second, they give rise to meaningful heterogeneity when not embedded in the right setup (more on this below) and would require tracking distributions in the model. Rotemberg (1982)-type adjustment costs are therefore currently experiencing a renaissance.[2]

However, it is quite difficult to attach a structural interpretation to the Rotemberg adjustment cost parameter, because there is no natural equivalent in

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the data. In contrast, for the Calvo approach various papers have computed average price durations, for example, Bils and Klenow (2004) and Nakamura and Steinsson (2008). The literature on *price* rigidities has therefore regularly made use of the first-order equivalence of Rotemberg- and Calvo-type adjustment frictions[3] by translating the Rotemberg adjustment costs to an implied Calvo price duration via the slope of the New Keynesian Price Phillips Curve.[4] However, such guidance for Rotemberg *wage* adjustment costs is still missing, despite good estimates for wage durations being available for both the USA [Taylor (1999) and Barattieri et al. (2014)] and the euro area [Le Bihan et al. (2012)]. This is unfortunate, as there has recently been a renewed focus on the importance of wage rigidities [e.g. Galí (2011); Barattieri et al. (2014), and Born and Pfeifer (2017)].

The present study closes this gap by systematically assessing the mapping between Calvo and Rotemberg wage rigidities in a prototypical medium-scale New Keynesian model including fiscal policy.[5] A particular goal is to provide guidance for researchers working on nonlinear New Keynesian DSGE models with wage rigidities. We focus especially on how (i) the other deep parameters of the model and (ii) the assumed labor market structure and insurance scheme in the model affect this mapping. For example, it greatly matters whether households supply idiosyncratic labor services and insurance is conducted via state-contingent securities as in Erceg et al. (2000) (EHL henceforth) or whether insurance takes place inside of a large family and a labor union supplies distinct labor services as in Schmitt-Grohé and Uribe (2006b) (SGU henceforth).[6] We also consolidate the results in the literature by providing a systematic overview of analytic expressions for the slope of the New Keynesian Wage Phillips Curve arising in the EHL[7] and SGU setups when using different utility functions with and without consumption habits. It turns out that the preference specification used only influences the slope of the New Keynesian Wage Phillips Curve in the EHL setup.

The study most related to this part of the paper is unpublished work by Schmitt-Grohé and Uribe (2006a), who compare the slope of the New Keynesian Wage Phillips Curve arising under the EHL and the SGU setups with Calvo wage setting. However, they do not consider Rotemberg wage setting, nor the role of fiscal policy or fixed costs in this mapping.

While the first part of the paper is concerned with the identical first-order dynamics in the Rotemberg and Calvo wage-setting frameworks, at higher order the two frameworks generally differ. In the second part of the paper, we investigate some of these differences by analyzing the welfare implications of different types of wage rigidities in a business cycle context. We theoretically show that if the steady state is efficient, welfare conditional on zero initial wage dispersion is identical under Calvo and Rotemberg wage setting. In contrast, from an unconditional welfare perspective the welfare losses under Calvo wage setting are bigger. We also show numerically that if the steady state is not efficient, then Calvo wage setting tends to generate larger welfare losses. These results mirror the findings of Nisticó (2007), Lombardo and Vestin (2008), and Damjanovic and Nolan (2011) for the case of price stickiness. We also investigate the welfare differences arising between the EHL and SGU wage-setting schemes.

The paper proceeds as follows. Sections 2 and 3 consider the EHL and SGU setups, respectively. Section 4 provides a numerical comparison. Section 5 performs the welfare analysis. Section 6 concludes. An appendix with detailed derivations and accompanying computer codes is available online.

2. NEW KEYNESIAN PHILLIPS CURVE IN THE EHL SETUP

In this section we lay out the respective prototypical household setups used in EHL and then derive the slope of the New Keynesian Wage Phillips Curve under Calvo and Rotemberg wage setting. In the background, but not of interest here, there are a continuum of firms producing differentiated intermediate goods and a final good firm bundling intermediate goods to a final good. As long as price dispersion (Calvo case) or price adjustment costs (Rotemberg case) are zero in steady state, prices can be sticky without affecting the results that only depend on the first-order dynamics, that is, all sections except Section 5.3.[8] In addition, there is a fiscal authority that finances government spending with distortionary labor and consumption taxation and transfers and a monetary authority conducting monetary policy, for example, according to a Taylor-type interest rate rule.

As long as price dispersion/price adjustment costs are zero in steady state, they could easily be added without affecting the conclusions derived.[9]

2.1. Setup

Following EHL, we assume that the economy is populated by a continuum of monopolistically competitive, infinitely lived households $j, j \in [0, 1]$, supplying differentiated labor services N_t^j at wage W_t^j to intermediate goods producers who aggregate them into a composite labor input N_t^d with cost W_t using a Dixit and Stiglitz (1977) aggregator.

2.2. Calvo Wage Setting

In case of Calvo wage setting, the household is not able to readjust its wage in any given period with probability θ_{w}. Therefore, it chooses today's optimal wage W_t^* to maximize the expected utility over the states of the world where this wage is operative:

$$\max_{W_t^*} V_t = E_t \sum_{k=0}^{\infty} (\beta\theta_{\mathrm{w}})^k \, U\left(C_{t+k|t}^j, N_{t+k|t}^j, \cdot\right), \tag{1}$$

where V is the utility function and U is the felicity function with partial derivatives $U_C > 0$ and $U_N < 0$. The dot denotes additional variables potentially entering the felicity function (e.g., lagged consumption in the case of habits), and where

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$0 < \beta < 1$ is the discount factor. The subscript $t+k|t$ indicates a variable in period $t+k$ conditional on having last reset the wage at time t. When choosing the optimal wage W_t^*, the household does so taking into account the demand for its labor services:

$$N_{t+k|t}^{j} = \left(\frac{W_{t+k|t}^{j}}{W_{t+k}} \right)^{-\varepsilon_{\mathrm{w}}} N_{t+k}^{d}\,, \tag{2}$$

where the wage operative in period $t+k$, $W_{t+k|t}^{j}$ is given by the originally chosen wage W_t^* times a term $\Gamma_{t,t+k}^{\mathrm{ind}}$ that reflects the indexing of wages to (past) inflation:

$$W_{t+k|t}^{j} = \Gamma_{t,t+k}^{\mathrm{ind}} W_t^*\,. \tag{3}$$

We keep this term generic to encompass the varying indexing schemes in the literature and only require that there is full indexing in steady state, that is, $\Gamma_k^{\mathrm{ind}} = \Pi^k$ (omitted time indices denote steady-state values).[10] Note that $\Gamma_{t,t}^{\mathrm{ind}} = 1$. The final constraint of this problem is the budget constraint[11]:

$$\left(1 + \tau_{t+k}^{c}\right) P_{t+k} C_{t+k|t}^{j} = \left(1 - \tau_{t+k}^{n}\right) W_{t+k|t}^{j} N_{t+k|t}^{j} + X_{t+k}, \tag{4}$$

where the household earns income from supplying differentiated labor N_t^j at the nominal wage rate W_t^j, which is taxed or subsidized at rate τ_t^n, and spends its income on consumption C_t^j, priced at the price of the final good P_t and taxed at rate τ_t^c. In this budget constraint, all additive terms that drop from the current optimization problem when taking the derivative with respect to W_t^* (e.g. capital income or transfers) have been lumped together in X_t.

Define the after-tax marginal rate of substitution as

$$\mathrm{MRS}_{t+k|t} = -\frac{\left(1 + \tau_{t+k}^{c}\right)}{\left(1 - \tau_{t+k}^{n}\right)} \frac{U_{N,t+k|t}}{V_{C,t+k|t}}, \tag{5}$$

where subscripts C and N denote partial derivatives, and the index j has been suppressed.[12] The well-known optimality condition for the optimal wage W_t^* prescribes that households set a desired markup over the weighted average of expected future marginal rates of substitution. In its log-linearized version, it yields

$$\hat{W}_t^* = (1 - \beta\theta_{\mathrm{w}}) \sum_{k=0}^{\infty} (\beta\theta_{\mathrm{w}})^k E_t \left[\widehat{\mathrm{MRS}}_{t+k|t} + \hat{P}_{t+k} - \hat{\Gamma}_{t,t+k}^{\mathrm{ind}} \right], \tag{6}$$

where hats denote percentage deviations from steady state. In order to derive the New Keynesian Wage Phillips Curve, one needs to express the previous equation recursively and aggregate over households j. Aggregation in particular implies replacing the idiosyncratic marginal rate of substitution $\widehat{\mathrm{MRS}}_{t+k|t}$ by an expression not depending on the initial period in which household j last reset the wage.

Log-linearizing (5) around the deterministic steady state, and combining it with the assumption of complete markets and equal initial wealth, yields

$$\widehat{\mathrm{MRS}}_{t+k|t} = \widehat{\mathrm{MRS}}_{t+k} + \underbrace{\left[-\frac{V_{CN} \times N}{V_{CC} \times C}\varepsilon_c^{\mathrm{mrs}} + \varepsilon_n^{\mathrm{mrs}}\right]}_{\equiv \varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k}\right), \qquad (7)$$

where $\varepsilon_n^{\mathrm{mrs}}$ and $\varepsilon_c^{\mathrm{mrs}}$ denote the steady-state elasticities of the marginal rate of substitution with respect to labor and consumption, respectively, and $\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}$ is the total elasticity of the MRS. The latter simplifies to $\varepsilon_n^{\mathrm{mrs}}$ in the case of additively separable preferences as in Erceg et al. (2000), because $V_{CN} = 0$. $\widehat{\mathrm{MRS}}_{t+k}$ is the average MRS in the economy.

Using equation (7), the linearized law of motion for the aggregate wage level, and defining wage inflation $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$, the New Keynesian Wage Phillips Curve follows after some tedious algebra as

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(1-\theta_{\mathrm{w}})(1-\beta\theta_{\mathrm{w}})}{\theta_{\mathrm{w}}(1+\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}})}\hat{\mu}_t^w - \frac{\beta\theta_{\mathrm{w}}}{1-\theta_{\mathrm{w}}}E_t\hat{\Gamma}_{t,t+1}^{\mathrm{ind}} + \frac{\theta_{\mathrm{w}}}{1-\theta_{\mathrm{w}}}\hat{\Gamma}_{t-1,t}^{\mathrm{ind}}, \qquad (8)$$

where $\hat{\mu}_t^w$ defines the deviation of the wedge between the average marginal rate of substitution and the real wage from its long-run value, that is, the steady-state markup:

$$\hat{\mu}_t^w \equiv \left(\hat{W}_t - \hat{P}_t\right) - \widehat{\mathrm{MRS}}_t. \qquad (9)$$

Equation (8) has the familiar intuition that if $\hat{\mu}_t^w < 0$, the wage markup is below its long-run value, inducing wage setters ceteris paribus to adjust wages upwards, leading to wage inflation.

2.3. Rotemberg Wage Setting

In case of Rotemberg wage setting, the problem of household j is choosing W_t^j to maximize

$$V_t = E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}^j, N_{t+k}^j, \cdot\right), \qquad (10)$$

taking into account the demand for its labor variety:

$$N_{t+k}^j = \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_{\mathrm{w}}} N_{t+k}^d \qquad (11)$$

and subject to the budget constraint:

$$\begin{aligned}&\left(1+\tau_{t+k}^c\right)P_{t+k}C_{t+k}^j \\ &\quad = \left(1-\tau_{t+k}^n\right)W_{t+k}^j N_{t+k}^j - \frac{\phi_{\mathrm{w}}}{2}\left(\frac{1}{\Gamma_{t+k-1,t+k}^{\mathrm{ind}}}\frac{W_{t+k}^j}{W_{t+k-1}^j} - 1\right)^2 \Xi_{t+k} + X_{t+k}.\end{aligned} \qquad (12)$$

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Here, the second-to-last term represents the quadratic Rotemberg costs of adjusting the wage, with ϕ_{w} being the Rotemberg wage adjustment cost parameter. The costs are proportional to the nominal adjustment cost base Ξ_{t+k} and arise whenever wage changes differ from the indexed inflation rate $\Gamma^{\mathrm{ind}}_{t+k-1,t+k}$.[13] X_{t+k} again captures additive terms not related to the current optimization problem. After imposing symmetry and making use of the definition of the after-tax MRS, equation (5), the resulting FOC can be written as

$$\begin{aligned}
0 = {} & \varepsilon_{\mathrm{w}} \frac{\mathrm{MRS}_t}{\frac{W_t}{P_t}} \left(1-\tau_t^n\right) \\
& + \left\{ (1-\varepsilon_{\mathrm{w}}) \left(1-\tau_t^n\right) - \phi_{\mathrm{w}} \left(\frac{1}{\Gamma^{\mathrm{ind}}_{t-1,t}} \Pi_{w,t} - 1 \right) \Pi_t \frac{1}{N_t} \frac{1}{\Gamma^{\mathrm{ind}}_{t-1,t}} \frac{\frac{\Xi_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}} \right\} \\
& + E_t \beta \frac{V_{C,t+1}}{V_{C,t}} \frac{\left(1+\tau_t^c\right)}{\left(1+\tau_{t+1}^c\right)} \frac{1}{N_t} \frac{1}{\frac{W_t}{P_t}} \left\{ \phi_{\mathrm{w}} \left(\frac{1}{\Gamma^{\mathrm{ind}}_{t,t+1}} \Pi_{w,t+1} - 1 \right) \frac{1}{\Gamma^{\mathrm{ind}}_{t,t+1}} \Pi_{w,t+1} \frac{\Xi_{t+1}}{P_{t+1}} \right\}.
\end{aligned} \tag{13}$$

Linearizing, equation (13), around the steady state and making use of the definition of $\hat{\mu}_t^w$, equation (9), yields

$$\hat{\Pi}_{w,t} = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(\varepsilon_{\mathrm{w}}-1)\,(1-\tau^n)\aleph}{\phi_{\mathrm{w}}} \hat{\mu}_t^w, \tag{14}$$

where $\aleph \equiv \frac{N \times W}{\Xi}$ denotes the steady-state share of the wage bill in the adjustment cost base.[14] This term appears, because, together with ϕ_{w}, it determines the costliness of wage changes. For a given ϕ_{w}, a higher adjustment cost base increases the effective costs of adjusting the wage, thereby decreasing the responsiveness of wage inflation to the wage gap $\hat{\mu}_t^w$. It is notable that despite allowing for indexing the Rotemberg framework does not give rise to a hybrid wage Phillips Curve with a backward-looking inflation term. The reason is that all firms in the Rotemberg wage adjustment framework adjust their wage each period. In contrast, in the Calvo framework, there are non-adjusters whose wage evolves according to the indexing process, which gives rise to a backward-looking term. Most papers assume that wage adjustment costs are proportional to either current or steady-state output.[15] Thus, the real steady-state adjustment cost base Ξ/P is equal to output Y, which is produced via a production function of the type $Y = F(K,N) - \Phi$, where F is a constant returns to scale production function and $\Phi \geq 0$ denotes fixed costs in production. The literature typically either abstracts from fixed costs, that is $\Phi = 0$, or sets them to the value of monopolistic pure profits so that there is no incentive for entry or exit in steady state. In that case, $\Phi = \varepsilon_{\mathrm{p}}^{-1} Y$, where $\varepsilon_{\mathrm{p}} > 0$ is the elasticity of substitution between monopolistically competitive intermediate goods firms. Steady-state output then is $Y = \frac{\varepsilon_{\mathrm{p}}-1}{\varepsilon_{\mathrm{p}}} F(K,N)$.

With firms choosing a gross markup of $\varepsilon_{\mathrm{p}}/(\varepsilon_{\mathrm{p}}-1)$ over marginal cost, $\aleph$ is given by

$$\aleph = \frac{WN}{\Xi} = \frac{\frac{\varepsilon_{\mathrm{p}}-1}{\varepsilon_{\mathrm{p}}} F_N N}{Y} = \begin{cases} \frac{\varepsilon_{\mathrm{p}}-1}{\varepsilon_{\mathrm{p}}}\,(1-\alpha)\,, & \text{if } \Phi = 0\,, \\ (1-\alpha)\,, & \text{if } \Phi = \varepsilon_{\mathrm{p}}^{-1} Y\,. \end{cases} \tag{15}$$

Here, $1-\alpha$ denotes the steady-state elasticity of the production function with respect to labor, for example, the labor exponent in a Cobb–Douglas production function. Expression (15) shows that the relevant steady-state labor share $\aleph$ is bigger in case of fixed costs, because net output Y in the denominator only includes capital and labor payments, while in case of no fixed costs, it also includes pure profits. Hence, the slope of the Wage Phillips Curve is ceteris paribus flatter in the absence of fixed costs.

2.4. Mapping Calvo Durations to Rotemberg Adjustment Costs

Comparing the slopes of the two Wage Phillips Curves, equations (8) and (14), yields

$$\frac{(1-\theta_{\mathrm{w}})\,(1-\beta\theta_{\mathrm{w}})}{\theta_{\mathrm{w}}\,(1+\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}})} = \frac{(\varepsilon_{\mathrm{w}}-1)\,(1-\tau^{n})\aleph}{\phi_{\mathrm{w}}}, \tag{16}$$

from which the Rotemberg wage adjustment cost parameter $\phi_{\mathrm{w}}^{\mathrm{EHL}}$ in the EHL framework implied by a particular Calvo wage duration θ_{w} can be inferred as

$$\phi_{\mathrm{w}}^{\mathrm{EHL}} = \frac{(\varepsilon_{\mathrm{w}}-1)\,(1-\tau^{n})\aleph}{(1-\theta_{\mathrm{w}})\,(1-\beta\theta_{\mathrm{w}})}\theta_{\mathrm{w}}\left(1+\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}\right). \tag{17}$$

The left-hand side of equation (16) shows that, similar to the case of the New Keynesian Price Phillips curve, the discount factor β and the Calvo wage duration θ_{w} determine the slope of the Wage Phillips Curve in the Calvo case. But there is an additional correction factor in the denominator of equation (16) that is a function of the elasticity of substitution ε_{w} and the total elasticity of the marginal rate of substitution, $\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}$. This correction factor arises from the EHL setup in the Calvo case due to the idiosyncratic marginal rate of substitution being used to evaluate the labor–leisure trade-off when deciding on the new wage, while the New Keynesian Wage Phillips Curve is written in terms of the average marginal rate of substitution. As equation (7) shows, the idiosyncratic MRS of a wage re-setter is equal to the average MRS plus a correction factor accounting for differences in hours worked relative to the average. These differences in hours worked, in turn, arise from the reset wage W_t^* differing from the aggregate one [see the labor demand equation (2)]. Consequently, this additional correction factor drops out when either the total elasticity of the MRS $\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}$ is zero, or the substitution elasticity ε_{w} is zero (or both). Both these cases result in the idiosyncratic MRS being equal to the average one. In the first case, the idiosyncratic MRS is completely unresponsive to differences in hours worked arising from wage stickiness. In the

TABLE 1. Elasticity $\varepsilon_{\text{tot}}^{\text{mrs}}$ for different felicity functions

	$U(C,N)$	$\varepsilon_{\text{tot}}^{\text{mrs}}$	Habits
Add. separable	$\frac{C^{1-\sigma}-1}{1-\sigma} - \psi \frac{N^{1+\varphi}}{1+\varphi}$	φ	✓
GHH (1988)	$\frac{\left(C-\psi N^{1+\varphi}\right)^{1-\sigma}-1}{1-\sigma}$	φ	✓
Add. sep., log leisure	$\frac{C^{1-\sigma}-1}{1-\sigma} + \psi \log(1-N)$	$\frac{N}{1-N}$	✓
Multipl. separable	$\frac{\left(C^{\eta}(1-N)^{1-\eta}\right)^{1-\sigma}-1}{1-\sigma}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\right] \times \frac{N}{1-N}$	✓[a]

Notes: Total elasticity of the after-tax marginal rate of substitution, $\varepsilon_{\text{tot}}^{\text{mrs}}$, for additively separable preferences in consumption and hours worked (first row), for Greenwood et al. (1988)-type preferences (second row), additively separable preferences in consumption and log leisure (third row), and multiplicative preferences (fourth row). The last column indicates whether the computed elasticity is robust to the inclusion of internal or external habits in consumption of the form $C_t - \phi_c C_{t-1}$.
[a]For multiplicatively separable preferences, the resulting expression becomes somewhat more complex, see Appendix C.1.2 in Supplementary Material.

second case, the j labor services are perfect complements so that even arbitrarily large wage differences do not translate to any differences in hours worked.

Table 1 displays the respective expressions for $\varepsilon_{\text{tot}}^{\text{mrs}}$ for different felicity functions (see Appendix C in Supplementary Material for details). In case of standard additively separable preferences and for Greenwood et al. (1988) preferences, $\varepsilon_{\text{tot}}^{\text{mrs}}$ simply corresponds to the inverse Frisch elasticity parameter φ. For additively separable preferences with log leisure, the total elasticity is pinned down by the ratio of hours worked to leisure. For multiplicatively separable Cobb–Douglas-type preferences, $\varepsilon_{\text{tot}}^{\text{mrs}}$ depends on the degree of risk aversion, the weight of leisure in the utility function, and the ratio of hours worked to leisure.

With Frisch elasticity estimates ranging from 0.75 using micro data [Chetty et al. (2011)] to 2–4 using macro data [e.g., Smets and Wouters (2007) and King and Rebelo (1999)] as well as a share of hours worked in total time of 0.2–0.33, plausible values for the elasticity range between 0.25 and 1.5. With multiplicative preferences, realistic calibrations are in the same range as those obtained for separable preferences. For example, Backus et al. (1992) use $\sigma = 2$, $\eta = 0.34$, and $N/(1-N) = 0.5$ so that $\varepsilon_{\text{mrs}} \approx 0.75$.[16]

For the Rotemberg case on the right-hand side of equation (16), the slope depends on the elasticity of substitution, the Rotemberg adjustment cost parameter ϕ_{w}, and on the share of the wage bill in the adjustment cost tax base $\aleph$. In contrast to the previously considered Calvo case, the slope of the Wage Phillips Curve with Rotemberg wage setting is decreasing in the labor tax rate τ^n. The reason is that the labor tax rate drives a wedge between the real wage and the marginal rate of substitution. In the limit case of $\tau^n \to 1$, it does not pay off for the household to invest any resources into changing the nominal wage. Wage inflation then becomes completely decoupled from $\hat{\mu}_t$.[17]

Two remarks are in order. The first, technical one, is that Rotemberg wage adjustment cost estimates from papers abstracting from labor taxes cannot be translated directly to models with such taxes, because they will correspond to a flatter Phillips curve than intended. The second point is an economic one. If one believes that the Rotemberg price adjustment cost parameter is structural, then equation (14) implies that permanent increases in labor taxes can flatten the Wage Phillips Curve. Therefore, if presumed permanent, the gradual increase of labor taxes in the USA from below 15% before 1960 to its new plateau of about 23% is, ceteris paribus, associated with a flattening of the Wage Phillips Curve of 8% points in this framework.[18]

3. NEW KEYNESIAN PHILLIPS CURVE IN THE SGU SETUP

In this section we first derive the slope of the New Keynesian Wage Phillips curve in the Schmitt-Grohé and Uribe (2006b) setup under Calvo wage setting and under Rotemberg wage setting and then map them into each other.

3.1. Setup

Schmitt-Grohé and Uribe (2006b) assume that the economy is populated by a household with a continuum of members that supply the same homogenous labor service N_t, have the same consumption level due to insurance within the household, and work the same amount of hours. This contrasts with EHL, where households supply differentiated labor services and insurance takes place via complete markets.[19] The homogenous labor input in the SGU setup is supplied to a labor union that takes its members' utility into account and acts as a monopoly supplier of a continuum of j differentiated labor services N_t^j. These differentiated labor services are bundled into a composite labor input by intermediate goods producers exactly as in the EHL setup in Section 2.

The household has lifetime utility function:

$$V_t = E_t \sum_{k=0}^{\infty} \beta^k U\big(C_{t+k}, N_{t+k}, \cdot\big) , \tag{18}$$

where $N_{t+k} = \int_0^1 N_{t+k}^j \, dj$ is the market clearing condition assuring that total hours worked across all markets equal the supply by households. The household's nominal budget constraint is

$$\big(1 + \tau_{t+k}^c\big) P_{t+k} C_{t+k} = \big(1 - \tau_{t+k}^n\big) \int_0^1 W_{t+k}^j N_{t+k}^j \, dj + X_{t+k} , \tag{19}$$

where the household earns income from differentiated labor N_{t+k}^j at the nominal wage rate W_{t+k}^j through the labor services supplied by the union and X_{t+k} again captures unrelated additive terms.

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3.2. Calvo Wage Setting

The labor union chooses the optimal wage W_t^* in all labor markets where it is able to reoptimize in order to maximize its members' utility, equation (18). It takes into account the demand for labor variety j, equation (2), and the relevant part of the budget constraint (19):

$$\left(1+\tau^c_{t+k}\right)P_{t+k}C_{t+k} = \left(1-\tau^n_{t+k}\right)W^{\varepsilon_{\mathrm{w}}}_{t+k}N^d_{t+k}\theta^k_{\mathrm{w}}\left(\Gamma^{\mathrm{ind}}_{t,t+k}W^*_t\right)^{1-\varepsilon_{\mathrm{w}}}. \tag{20}$$

The latter makes use of the fact that, at each point in time $t+k$, the union is able to reset the wage in a fraction $1-\theta_{\mathrm{w}}$ of labor markets, which therefore become irrelevant for the wage-setting decision at time t. This leaves a fraction θ^k_{w} of labor markets where the time t optimal wage W_t^* is still active. Taking the FOC, the New Keynesian Wage Phillips Curve follows after some tedious algebra as

$$\hat{\Pi}^w_t = \beta E_t \hat{\Pi}^w_{t+1} - \frac{(1-\beta\theta_{\mathrm{w}})(1-\theta_{\mathrm{w}})}{\theta_{\mathrm{w}}}\hat{\mu}^w_t - \frac{\beta\theta_{\mathrm{w}}}{1-\theta_{\mathrm{w}}}E_t\hat{\Gamma}^{\mathrm{ind}}_{t,t+1} + \frac{\theta_{\mathrm{w}}}{1-\theta_{\mathrm{w}}}\hat{\Gamma}^{\mathrm{ind}}_{t-1,t}. \tag{21}$$

Comparing the slope of the Wage Phillips Curve in (21) to the one of EHL in (8), the EHL slope is smaller by a factor of $\left(1+\varepsilon_{\mathrm{w}}\varepsilon^{\mathrm{mrs}}_{\mathrm{tot}}\right)^{-1}$. The reason is that in the SGU setup, due to the family structure where everyone consumes the same and works the same hours, wage re-setters use the average MRS to evaluate the labor leisure trade-off, not the idiosyncratic one. Hence, no correction factor is needed.

3.3. Rotemberg Wage Setting

The Rotemberg problem of the labor union is similar to the household wage-setting problem in the EHL case. The budget constraint is given by

$$(1+\tau^c_t)P_tC_t = \left(1-\tau^n_t\right)\int_0^1 W^j_tN^j_t\,dj - \frac{\phi_{\mathrm{w}}}{2}\int_0^1\left(\frac{1}{\Gamma^{\mathrm{ind}}_{t-1,t}}\frac{W^j_t}{W^j_{t-1}}-1\right)^2 dj\ \Xi_t + X_t. \tag{22}$$

Following the steps outlined in Section 2.3, it can be verified that this leads to the same Wage Phillips Curve as in the EHL case:

$$\hat{\Pi}_{w,t} = \beta E_t\hat{\Pi}_{w,t+1} - \frac{(\varepsilon_{\mathrm{w}}-1)(1-\tau^n)\aleph}{\phi_{\mathrm{w}}}\hat{\mu}^w_t. \tag{14}$$

3.4. Mapping Calvo Durations to Rotemberg Adjustment Costs

Comparison of the slopes of the two Wage Phillips Curves, equations (21) and (14), yields an expression for the Rotemberg parameter ϕ_{w} implied by a Calvo wage duration θ_{w} in the SGU setup:

$$\phi^{\mathrm{SGU}}_{\mathrm{w}} = \frac{(\varepsilon_{\mathrm{w}}-1)(1-\tau^n)\aleph}{(1-\theta_{\mathrm{w}})(1-\beta\theta_{\mathrm{w}})}\theta_{\mathrm{w}}, \tag{23}$$

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TABLE 2. Implied Rotemberg adjustment cost parameters ϕ_{w} (quarterly model)

	$\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}=0.25$	$\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}=1$	$\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}=1.5$	$\beta=0.985$	$\beta=0.99$	$\beta=0.995$
SGU	61.36	61.36	61.36	60.48	61.36	62.27
EHL	230.10	736.31	1073.79	725.74	736.31	747.19
	$\varepsilon_{\mathrm{w}}=6$	$\varepsilon_{\mathrm{w}}=11$	$\varepsilon_{\mathrm{w}}=21$	$\tau^n=0$	$\tau^n=0.21$	$\tau^n=0.4$
SGU	30.68	61.36	122.72	77.67	61.36	46.60
EHL	214.76	736.31	2699.81	932.04	736.31	559.22
	$\Phi=\varepsilon_{\mathrm{w}}^{-1}Y$	$\Phi=0$				
SGU	61.36	55.78				
EHL	736.31	669.37				

Notes: Implied Rotemberg wage adjustment cost parameter ϕ_{w} that corresponds to an implied Calvo wage duration of four quarters ($\theta_{\mathrm{w}}=0.75$) for different parameter values in the SGU and EHL frameworks. All other parameters are kept at their baseline value: $\beta=0.99$, $\tau^n=0.21$, $\varepsilon_{\mathrm{w}}=\varepsilon_{\mathrm{p}}=11$, $\alpha=0.3$, $\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}=1$, $\Phi=\varepsilon_{\mathrm{w}}^{-1}Y$.

which differs from the EHL case, equation (17). The latter has an additional term $(1+\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}})$ arising from the idiosyncratic MRS being used to evaluate the labor leisure trade-off when deciding on the new optimal wage instead of the aggregate one.

4. COMPARISON OF EHL- VERSUS SGU-STYLE INSURANCE SCHEMES

4.1. Rotemberg Wage Adjustment Costs

Table 2 shows the implied Rotemberg wage adjustment cost parameter corresponding to an implied Calvo wage duration of four quarters ($\theta_{\mathrm{w}}=0.75$) for different parameter values in the SGU and EHL frameworks at quarterly frequency. All parameters except for the one under consideration are kept at their baseline values. For the baseline calibration we choose a discount factor of $\beta=0.99$, corresponding to a 4% real interest rate. The labor tax rate is set to 0.21, which is the mean US effective tax rate over the sample 1960Q1:2015Q4, computed following Jones (2002). The substitution elasticities are set to $\varepsilon_{\mathrm{w}}=\varepsilon_{\mathrm{p}}=11$, implying a steady-state markup of 10%. $\aleph$ is set to 2/3, corresponding to an exponent of capital in a Cobb–Douglas production function of $\alpha=0.3$ and the presence of fixed costs that make steady-state firm profits 0. The total elasticity of the marginal rate of substitution, $\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}$, is set to 1 as is the case with additively separable preferences and an inverse Frisch elasticity of $\varphi=1$.

As can be seen in the rows labeled SGU and EHL, the particular household setup assumed makes a big difference due to the multiplicative $\left(1+\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}\right)$ factor appearing in the EHL setup. For our baseline parameterization, this factor amounts to $1+11\times 1=12$. This factor is also what makes the slope of the Wage Phillips Curve increase (almost) proportionally with the total elasticity of the marginal rate of substitution, $\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}$, in the EHL setup (second row, left panel).

In contrast, $\varepsilon_{\text{tot}}^{\text{mrs}}$ does not affect the slope in the SGU case (first row, left panel). The implied Rotemberg parameter increases proportionally in the elasticity of substitution between goods ε_{w} for the SGU setup (third row, left panel). However, it increases overproportionally in the EHL setup (fourth row, left panel). Increasing ε_{w} by a factor of 3.5 from 6 to 21 results in an increase of the implied ϕ_{w} by a factor of 12.6. Assuming the absence of fixed costs, $\Phi = 0$, hardly changes the implied cost parameter in both setups for plausible calibrations (fifth and sixth rows, left panel). The first two rows of the right panel of Table 2 show that the effect of varying the discount factor β is relatively minor in both setups. Finally, the third and fourth rows of the right panel show that the steady-state labor tax rate τ^n significantly impacts the implied Rotemberg costs parameter as already discussed in Section 2.4.

4.2. Business-Cycle Dynamics Under Calvo Wage Setting: Monetary Policy Shock Example

To gauge the economic significance of the difference in the Wage Phillips Curve implied by the SGU- versus EHL-style insurance schemes under Calvo wage setting, we explore the impulse response functions (IRFs) to a monetary policy shock in the quarterly benchmark New Keynesian model with sticky prices and wages à la Calvo outlined in Galí (2015, Chapter 6). As we will rely on this model setup for our welfare analysis in the next section, we also introduce a supply shock in the form of a total factor productivity shock and a demand shock in the form of a preference shock. We deliberately keep the exposition at a minimum and refer to the textbook chapter for details.

The intermediate goods of firm $i \in [0, 1]$ are produced using the production function

$$Y_t^i = A_t \left(N_t^i\right)^{1-\alpha}, \tag{24}$$

with $0 < \alpha < 1$ measuring the decreasing returns to scale, A_t being an AR(1) exogenous technology shock process with mean 1, and N_t^i a Dixit–Stiglitz aggregate of differentiated labor services $N_t^{i,j}$, $j \in [0, 1]$ with substitution elasticity ϵ_{w}. There are no fixed costs of production. The final good is a Dixit–Stiglitz aggregate of the intermediate goods with substitution elasticity ϵ_{p}.

Household member $j \in [0, 1]$ has the felicity function:

$$U_t^j = Z_t \left(\frac{\left(C_t^j\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi} \right), \tag{25}$$

where σ is the risk aversion parameter and φ is the inverse Frisch elasticity. Z_t is an AR(1) exogenous demand shock process with mean 1. Due to the assumption of risk sharing via complete markets (EHL) or within the large family (SGU), the consumption level is the same for all j, that is, $C_t^j = C_t \; \forall j$. In addition, in the SGU case, all household members supply the same labor service to a labor union, so that $N_t^j = N_t \; \forall j$.

TABLE 3. Parameter values

Parameter	Value	Description
α	0.250	Capital share
β	0.990	Discount factor
σ	1.000	Inverse elasticity of intertemporal substitution
φ	5.000	Inverse Frisch elasticity
ϕ_π	1.500	Inflation feedback Taylor Rule
ϕ_y	0.125	Output feedback Taylor Rule
ϵ_{p}	9.000	Substitution elasticity intermediate goods
θ_{p}	0.750	Calvo parameter price setting
ϵ_{w}	4.500	Substitution elasticity labor services
θ_{w}	0.750	Calvo parameter wage setting
Π	1	Steady-state gross price inflation
ρ_a	0.900	Autocorrelation technology shock
ρ_ν	0.500	Autocorrelation monetary policy shock
ρ_z	0.500	Autocorrelation demand shock

Monetary policy is conducted using a Taylor rule of the form

$$R_t = \frac{1}{\beta}\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y} e^{\nu_t}, \tag{26}$$

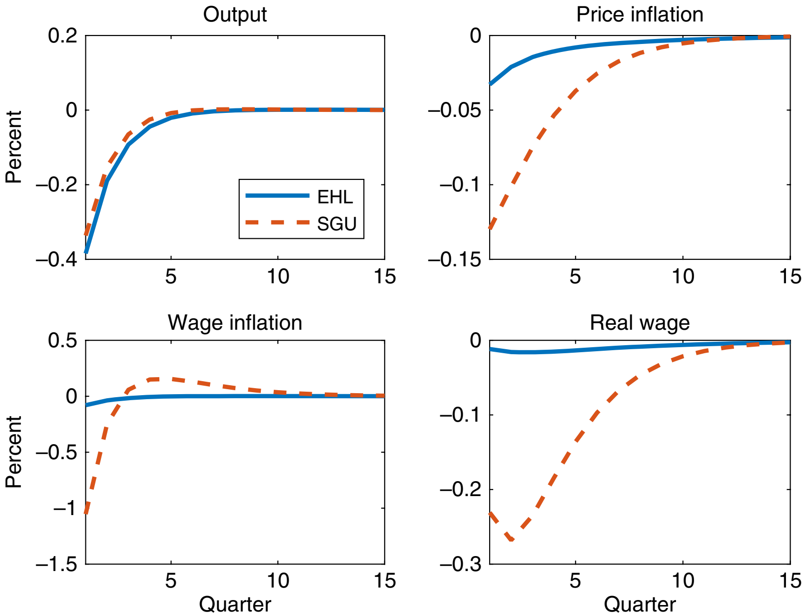
where ν_t is a mean zero AR(1) exogenous monetary policy shock process, and Π and Y denote steady-state price inflation and output, respectively. The calibration is summarized in Table 3. The model is solved using first-order perturbation techniques.

Figure 1 displays the results of a 1% point (annualized) monetary policy shock. The blue solid line shows the IRFs from the Galí (2015, Chapter 6.2.1) baseline EHL-type model. The red dashed line displays the IRFs from the SGU-type model with the same Calvo wage stickiness parameter, which implies a steeper wage Phillips Curve. As a consequence, output movements are very similar in both setups, but the responses of wage and price inflation as well as the real wage are stronger.

5. WELFARE IMPLICATIONS

Calvo and Rotemberg price and wage setting are identical up to first order in the absence of trend inflation.[20] However, the two price-/wage-setting models differ at higher order, potentially giving rise to different welfare implications as welfare computations generally require at least a second-order approximation [see, e.g., Woodford (2002)].

For the price stickiness case, Nisticó (2007) has shown that, conditional on initial price dispersion being zero, welfare up to second order is identical in the two price stickiness frameworks if the steady state is efficient. Lombardo and



Notes: blue solid line: EHL-type model; red dashed line: same model but with SGU-style insurance scheme. IRFs are in %.

FIGURE 1. IRFs to 1% point (annualized) monetary policy shock under Calvo at time 1 (Galí, 2015, Chapter 6.2.1).

Vestin (2008) have shown that, up to second order, Calvo price setting (i) generates higher unconditional welfare losses than Rotemberg price setting even in an efficient steady state and (ii) that for realistic model calibrations, conditional welfare losses of Calvo price setting are also higher in a distorted steady state. In the following, we investigate whether these findings also hold for wage stickiness.

5.1. Setup

Our welfare analysis is based on the canonical model setup of Galí (2015, Chapter 6) described in the previous section, except that we now consider the four different versions of the labor market discussed above.

The relevant felicity function U_t is an aggregate over all agents j in the economy. In the case of the EHL insurance scheme and Calvo price rigidities, we get

$$U_t = \int_0^1 U_t^j dj = Z_t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{N_t}{S_t^W}\right)^{1+\varphi}}{1+\varphi} X_t^W \right), \tag{27}$$

where S_t^W and X_t^W are auxiliary variables related to wage dispersion whose recursive laws of motion are given in Appendix D in Supplementary Material.

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These terms reflect the fact that wage dispersion in the EHL framework results in cross-sectional differences in hours worked that are immediately welfare reducing.

For the SGU Calvo and the two Rotemberg setups, we obtain

$$U_t = \int_0^1 U_t^j dj = Z_t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{28}$$

as the symmetric equilibrium of the Rotemberg wage setting scheme and the fact that household members supply a homogeneous labor good to unions in the SGU Calvo case considerably simplify aggregation.

5.2. Theoretical Results

For our theoretical results, we first make an assumption that allows us to compare welfare across the four different frameworks while assigning them the same first-order dynamics.

ASSUMPTION 1. (Identical first-order dynamics) *Assume that the slope of the linearized Wage Phillips Curve is identical in the four setups.*

Our first result concerns welfare under Rotemberg wage setting in the EHL and SGU frameworks. As the two setups are isomorphic, they also result in the same welfare losses:

PROPOSITION 1. (Welfare under Rotemberg wage setting) *Under Assumption 1, the welfare losses from Rotemberg wage stickiness are identical in the EHL and SGU setups.*

Proof. The result immediately follows from the identical aggregate utility function (28) and the identical first-order condition governing the wage-setting dynamics [see equation (B.16) in Supplementary Material], which proves that the two setups are isomorphic. ■

We now introduce an additional assumption that allows for clean analytical results and a better comparison to the findings from the price setting literature. We will relax it in the next section.

ASSUMPTION 2. (Efficient steady state) *Assume that the steady state is efficient, that is, appropriate subsidies counteract the monopolistic distortion in the goods and labor markets and there is no trend inflation.*[21]

Our second result confirms that the finding of identical conditional welfare losses from inflation variability in the Calvo versus Rotemberg price setting literature transfers to the welfare losses from wage inflation variability in the Calvo versus Rotemberg wage setting:

PROPOSITION 2. (Conditional welfare) *Under Assumptions 1 and 2, conditional on initial wage dispersion being zero in the Calvo wage-setting framework,*

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welfare losses from wage stickiness are identical up to second order in the Rotemberg and Calvo wage-setting framework for both the EHL and SGU setups.

Proof. See Appendix D in Supplementary Material. ■

However, this proposition only refers to conditional welfare. As is well known, the Calvo framework introduces an additional state variable in the form of wage dispersion. This additional state variable is a source of fundamental differences between the two frameworks. Wage dispersion in the stochastic equilibrium is on average different from zero, with its unconditional mean increasing in the variance of wage inflation. Thus, from an unconditional perspective, the on-average non-zero wage dispersion in the Calvo framework causes welfare losses that are not present in the Rotemberg framework. The next proposition summarizes the differences in unconditional welfare:

PROPOSITION 3. (Unconditional welfare) *Even if Assumptions 1 and 2 are satisfied, the Calvo wage-setting framework up to second order produces higher unconditional welfare losses than the Rotemberg framework.*

Proof. See Appendix D in Supplementary Material. ■

This result is also not surprising as it again mirrors the findings from the price setting literature [see Lombardo and Vestin (2008)]. Our last proposition concerns the different unconditional welfare implications of the SGU and EHL frameworks. The central difference between the two is whether households supply an idiosyncratic labor service and thus whether the idiosyncratic or aggregate marginal rate of substitution is used to value the real wage. We already saw in Section 3 that this distinction does not matter in the symmetric Rotemberg equilibrium, because all workers are alike and the idiosyncratic MRS coincides with the aggregate one. The situation is different with Calvo wage setting where wage dispersion also creates a heterogeneity in hours worked that is welfare-relevant. This can be seen from the presence of the wage dispersion terms S_t^w and X_t^w in the aggregate felicity function in equation (28). The negative aggregate utility effect from a dispersion in hours worked in the EHL setup comes on top of the welfare loss caused by the inefficiency in production introduced by wage dispersion, which appears in both the EHL and SGU frameworks. Thus, for a given amount of initial wage dispersion, the welfare losses in the SGU framework are bigger. However, under Assumption 1, the EHL framework produces a smaller unconditionally expected wage dispersion, because the unconditional mean of wage dispersion is a function of the Calvo parameter. As the slope of the Wage Phillips Curve in the EHL framework has an additional term $1 + \varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}}$ compared to the SGU framework, the required amount of Calvo wage stickiness to generate a particular slope of the Wage Phillips Curve is lower in the EHL setup than in the SGU setup. It turns out that for an efficient steady state the latter effect on the average wage dispersion always dominates the additional utility effect. Only if workers either do not dislike fluctuations in hours worked, that is, if the Frisch elasticity of labor supply is infinite, or labor services are perfect complements ($\varepsilon_{\mathrm{w}} = 0$) so that there are no differences in demand across varieties, are the welfare losses under SGU and EHL Calvo identical. In this case, the mean wage dispersion is the same, because the Calvo parameters coincide, and the dispersion in hours worked is only welfare-relevant via its effect on aggregate output, which is identical in both setups.

PROPOSITION 4. (Unconditional welfare: SGU vs. EHL) *Under Assumptions 1 and 2, the unconditional welfare losses under Rotemberg wage setting are identical in the EHL and SGU frameworks, but the losses from Calvo wage setting in the SGU case are bigger than in the EHL case if $\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}} > 0$. If $\varepsilon_{\mathrm{w}}\varepsilon_{\mathrm{tot}}^{\mathrm{mrs}} = 0$, the welfare losses are identical.*

Proof. The first part regarding the Rotemberg wage setting immediately follows from Proposition 1. For the Calvo case, see Appendix D in Supplementary Material. ■

After these qualitative results, we will next turn to a numerical evaluation of the welfare differences, which will also allow us to relax Assumption 2.

5.3. Numerical Evaluation

The exercise we conduct is in the spirit of Galí (2015, Chapter 6.5). We consider a number of simple monetary policy rules and quantitatively evaluate their impact on welfare, differentiating between Calvo and Rotemberg wage rigidities and EHL and SGU insurance schemes. In the experiments, fluctuations are driven by either the technology shock process A_t or the demand shock process Z_t, each with innovations with 1% standard deviation. The former shock is one that affects natural output, while the latter is a pure demand shock that can in principle be fully stabilized by monetary policy. The model calibration is shown in Table 3. We take the EHL Calvo case as the benchmark, resulting in a slope of the Wage Phillips Curve of 0.0037 that we keep fixed throughout.

Denote with V_0 expected lifetime utility in a particular specification:

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) . \tag{29}$$

We use as our welfare measure the fraction of flex-price consumption a household would be willing to give up in order to be indifferent to living under the alternative specification.[22] Thus, the permanent consumption loss λ is implicitly defined by

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t U\left((1-\lambda)C_t^{\mathrm{nat}}, N_t^{\mathrm{nat}}\right) , \tag{30}$$

where the superscript indicates the natural level of the variables in the flex-price economy. In line with the discussion in the previous section, we consider both the conditional version λ_{cond} of this measure where expectations are taken conditional on being in a steady state with zero price and wage dispersion as well as the unconditional version λ_{unc}.

In our evaluation of welfare differences across the four different labor market specifications, we follow Galí (2015) and consider two different sets of monetary policy. The first set are "strict targeting rules" that require either price inflation or wage inflation or a composite inflation measure to be zero at all times:

$$\Pi_t^k = 0, k \in \{p, w, c\}, \qquad \textbf{(31)}$$

where the composite inflation measure is a weighted average:

$$\Pi_t^c \equiv \left(\Pi_t^p\right)^{\vartheta} \left(\Pi_t^w\right)^{1-\vartheta} \qquad \textbf{(32)}$$

and the weight $\vartheta \equiv \frac{\Lambda_{\text{p}}}{\Lambda_{\text{p}}+\Lambda_{\text{w}}}$ is given by the relative slopes of the linearized Price and Wage Phillips Curves, Λ_{p} and Λ_{w}, respectively.

The second set consists of Taylor rule type "flexible targeting rules" that take the form

$$R_t = R\left(\Pi_t^k\right)^{1.5}, \; k \in \{p, w, c\}, \qquad \textbf{(33)}$$

where R denotes the steady-state nominal interest rate. The model is solved using second-order perturbation techniques in Dynare 4.5.4 [Adjemian et al. (2011)]. Unconditional lifetime utility is computed as the theoretical mean based on the first-order terms of the second-order approximation to the nonlinear model,[23] resulting in a second-order accurate welfare measure [see e.g. Kim et al. (2008)]. Conditional welfare is evaluated at the deterministic steady state.

Table 4 displays the results for the case of an efficient steady state. The upper left panel displays the results from the EHL Calvo setup, which corresponds exactly to the case considered in Galí (2015, Table 6.1).[24] The table displays both the unconditional and conditional (on the deterministic steady state) permanent consumption equivalent relative to the flex-price equilibrium.

Consistent with our theoretical results, all four setups produce the same conditional welfare losses when the steady state is efficient. We also see that the two Rotemberg setups produce the same unconditional welfare losses, which are smaller than the unconditional losses under Calvo. Among the Calvo setups, the EHL framework produces smaller unconditional losses than the SGU framework. This is driven by the fact that the SGU framework requires a Calvo wage adjustment parameter $\theta_{\text{w}} = 0.9458$ to generate the same Wage Phillips Curve slope as the EHL framework with $\theta_{\text{w}} = 0.75$. Despite these qualitative differences, the quantitative differences in unconditional welfare between the best (Rotemberg) and worst setups (Calvo SGU) are small, amounting to just 0.06% of consumption under the worst policy (strict price targeting under technology shocks).

Table 5 displays the results for the case of an inefficient steady state where monopolistic competition in goods and labor markets drives a wedge between the marginal rate of substitution and the marginal product of labor.[25]

Three things are notable. First, in line with Proposition 1, the conditional and unconditional welfare losses from Rotemberg wage setting are identical in the

TABLE 4. Welfare: Efficient steady state

	EHL Calvo						EHL Rotemberg					
	Strict targeting			Flexible targeting			Strict targeting			Flexible targeting		
	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.
	Technology shock											
λ_{unc}	0.802	0.041	0.035	0.489	0.313	0.315	0.792	0.041	0.035	0.482	0.309	0.311
λ_{cond}	0.773	0.038	0.032	0.450	0.295	0.297	0.773	0.038	0.032	0.450	0.295	0.297
	Demand shock											
λ_{unc}	0.000	0.000	0.000	0.062	0.069	0.067	0.000	0.000	0.000	0.062	0.068	0.067
λ_{cond}	0.000	0.000	0.000	0.061	0.067	0.066	0.000	0.000	0.000	0.061	0.067	0.066
	SGU Calvo						SGU Rotemberg					
	Technology shock											
λ_{unc}	0.849	0.041	0.035	0.527	0.331	0.334	0.792	0.041	0.035	0.482	0.309	0.311
λ_{cond}	0.773	0.038	0.032	0.450	0.295	0.297	0.773	0.038	0.032	0.450	0.295	0.297
	Demand shock											
λ_{unc}	0.000	0.000	0.000	0.064	0.071	0.070	0.000	0.000	0.000	0.062	0.068	0.067
λ_{cond}	0.000	0.000	0.000	0.061	0.067	0.066	0.000	0.000	0.000	0.061	0.067	0.066

TABLE 5. Welfare: Inefficient steady state

	EHL Calvo						EHL Rotemberg					
	Strict targeting			Flexible targeting			Strict targeting			Flexible targeting		
	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.
	Technology shock											
λ_{unc}	0.794	0.041	0.035	0.482	0.309	0.311	0.690	0.041	0.034	0.392	0.265	0.265
λ_{cond}	0.767	0.038	0.032	0.454	0.296	0.297	0.672	0.038	0.032	0.373	0.255	0.255
	Demand shock											
λ_{unc}	0.000	0.000	0.000	0.062	0.068	0.067	0.000	0.000	0.000	0.057	0.061	0.061
λ_{cond}	0.000	0.000	0.000	0.061	0.067	0.066	0.000	0.000	0.000	0.057	0.061	0.060
	SGU Calvo						SGU Rotemberg					
	Technology shock											
λ_{unc}	0.744	0.041	0.034	0.441	0.289	0.291	0.690	0.041	0.034	0.392	0.265	0.265
λ_{cond}	0.687	0.038	0.032	0.389	0.263	0.264	0.672	0.038	0.032	0.373	0.255	0.255
	Demand shock											
λ_{unc}	0.000	0.000	0.000	0.060	0.065	0.064	0.000	0.000	0.000	0.057	0.061	0.061
λ_{cond}	0.000	0.000	0.000	0.057	0.062	0.061	0.000	0.000	0.000	0.057	0.061	0.060

EHL and SGU setups. Second, the differences between Calvo and Rotemberg wage setting are now more pronounced, reaching a consumption equivalent difference of 0.1% between EHL Calvo and Rotemberg for strict price targeting with technology shocks. Third, the welfare ranking between EHL Calvo and SGU Calvo reverses compared to the efficient steady state, with SGU Calvo producing smaller welfare losses than EHL. That difference is particularly pronounced conditional on being in the deterministic steady state. Here, welfare under SGU Calvo is very close to the one in the Rotemberg setup, while EHL Calvo has losses that are higher by up to 0.08% of flex-price consumption.

6. CONCLUSION

We have provided applied researchers with guidance on how to translate a Calvo wage duration into an implied Rotemberg wage adjustment cost parameter by using the equivalence of their setups at first order. In doing so, we have shown that both the presence of labor taxation and the assumed household insurance scheme matter greatly for this mapping, giving rise to differences of up to one order of magnitude. Our results account for the inclusion of wage indexing, habit formation in consumption, and the presence of fixed costs in production, features commonly used in medium-scale New Keynesian DSGE models.

In the second part of the paper we investigated the second-order implications of Rotemberg versus Calvo wage setting by turning to a welfare evaluation. Despite first-order equivalence, Calvo and Rotemberg wage settings generally produce different welfare implications, which are a second-order property. We showed that both wage-setting schemes are equivalent conditional on initial wage dispersion being zero and the steady state being efficient. If these assumptions are not satisfied, Calvo wage setting generally produces higher welfare losses.

The present work restricted its analysis of the model dynamics and welfare implications implied by the different wage-setting schemes to the case where the first-order dynamics were identical. An interesting avenue for future research would be to further investigate the implications of trend inflation. In case of incomplete indexing, non-zero steady-state Calvo wage dispersion and Rotemberg resource costs give rise to differences in first-order dynamics. Based on the work of Ascari and Rossi (2012) for price setting, it stands to be expected that the different wage-setting schemes give rise to significant differences in, for example, welfare and determinacy properties. We conjecture that something similar applies in case of full indexing to steady-state inflation, because the linear dynamics differs due to the purely forward-looking Rotemberg wage setting not giving rise to a hybrid Wage Phillips Curve.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100518000615.

NOTES

1. Exceptions are risk shock models like Christiano et al. (2014) and Dmitriev and Hoddenbagh (2017) where first-order approximations are sufficient.

2. Examples of nonlinear models with Rotemberg price adjustment costs include Mumtaz and Zanetti (2013); Basu and Bundick (2017), and Plante et al. (2018), while Heer et al. (2012); Fernández-Villaverde et al. (2015); Nath (2015); Born and Pfeifer (2017), and Hagedorn et al. (2018) (also) consider wage adjustment costs. Richter and Throckmorton (2016) have recently argued for using Rotemberg-type price adjustment costs to improve the model fit, not just for computational convenience.

3. This approach can also be justified when using nonlinear methods, because the first-order approximation is only used to generate one restriction required to pin down one parameter. The equivalence, however, does not hold in case of trend inflation and incomplete indexing [see Ascari and Sbordone (2014, for a review)].

4. Early works include Roberts (1995); Keen and Wang (2007), and Nisticó (2007). This literature has also shown that the same value of the Rotemberg adjustment cost parameter can have very different economic effects, depending on the value of other structural parameters like the discount factor or the substitution elasticity.

5. Earlier studies like, for example, Chugh (2006, Appendix A) and Faia et al. (2014, footnote 29) provide the particular mapping applying in their respective model settings when calibrating the Rotemberg wage adjustment costs, but do not demonstrate how it would translate to more general models.

6. While the former is more prominent, the latter has been used, for example, in Trigari (2009); Pariés et al. (2011), and Born et al. (2013).

7. An early precursor of this work is Sbordone (2006).

8. If nominal rigidities affect the deterministic steady state, there would be interaction effects between price and wage rigidities affecting the linear dynamics that would need to be taken into account.

9. In case nominal rigidities affect the deterministic steady state, there would be interaction effects between price and wage rigidities.

10. Our formulation encompasses, for example, the partial indexation scheme of Smets and Wouters (2007), which is of the form $\Gamma^{ind}_{t,t+k} = \prod_{s=1}^{k} \Pi^{\iota}_{t+s-1}\Pi^{1-\iota}$, where $\iota \in [0, 1]$ denotes the degree of indexing to past inflation and Π without subscript denotes steady-state inflation. Another indexing scheme nested is the one by Christiano et al. (2005), who use full indexation to past inflation with $\iota = 1$. The absence of indexing is characterized by $\iota = 0$ and $\Pi = 1$ so that $\Gamma^{ind}_{t,t+k} = 1 \; \forall \, k$.

11. Throughout the paper we assume that the budget constraint holds with equality.

12. This formulation allows for non-time separable utility in consumption as introduced by habits, but excludes habits in leisure [e.g. Uhlig (2007)].

13. Papers typically assume full indexing with the steady-state inflation rate, that is $\Gamma^{ind}_{t+k-1,t+k} = \Pi$. Some papers like Fernández-Villaverde et al. (2015) use the formulation $\tilde{\phi}_w/2 \times (W^j_{t+k}/W^j_{t+k-1} - \Pi)^2$. Our specification is equivalent to this formulation with $\phi_w = \tilde{\phi}_w \Pi^2$. An exception using Rotemberg price setting with incomplete indexing is Bilbiie et al. (2014).

14. Note that while we allow tax rates to vary, tax rate changes only have a direct effect on the Wage Phillips Curve via their effect on the after-tax marginal rate of substitution.

15. For the purpose of this paper it is only important that this term is exogenous from the perspective of the wage-setting household so that the effects of household decisions on it are not internalized.

16. The lower bound is obtained with $\varepsilon^{mrs}_{tot} = 0$ for $\sigma = 0$, reaches $\varepsilon^{mrs}_{tot} = N/(1 - N) = 0.5$ for $\sigma = 1$ (i.e., the additively separable case) and then keeps increasing.

17. The same does not hold true for the time-dependent Calvo wage adjustment. Whenever the household is allowed to reset its wage, it can do so costlessly. For that reason, as shown in equation (17), the wage adjustment cost parameter ϕ_w implied by a particular Calvo duration, which appears in the denominator of equation (14), is decreasing at rate $(1 - \tau^n)$, canceling the overall effect of τ^n.

18. Appendix A.2 in Supplementary Material shows that the coefficient on labor tax rate *changes* in the linearized wage PC in the Rotemberg case is always 0. Thus, under both the Calvo and the Rotemberg cases, the direct effect of changes in tax rates on the wage Phillips Curve only comes via its effect on the after-tax MRS and is therefore identical under both setups.

19. Galí (2015) provides a different microfoundation of the EHL setup. He assumes a household with j members, each supplying a differentiated labor service, who are perfectly insured within the family. He then pairs this with j labor unions responsible for the wage setting in market j. Because unions only take the utility of their members into account, that is, use the idiosyncratic MRS, this setup is isomorphic to EHL.

20. In the presence of trend inflation, differences arise already at first order. As shown in Section 2.3, even in case of full indexing occurring in steady state, there is a difference in the linear dynamics between Calvo and Rotemberg wage settings due to the latter not giving rise to a hybrid Wage Phillips Curve. In case of trend inflation without or with only incomplete indexing in steady state, there are additional differences [see, e.g., Ascari and Rossi (2012)]. Analyzing the issues arising from differences in linear dynamics is beyond the scope of the present paper.

21. We assume that firms are paid a subsidy to counteract the distortion in the product market, while households pay a tax on their wage to counteract the distortion in the labor market. Due to the slope of the Wage Phillips Curve in the Rotemberg case depending on labor taxes, it matters how the subsidy to counteract monopolistic distortions is introduced. In the Calvo framework, one could simply assume that firms are paid a subsidy that counteracts both the monopolistic distortion in the product and in the labor market [see, e.g., Galí (2015, Chapter 6)].

22. Note that in the previous subsection, we used welfare losses relative to the steady-state allocation, while here we rely on differences relative to the flex-price allocation. However, as the steady state and the flex-price allocation are the same in all four setups, this distinction is inconsequential and involves only a renormalization.

23. The nonlinear first-order conditions can be produced from the replication files by making use of Dynare's LaTeX capabilities.

24. Table 7 of Appendix D in Supplementary Material displays the variance of log price inflation, log wage inflation, and of the log output gap, because with an efficient steady-state welfare up to second order can be expressed as a linear function of these three terms only.

25. Welfare evaluation in this case requires a full second-order approximation to the model dynamics [see e.g. Woodford (2002)]. Hence, the type and strength of price rigidities matters for the results, because (i) Calvo and Rotemberg price settings are not isomorphic at second order and (ii) there will be interaction effects between price and wage rigidities.

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