JFM RAPIDS
journals.cambridge.org/rapids

Pressure statistics in self-similar freely decaying isotropic turbulence

Marcello Meldi† **and Pierre Sagaut**

Institut Jean Le Rond d'Alembert, UMR 7190, 4 Place Jussieu, Case 162, Université Pierre et Marie Curie, Paris 6, F-75252 Paris CEDEX 5, France

(Received 27 July 2012; revised 21 November 2012; accepted 14 December 2012; first published online 7 February 2013)

The time evolution of pressure statistics in freely decaying homogeneous isotropic turbulence (HIT) is investigated via eddy-damped quasi-normal Markovian (EDQNM) computations. The present results show that the time decay rate of pressure-based statistical quantities, such as pressure variance and pressure gradient variance, are sensitive to the breakdown of permanence of large eddies. New formulae for the associated time-decay exponents are proposed, which extend previous relations proposed in Lesieur, Ossia & Metais (*Phys. Fluids*, vol. 11, 1999, p. 1535). Particular attention is paid to finite-Reynolds-number (FRN) effects on the pressure spectrum and pressure statistics. The results also suggest that $Re_\lambda = O(10^4)$ must be considered to observe a one-decade inertial range in the pressure spectrum with Kolmogorov $-7/3$ scaling. This threshold value is larger than almost all existing direct numerical simulation (DNS) and experimental data, justifying the discussion about other possible scaling laws. The $-5/3$ slope reported in some DNS data is also recovered by the EDQNM model, but it is observed to be a low-Reynolds-number effect. Another important result is that FRN effects yield a departure from asymptotic theoretical behaviours which appear similar to some effects attributed to intermittency by most authors. This is exemplified by the ratio between pressure-based and velocity-based Taylor microscales. Therefore, high-Reynolds-number DNS or experiments such that $Re_\lambda = O(10^4)$ would be required in order to remove FRN effects and to analyse pure intermittency effects.

Key words: acoustics, isotropic turbulence, turbulence theory

1. Introduction

The free decay of homogeneous isotropic turbulence (HIT) is still a timely subject of research due to its relevance in understanding the physical behaviour of turbulent flows and its implications in turbulence modelling. The evolution in time of the physical quantities associated with the energy spectrum, such as the turbulent kinetic

† Email address for correspondence: marcellomeldi@gmail.com

doi:10.1017/jfm.2012.629

	$n_{\overline{u^2}}$	n_l	n_ε
Power-law exponent	$-\dfrac{2(\sigma-\alpha+1)}{\sigma-\alpha+3}$	$\dfrac{2}{\sigma-\alpha+3}$	$-\dfrac{3(\sigma-\alpha)+5}{\sigma-\alpha+3}$

TABLE 1. Analytical formulae derived by Comte-Bellot & Corrsin (1966) for power-law exponents in HIT decay at high Re_λ. σ is such that $E(k\rightarrow 0,t)\propto k^\sigma$. Coefficient α accounts for breakdown of permanence of large eddies, with $\alpha=\max[0, 0.65(\sigma-3.2)]$.

energy $\overline{u^2}$, the integral length scale l and the energy dissipation rate ε, can be described by power laws in HIT decay. After the seminal work by Taylor (1935), several comprehensive reviews have been published (e.g. Batchelor 1953; Hinze 1975; Davidson 2004; Sagaut & Cambon 2008), but the full agreement of the scientific community about some basic aspects, such as the quantification of the power-law exponent related to the decay of $\overline{u^2}$, has not been reached at the present time.

It appears that initial conditions drive the emergence and evolution of different stable decay regimes. The two classical cases investigated in the literature are referred to as Saffman turbulence ($E(k\rightarrow 0,0)\propto k^2$) and Batchelor turbulence ($E(k\rightarrow 0,0)\propto k^4$). In a famous work by Comte-Bellot & Corrsin (1966), analytical formulae are derived to recover the power-law exponents as a function of the slope of the energy spectrum at the large scales σ. These formulae, which are reported in table 1, have to be corrected by a coefficient α to take into account the breakdown of 'permanence of large eddies' (i.e. $E(k,t)=E(k,0), k\ll k_l$) due to non-local energy transfers (Eyink & Thomson 2000; Lesieur 2008). For integer values of $\sigma, \alpha=0$ for $\sigma=1,2,3$ and $\alpha\approx 0.52$ for $\sigma=4$. These formulae have been extended to non-integer values of σ by Meldi & Sagaut (2012): $\alpha=\max[0, 0.65(\sigma-3.2)]$.

The attention of the scientific community on HIT decay has been mostly focused on the energy transfer and on the decay law of the velocity-based statistical quantities. Also, a limited number of papers in the literature are devoted to the analysis of the pressure spectrum and the related statistics. Some early works, e.g. Heisenberg (1948), investigated the pressure gradient variance $\overline{(\nabla p)^2}$ and Batchelor (1951) did background work starting from the joint Gaussian assumption (JGA) of the velocity field. In his work, Batchelor highlighted the correlation between the pressure and velocity fields. Without using the JGA hypothesis, Hill & Wilczak (1995) derived a theory relating the pressure structure function to fourth-order velocity structure functions. The statistics related to the energy spectrum being extremely sensitive to its shape at large energetic scales, i.e. near the spectrum peak, we can assume that the pressure statistics should exhibit a significant sensitivity to the shape of the energy spectrum.

After the publication of a number of papers studying pressure statistics through experiments (Uberoi & Corrsin 1953; Pearson & Antonia 2001; Tsuji & Ishihara 2003) and direct numerical simulation (DNS) (Schumann & Patterson 1978; Kim & Antonia 1993; Pumir 1994; Gotoh & Fukayama 2001; Yeung, Donzis & Sreenivasan 2012), several questions remain open.

A first question concerns the time decay rate of pressure-related statistical moments, such as $\overline{(\nabla p)^2}$ and $\overline{p^2}$. Formulae for associated time-decay exponents were proposed in Lesieur, Ossia & Metais (1999) neglecting the influence of the breakdown of the permanence of large eddies. Since this phenomenon is known to have a dramatic effect on time evolution of velocity-related statistical moments, its influence on pressure-based quantities needs to be addressed.

Reference	Re_λ	DNS/Exp.	E_p slope	Inertial region length
Schumann & Patterson (1978)	<35	DNS	NA	
Kim & Antonia (1993)	53	DNS	$-7/3$	<1 decade
Pumir (1994)	21.6–77.5	DNS	$-7/3$	≪1 decade
Cao *et al.* (1999)	103–218	DNS	$-5/3$	<1 decade
Gotoh & Rogallo (1999)	39–172	DNS	$-5/3$, $Re_\lambda \leqslant Re_c$	1 decade
Lesieur *et al.* (1999)	21–235	DNS/LES	NA	
Vedula & Yeung (1999)	21–235	DNS	$-5/3$	1 decade
Gotoh & Fukayama (2001)	38–478	DNS	$-7/3$, $-5/3$ bump	≪1 decade
Pearson & Antonia (2001)	40–4250	Exp.	NA	
Tsuji & Ishihara (2003)	200–1200	Exp.	$-7/3$, $Re_\lambda \geqslant 600$	<1 decade
Donzis *et al.* (2012)	8–1000	DNS	NA	
Yeung *et al.* (2012)	140–1000	DNS	NA	

TABLE 2. Slope of the pressure spectrum E_p in the inertial region recovered in numerical and experimental studies reported in the literature. An approximate estimation of the length of the inertial region, expressed in decades, is also reported. NA denotes references in which the pressure variance spectrum is not available.

A second key issue is the origin of the observed departure of some pressure-based statistical quantities from their theoretical asymptotic behaviours. These asymptotic behaviours are usually derived by considering Taylor-microscale Reynolds numbers of either $Re_\lambda \rightarrow 0$ or $Re_\lambda \rightarrow +\infty$, and rely on the JGA hypothesis. Among the most well-known results is the existence of a $k^{-7/3}$ inertial range in the pressure variance spectrum at small scales at very large Reynolds number, which is tied to the existence of Kolmogorov's $k^{-5/3}$ inertial range in the kinetic energy spectrum. This $-7/3$ spectral slope has not yet been observed over one full decade in either DNS or experimental results (see table 2), while it is not observed at all in many cases. A striking phenomenon is that a $-5/3$ slope for the pressure variance spectrum as been reported in several DNS results (Gotoh & Rogallo 1999; Cao, Chen & Doolen 1999; Vedula & Yeung 1999). While the possible role of intermittency of pressure fluctuations and the related statistical quantities in the departure from JGA-based predictions has been addressed by many authors, e.g. Donzis, Sreenivasan & Yeung (2012), the influence of finite-Reynolds-number (FRN) effects has received much less attention. FRN has recently been proved to have very significant effects on velocity statistical moments, e.g. Bos *et al.* (2012) and Tchoufag, Sagaut & Cambon (2012). These effects are very similar to possible intermittency effects on anomalous scaling laws. Therefore, the analysis of FRN effects on pressure-based statistical quantities is important, since most existing data (see table 2) have been obtained at Reynolds numbers at which strong FRN effects exist on the velocity field.

In the present work, time evolution of the pressure spectrum in freely decaying HIT, as well as the related statistical quantities, are addressed using an eddy-damped quasi-normal Markovian (EDQNM) model. Since the EDQNM model is known to be among the most reliable tools for HIT analysis at all Reynolds numbers and is free of any intermittency effects, it enables FRN effects to be isolated.

The paper is organized as follows: in § 2 the EDQNM model is briefly recalled, and the setup of the numerical simulations is described. The time evolution of the pressure spectrum and the validity of the Kolmogorov scaling for pressure are investigated in § 3. The influence of viscous effects is investigated, and a threshold value for Re_λ to obtain a one-decade $k^{-7/3}$ range is identified. Section 4 displays results dealing with the power-law decay of the statistical quantities related to the pressure. An extension of Lesieur's relations is proposed, which accounts for the breakdown of permanence of large eddies. In § 5 the ratio between the Taylor microscales related to the pressure and velocity fields is addressed, the results being extensively compared with those reported in the literature. The emphasis is on FRN effects, and their similarity with intermittency effects at Reynolds numbers considered by most authors. Conclusions are given in § 6.

2. EDQNM model and setup of the simulations

The EDQNM model is a quasi-normal closure based on the discretization of the Lin equation, which is the spectral counterpart of the Kármán–Howarth equation. The model accurately describes the triadic energy transfer in wavenumber space and can be used to evaluate several statistics in turbulence, up to three-point fourth-order correlations. It has proved to be a reliable, robust and efficient method to investigate HIT free decay (e.g. Meldi, Sagaut & Lucor 2011; Tchoufag *et al.* 2012). The works of Orszag (1970), Lesieur (2008) and Sagaut & Cambon (2008) provide an exhaustive discussion.

A number of sets of computations have been performed, imposing as initial condition $10^3 \leqslant Re_\lambda \leqslant 2 \times 10^6$. Defining $k_l(0)$ the initial position of the peak of the energy spectrum, the largest resolved scale is $k_0 = 10^{-10}\, k_l(0)$. The minimum resolution is set to $k_{max} = 10/\eta(0)$, where $\eta(0)$ is the initial value of the Kolmogorov scale. The initial energy spectrum is a simplified version the energy spectrum formulated by Pope (2000):

$$E(k) = \begin{cases} A\, k^{\sigma}, & kl \ll 1, \\ C_k\, \varepsilon^{2/3} k^{-5/3} f_l(kl), & kl \gg 1, \end{cases} \tag{2.1}$$

with

$$f_l(kl) = \left(\frac{kl}{[(kl)^{c_1} + c_2]^{1/c_1}} \right)^{5/3+\sigma}. \tag{2.2}$$

The free parameter c_1 in (2.2) is set to 1.5, while the parameter c_2 is computed in order to recover $l(0) = 1$. For each set, four computations for integer values of the parameter $\sigma = 1, 2, 3$ and 4 have been carried out to investigate the sensitivity of the pressure statistics to details of the kinetic energy spectrum. Combining the results of the computations for each value of σ, it is possible to describe HIT decay in the range $5 \leqslant Re_\lambda \leqslant 5000$. All the results are referred to the normalized time scale $\tau = t/t_0$, with $t_0 = \varepsilon(0)^{-1/3}\, l(0)^{2/3}$ being the initial characteristic turnover time.

The EDQNM model has been used to compute the triadic energy transfer and, simultaneously, to compute the pressure spectrum E_p. Starting from the relation between the pressure fluctuations and the fourth-order velocity correlations, the application of the JGA approximation allows the following form of the pressure spectrum to be recovered:

$$E_p(k) = \frac{k^2}{4\pi} \int_{\boldsymbol{r}+\boldsymbol{q}=\boldsymbol{k}} E(r)\, E(q)\, \frac{\sin^4\beta}{r^4}\, \mathrm{d}\boldsymbol{q}, \tag{2.3}$$

where $[\boldsymbol{k}, \boldsymbol{r}, \boldsymbol{q}]$ is the basis used to compute the energy triadic interactions in the spectral space, and β is the angle facing $\boldsymbol{r}$ in the triangle formed by the three vectors. The mean-square pressure fluctuation $\overline{p^2}$, the pressure integral length l_p and the pressure gradient $\overline{(\nabla p)^2}$ have been respectively recovered as:

$$\overline{p^2} = \int_0^\infty E_p(k)\, \mathrm{d}k, \tag{2.4}$$

$$l_p = \frac{\pi}{2} \frac{\displaystyle\int_0^\infty E_p(k)\, k^{-1}\, \mathrm{d}k}{\displaystyle\int_0^\infty E_p(k)\, \mathrm{d}k}, \tag{2.5}$$

$$\overline{(\nabla p)^2} = \int_0^\infty k^2\, E_p(k)\, \mathrm{d}k. \tag{2.6}$$

These quantities have been sampled in the range $5 \leqslant Re_\lambda \leqslant 5000$ and the power-law exponents have been recovered by local polynomial fitting.

3. Pressure spectrum and Kolmogorov scaling law

The characteristics of the computed pressure spectrum $E_p(k, \tau)$ are investigated in the present section. We will first restrict our analysis to the case of $Re_\lambda \geqslant 10^3$ and to Saffman and Batchelor turbulence. The two decay regimes are associated with a value of the parameter $\sigma = 2$ and $\sigma = 4$, respectively.

The evolution in time of the pressure spectrum is shown in figures 1(*a*) and 1(*b*) for the two considered cases. At the large scales, we can observe the presence of an extended range for which $E_p(k, \tau) = A_p(\tau)k^2$. This range, which has been predicted theoretically and observed numerically by Lesieur *et al.* (1999), exhibits a constant slope coefficient 2 which is independent of the parameter σ. Conversely, the evolution in time of the coefficient $A_p(\tau)$ is governed by the initial conditions enforced on the energy spectrum since $A_p(\tau) \simeq (8/15) \int_0^{+\infty} (E^2(k, \tau)/k^2)\, \mathrm{d}k$, yielding a strong sensitivity to σ. This point will be detailed in § 4. In figure 1, we can also observe that the peak of the pressure spectrum evolves in time with the same power law as the energy-spectrum integral length l, which is shown as a vertical line in the plot. Indeed, the results recovered by the EDQNM model show that the ratio l_p/l is constant after the initial transient fades and it is equal to $l_p/l = 0.539$. This result is in agreement with the results reported in Batchelor (1951), who predicted a ratio of 0.54 between the two integral scales.

Let us now consider the compensated pressure spectrum $E_p^s = E_p/(\varepsilon^{4/3}\, k^{-7/3})$, which is displayed in figure 2(*a*) for Saffman turbulence. The results of Batchelor turbulence simulations are omitted, the information deducible being the same as observed for Saffman turbulence. In this case, we consider pressure spectra in the range $100 \leqslant Re_\lambda \leqslant 10^6$. At very high Re_λ, a plateau in the inertial range is observed for more than three decades. A much less developed plateau has been observed as well in the DNS results by Pumir (1994). Moreover, a bump in the compensated pressure

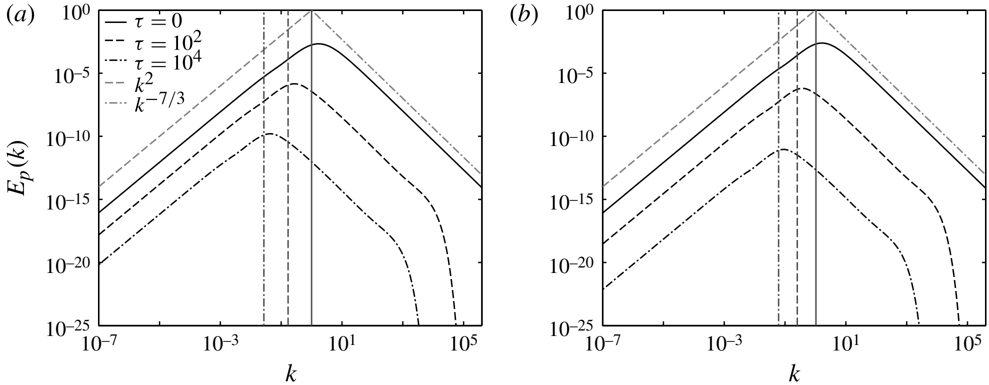

FIGURE 1. Evolution of the pressure spectrum $E_p(k, \tau)$ in time for Saffman turbulence (*a*) and Batchelor turbulence (*b*). Two ranges at $E_p \propto k^2$ and $E_p \propto k^{-7/3}$ are clearly observable at large scales and in the inertial range, respectively. The vertical lines represent the inverse of the magnitude of the integral length scale l at $\tau = 0, 10^2, 10^4$ (continuous, dashed, dashed-dotted vertical lines, respectively).

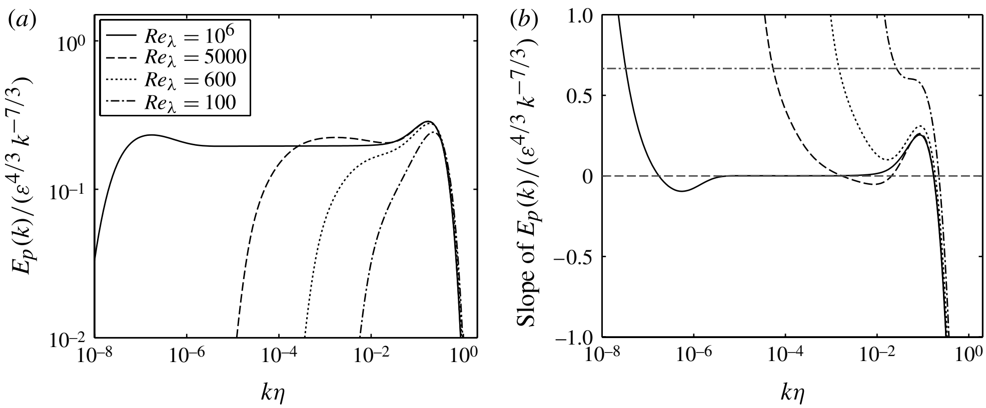

FIGURE 2. Compensated pressure spectrum $E_p^s = E_p/(\varepsilon^{4/3}\, k^{-7/3})$ (*a*) and local slope of the compensated pressure spectrum (*b*) in the case of Saffman turbulence. Horizontal lines in (*b*) show slopes of 0 and 2/3).

spectrum is observed at scales close to the dissipation region. This phenomenon, which is classically referred to as the *bottleneck effect* when dealing with the energy spectrum, has also been observed in the pressure spectrum by Gotoh & Fukayama (2001). The slope of this bump, computed by EDQNM results, is $\sim 1/4$ at very high Reynolds numbers and progressively increases up to 3/10 for $Re_\lambda \approx 600$. This behaviour can be observed in figure 2(*b*), where the local slope of the pressure spectra is reported. Analysing pressure spectra at progressively decreasing Re_λ, the plateau of the compensated spectra becomes less and less visible, and the bump region degenerates into a small secondary plateau at very low Reynolds number. Moreover, the slope becomes progressively steeper at lower Re_λ, with an asymptotic value of $-5/3$.

The global picture is therefore the following: at very large Re_λ, the pressure spectrum obeys the $-7/3$ Kolmogorov scaling and exhibit a kind of bottleneck near the Kolmogorov scale. At moderate Re_λ, the Kolmogorov inertial range is no longer present, but the bottleneck near the Kolmogorov scale evolves into a small (less than one-decade long) $-5/3$ range. This trend, which seems to be independent of the shape of the energy spectrum, can be clearly appreciated in figure 2(*b*) for moderate Reynolds numbers. Tsuji & Ishihara (2003) argued that most of the results from DNS studies reported in literature did not match with the Kolmogorov scaling because the Re_λ investigated was not sufficiently high to observe a fully developed plateau of the compensated spectrum. Tsuji & Ishihara (2003) indicate a minimum limit of $Re_\lambda = 600$ to recover a plateau. An extensive investigation of the present results supports the discussion by Tsuji & Ishihara (2003) and suggests that a minimum limit of $Re_\lambda \geqslant 10^4$ seems necessary to observe the Kolmogorov scaling. A higher Reynolds number is required to observe this scaling on the pressure spectrum than on the energy spectrum. This is justified by the fact that the pressure spectrum is related to the fourth-order two-point velocity correlation, while the energy spectrum is related to the second-order one. Indeed, the EDQNM results proposed are not reachable by present DNS, due to the prohibitive amount of computational resources needed to simulate a flow at $Re_\lambda = 10^6$.

4. Power-law exponents driving the decay of the pressure statistics

The time evolution of pressure statistics is now investigated. Starting from the assumption $Re_\lambda \rightarrow \infty$, Lesieur *et al.* (1999) derived that the decay of the statistical quantities based on the pressure spectrum is related to that associated to the energy spectrum, and their decay can also be described by power laws. In particular, the power-law exponents for pressure statistics are related to the ones governing the decay law of $\overline{u^2}$, l and ε, and they exhibit a sensitivity to the parameter σ. Denoting by n_Q the power-law exponent of a quantity Q, the expressions derived by Lesieur *et al.* (1999) extended here to account for breakdown of permanence of large eddies are (for large Re_λ):

$$n_{\overline{p^2}} = 2\,n_{\overline{u^2}} = -4\frac{(\sigma-\alpha+1)}{\sigma-\alpha+3}, \tag{4.1}$$

$$n_{l_p} = n_l = \frac{2}{\sigma-\alpha+3}, \tag{4.2}$$

$$n_{A_p} = 3 + \frac{7}{2}\,n_{\overline{u^2}} = 3 - 7\frac{(\sigma-\alpha+1)}{\sigma-\alpha+3}. \tag{4.3}$$

Moreover, Batchelor (1951) derived a relation between the pressure gradient and the dissipation rate: $n_{\overline{(\nabla p)^2}} = 1.5\,n_\varepsilon = -(9(\sigma-\alpha)+15)/2(\sigma-\alpha+3)$.

The evolution in time of the power-law coefficients for the four quantities under consideration is displayed in figure 3 for $\sigma = 1, 2, 3, 4$. For all the cases, $Re_\lambda \geqslant 10^3$ and the ratio $k_l(\tau)/k_0 \geqslant 10^5$. The results recovered are in excellent agreement with the theoretical results by Batchelor (1951) and Lesieur *et al.* (1999), a maximum error of 0.5 % being observed in the prediction of the power-law exponent of $\overline{(\nabla p)^2}$.

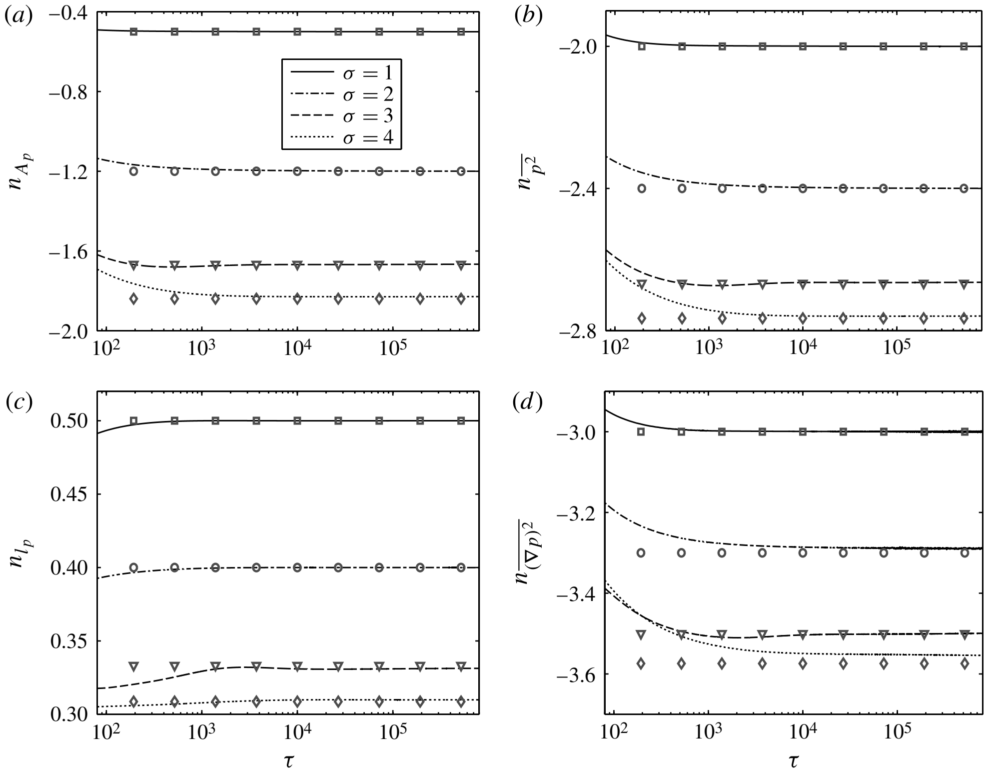

FIGURE 3. Evolution in time of the power-law exponents of the physical quantities associated to the pressure spectrum. The pressure spectrum coefficient $A_p = E_p k^{-2}$ (*a*), the mean-square pressure fluctuation $\overline{p^2}$ (*b*), the pressure spectrum integral length scale l_p (*c*) and the pressure gradient $\overline{(\nabla p)^2}$ (*d*) are respectively represented. The four curves represent the power-law exponent value for $\sigma = 1, 2, 3, 4$. Symbols refer to the theoretical formulae by Batchelor (1951) and Lesieur *et al.* (1999), with a proposed correction coefficient α for $\sigma = 4$.

In the particular case of Batchelor turbulence ($\sigma = 4$), we recall that the numerical results are in agreement with the theoretical formulae only if the correction term $\sigma_{eff} = 4 - \alpha$, $\alpha = \max[0, 0.65(\sigma - 3.2)]$, is considered to represent the effects of non-local triadic interactions (see Eyink & Thomson 2000; Meldi & Sagaut 2012). Due to the simplified initial energy spectrum enforced, the numerical results converge to the theoretical values after a long transitory, which can be estimated in $10^4 t_0$ units. Nevertheless, the high resolution achievable by the EDQNM model, combined with the high initial Re_λ enforced, allow us to study the HIT decay at high Re_λ over very long decay times.

The time evolution of the coefficient A_p, which has been derived at low wavenumbers by the formula $A_p = E_p k^{-2}$, is consistent with the formula (4.3) as long as the pressure spectrum peak is at least three decades away from the wavenumber at which A_p is computed. When the peak of the pressure spectrum moves closer, the decay of the coefficient A_p is progressively affected by its vicinity.

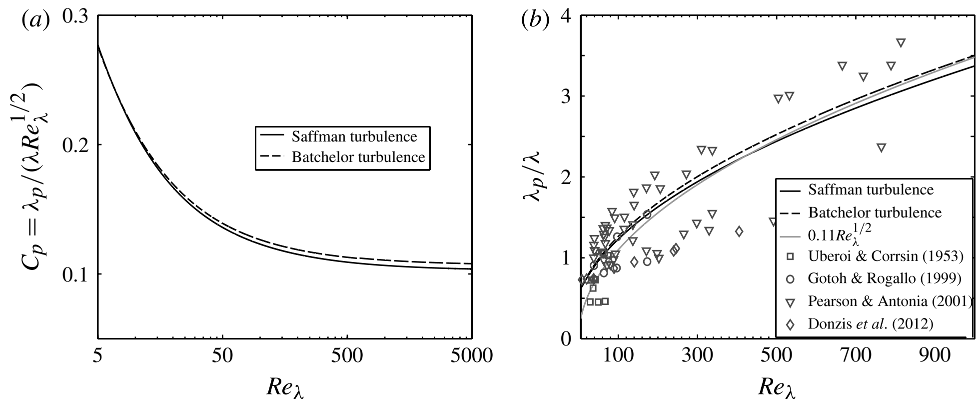

FIGURE 4. Coefficient $(\lambda_p/\lambda)/Re_\lambda^{1/2}$ in the high-Re_λ formula by Batchelor (1951) (*a*) and ratio of the Taylor microscales λ_p/λ for moderate to low Re_λ (*b*). Experimental data by Uberoi & Corrsin (1953) and Pearson & Antonia (2001) and numerical DNS results by Gotoh & Rogallo (1999) and Donzis *et al.* (2012) are also shown.

5. Decay of the Taylor microscales related to the energy and pressure spectra

A significant effort has been made by the scientific community over many decades (e.g. Batchelor 1951; Vedula & Yeung 1999; Pearson & Antonia 2001) to find a relation between the decay of the Taylor microscale $\lambda^2 = 15\nu\overline{u^2}/\varepsilon$ and the equivalent scale for the pressure spectrum $\lambda_p^2 = \rho^2\left(\overline{u^2}\right)^2/\overline{(\nabla p)^2}$. In the early work of Batchelor (1951), which relied on the JGA hypothesis, the following relations have been derived in the cases of high and low Re_λ:

$$\begin{cases} \lambda_p/\lambda = 0.11Re_\lambda^{1/2}, & Re_\lambda \to \infty, \\ \lambda_p/\lambda = 0.81, & Re_\lambda \to 0. \end{cases} \tag{5.1}$$

The law recovered at high Re_λ can also be deduced by dimensional analysis. If λ, λ_p and Re_λ are described by classical power laws, (5.1) can be rewritten as:

$$\tau^{0.5\left(2n_{\overline{u^2}}-1.5(n_\varepsilon)\right)}\tau^{-0.5} \propto \tau^{0.5\left(n_{\overline{u^2}}-0.5(n_\varepsilon)\right)} \tag{5.2}$$

substituting $n_{\overline{u^2}}$ and n_ε with the corresponding Comte-Bellot & Corrsin (1966) formulae values reported in table 1, it is easy to verify that the terms on the left and right of (5.2) decay with the same power-law exponent, for all of $\sigma \in [1, 4]$. Moreover, if $\sigma = 1$, the power-law exponent is 0 and the ratio between λ_p and λ is constant in time.

We now investigate the behaviour of the coefficient $C_p = (\lambda_p/\lambda)/Re_\lambda^{1/2}$. If the decay power laws are exactly recovered in EDQNM simulations, the coefficient C_p will be an invariant in HIT decay at high Re_λ. The value of this invariant may exhibit a sensitivity to the parameter σ. The results are shown as a function of Re_λ in figure 4(*a*), considering the classical decay regimes of Saffman and Batchelor turbulence. For very high Re_λ, the value of C_p is almost constant, in agreement with Batchelor (1951). However, the value recovered by the EDQNM simulations is smaller. In the range $1000 \leqslant Re_\lambda \leqslant 5000$, $C_p \in [0.103, 0.106]$ for Saffman turbulence

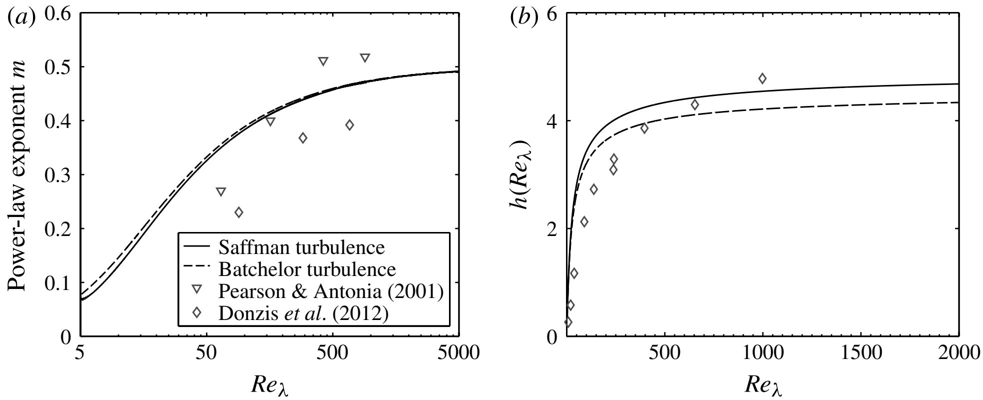

FIGURE 5. Power-law exponent m in the relation $\lambda_p/\lambda = C_p Re_\lambda^m$ (a) and function $h(Re_\lambda) = (\nu^{1/2}\,\overline{(\nabla p)^2})/(3\rho^2\,\varepsilon^{3/2})$ (b), computed by the EDQNM model and compared with the experimental data by Pearson & Antonia (2001) (in a only) and the DNS data by Donzis *et al.* (2012). The data are locally derived by the analysis of the results reported in the references.

and $C_p \in [0.107, 0.111]$ for Batchelor turbulence. Approaching values of $Re_\lambda \leqslant 10^3$, C_p increases faster, and the ratio λ_p/λ diverges from the high-Reynolds-number behaviour $\approx 0.11 Re_\lambda^{1/2}$. Interestingly (see figure 4b) the curves λ_p/λ for Saffman and Batchelor turbulence converge toward a very similar value, which is approximately $\lambda_p/\lambda = 0.62$, for low Re_λ. This result is in qualitative agreement with Batchelor (1951) and in very good agreement with the experimental results of Uberoi & Corrsin (1953) and Pearson & Antonia (2001).

Several DNS studies in the literature have shown a departure from the JGA behaviour $\lambda_p/\lambda = C_p Re_\lambda^m$, $m = 0.5$, which is usually attributed to intermittency. Vedula & Yeung (1999) and Donzis *et al.* (2012) reported a better-fit power law of $m = 0.25$ and $m = 0.34$, respectively. The exponent m, computed by the EDQNM model, is reported in figure 5(a). The results point out that the asymptotic limit $m = 0.5$ is approached but not reached at $Re_\lambda = 5000$, and the values observed in the cited DNS studies are matched in the range $30 \leqslant Re_\lambda \leqslant 100$. Recalling that the EDQNM model does not account for intermittency, we can conclude that the FRN effects play a very important role at moderate Re_λ and cannot be neglected even at $Re_\lambda = O(10^3)$. Similar FRN effects have been observed when considering Kolmogorov $-4/5$ law by Tchoufag *et al.* (2012), and in velocity increments by Bos *et al.* (2012). In the latter case, it is shown that anomalous exponents very similar to those predicted by classical intermittency theories can be due solely to the viscous effects. A confirmation is given in figure 5(b) by the analysis of the function $h(Re_\lambda) = (\nu^{1/2}\,\overline{(\nabla p)^2})/(3\rho^2\,\varepsilon^{3/2})$: the DNS results at moderate Re_λ by Donzis *et al.* (2012) match the present EDQNM results.

6. Conclusions

The time evolution of the pressure spectrum and the related statistics have been investigated by EDQNM simulations in the range $5 \leqslant Re_\lambda \leqslant 5000$. The pressure spectrum has been analysed by considering $\sigma = 1, 2, 3, 4$ to investigate its sensitivity with respect to the shape of the initial energy spectrum.

The analysis of the sampled pressure spectra confirms the presence of two ranges. The first one, at the large scales, is $E_p(k,\tau) = A_p(\tau)\,k^2$. Its slope does not depend on σ, but the time-decay exponent of the coefficient $A_p(\tau)$ is governed by σ (Lesieur *et al.* 1999). The second range, which represents the inertial region, can be approximated as $E_p(k) \propto \varepsilon^{4/3}\,k^{-7/3}$. This range becomes progressively less clear at moderate Re_λ, as it merges with a pseudo-bottleneck region close to the Kolmogorov scale. We can argue that the presence of a short range exhibiting a $-5/3$ scaling near the Kolmogorov scale, which has been reported in several DNS, originates in the pseudo-bottleneck and is due to finite-Reynolds-number effects.

For high Re_λ, the computed statistical quantities are in very good agreement with the theoretical asymptotic framework proposed by Batchelor (1951) and Lesieur *et al.* (1999). The time evolution of the coefficient $A_p(\tau)$ is consistent with the proposed formula only if it is computed at a wavenumber which is at least three decades away from the pressure spectrum peak. However, this resolution requirement is prohibitive for a high-Re_λ DNS at the present time. New formulae for time decay exponents for pressure variance and pressure gradient variance, that account for the breakdown of permanence of large eddies, are observed to be in very good agreement with the present EDQNM results.

Finally, the ratio between the Taylor microscales λ_p/λ has been investigated to obtain insight into finite-Reynolds-number effects. The trends predicted at both very high and very low Re_λ by Batchelor (1951) have been observed. For the high-Re_λ case, the coefficient $C_p = \lambda_p/(\lambda\,Re_\lambda^{1/2}) \in [0.105, 0.11]$ and exhibits a low sensitivity to the parameter σ. The ratio λ_p/λ converges toward a universal value close to 0.62 at very low Re_λ. Moreover, the comparison with DNS data confirms that FRN effects are significant for $Re_\lambda \leqslant 10^4$, and that they lead to deviation of the results from the theoretical JGA behaviour. The observed deviations share many features with those classically attributed to intermittency effects. As an example, they yield the occurrence of anomalous exponents, e.g. for λ_p/λ in the present study. Therefore, flows such that $Re_\lambda = O(10^4)$ should be considered, to analyse pure intermittency effects on pressure.

References

BATCHELOR, G. K. 1951 Pressure fluctuations in isotropic turbulence. *Proc. Camb. Phil. Soc.* **47**, 359–374.

BATCHELOR, G. K. 1953 *The Theory of Homogeneous Turbulence*. Cambridge University Press.

BOS, W. J. T., CHEVILLARD, L., SCOTT, J. F. & RUBINSTEIN, R. 2012 Reynolds number effect on the velocity increment skewness in isotropic turbulence. *Phys. Fluids* **24**, 015108.

CAO, N., CHEN, S. & DOOLEN, G. D. 1999 Statistics and structures of pressure in isotropic turbulence. *Phys. Fluids* **11**, 2235.

COMTE-BELLOT, G. & CORRSIN, S. 1966 The use of a contraction to improve the isotropy of grid-generated turbulence. *J. Fluid Mech.* **25**, 657–682.

DAVIDSON, P. A. 2004 *Turbulence. An Introduction for Scientists and Engineers*. Oxford University Press.

DONZIS, D. A., SREENIVASAN, K. R. & YEUNG, P. K. 2012 Some results on the Reynolds number scaling of pressure statistics in isotropic turbulence. *Physica* D **241**, 164–168.

EYINK, G. L. & THOMSON, D. J. 2000 Free decay of turbulence and breakdown of self-similarity. *Phys. Fluids* **12**, 477–479.

GOTOH, T. & FUKAYAMA, D. 2001 Pressure spectrum in homogeneous turbulence. *Phys. Rev. Lett.* **86**, 3775–3778.

GOTOH, T. & ROGALLO, R. S. 1999 Intermittency and scaling of pressure at small scales in forced isotropic turbulence. *J. Fluid Mech.* **396**, 257–285.

HEISENBERG, W. 1948 Zur statischen Theorie der Turbulenz. *Z. Phys.* **124**, 628–657.

HILL, R. J. & WILCZAK, J. M. 1995 Pressure structure functions and spectra for locally isotropic turbulence. *J. Fluid Mech.* **296**, 247–269.

HINZE, J. O. 1975 *Turbulence*. McGraw-Hill.

KIM, J. & ANTONIA, R. A. 1993 Isotropy of the small scales of turbulence at low Reynolds number. *J. Fluid Mech.* **251**, 219–238.

KOLMOGOROV, A. N. 1941 On the degeneration of isotropic turbulence in an incompressible viscous fluid. *Dokl. Akad. Nauk SSSR* **31**, 538–541.

LESIEUR, M. 2008 *Turbulence in Fluids*. Springer.

LESIEUR, M., OSSIA, S. & METAIS, O. 1999 Infrared pressure spectra in two- and three-dimensional isotropic incompressible turbulence. *Phys. Fluids* **11**, 1535.

MELDI, M. & SAGAUT, P. 2012 On non-self similar regimes in homogeneous isotropic turbulence decay. *J. Fluid Mech.* **711**, 364–393.

MELDI, M., SAGAUT, P. & LUCOR, D. 2011 A Stochastic view of isotropic turbulence decay. *J. Fluid Mech.* **668**, 351–362.

ORSZAG, S. A. 1970 Analytical theories of turbulence. *J. Fluid Mech.* **41**, 363–386.

PEARSON, B. R. & ANTONIA, R. A. 2001 Reynolds-number dependence of turbulent velocity and pressure increments. *J. Fluid Mech.* **444**, 343–382.

POPE, S. B. 2000 *Turbulent Flows*. Cambridge University Press.

PUMIR, A. 1994 A numerical study of pressure fluctuations in three-dimensional, incompressible, homogeneous, isotropic turbulence. *Phys. Fluids* **6**, 2071.

SAGAUT, P. & CAMBON, C. 2008 *Homogeneous Turbulence Dynamics*. Cambridge University Press.

SCHUMANN, U. & PATTERSON, G. S. 1978 Numerical study of pressure and velocity fluctuations in nearly isotropic turbulence. *J. Fluid Mech.* **88**, 685–709.

TAYLOR, G. I. 1935 Statistical Theory of Turbulence. *Proc. R. Soc. Lond.* A **151**, 421–444.

TCHOUFAG, J., SAGAUT, P. & CAMBON, C. 2012 Spectral approach to finite Reynolds number effects on Kolmogorov 4/5 law in isotropic turbulence. *Phys. Fluids* **24**, 015107.

TSUJI, Y. & ISHIHARA, T. 2003 Similarity scaling of pressure fluctuation in turbulence. *Phys. Rev.* E **68**, 026309.

UBEROI, M. S. & CORRSIN, S. 1953 Diffusion of heat from a line source in isotropic turbulence. *NACA Rep.* 1142.

VEDULA, P. & YEUNG, P. K. 1999 Similarity scaling of acceleration and pressure statistics in numerical simulations of isotropic turbulence. *Phys. Fluids* **11**, 1208.

YEUNG, P. K., DONZIS, D. A. & SREENIVASAN, K. R. 2012 Dissipation, enstrophy and pressure statistics in turbulence simulations at high Reynolds numbers. *J. Fluid Mech.* **700**, 5–15.