# Laser excitation of terahertz surface plasma wave over a hollow capillary plasma

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#### Abstract

Two collinear laser pulses of finite spot size propagating through a capillary plasma, modeled as a hollow plasma cylinder, are shown to produce beat frequency terahertz (THz) surface plasmons at the inner surface. The evanescent laser fields in the plasma impart oscillatory velocity to electrons and exert a beat ponderomotive force on them. The static component of the ponderomotive force inhibits plasma from filling the vacuum region while the beat frequency component produces a nonlinear current ( $\vec{J}^{NL}$ ) that drives the difference frequency THz surface plasma wave (SPW). Phase matching for the THz surface wave excitation is achieved when the group velocity of the lasers equals the phase velocity of the beat frequency SPW. At laser intensities of ~10<sup>14</sup> W/cm<sup>2</sup> at 10 µm wavelength, one may attain normalized surface wave amplitude ~ 0.03.

Keywords: Capillary plasma; Phase matching; Plasmons; Ponderomotive; Terahertz

# 1. INTRODUCTION

The generation of terahertz (THz) radiation by high power laser-surface interaction has attracted considerable attention in recent years. Structured metal surfaces, such as spoof, are employed for this purpose as they slow down the group velocity of surface plasmons (SPs) and bring the asymptotic SP frequency into the THz range (Schultz et al., 2000; Jeon & Grischkowsky, 2006; Williams et al., 2008). When a laser is impinged on the surface, its mode converts to SP and the latter produces photoelectrons. These electrons get accelerated in the ponderomotive potential associated with the evanescent field producing short pulse THz radiation. Kadlec et al. (2005) have experimentally observed THz generation in gold films of variable thickness. For thickness  $\sim 150\,\text{nm}$  and incident angle  $\sim 45^\circ,$  the measured THz field attains peak values of  $\sim 4 \text{ KV/cm}$ . The experiments also reveal that emitted THz field is suppressed for thickness below 100 nm.

Welsh *et al.* (2007) have reported THz emission from gold-coated glass gratings having a period suitable for phase matched excitation of SPs. Using 0.8  $\mu$ m intense optical pulses they saw THz emission from nanostructured metal surfaces only for *p*-polarized pump light with a strong peak when the angle of incidence of the pump satisfied the

*k*-matching condition for laser mode conversion into SPs. The pump energy density in their experiments reached values up to  $3.5 \text{ mJ/cm}^2$ .

Kumar and Tripathi (2013) have proposed a planar structure, comprising a dielectric plate coated on top with an ultra-thin metal film, for nonlinear mixing of lasers and the generation of THz surface plasma wave (SPW). The structure supports THz SPW with SP resonance, controllable by film thickness, in the THz domain. The lasers exert a beat frequency ponderomotive force on film electrons that drives the THz SPW. Suizu and Kawase (2007) have theoretically studied surface-emitted and collinear phase-matched THz generation in a conventional optical fiber. The third-order nonlinear effect, four-wave mixing, is used to generate THz that radiates out from the surface of the fiber, perpendicular to the direction of the optical beams.

Due to wide range of applications (Taton *et al.*, 2000; Kalkbrenner *et al.*, 2001; Ditlbacher *et al.*, 2002; Li *et al.*, 2003) SPs have drawn vigorous attention in recent years. SPs are ideally suited as sensors, they propagate along the surface between a conductor and a dielectric or conductor and air with fields peaking at the interface and falling off exponentially away from it in either medium (Barnes *et al.*, 2003; Steinhauer & Kimura, 2003; Kalmykov *et al.*, 2006; Mork *et al.*, 2014). A tiny trace of a gas or DNA on the interface produces a measurable change in the propagation constant when one uses the attenuated total reflection configuration (Kretschmann & Raether, 1968; Kretschmann, 1971).

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In this paper, we examine the efficacy of capillary plasma for laser excitation of THz surface plasma wave. Capillary plasma could be produced in two ways (Kaganovich *et al.*, 1997; Nitikant & Sharma, 2005; Kameshima *et al.*, 2009; Genoud *et al.*, 2011) one, by electrical discharge of a hollow capillary filled with a gas and second, by the irradiation of a low spot size hollow capillary by a laser. In the second, hollow plasma waveguide is created naturally. The laser ionizes the capillary and laser ponderomotive force keeps the plasma away from the axial region. Thus we model the capillary as a hollow plasma cylinder. An electron depleted ion channel created by a laser irradiation of gas jet target has similar electron density profile. Gas filled metallic waveguide is not practical as it cannot sustain a plasma for long without suffering damage to its wall.

Two collinear Gaussian laser beams with frequencies much above the plasma frequency propagate along the cylinder axis and extending radially outward into the plasma region. The lasers impart oscillatory velocity to plasma electrons and exert a ponderomotive force on them at the beat frequency. The ponderomotive force has a transverse component that drives nonlinear current, producing THz radiation. The lasers also exert a static radial ponderomotive force that keeps the plasma from creeping in.

The situation resembles high power laser propagation in gas jet plasmas. The laser ponderomotive force pushes the electrons radially outward creating a fully electron evacuated channel (Akhmanov et al., 1967, 1968; Sodha et al., 1976; Esarey et al., 1997). The space charge thus created holds the electrons and pulls the ions outward, causing ambipolar diffusion of the plasma. The plasma density is minimum on the laser axis and monotonically falls off away from it. In a two-stage acceleration process using three-dimensional particle in cell simulations Naseri et al. (2012) have shown that for laser powers above the threshold, the laser pulse propagates in a single mode through the electron-free channel for a time of the order of 1 picosecond, the steep laser front excites a surface wave along the boundaries of the ion channel. This surface wave traps the electrons at the channel wall and preaccelerates them to relativistic energies, after having gained sufficient energies these electrons are further accelerated by the longitudinal electric field of the surface wave.

A novel feature of the scheme is that the phase matching condition between the beating lasers and the THz is exactly satisfied in certain parameter regime. In Section 2, we derive the dispersion relation for SPW. In Section 3 we study SPW THz excitation and obtain the amplitude. The results are discussed in Section 4.

### 2. SPW DISPERSION RELATION

Consider a hollow plasma cylinder of inner radius a with electron density

$$n_0 = 0 \text{ for } r < a,$$
$$= n_0 \text{ for } r > a.$$

A SPW propagates over the inner surface with electric field

$$E_z = A(r)e^{-i(\omega t - k_z \hat{z})}.$$
(1)

Using this in the wave equation one obtains the equation governing A

 $A = A'_0 K_0(\alpha_{\text{II}} r)$  for r > a,

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \left(\frac{\omega^2}{c^2} \varepsilon - k_z^2\right) A = 0, \qquad (2)$$

giving

$$A = A_0 I_0(\alpha_1 r) \text{ for } r < a,$$

(3)

where

$$\varepsilon = 1 \text{ for } r < a,$$

$$\varepsilon = \varepsilon_{p} = \left(1 - \omega_{p}^{2}/\omega^{2}\right) \text{ for } r > a,$$

$$\alpha_{I}^{2} = k_{z}^{2} - \frac{\omega^{2}}{c^{2}},$$

$$\alpha_{II}^{2} = k_{z}^{2} - \frac{\omega^{2}}{c^{2}} \left(1 - \frac{\omega_{p}^{2}}{\omega^{2}}\right) = \alpha_{I}^{2} + \frac{\omega_{p}^{2}}{c^{2}},$$
(4)

where,  $\omega_{\rm p} = (n_0 e^2 / m \varepsilon_0)^{1/2}$ , -e, and *m* are the electronic charge and mass. Using  $E_z$  in the relevant Maxwell's equations one obtains  $\vec{H} = \hat{\Phi} H_{\Phi}$  and  $E_{\rm r}$ 

$$H_{\phi} = \frac{\omega}{k_z} \varepsilon_0 \varepsilon E_{\rm r}.$$
 (5)

$$E_{\rm r} = -\frac{ik_z}{(k_z^2 - \omega^2 \varepsilon/c^2)} \frac{\partial}{\partial r} E_z.$$
 (6)

Employing the continuity of  $E_z$  and  $H_{\phi}$  at r = a, we obtain the SPW dispersion relation

$$\frac{I_{1}(\alpha_{\mathrm{I}}a)}{I_{0}(\alpha_{\mathrm{I}}a)} = -\frac{\alpha_{\mathrm{I}}}{\alpha_{\mathrm{II}}} \frac{K_{1}(\alpha_{\mathrm{II}}a)}{K_{0}(\alpha_{\mathrm{II}}a)} \varepsilon_{\mathrm{p}}.$$
(7)

For  $\alpha_{I}a \gg 1$  and  $\alpha_{II}a \gg 1$ , this dispersion relation reduces to the usual SP dispersion relation over a planar surface

$$k_z = \frac{\omega}{c} \left( \frac{\varepsilon_{\rm p}}{\varepsilon_{\rm p} + 1} \right)^{1/2},\tag{8}$$

The power carried by the SPW is

$$P_{\rm s} = \pi \int E_{\rm r}^* H_{\rm \phi} \, r dr,$$
  
$$= \frac{\pi \varepsilon_0 k_z \omega}{\alpha_{\rm I}^2} A_0^2 \bigg[ \int_0^a I_1^2(\alpha_{\rm I} r) r dr + \frac{\varepsilon_{\rm p} \alpha_{\rm I}^2}{\alpha_{\rm II}^2} \frac{I_0^2(\alpha_{\rm I} a)}{K_0^2(\alpha_{\rm II} a)} \int_a^\infty K_1^2(\alpha_{\rm II} r) r dr \bigg]. \tag{9}$$

We have solved Eq. (7) numerically for the following parameters:  $a = 2 \,\mu\text{m}$ ,  $\omega_{\text{p}}a/c = 0.2$ , and  $\varepsilon_{\text{L}} = 9$  (corresponding to gold). In Figure 1 we have plotted the dispersion relation; we see that the frequency asymptotically approaches  $\omega = 0.71 \,\omega_{\text{p}}$ .

## 3. EXCITATION OF THz SPW

Let two lasers propagate through the hollow plasma cylinder in the cylindrically symmetric fundamental mode (cf. Fig. 2). The axial components of the laser fields satisfying wave equation [Eq. (2)] can be written as

$$E_{iz} = A_i e^{-i(\omega_j t - k_{jz} z)}$$

$$A_{j} = A_{0j}J_{0}(\delta_{j}r), \text{ for } r > a, = A_{0j}\frac{J_{0}(\delta_{j}a)}{K_{0}(\beta_{j}a)}K_{0}(\beta_{j}r),$$

$$= A_{0j}\frac{J_{0}(\delta_{j}a)}{K_{0}(\beta_{j}a)}K_{0}(\beta_{j}r), \text{ for } r < a,$$
(10)



Fig. 1. Plot of dispersion relation of SPW for the parameters  $a = 2 \mu m$ ,  $\omega_p a / c = 0.2$ , and  $\varepsilon_L = 9$ .



Fig. 2. Schematic of beat wave excitation of THz SPW over a hollow plasma cylinder.

where j = 1, 2 and  $\delta_j = (\omega_j^2 - k_{jz}^2 c^2)^{1/2}/c$ ,  $\beta_j = (\omega_p^2 + k_{jz}^2 c^2 - \omega_j^2)^{1/2}/c$ ,  $\beta_j = (\omega_p^2 + k_{jz}^2 c^2 - \omega_j^2)^{1/2}/c$ . Using Maxwell's equations one can deduce  $E_{jr}$  and  $H_{j\phi}$ . These turn out to be the same as given by Eqs. (5) and (6) with  $\omega$ ,  $\varepsilon$  replaced by  $\omega_j$ , and  $\varepsilon_j = (1 - \omega_p^2 / \omega_j^2)$ . Employing the boundary conditions on  $E_z$ ,  $H_{\phi}$ , and  $H_{\phi}$  at r = a one obtains the dispersion relation, for laser of frequency  $\omega_1$ 

$$\frac{J_0'(\delta_1 a)}{J_0(\delta_1 a)} = -\frac{\delta_1}{\beta_1} \left( 1 - \frac{\omega_p^2}{\omega_1} \right) \frac{K_0'(\beta_1 a)}{K_0(\beta_1 a)}.$$
 (11)

From the expressions for  $\delta_1$  and  $\beta_1$  one may write  $\beta_1^2 = \omega_p^2/c^2 - \alpha_1^2$ . Using this in Eq. (11) one gets an equation that determines  $\delta_1 c/\omega_p$  explicitly in terms of  $a\omega_p/c$  and  $\omega_1/\omega_p$ . For  $\omega_1/\omega_p \gg 1$ ,  $\delta_1 c/\omega_p$  is fairly independent  $\omega_1/\omega_p$ . We call this value of  $\delta_1 c/\omega_p$  as  $\alpha_{\perp} c/\omega_p$ , which is fixed for a particular mode, irrespective of  $\omega_1/\omega_p$ . Thus the dispersion relation for transverse magnetic (TM) mode reduces to  $\omega_j^2 = \alpha_{\perp}^2 c^2 + k_{jz}^2 c^2$ . For  $a\omega_p/c = 4$ , one obtains  $\alpha_{\perp} c/\omega_p = 0.7$ , hence  $\omega_j^2/\omega_p^2 = 0.49 + k_{jz}^2 c^2/\omega_p^2$ . The field structure is quite similar to Gaussian. The power carried by a laser is

$$P_{\rm L} = \pi \int E_{\rm r}^* H_{\phi} \, r dr,$$
  

$$\cong \frac{\pi \epsilon_0 k_{1z} \omega_1}{\alpha_{\perp}^2} A_{01}^2 \left[ \int_0^a J_1^2(\alpha_{\perp} r) r dr + \frac{\alpha_{\perp}^2}{\beta_1^2} \frac{J_0^2(\alpha_{\perp} a)}{K_0^2(\beta_1 a)} \int_a^{\infty} K_1^2(\beta_1 r) r dr \right].$$
(12)

The lasers impart oscillatory velocities to electrons

$$\vec{v}_1 = \frac{e\vec{E}_1}{mi\omega_1},$$
  
$$\vec{v}_2 = \frac{e\vec{E}_2}{mi\omega_2}.$$
 (13)

They also exert a beat ponderomotive force on electrons at the beat frequency, that goes as  $e^{-i(\omega t - k_z z)}$  with  $\omega = \omega_1 - \omega_2$ ,  $k_z = k_{1z} - k_{2z}$ .

$$\vec{F}_{\mathbf{p}_{\omega_1-\omega_2}} = e\nabla\phi_{\mathbf{p}} = -\frac{m}{2}\nabla(\vec{v}_1\cdot\vec{v}_2^*). \tag{14}$$

The ponderomotive force imparts oscillatory velocity to electrons

$$\vec{v}_{\omega}^{\mathrm{NL}} = -\frac{\vec{F}_{p_{\omega_1-\omega_2}}}{mi\omega} = \frac{\nabla(\vec{v}_1 \cdot \vec{v}_2^*)}{2i\omega},\tag{15}$$

giving a nonlinear current

$$\vec{J}^{\rm NL} = -n_e e \, \vec{v}_{\omega}^{\rm NL} \quad \text{for } r > a,$$
  
= 0 for  $r < a.$  (16)

The wave equation governing the propagation of THz SPW is

$$\nabla^2 \vec{E} - \nabla (\nabla \cdot \vec{E}) + \left(\frac{\omega^2 - \omega_p^2}{c^2}\right) \vec{E} = -\frac{i\omega}{c^2 \varepsilon_0} \vec{J}^{\rm NL}, \qquad (17)$$

 $\nabla \cdot$  of the above equation gives

$$\left(\frac{\omega^2 - \omega_{\rm p}^2}{c^2}\right) \nabla \cdot \vec{E} = -\frac{i\omega}{c^2 \varepsilon_0} \vec{J}^{\rm NL}.$$
(18)

Hence the Z-component of Eq. (17) can be written as

$$\nabla^2 E_z + \left(\frac{\omega^2 - \omega_p^2}{c^2}\right) E_z = -\frac{i\omega}{c^2 \varepsilon_0} J_z^{\text{NL}} + \nabla_z \left(-\frac{i\omega \left(\nabla \cdot \vec{J}^{\text{NL}}\right)}{\varepsilon_0 (\omega^2 - \omega_p^2)}\right).$$

For  $k_z a > 1$ , this equation takes the form

$$\nabla^{2} E_{z} + \left(\frac{\omega^{2} - \omega_{p}^{2}}{c^{2}}\right) E_{z} = \frac{m\omega c}{e} \frac{\omega_{p}^{2} i k_{z} (\vec{v}_{1} \cdot \vec{v}_{2}^{*})}{2c^{3} \omega (\omega^{2} - \omega_{p}^{2})} \times \left(\omega^{2} - \omega_{p}^{2} - k_{z}^{2} c^{2} - \frac{8c^{2}}{r_{0}^{2}} + \frac{16c^{2}}{r_{0}^{4}}\right),$$
(19)

where,  $k_z = k_{1z} - k_{2z}$  and  $r_0$  is the radial half width of the laser, for getting Eq. (19), we have used

$$\nabla \left( \vec{J}^{\mathrm{NL}} \right) \cong -\frac{n_0 e}{2i\omega} \nabla^2 (\vec{v}_1 \cdot \vec{v}_2^*),$$
$$= -\frac{n_0 e}{2i\omega} \left( -k_z^2 + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) (\vec{v}_1 \cdot \vec{v}_2^*).$$

The *z* dependence of the right hand side (RHS) of Eq. (19) is  $e^{i(k_{1z}-k_{2z})z}$ , for  $\omega_1$ ,  $\omega_2 \gg \omega_p$ ,  $\alpha_{\perp}c$ ,  $k_{1z}-k_{2z} = \omega/v_{g1}$ , where  $v_{g1} = c(1 - \alpha_{\perp}^2 c^2/2\omega_1^2)$ , thus

$$k_z = k_{1z} - k_{2z} = \frac{\omega}{c} \left( 1 + \frac{\alpha_{\perp}^2 c^2}{2\omega_1^2} \right) = \frac{\omega}{c} \left( \frac{\varepsilon_p}{\varepsilon_p + 1} \right)^{1/2}.$$
 (20)

For phase matching  $k_z$  must equal the parallel wave number of THz SPW. For  $\alpha_{I}a \gg 1$ ,  $I_0(\alpha_{I}a) = e^{\alpha_{I}a}/\sqrt{2\pi\alpha_{I}a}$ ,  $I_0(\alpha_{I}a) = I_0$ , and  $K_0(\alpha_{II}a) = e^{-\alpha_{II}a}\sqrt{\pi/2\alpha_{II}a}$ , so we have

$$1 = -\frac{\alpha_{\rm I}}{\alpha_{\rm II}}\varepsilon,$$
$$k_z = \frac{\omega}{c} \left(\frac{\varepsilon_{\rm p}}{\varepsilon_{\rm p}+1}\right)^{1/2},$$

using this value of  $k_z$  in Eq. (20) one obtains the requisite value of  $\varepsilon_p$ ,  $\varepsilon_p = -(\omega_p^2/\alpha_\perp^2 c^2) \cdot (2\omega_1^2/\omega_p^2) - 1$ .



Fig. 3. Plot of  $\omega/\omega_p$  versus  $\omega_1/\omega_p$  (THz frequency vs., laser frequency).

Thus the phase matching condition for resonant excitation of the THz SPW by the lasers turns out to be

$$\frac{\omega_p^2}{\omega^2} = 2 + \frac{4\omega_1^2}{\omega_p^2}.$$
(21)

From the above equation we plot  $\omega/\omega_p$  versus  $\omega_1/\omega_p$ , in Figure 3, we see that the THz frequency falls off with the increasing laser frequency.

Under exact phase matching we solve Eq. (17) iteratively. First we ignore the RHS, then solution of this equation is

$$E_z = A\vec{F}(r) e^{-i(\omega t - k_z z)},$$

where

$$F = I_0(\mathfrak{a}_{\mathrm{I}} r) \text{ for } r < a,$$

$$F = \frac{I_0(\mathfrak{a}_{\mathrm{I}}a)}{K_0(\mathfrak{a}_{\mathrm{II}}a)} \cdot K_0(\mathfrak{a}_{\mathrm{II}}r) \text{ for } r > a.$$

Substituting  $E_z$  from above in Eq. (17) and multiplying the resulting equation by Frdr and integrating it from 0 to  $\infty$ . With finite RHS we assume that the radial mode structure remains the same but A becomes a function of z, from Eq. (16) we have

$$2ik_{z}c^{2}\frac{\partial A}{\partial z} + A\left(\omega^{2} - \omega_{m}^{2}\right)\left(P_{1} + \frac{I_{0}^{2}(\alpha_{I}a)}{K_{0}^{2}(\alpha_{II}a)}P_{2}\right)$$
  
$$= \frac{m\omega c}{e}\frac{\omega_{p}^{2}ik_{z}}{2\omega(\omega^{2} - \omega_{p}^{2})}\frac{I_{0}(\alpha_{I}a)}{K_{0}(\alpha_{II}a)}\frac{e^{2}A_{01}A_{02}}{m^{2}\omega_{I}\omega_{2}}\frac{J_{0}(\alpha_{I}a)}{K_{0}(\beta_{I}a)}\frac{J_{0}(\alpha_{2}a)}{K_{0}(\beta_{2}a)}$$
$$\times \left[\left(\omega^{2} - \omega_{p}^{2} - k_{z}^{2}c^{2} - \frac{8c^{2}}{r_{0}^{2}}\right)P_{3} + \frac{16c^{2}}{r_{0}^{4}}P_{4}\right],$$

where  $\omega_m$  is the eigen frequency of the TM mode THz wave,  $\omega^2 - \omega_m^2 \simeq \beta \omega_p^2$ , where  $\beta \le 1$  is a parameter that accounts for the waveguide effects.

$$\frac{eA}{m\omega c} = \frac{z\omega}{c} \frac{\omega_{\rm p}^2}{4c^2\omega^2(\omega^2 - \omega_{\rm p}^2)} \frac{I_0(\alpha_{\rm I}a)}{K_0(\alpha_{\rm II}a)} \frac{e^2 A_{01} A_{02}}{m^2 \omega_{\rm I} \omega_2} \frac{J_0(\alpha_{\rm I}a)}{K_0(\beta_{\rm I}a)} \frac{J_0(\alpha_{\rm 2}a)}{K_0(\beta_{\rm 2}a)} \\ \times \left[ \left( \omega^2 - \omega_{\rm p}^2 - k_z^2 c^2 - \frac{8c^2}{r_0^2} \right) P_3 + \frac{16c^2}{r_0^4} P_4 \right],$$
(22)

where

$$P_{1} = \int_{0}^{a} I_{0}^{2}(\alpha_{\Pi}r)rdr,$$

$$P_{2} = \int_{a}^{\infty} K_{0}^{2}(\alpha_{\Pi}r)rdr,$$

$$P_{3} = \int_{a}^{\infty} \left[ K_{0}(\beta_{1}r)K_{0}(\beta_{2}r) + \frac{k_{1z}k_{2z}}{\beta_{1}\beta_{2}}K_{0}'(\beta_{1}r)K_{0}'(\beta_{2}r) \right]rdr,$$

$$P_{4} = \int_{a}^{\infty} \left[ K_{0}(\beta_{1}r)K_{0}(\beta_{2}r) + \frac{k_{1z}k_{2z}}{\beta_{1}\beta_{2}}K_{0}'(\beta_{1}r)K_{0}'(\beta_{2}r) \right]rdr.$$

The power conversion efficiency may be written as

$$\eta = \frac{P_{\text{THz}}}{P_{\text{L}}} = \frac{\frac{\pi\epsilon_0 k_z \omega}{\alpha_1^2} A_0^2 \left[ \int_0^a I_1^2(\alpha_1 r) r dr + \frac{\epsilon_p \alpha_1^2}{\alpha_{\text{II}}^2} \frac{I_0^2(\alpha_1 a)}{K_0^2(\alpha_{\text{II}} a)} \right]}{\frac{\pi\epsilon_0 k_{1z} \omega_1}{\alpha_{\perp}^2} A_{01}^2 \left[ \int_0^a J_1^2(\alpha_{\perp} r) r dr + \frac{\alpha_{\perp}^2}{\beta_1^2} \frac{J_0^2(\alpha_{\perp} a)}{K_0^2(\beta_1 a)} \right]}{\times \int_a^\infty K_1^2(\beta_1 r) r dr} \right].$$
(23)

We have carried out numerical calculations for Eq. (22) using MATHEMATICA with the parameters:  $a_1 = 0.1$ ,  $a_2 = 0.1$ ,  $k_z c/\omega_p = 0.3$ ,  $r_0 \omega_p/c = 1$ , and  $z \omega/c = 100$ . In Figure 4 we have plotted the normalized SPW THz amplitude  $|eA/m\omega c|$  as a function of  $\omega_p a/c$  at  $\omega_p a/c = 0.3$  (red line) and  $\omega_p a/c = 0.6$  (green dashed). As the plasma frequency is raised the



**Fig. 4.** Normalized SPW THz amplitude  $|eA/m\omega c|^2$  as a function of  $\omega_p a/c$  for  $r_0 \omega_p/c = 1$  and  $k_z c/\omega_p = 0.3$  at  $\omega_p a/c = 0.3$ , 0.6.



**Fig. 5.** Power conversion efficiency  $(P_{THz}/P_L)$  as a function of  $\omega_p a/c$  for  $\omega/\omega_p = 0.6$  and  $\omega_1/\omega_p \approx 100$ .

amplitude falls monotonically because of the low coupling at higher frequencies. The THz amplitude has an inverse relationship with the laser frequencies  $\omega_1$  and  $\omega_2$ , hence decreases as the laser frequencies increase. Also the variation of THz amplitude with the laser beam width is explicit from the figure as the radius of the plasma channel is of the order of the laser beam width. After solving Eq. (23), we plot the efficiency  $(P_{\text{THz}}/P_{\text{L}})$  as a function of  $\omega_{\text{p}}a/c$  for the parameters:  $\omega/\omega_{\text{p}} = 0.6$  and  $\omega_1/\omega_{\text{p}} \approx 100$  (Fig. 5). We see that efficiency falls off as the normalized size of the capillary plasma increases. The reason is that the power of the laser increases as the square of the radius, whereas the power of the THz goes linearly with *a*.

#### 4. DISCUSSION

Laser beat wave excitation of SPW on a hollow plasma cylinder has several notable features. The processes is resonant (phase matched) when the phase velocity of the SPW equals the group velocity of lasers. This demands  $\omega_1/\omega_p \sim \omega_p/\omega$ , as  $\omega_1$  increases  $\omega$  decreases Under phase matched condition, THz power scales linearly with distance, up to a distance of the order of attenuation length,  $(k_{zi})^{-1}$  and then saturates. The THz SPW amplitude decreases with increasing  $a\omega_p/c$ as the evanescent laser field in the plasma decreases. However, the THz amplitude increases when THz frequency increases.

In the present formalism we have neglected the collisions in the electron response. If one includes the collisional effects,  $\varepsilon_{eff}$  gets modified to  $\varepsilon_{eff} = 1 - \omega_p^2 (1 - i\nu/\omega)/\omega^2$ , where  $\nu$  is the electron collision frequency, this leads to imaginary part in  $k_{zi} \sim (\nu/\omega)(\omega_p^2/\omega^2)k_{zr}$ . For  $z > k_{zi}^{-1}$ , one must replace z by  $k_{zi}^{-1}$  in Eq. (22). Thus the amplitude of THz saturates to this value.

### ACKNOWLEDGEMENTS

The authors are grateful to Prof. C. S. Liu (University of Maryland) for valuable discussions.

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