Properly Extensive Quantities

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This article introduces and motivates the notion of a "properly extensive" quantity by means of a puzzle about the reliability of certain canonical length measurements. An account of these measurements' success, I argue, requires a modally robust connection between quantitative structure and mereology that is not mediated by the dynamics and is stronger than the constraints imposed by "mere additivity." I outline what it means to say that length is not just extensive but properly so and then briefly sketch an application of proper extensiveness to the project of providing a reductive ground for metric quantitative structure.

1. Introduction

1.1. Physical Quantities. Physical quantities—like mass, charge, volume, and length—are associated with a class of determinate *magnitudes* or *values*, each member of which is a property or relation itself. So when an object possesses mass, charge, or length, it always instantiates one particular magnitude of that quantity. Magnitudes are commonly represented in science and in everyday practice with mathematical entities, like numbers and vectors (e.g., 2.5 kg, 7 C [coulombs], 2π m).

These mathematical representations are appropriate because they faithfully represent these magnitudes, or the objects that instantiate them, as exhibiting certain structural features. This article introduces a phenomenon that I call "proper extensiveness." Proper extensiveness is one way the structure exhibited by quantity's magnitudes can influence the mereological (parthood) structure of their worldly instances.

In the next two sections, I provide motivations for positing proper extensiveness and argue that it does not depend on dynamics. Section 2 in-

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troduces a puzzle about explaining the reliable success of paradigm length measurements. The worry is that no explanation that essentially appeals to dynamics can account for the success of these measurements. The best explanation for this success, I argue, requires a predynamic but modally robust connection between quantitative structure and mereology.

Section 3 outlines two candidate connections, one commonly known as "additivity" and a strictly stronger connection, the aforementioned proper extensiveness. I show that only proper extensiveness is sufficient to underwrite the explanation of the length measurement presented in section 2. Also, taking length to be properly extensive rather than merely additive better accords with our modal intuitions involving the quantity.

I conclude and briefly sketch an application of the notion of proper extensiveness to the problem of quantity (i.e., the problem of finding a nonmetrical ground for metrical quantitative structure). The constraints properly extensive quantities put on mereological structure render toothless a common objection many potential solutions to this problem face.

1.2. Quantitative Structure. We can understand "quantitative structure" in terms of a system of structuring relations. We can represent these relations as holding between a quantity's magnitudes or between the instances of those magnitudes.

Some of these relations are *metrical*—we say "this pumpkin is precisely 8.73 times as massive as that gourd" when talking about objects and "1.5 m is 10 times as much as 15 cm" when talking about magnitudes. Others are *submetrical*. Let me introduce two relations that handily express the submetrical structure we intuitively apply to one-dimensional unsigned scalar quantities, that is, things like mass, length, and volume (and unlike charge, velocity, and spin).[1]

We say "this pumpkin is less massive than that table" and "22 m^3 is less than 22.1 m^3," when talking about the *ordering* on (in these cases) massive objects and determinate magnitudes of volume, respectively.

Let '$\prec$' denote a two-place relation symbolizing the intuitive "less than" relation over magnitudes, Q_i, of some quantity, Q. Intuitively $Q_a \prec Q_b$ when Q_a is "less than" Q_b. When an object, x, instantiates a mass magnitude that bears $\prec$ to the magnitude instantiated by another object y, we say that x is less massive than y.

We say "this stick is as long as that pencil and this highlighter put together" and "12 kg is the sum of 7 kg and 5 kg," when talking about the

1. By "one-dimensional scalar" quantity, I mean one that is intuitively gradated along only one axis, does not involve any notion of direction, and does not employ "signed" categories (like "positive" or "negative"), where two magnitudes might have the same "degree" but differ in "sign."

concatenation or *summation* structure on (in these cases) lengthy objects and mass magnitudes, respectively.

Let '$\oplus$' denote a three-place relation over the Q_is that serves to map two magnitudes to a third magnitude that is their "sum." So when $\oplus(Q_a, Q_b, Q_c)$, we say Q_c is the sum of Q_a and Q_b, and we write $Q_a \oplus Q_b = Q_c$. When $\oplus$ obtains between three length magnitudes instantiated by objects *x*, *y*, and *z*, respectively, we say that *z* is as long as *x* and *y* taken together.

2. Quantities and the World. The primary way that we gain epistemic access to facts about quantities is by performing measurements. For our purposes, a "*Q* measurement" is a physical procedure performed on certain objects, *a* and *b* (there need not be just two), which instantiate magnitudes of the quantity, *Q*. Measurements have a *ready state*—a specification of the state of the measurement apparatus and of *a* and *b* relative to that apparatus—as well as a set of mutually incompatible possible *outcomes*. Outcomes include things like the positions of a pointer, the relative positions of plates on a balance scale, or a distribution of illuminated pixels on a readout screen.

Call a token *Q* measurement, performed on *a* and *b*, successful if the occurrence or nonoccurrence of each outcome is reliably correlated with the obtaining or nonobtaining of a distinct (mutually incompatible) quantitative relation between *a* and *b* (or between the magnitudes of *Q* they instantiate). A successful *Q* measurement generates a counterfactually robust correlation between its outcomes and the quantitative facts—that is, it renders true conditionals of the form "if *a* had stood in R_i to *b* (at the time of our measurement), then outcome O_i would have occurred."

Such robust correlations, when they occur, cry out for explanation. Many such explanations appeal to the role of *Q* in the dynamics (case 1, below, provides an example). However, certain paradigmatic length measurements do not admit of explanation by such means yet may still be successful. Case 2 describes one such successful length measurement and offers an intuitive, nondynamic explanation for its success. The rub is that this explanation requires that we posit a substantive connection—not mediated by the dynamics—between length's quantitative structure and the mereology of lengthy physical entities.

2.1. Case 1: Weights on a Scale. We want to measure the ordering structure of a pair of massive objects, *a* and *b* (i.e., to determine which, if either, is more massive than the other). To do this, we set up a balance scale, with two plates suspended from opposite ends of a rigid bar, itself balanced at its center on a rigid vertical stand. The ready state for the scale is with the bar parallel to the ground and with *a* and *b* positioned on opposing plates. To perform this measurement, we release the plates and wait a moment or two. The possible outcomes are *a*'s plate is lower than *b*'s plate, *b*'s plate is lower than *a*'s plate, or the bar is parallel to the ground.[2]

Suppose we run this measurement and get the first outcome—*a*'s plate is lower. Suppose further that *a* is more massive than *b* and that if *a* had been less massive than (just as massive as) *b*, the second (third) outcome would have obtained. That is, we have performed a successful length measurement on *a* and *b*. In this particular case, what explains our measurement's success?

No mystery here. Mass's quantitative structure plays a certain role in the dynamic laws of motion and gravitation, which govern the evolution of the measurement apparatus. Objects that are more massive experience a greater force pulling them toward the earth. After we set the scale up in its ready state, the weights on the scale are impressed by gravitational forces, as dictated by the physical laws. The downward forces on the plates will unbalance a properly calibrated balance scale just in case the objects differ in mass, with the more massive object being pulled more forcefully. Thus, the dynamic laws come together with the quantitative facts and the physical makeup of the scale to bring about one of the three outcomes in a way that is reliably correlated with the "less massive than" relation. Call a successful measurement with an explanation of this sort a *dynamic* measurement.

2.2. Case 2: Aligning Rods. We want to measure the ordering structure for a pair of lengthy objects, in this case straight rigid rods. To do this, we adjust the rods so that they are parallel and lay them side by side. We then align them at one endpoint—that is, while keeping them parallel, positioning one endpoint of rod *a* such that it is immediately adjacent to the endpoint on the same side of rod *b*. This is the ready state. There are three possible outcomes, as before: rod *a* extends past rod *b*, rod *b* extends past rod *a*, or neither rod extends past the other (where "extending past," for these rods, just means one rod having a part that is not adjacent to any part of the other rod).[3] We observe which of the rods, if either, extends past the other and conclude that that rod is longer.

Suppose we perform this measurement and get the second outcome—rod *b* extends past rod *a*. Let us also suppose that this measurement is successful, that is, that *b* is, in fact, longer than *a* and that if *b* had not been longer than *a*, then *b* would not have extended past *a*, and so on. What explains this success?

2. One might worry that "lower" is a quantitative notion. However, it is not a matter of any quantitative relations between *a* and *b* and, in particular, is not a fact about *a* and *b*'s masses.

3. That is, adjacent in a direction orthogonal to *a* and *b*.

In contrast with the previous case, we cannot appeal to length's role in the dynamics to explain our measurement's success because this measurement has no temporal component. The procedure's ready state (*a* and *b* laid flush against each other and aligned at one endpoint) is simultaneous with its outcome (*b*'s extending past *a*). Certainly the dynamics may play a role in our observing the outcome after the measurement and in our positioning the rods before the measurement, but it plays no role in evolving the system from the ready state to the particular outcome. This means that the success of this measurement, and the reliable correlation between its nonquantitative outcome and the quantitative facts, cannot be dependent on the dynamics of length or any other quantity. Indeed, this length measurement could succeed even in a world governed by no dynamic laws, which exists only for one moment—as long as, at that moment, the rods *a* and *b* are situated in the right way.

2.3. The Problem of Nondynamic Measurement. Despite its nondynamic nature, there is nothing especially mysterious about the success of this length measurement. What is going on, intuitively, is something like this: *b* extends past *a*. So while there is a part of *b* that is perfectly aligned with *a*, there is also a remainder (i.e., another part of *b* that has no part that is adjacent to any part of *a*). Call the first part *x* and the second part, the remainder, *y*. The existence of such parts does not yet establish that *b* is longer than *a*. For that we need two bridge principles connecting the mereology and the quantitative facts.

1. If two rods are laid side by side such that neither extends past either endpoint of the other, then they are as long as each other.[4]
2. A rod must be longer than any of its proper "rod segments."[5]

4. Premise 1 approximates something akin to Euclid's Common Notion 4: "Things which coincide with one another are equal to one another" (1908, 155). Since material bodies cannot interpenetrate, the closest to coinciding we can practically achieve is alignment without remainder, i.e., being laid side by side with neither extending beyond the other. There is much more to be said about this premise and why it is reliable, but that would take us beyond the scope of this article.

5. Premise 2 makes use of the notion of a "rod segment." This is not ideal, but it is important to recognize that the more natural-sounding principle: "a rod must be longer than any of its proper parts" has some unfortunate exceptions. A 3 m rod could be cut "lengthwise," so to speak, and thus divide into two 3 m parts, or cut into parts that, intuitively, have no length at all but are just spatially disconnected bits of rod. The notion of "rod segment" is meant to rule such cases out. If the reader is still worried that a rod could be as long as one of its rod segments, perhaps with thoughts of a rod segment that is just the rod itself minus some length-less slice at one endpoint, we can add premise 3: If a rod can be partitioned into two rod segments, it is longer than each of them. Premise 3 relies on the idea that an infinitely

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Premise 1 establishes that *a* is as long as *x*. Premise 2 establishes that *b* is longer than *x*. Together they establish that, in situations like our length measurement above, *b* is longer than *a*.

According to this explanation, the outcome (*b* extending past *a*) and the quantitative facts (*b* being longer than *a*) are correlated but not because of length's role in the dynamics. Rather, they are correlated because of certain constraints on the possible lengths of objects given their mereological structure and relations and the possible mereological structure of objects given their lengths and length relations. This connection between quantitative structure and mereology shows up at two points in the explanation.

The first is obvious. Premise 2 establishes that a rod bears a certain quantitative relation (longer than) to every member of a certain special subclass of its parts. The second is more nuanced and involves premise 1. The explanation of the success of a length measurement of *a* and *b*, such that *b* extends past *a*, was presented as *fully general*. That is, for any rod shorter than *b*, which is measured against it in this way, *b* must have a proper part to be perfectly aligned with that rod. By 1 this implies that *b* has a proper part that is as long as that shorter rod, for any such rod shorter than *b*. Here the generality of this explanation depends on substantial constraints on the parts of *b* and the lengths of those parts.

Premises 1 and 2 are approximately true, but we do not need to tether our explanation to the nature of something as derivative and clunky as the notion of a concrete, straight, macroscopic material rod (and the "rod segments" that make it up). If we want a truly rigorous and general explanation, we will need to give it in terms of the fundamental entities and properties in the vicinity.

Let us say that length is, fundamentally, a property of one-dimensional, open (i.e., nonlooped) paths through space-time. To the extent that a concrete material rod can be said to have length, it has its length derivatively, in virtue of occupying a region containing certain, properly oriented, spatiotemporal paths of that length. For the remainder of this article, I concern myself with length as a property of substantival spatiotemporal paths.

We can capture the significance of premise 2 and of the generality assumption in one principle:

$2'$. For all paths *x* of length L_n, and for all lengths $L_m \neq L_n$, *x* has a proper part of length L_m if and only if $L_m \prec L_n$.

Principle $2'$ puts very strong constraints on the sorts of parts lengthy objects can have and on the possible lengths those parts can have. Analo-

thin slice off the end point of a rod is not a rod segment (even if its complement is). Once we do away with talk of rods in favor of talk of spatiotemporal paths, we can avoid this sort of ambiguity.

gously to 2, $2'$ implies that a given path is as long or longer than all of its lengthy parts. Analogously to the assumption about generality, $2'$ implies that a given path of length L_n must have a lengthy proper part corresponding to every length property bearing $\prec$ to L_n.

The only explanation for the reliable success of synchronic length measurement on offer requires a principle like $2'$. But neither the physical details of the measurement procedure nor the dynamic laws governing the system entail such a principle. If this explanation is a good one, then our metaphysics of length (and quantities like it) must be able to account for $2'$.

3. Constraining the World. In this section, I distinguish two phenomena, "additivity" and "proper extensiveness." I argue that the former is not, while the latter is, sufficient to underwrite the explanation outlined in the last section.

3.1. Additivity. An additive quantity, Q, is one where the specific magnitude of Q instantiated by a mereologically composite object is determined by the Q-magnitudes of its parts.[6] For instance, mass and length are additive quantities: 2 kg and 3 kg stand in $\oplus$ to 5 kg (2 kg $\oplus$ 3 kg = 5 kg). Composites of massive objects "inherit" their masses from their parts, so the mereological sum of a nonoverlapping pair of objects weighing 2 kg and 3 kg must weigh 5 kg.[7] The inheritance analogy is a powerful one, as it indicates both the strength and—we will see—limitations of this connection.

Additive quantities necessarily satisfy the following conditionals. They hold for any magnitudes, Q_i (of the same additive quantity), that satisfy the antecedent. The mereological relations used are these: $O(x, y)$ for overlap; $(x, y)C(z)$ for a three-place composition relation, with the third relatum being the fusion of the first two; and $P(x, y)$ for parthood.

Additive $\prec$: $(Q_m \prec Q_n) \rightarrow \forall x \forall y((Q_n(x) \wedge Q_m(y)) \rightarrow \neg P(x, y))$.

Additive $\oplus$: $(Q_m \oplus Q_n = Q_r) \rightarrow \forall x \forall y \forall z((Q_m(x) \wedge \neg O(x,y) \wedge (x,y)C(z)) \rightarrow (Q_r(z) \leftrightarrow Q_n(y)))$.

In the case of mass, Additive $\prec$ says that no massive object can have a part that is more massive than it. Additive $\oplus$ says that the fusion of any two

6. For simplicity of presentation, I assume mereological universalism.

7. Technically an overlapping pair could satisfy these conditions if they have negligible overlap. Usually overlap is considered negligible when it instantiates the "zero magnitude," like 0 m or 0 kg. However, if one's metaphysics of quantity does not include zero magnitudes (cf. Balashov 1999) the notion of negligible overlap must be obtained at in a different way.

nonoverlapping massive objects has, as its mass, the "sum" of their respective mass magnitudes. These conditionals (on the assumption that $\oplus$ is commutative) fully specify the mereological significance of additivity. These conditionals are modally robust.[8]

Suppose *pumpkin* is a 5 kg object composed out of nonoverlapping parts *body* and *stem*. If we consider a possibility where stem is 2 kg heavier than it in fact is, we readily (often automatically) infer that, at this world, pumpkin is 2 kg heavier as well. Indeed, it is difficult, or at least very awkward, to conceive of additivity failing here, that is, where body and pumpkin have their actual masses but stem is 2 kg heavier than it is at the actual world.

3.2. Additivity and Measurement. The reason additivity cannot explain the success of synchronic length measurement is well illustrated by the "inheritance" analogy. Additivity says that an object's length is determined by the lengths of its parts. However, Additive $\oplus$ and Additive $\prec$ are entirely silent on whether a given object has parts (lengthy or otherwise). This means that length's additivity cannot itself account for the truth of $2'$.

Since Additive $\prec$ and Additive $\oplus$ never imply that a given object must have parts of some kind, they are consistent with a pair of objects, *a* and *b*, instantiating magnitudes, Q_a and Q_b (of some additive quantity), where $Q_a \prec Q_b$ yet both *a* and *b* are mereological simples. There is nothing obviously wrong with this possibility if Q is mass. On the ordinary understanding of most particle theories, elementary particles are mereologically simple. And there is no prohibition on elementary particles differing in mass. However, the analogous possibility for lengthy entities is flatly inconsistent with $2'$. To see this, realize that it is consistent with the dictates of additivity that there be two lengthy objects, *a* and *b*, of lengths 2 m and 5 m, respectively, where *b* has no proper part as long as *a* (i.e., 2 m long) because *b* is a mereological simple. Mere additivity, then, cannot explain the reliable and general success of synchronic length measurement.

3.3. Proper Extensiveness. My contention is that certain physical quantities—length, volume, and temporal duration among them—put stronger constraints on the mereological structure of the world than merely additive quantities do. This feature accounts for why those possibilities presented in the previous section—which are consistent with additivity but inconsistent with $2'$—fail to characterize the modality of length and lengthy objects.

Physical quantities can be grouped into the additive and the nonadditive (sometimes called "intensive") quantities. The class of additive quantities, I

8. The strength of that necessity (nomological, metaphysical, etc.) may differ from quantity to quantity. Some quantities may constrain in virtue of the dynamics, while others do not.

maintain, can be further divided into the merely additive quantities and the properly extensive quantities. As such, properly extensive quantities also satisfy Additive $\prec$ and Additive $\oplus$: 2 m and 3 m stand in $\oplus$ to 5 m (i.e., 2 m $\oplus$ 3 m = 5 m). Length is additive, so the fusion of two nonoverlapping objects of length 2 m and 3 m laid end to end (in the right way) will be 5 m long. If length were merely additive, that would be the end of the story. If we suppose that length is also properly extensive, we can say more: since 2 m $\oplus$ 3 m = 5 m, any 5 m path must admit of a partition (no overlap) into a 2 m part and a 3 m part. That is, properly extensive quantities also necessarily satisfy:[9]

Extensive $\prec$: $(Q_m \prec Q_n) \rightarrow \forall x\,(Q_n(x) \rightarrow \exists y\,(y \neq x \wedge Q_m(y) \wedge P(y,x)))$.

Extensive $\oplus$: $(Q_m \oplus Q_n = Q_r) \rightarrow \forall x\,(Q_r(x) \leftrightarrow \exists y\,\exists z(Q_m(y) \wedge Q_n(z) \wedge \neg O(y,z) \wedge (y,z)C(x)))$.

In the case of length, what Extensive $\prec$ says is that every spatial path of a given length L_n, such that $L_m \prec L_n$, has an interval (which is to say, a part that is itself a path) of length L_m. Extensive $\oplus$ says a path can instantiate a length magnitude L_a such that $L_b \oplus L_c = L_a$, if and only if it has two nonoverlapping parts that respectively instantiate those magnitudes. This is a very powerful condition because it says that, given the quantitative facts, instantiating a given length magnitude, L_a, necessarily requires that you have parts with certain lengths, bearing certain mereological relations to one another.

In order to support our explanation of synchronic length measurement in terms of the existence of a remainder, length's proper extensiveness needs to imply:

2′. For all objects x of length L_n, and for all lengths $L_m \neq L_n$, x has a proper part of length L_m if and only if $L_m \prec L_n$.

By Extensive $\prec$, we get that if $L_m \prec L_n$, then x has a part of length L_m, and by Additive $\prec$, we get that if x has a proper part of length L_m, then L_m must either $= L_n$ or $\prec L_n$ (which, given the assumption that $L_m \neq L_n$, implies that

9. Both mere additivity and proper extensiveness involve principles that concern objects "put together in the right way." For quantities like mass or volume, the formula $\neg O(x,y) \wedge (x,y)C(z)$ accurately describes this condition. To apply to length, these conditions must be more stringent. The conditions for length would be something like this: a and b are both intervals of path c, which is their mereological fusion, and a and b either do not overlap or have a length-less overlap (either with 0 m length or without length, depending on what we want to say about the lengths of unextended points).

$L_m \prec L_n$). These, together, entail 2′. Proper extensiveness is able to explain the reliable success of synchronic length measurements.

4. Conclusion

4.1. The Significance of Proper Extensiveness. In this section I briefly survey some other interesting features of proper extensiveness and then gesture toward a very significant application of the notion. Some of our central intuitions regarding physical quantities like length, volume, and temporal duration—specifically those concerning how the mereological structure of the world reflects the quantitative structure of the properties instantiated at it—already suggest a tacit commitment to something like proper extensiveness. One striking consequence of taking length to be properly extensive illustrates this quite well. Suppose we discover a path through space that had a nonzero length, L_u, but no proper subpaths (i.e., no proper parts that are paths). According to Extensive $\prec$, this implies that there are no length magnitudes $\prec L_u$ (except the zero magnitude, 0 m, if there is such a thing)—meaning that the quantity, length, is discrete (best represented by the natural numbers plus zero) and that L_u is its unit length.

This result very closely accords with our intuitive expectations about what the physical world can tell us about length. We do not hear metaphysicians raise concerns when physicists run together the possibility that there is a smallest nonzero length (alternatively, that the quantity length is discrete) with the possibility that there are shortest possible paths (alternatively, that space is discrete). Indeed, many discussions of length readily use "shorter than" and "as long as a proper subinterval of" interchangeably. Similar points can be made for area, volume, and temporal duration. The pervasiveness of this line of thought disguises the substantive metaphysical commitments it requires. It is important to stress again that these commitments simply do not hold sway for merely additive quantities. According to a very common understanding of mass, there could very well be two simples (objects without proper parts) with differing, nonzero, masses. Mass, then, would be merely additive.[10] When entertaining the epistemic possibility that, for example, the

10. The fact that mass is closely associated with a certain dynamic role is good evidence that it is not properly extensive, since we standardly think that the same dynamic role in gravitation or inertia could be played equally well by a mereological complex or a simple. However, for all we know it may turn out that mass more closely aligns with earlier conceptions of mass as the "measure of matter." If that is right, to say that *a* is less massive than *b* is to say that *a* has less matter making it up than *b*. One way to draw out this understanding would be to treat mass as properly extensive and to expect its instances to obey the associated mereological constraints (i.e., if *b* has more matter making it up than *a* does, then *b* should have a part that has exactly as much matter making it up as *a* does).

electron is a point particle (without spatial extension and, it is presumed, mereologically simple), we do not at all expect every other elementary particle to therefore be exactly as massive as the electron. However, that is precisely the sort of conclusion we should reach in the analogous scenario for properly extensive quantities like length or volume.

I have offered two considerations in favor of a distinguishing, among the additive quantities, the merely additive and the properly extensive. This distinction accords with our intuitions about the modal mereology of quantities like mass and length, and proper extensiveness is necessary to adequately explain the reliable success of paradigm length measurements. I do not pretend to offer a reductive account of proper extensiveness or of how this constraining of the mereology is supposed to be achieved. For our purposes, it suffices to say that some quantities are properly extensive and that they constrain mereology in a modally robust way that is independent of the dynamic laws.

4.2. Application: The Problem of Quantity. In the previous two sections I have argued in favor of positing a distinction among the additive quantities into the merely additive and the properly extensive. I would like to close by gesturing in the direction of a significant potential application of this distinction. The problem of quantity is the problem of explaining how a quantity's metrical structure—the structure we represent with ratios, like when we say "this pumpkin is precisely 8.73 times as massive as that gourd"—can arise out of a nonmetrical basis.

A popular approach involves attempting to reduce facts about metric structure to facts about the world satisfying the right measurement-theoretic axioms.[11] Measurement theory is a formal discipline that involves rationalizations, formalizations, and defenses of empirical measurement practices. The game of measurement theory is to take a domain of material objects, which instantiate different magnitudes of some *Q*, posit some axioms that these objects obey, and then prove theorems which imply that *Q* can be faithfully represented (up to a point) with a certain mathematical structure (e.g., the real numbers; cf. Krantz et al. 1971). Some of the axioms needed to prove these theorems impose certain requirements on the size and structure of the domain itself. They require that the domains be well populated (existence axiom) and that there is ample variation in which magnitudes of *Q* are instantiated therein (richness axiom). The satisfaction of such requirements is a contingent matter. If there are not enough objects, or if they do not instantiate enough different magnitudes, these axioms fail to be satisfied.

This means that measurement-theoretic solutions to the problem of quantity have to contend with a powerful contingency worry. Our account of the

11. Field (1980) is the most famous account along these lines.

ground of metric structure ought not be contingent on how well populated the world is.[12] This is where proper extensiveness comes in. Suppose that L_x is a length magnitude, instantiated by a path, p. Extensive $\prec$ implies that p will have at least as many proper parts as there are length magnitudes that bear $\prec$ to L_x. Similarly, Extensive $\oplus$ implies that p will admit of a partition into parts of length L_y and L_z, for every such pair of length magnitudes such that $L_y \oplus L_z = L_x$.

That is, the instances of a properly extensive quantity necessarily, by virtue of the constraints it puts on their mereology, constitute a well-populated and variegated domain. Would the resulting domain satisfy the relevant existence and richness axioms? I think this can be shown, but there is no room to do so here. However, if true, it would allow for a uniquely elegant and principled solution to the problem of quantity, insofar as it applies to properly extensive quantities.

A result of this kind, if it works, is important for other reasons as well. A unique account of metric structure that only works when applied to properly extensive quantities speaks to the metaphysical depth of the distinction between properly extensive and additive quantities.

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12. Various theorists have proposed ad hoc solutions to avoid this contingency problem. Mundy (1987) gives up on the domain of massive objects and instead applies measurement-theoretic tools to the domain of mass magnitudes, while Arntzenius and Dorr (2012) avoid the contingency problem by positing additional, well-populated, substantival physical spaces.