# A computer environment for polymodal music

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**KYKLOS, an algorithmic composition program, is presented here. It generalises musical scales for use in composition as well as in performance. The sonic output of the system is referred to as *polymodal music* since it consists of four independent voices playing 'synthetic modes'. KYKLOS is suitable for computer-assisted composition because it generates MIDI files which can be altered later by the composer. It can equally well be used in live performance for dynamic modification of parameters enabling realtime musical control.**

## 1. INTRODUCTION

Computer music literature contains a great number of works investigating the creative potential of mathematical structures applied to composition. Further, mathematical structures in computer music have become not only an occasional tool for generating new music, but common starting points to extract basic material and to develop new ideas. Early investigations on mathematical structures in music studied scales and modes using combinatorics, Fibonacci series and the golden mean in order to understand compositional processes based on modal concepts (Barbour 1929). The analysis of Bartok's works is another example of this line of investigation (Lendvai 1968, Bachman and Bachman 1979).

Recently, we postulated that many mathematical applications in computer music can be understood as *sound functors* (Manzolli and Maia Jr 1998, Maia Jr, do Valle and Manzolli 1998). Here, we continue this exploration on mathematical structures in music. We propose a model for generating scales and modes and a compositional environment named KYKLOS conceived to work out this sound material. It is an interactive tool for composition.

In the following sections we start with basic musical and mathematical concepts and introduce the algorithmic mechanism used. Further, we present an interactive concept used to create a compositional environment that is based on a polyphony paradigm. We also describe a graphic interface developed at NICS and the general functions of KYKLOS.

## 2. MUSICAL PRELIMINARIES

It is well known that 'from roughly 800 to 1500 the Gregorian Modes formed the basis for nearly all western music. Since the music of this period was primarily vocal, the modes reflect the many influences and accommodations of this medium of expression.' (Benward 1981). These modes totalled twelve (eight Greek modes and four others created in the Renaissance) and included not only the major and minor modes, but also several others which have not as strong a sense of gravitation to a tonic note as is the main characteristic of the modern major–minor system.

Many composers advocated the use of modes in order to achieve a particular expression to their music. For example, it is well known that Beethoven in his *Missa Solemnis* and in some of his later quartets used Early Greek modes. Also Bartok, in several of his works, used pentatonic scales based on Fibonacci numbers. Messiaen introduced the 'modes de transposition limitée' (Messiaen 1944), as presented in figure 1.

The Italian composer, F. Busoni, 'described a method of forming scales by raising or lowering various tones of the scale of C major' (see Barbour 1929). He obtained 113 scales and this result was corrected later by Barbour (1929). As mentioned by Barbour, the number of possible scales is given by the combinatorial formula

$$C(p, 11) = 11!/(p-1)!(12-p)!,$$

where $p$ is the number of notes in the scale. For example, from 7-note scales we obtain 462 modes, from 5-note scales, 330 modes, and so on. Further, several 'exotic' scales are still in use around the world by different cultures and peoples. For example, some Japanese music is based on two pentatonic scales named *Miyako-bushi* and *Minyö* (see figure 2). There are also examples to be found in African, Latin American and Eastern European music (Fujie 1992).

Extending the discussion to 'synthetic scales' defined by Barbour (1929), we implemented an algorithmic system to expand the modal universe. This

**Figure 1.** Messiaen's second *mode à transpositions limitées* (*mélodiquement*).

**Figure 2.** The (a) *Minyö* and (b) *Miyako-bushi* scales.

program was called KYKLOS. Here we give only a simple model using cyclical permutations on scales and modes. From the point of view of sound functors (Manzolli and Maia Jr 1998), we simply mapped the well-known group of cyclical permutations on a set of sounds fixed *a priori*. In our example, this set is formed of generalised scales using a MIDI protocol. Of course, more complex mathematical and sound models can be used.

## 3. MATHEMATICAL APPROACH

We define a *mode of n notes* as any subset of $n$ notes, arranged in ascending order, extracted from the chromatic scale (C, C♯, D, D♯, E, F, F♯, G, G♯, A, A♯, B). For the mathematically oriented reader, these modes are nothing more than ordered subsets of the sequence (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11). Let us denote $\exists$ as the set of all modes, which is a finite set. Consider now $\exists_n$, the subset of $\exists$ which contains all modes of $n$ notes (or *n-modes* for short). So we have $\exists = U^{12}_{n=1}\exists_n$. Now, we can consider the operation of *cyclical permutations* on $\exists_n$. For example, if we take a mode of five notes, say (C, D, F, G, A), under cyclical permutations we get four other modes, namely (D, F, G, A, C), (F, G, A, C, D), etc. Mathematically this is obtained by the action of the cyclical group $Z_n$. From a musical point of view, this is nothing more than an analogy with the *Early Greek modes*. Under cyclical permutations, the set $\exists_n$ is partitioned in classes of equivalence whose elements are then the equivalent modes. The classes of equivalence are then denominated as *scales*. Loosely speaking, we may say that the modes are cyclical permutations of a particular scale. As in the Greek modes, the starting note of a particular scale's cyclical permutation gives the mode's name.

Although Messiaen was not a professional mathematician, he created an interesting problem in combinatorial analysis (see, for example, Read 1997). Read calculated the number of nonequivalent $n$-note scales under transposition – a Pólya-type problem in discrete mathematics. Table 1 shows all *modes de transposition limitée* proposed by Messiaen, i.e. $n$-note scales equivalent to at least one of their transpositions.

To calculate only the total number of modes associated with $n$-notes is a simpler problem than that presented by Read (see Barbour 1929). For any subset of the twelve tone set, our aim was to calculate all possible modes with KYKLOS. Following Barbour, these modes are here called *synthetic modes*, thus expanding the chromatic modal universe to its maximum size. In our implementation, we created a routine to calculate all these modes and to list them. In this sense, we present in the next section a simple computational solution to this problem.

## 4. COMPUTER IMPLEMENTATION

Using the above definitions it is possible to enumerate $n$-scales as a sequence of integers. Each value in the sequence gives the distance (in half tones) between two consecutive tones. For example, the sequence 3:2:2:3 is interpreted as a pentatonic scale, C-E♭-F-G-B♭, and as defined above it is a C mode. The same scale in F mode reads F-G-B♭-C-E♭. So, if we apply cyclical permutations, $(n-1)$-sequences of numbers should be interpreted as $n$-modes of tones. With this material at hand and an interactive graphic environment, KYKLOS becomes a tool for algorithmic composition. Our algorithmic implementation is described next.

An $n$-mode is defined as an array with $n-1$ integers $[a_1, a_2, \ldots, a_{n-1}]$. Each array generated at the $k$th step can be read as a number $a_1a_2a_3\ldots a_{n-1}$ in decimal representation, where $a_1$ is an integer between 1 and 9. We denote the number obtained at the $k$th step as $(a_1a_2a_3\ldots a_{n-1})^{(k)}$. The rules to implement the algorithm are as follows:

(1) $V_0 = (1, 1, 1, 1, \ldots, 1)$ (initial $n$-mode),
(2) $\sum a_i \leqslant 11$ with $i = 1, 2, \ldots, n-1$ (octave range constraint),

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**Table 1.** Number of all possible *modes de transposition limitée*.

| Number of notes / Symmetry | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 1 | 5 | 18 | 40 | 66 | 75 | 66 | 40 | 18 | 5 | 1 | |
| 2 | | | 1 | | 2 | | 3 | | 2 | | 1 | | |
| 3 | | | | 1 | | | 1 | | | 1 | | | |
| 4 | | | | | 1 | | | | 1 | | | | |
| 6 | | | | | | | 1 | | | | | | |
| 12 | 1 | | | | | | | | | | | | 1 |
| All scales | 1 | 1 | 6 | 19 | 43 | 66 | 80 | 66 | 43 | 19 | 6 | 1 | 1 |

**Table 2.** Relationship between number of notes and number of scales.

| Number of notes | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of scales | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 |

(3) $V_k = (a_1 a_2 a_3 \ldots a_{n-1})^{(k)}$
$< (b_1 b_2 b_3 \ldots b_{n-1})^{(k+1)}$
$= V_{k+1}$
where
$a_j \leqslant b_j,\ 1 \leqslant j \leqslant n-1,$
(4) $V_{max} = (13 - n,\ 1,\ 1,\ \ldots,\ 1).$

This algorithm obtains $C(p, 11)$ different scales in agreement with Barbour (1929) up to modes with 2 and 3 notes. This limitation is due to the decimal representation we have used in the algorithm. Table 2 displays the results. We include the modes from 2 to 11 notes (from Barbour 1929) for mathematical completeness.

## 5. INTERACTIVE SOUND MODEL

Western polyphony evolved through the use of the major and minor modes. The term 'modal' consequently refers to the type of melody and harmony that prevailed in the early and later Middle Ages. It is frequently used in opposition to tonal, which refers to the harmony based on the major–minor tonality, which came later (Machlis and Forney 1990). Before the establishment of tonality, the superimposition of modal melodies on multiple voices generated chords, thus creating harmony. This is a characteristic of Western music and distinguishes it from that of other civilisations. Using these observations as our paradigm, we developed an interactive computer system to expand the concept of polyphony to harmonic clusters. Thus, instead of searching for chords, we created a tool to produce harmonic complexity.

Using a set of parameters, we developed an algorithm to generate and control four independent voices. The voices differ from each other in the following properties: synthetic modes, rhythmic patterns, instrumentation and tempo. Using KYKLOS' graphic interface, a composer can explore many aspects of modal music in real time. The result of this process we call *polymodal music*.

As the name KYKLOS (Greek for cycles) indicates, cycles control the process used to generate and modify synthetic modes. Therefore, all modes are presented in ascending order and played in sequence originally. If this process were restricted to initial conditions, the composer could not change the mode's original order. Therefore, KYKLOS has a permutation tool based on a random process or any change input by the user (see figure 3).

Another of the system's attributes enables dynamic rhythmic control using strings written as sequences of small integers. Each number determines a proportional duration in relation to a voice tempo, and negative values represent rests.

In addition to the sound output produced in real time, there are two types of scores: MIDI file and parametric score. In the first case, sequences are recorded and processed later in any sequencer-like software. In the second, the parametric score stores changes made by the user on the graphic interface.

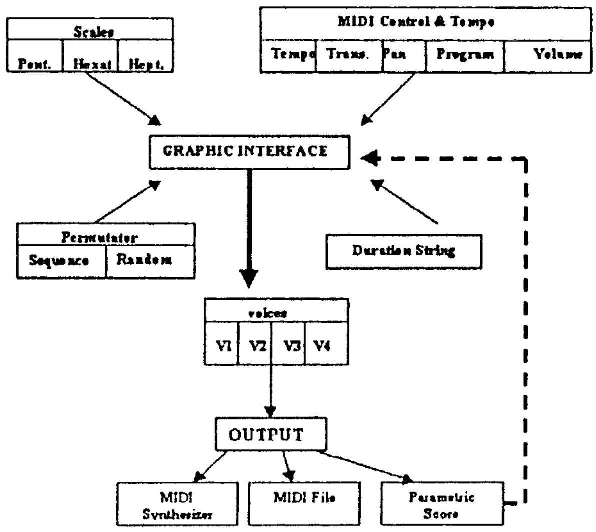


**Figure 3.** Diagram of the KYKLOS main functions and processes.

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This kind of score can be used in interactive performances to integrate a prerecorded sequence with live musicians (see figure 3).

## 6. KYKLOS ENVIRONMENT

The KYKLOS environment is based on a graphic interface written for a MS-Windows system and it is portable for any PC with any multimedia soundboard running under Windows 3.x, NT, 95 and 98. A diagram of the system functions, menus, control files and output is presented in figure 3.

KYKLOS initialisation uses a set of text files containing all precalculated synthetic modes. It also fills the voices' parametric array with MIDI controllers and other parameters such as pitch shift and voice starting note. As described above, KYKLOS' basic materials are synthetic modes varying from 5-notes to 11-notes. A composer can assign a different number of notes for each voice and consequently it is possible to choose any subset mode. Four voices play the chosen modes in a specific rhythmic pattern, volume, pan, MIDI program, pitch shift, permutation and tempo independently. These can all be changed by the user in real time.

## 7. CONCLUSION AND FURTHER DEVELOPMENTS

We have presented the program KYKLOS, whose potential is based on the set of all synthetic modes linked to realtime exploration of a graphic environment. The mathematical model described here could be implemented with other software tools for musical creation, such as MAX, for example. Nevertheless, the simultaneous creation of a mathematical model and a computer implementation will be useful for many composers and researchers, as well as enabling these tools to be used by the PC computer music community. We intend to provide a computer serial connection between KYKLOS and interfaces such as gloves, interactive tap shoes (Manzolli, Moroni and Matallo 1998) and the robot Khepera (for details, see the web page http://www.ini.unizh.ch:80/~jmb/roboser.html), developed at the NICS Gestures Interface Lab. These will enable the control of KYKLOS intuitively using body or machine motion.

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