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ASTIN BULLETIN

Volume 34 (2), 2004

AVRAM, F. & USABEL, M. A. *Ruin probabilities and deficit for the renewal risk model with phase-type interarrival times.* 315-332. This paper shows how the multivariate finite time ruin probability function, in a phase-type environment, inherits the phase-type structure and can be efficiently approximated with only one Laplace transform inversion. From a theoretical point of view, we also provide below a generalization of Thorin's formula (1971) for the double Laplace transform of the finite time ruin probability, by considering also the deficit at ruin; the model is that of a Sparre Andersen (renewal) risk process with phase-type interarrival times. In the case when the claims distribution is of phase-type as well, we obtain also an alternative formula for the single Laplace transform in time (or "exponentially killed probability"), in terms of the roots with positive real part of the Lundberg's equation, which complements Asmussen's representation (1992) in terms of the roots with negative real part.

BRAUN, C. *The prediction error of the chain ladder method applied to correlated run-off triangles.* 399-423. It is shown how the distribution-free method of Mack (1993) can be extended in order to estimate the prediction error of the Chain Ladder method for a portfolio of several correlated run-off triangles.

HERAS, A., GIL, J. A. & GARCÍA-PINEDA, P. *An application of linear programming to bonus malus system design.* 435-456. The purpose of this paper is to show how linear programming methodology can help us to design Bonus-Malus premium scales with some interesting theoretical and practical attributes. Examples of these properties are the financial equilibrium of the system, the monotonicity and proper variability of the premium scale, and the improvement of some efficiency measures such as the RSAL and the elasticity of the system. We will conclude that the use of the linear programming methodology makes possible a high degree of interaction between the designer and the mathematical model.

HESS, K. T. & SCHMIDT, K. D. *Optimal premium plans for reinsurance with reinstatements.* 299-313. The present paper is concerned with optimal premium plans for a reinsurance contract with reinstatements. In the reinsurance contract considered here, constant reinstatement premiums are due when the reinsurer's loss exceeds certain bounds. For this reinsurance contract we examine the existence of a premium plan which minimizes the expected squared difference between the loss and the total premium income of the reinsurer. We show that an optimal premium plan exists, that it is unique, and that it satisfies the net premium principle.

LANDSMAN, Z. M. *Second order Bayes prediction of functionals of exponential dispersion distributions and an application to the prediction of the tails.* 285-298. Second order Bayes estimators, being the main tool in second order optimal statistical theory, provide a natural basis for a new approach to the problem of the prediction of functions of expectation functional for members of an exponential dispersion family. A general formula, providing such prediction up to the term of the order $1/n$, is suggested and the application to the problem of the prediction of the tail of distributions is demonstrated. The results are

illustrated with normal and gamma claim sizes. The numerical experiment demonstrates the high effectiveness of the approach even for small sample sizes.

LEMAIRE, J. *Challenges to actuarial science in the 21st century*. 271-283. The 2004 ASTIN Colloquium was held in Bergen, Norway. Amongst many excellent invited talks and contributed papers (see the full report later in this volume) one of the highlights was the keynote closing address given by the Vice-Chairman of ASTIN, Jean Lemaire. The Editors thought that Jean's address was sufficiently original and thought provoking to merit wider exposure as an 'extended editorial' to all of the members of ASTIN and AFIR. We are very grateful to Jean for taking the time to provide us with a full text version of this address.

MORALES, M. *Risk theory with the generalized inverse Gaussian Lévy process*. 361-377. Dufresne *et al.* (1991) introduced a general risk model defined as the limit of compound Poisson processes. Such model is either a compound Poisson process itself or a strictly increasing Lévy process. Their construction is based on a non-negative non-increasing function Q that governs the jumps of the process. This function, it turns out, is the tail of the Lévy measure of the process. We discuss an illustration of their model using a generalized Inverse Gaussian (GIG) Lévy process. This increasing Lévy process has the gamma and the inverse Gaussian process as particular cases. Although mathematically more complex, the GIG Lévy process keeps some of the nice properties of the simpler gamma process.

OWADALLY, M. I. & HABERMAN, S. *The treatment of assets in pension funding*. 425-433. A recent survey of actuarial practitioners in North America shows that smoothed-market actuarial asset values are commonly used in funding valuations of defined benefit pension plans. Four methods of calculating such values are reported in the actuarial literature but only qualitative descriptions of the methods are given. This paper provides mathematical descriptions of the "average of market", "weighted average", "deferred recognition" and "write-up" actuarial values. They are shown to be based on either arithmetic or exponential smoothing. Provided the same form of smoothing is used, the four methods are equivalent.

PFEIFER, D. & NEŠLEHOVÁ, J. *Modeling and generating dependent risk processes for IRM and DFA*. 333-360. Modern Integrated Risk Management (IRM) and Dynamic Financial Analysis (DFA) rely in great part on an appropriate modeling of the stochastic behavior of the various risky assets and processes that influence the performance of the company under consideration. A major challenge here is a more substantial and realistic description and modeling of the various complex dependence structures between such risks showing up on all scales. In this presentation, we propose some approaches towards modeling and generating (simulating) dependent risk processes in the framework of collective risk theory, in particular w.r.t dependent claim number processes of Poisson type (homogeneous and non-homogeneous), and compound Poisson processes.

PITTS, S. M. *A functional approach to approximations for the individual risk model*. 379-397. A functional approach is taken for the total claim amount distribution for the individual risk model. Various commonly used approximations for this distribution are considered, including the compound Poisson approximation, the compound binomial approximation, the compound negative binomial approximation and the normal approximation. These are shown to arise as zeroth order approximations in the functional set-up. By taking the derivative of the functional that maps the individual claim distributions onto the total claim amount distribution, new first order approximation formulae are obtained as refinements to the existing approximations. For particular choices of input, these new approximations are simple to calculate. Numerical examples, including the well-known Gerber portfolio, are considered. Corresponding approximations for stop-loss premiums are given.

ALINK, S., LÖWE, M. & WÜTHRICH, M. V. *Analysis of the expected shortfall of aggregate dependent risks.* 25-43. We consider d identically and continuously distributed dependent risks X_1, \dots, X_d . Our main result is a theorem on the asymptotic behaviour of expected shortfall for the aggregate risks: there is a constant c_d such that for large u we have $E[\sum_{i=1}^d X_i | \sum_{i=1}^d X_i \leq -u] \sim -uc_d$. Moreover we study diversification effects in two dimensions, similar to our Value-at-Risk studies in [2].

BLADT, M. *A review on phase-type distributions and their use in risk theory.* 145-161. Phase-type distributions, defined as the distributions of absorption times of certain Markov jump processes, constitute a class of distributions on the positive real axis which seems to strike a balance between generality and tractability. Indeed, any positive distribution may be approximated arbitrarily closely by phase-type distributions whereas exact solutions to many complex problems in stochastic modeling can be obtained either explicitly or numerically. In this paper we introduce phase-type distributions and retrieve some of their basic properties through appealing probabilistic arguments which, indeed, constitute their main feature of being mathematically tractable. This is illustrated in an example where we calculate the ruin probability for a rather general class of surplus processes where the premium rate is allowed to depend on the current reserve and where claims sizes are assumed to be of phase-type. Finally we discuss issues concerning statistical inference for phase-type distributions and related functionals such as e.g a ruin probability.

DE FELICE, M. & MORICONI, F. *Market based tools for managing the life insurance company.* 79-111. In this paper we present an approach to market based valuation of life insurance policies, in the spirit of the NUMAT proposed by Hans Bühlmann (2002) in an editorial in the ASTIN Bulletin. We have experienced the valuation method for more than one decade, both as a pricing procedure applied to policy portfolios of leading insurance companies, and by including the valuation principles into several actuarial teaching activities.

Our interest is mainly focused here on participating policies that in Italy are characterized by contractually binding profit sharing rules. The problem of the fair valuation of the liabilities generated to the insurer by these contracts can be conveniently addressed using the methods of contingent claims pricing. These allow to price correctly the options embedded into the policies and to implement consistent plans of asset-liability management. The approach also provides a market based measurement of the value of business in force for outstanding policy portfolios and consistent assessments of the financial risk based capitals.

DEVOLDER, P. & DOMINGUEZ-FABIAN, I. *Fair valuation of various participation schemes in life insurance.* 275-297. Fair valuation is becoming a major concern for actuaries, especially in the perspective of IAS norms. One of the key aspects in this context is the simultaneous analysis of assets and liabilities in any sound actuarial valuation. The aim of this paper is to illustrate these concepts, by comparing three common ways of giving bonus in life insurance with profit: reversionary, cash or terminal. For each participation scheme, we compute the fair value of the contract taking into account liability parameters (guaranteed interest rate and participation level) as well as asset parameters (market conditions and investment strategy). We find some equilibrium conditions between all those coefficients and compare, from an analytical and numerical point of view, the systems of bonus. Developments are made first in the classical binomial model and then extended in a Black and Scholes economy.

DICKSON, D. C. M. & WILLMOT, G. E. *The density of the time to ruin in the classical Poisson risk model.* 45-60. We derive an expression for the density of the time to ruin in the classical risk model by inverting its Laplace transform. We then apply the result when the individual

claim amount distribution is a mixed Erlang distribution, and show how finite time ruin probabilities can be calculated in this case.

GARCIA, J. M. A. *Explicit solutions for survival probabilities in the classical risk model.* 113-130.

The purpose of this paper is to show that, for the classical risk model, explicit expressions for survival probabilities in a finite time horizon can be obtained through the inversion of the double Laplace transform of the distribution of time to ruin. To do this, we consider Gerber and Shiu (1998) and a particular value for their penalty function. Although other methods to address the problem exist, we find this approach, perhaps, more direct and simple. For the analytic inversion, we have applied twice, after some algebra, the Laplace complex inversion formula.

GAY, R. *Premium calculation for fat-tailed risk.* 163-188. When insurance claims are governed

by fat-tailed distributions considerable uncertainty about the value of the tail-index is often inescapable. In this paper, using the theory of risk aversion, a new premium principle (the power principle-analogous to the exponential principle for thin-tailed claims) is established and its properties investigated. Applied to claims arising from generalized Pareto distributions, the resultant premium is shown to be the ratio of the two largest expected claims, for which the ratio of the actual claims is an unbiased as well as a consistent estimator. Whereas thin-tailed claim premiums are determined largely by the first two moments of the claims distribution, fat-tailed claim premiums are determined by the first two extremes. The context of risk-aversion leads to a natural model for incorporating tail-index uncertainty into premiums, which nevertheless leaves the basic ratio structure unaltered. To illustrate the theory, possible 'premiums' for US hurricane data are examined, which utilize the consistent pattern of observed extremes.

HÜRLIMANN, W. *Excess of loss reinsurance with reinstatements revisited.* 211-238. The classical

evaluation of pure premiums for excess of loss reinsurance with reinstatements requires the knowledge of the claim size distribution of the insurance risk. In the situation of incomplete information, where only a few characteristics of the aggregate claims to an excess of loss layer can be estimated, the method of stop-loss ordered bounds yields a simple analytical distribution-free approximation to pure premiums of excess of loss reinsurance with reinstatements. It is shown that the obtained approximation is enough accurate for practical purposes and improves the analytical approximations obtained using either a gamma, translated gamma, translated inverse Gaussian or a mixture of the last two distributions.

KARLIS, D. *EM algorithm for mixed Poisson and other discrete distributions.* 3-24. Mixed

Poisson distributions are widely used in various disciplines including actuarial applications. The family of mixed Poisson distributions contains several members according to the choice of the mixing distribution for the parameter of the Poisson distribution. Very few of them have been studied in depth, mainly because of algebraic intractability. In this paper we will describe an EM type algorithm for maximum likelihood estimation for mixed Poisson distributions. The main achievement is that it reduces the problem of estimation to one of estimation of the mixing distribution which is usually easier. Variants of the algorithm work even when the probability function of the mixed distribution is not known explicitly but we have only an approximation of it. Other discrete distributions are treated as well.

LANDSMAN, Z. M. & VALDEZ, E. A. *Tail conditional expectations for exponential dispersion*

models. 189-209. There is a growing interest in the use of the tail conditional expectation as a measure of risk. For an institution faced with a random loss, the tail conditional expectation represents the conditional average amount of loss that can be incurred in a fixed period, given that the loss exceeds a specified value. This value is typically based on the quantile of the loss distribution, the so-called value-at-risk. The tail conditional expectation can therefore

provide a measure of the amount of capital needed due to exposure to loss. This paper examines this risk measure for “exponential dispersion models”, a wide and popular class of distributions to actuaries which, on one hand, generalizes the Normal and shares some of its many important properties, but on the other hand, contains many distributions of nonnegative random variables like the Gamma and the Inverse Gaussian.

LI, S. & GARRIDO, J. *Ruin probabilities for two classes of risk processes.* 61-77. We consider a risk model with two independent classes of insurance risks. We assume that the two independent claim counting processes are, respectively, Poisson and Sparre Andersen processes with generalized Erlang (2) claim inter-arrival times. The Laplace transform of the non-ruin probability is derived from a system of integro-differential equations. Explicit results can be obtained when the initial reserve is zero and the claim severity distributions of both classes belong to the K_n family of distributions. A relation between the ruin probability and the distribution of the supremum before ruin is identified. Finally, the Laplace transform of the non-ruin probability of a perturbed Sparre Andersen risk model with generalized Erlang (2) claim inter-arrival times is derived when the compound Poisson process converges weakly to a Wiener process.

NIELSEN, J. P. & SANDQVIST, B. L. *Proportional hazard estimation adjusted by continuous credibility.* 239-259. This paper extends the continuous credibility weighting introduced to hazard estimation in Hardy and Panjer (1998) and Nielsen and Sandqvist (2000) to the more general case, where the common basis is a proportional hazard model.

PITREBOIS, S., WALHIN, J. F. & DENUIT, M. *Bonus-malus systems with varying deductibles.* 261-274. Bonus-malus systems typically lead to high maluses when claims at fault are reported. Such penalties are often difficult to implement in practice. It is shown in this paper that this drawback may be avoided by combining a posteriori premium corrections with a deductible varying according to the level occupied in the scale.

STANFORD, D. A., AVRAM, F., BADESCU, A. L., BREUER, L., DA SILVA SOARES, A. & LATOUCHE, G. *Phase-type approximations to finite-time ruin probabilities in the Sparre-Andersen and stationary renewal risk models.* 131-144. The present paper extends the “Erlangization” idea introduced by Asmussen, Avram, and Usabel (2002) to the Sparre-Andersen and stationary renewal risk models. Erlangization yields an asymptotically-exact method for calculating finite time ruin probabilities with phase-type claim amounts. The method is based on finding the probability of ruin prior to a phase-type random horizon, independent of the risk process. When the horizon follows an Erlang- l distribution, the method provides a sequence of approximations that converges to the true finite-time probability as l increases. Furthermore, the random horizon is easier to work with, so that very accurate probabilities of ruin are obtained with comparatively little computational effort. An additional section determines the phase-type form of the deficit at ruin in both models. Our work exploits the relationship to fluid queues to provide effective computational algorithms for the determination of these quantities, as demonstrated by the numerical examples.

VISWANATHAN, K. S. & LEMAIRE, J. *Bonus-malus systems in a deregulated environment: forecasting market shares using diffusion models.* 299-319. In a deregulated insurance market, insurance carriers have an incentive to be innovative in their pricing decisions by segmenting their portfolios and designing new bonus-malus systems (BMS). This paper examines the evolution of market shares and claim frequencies in a two-company market, when one insurer breaks off the existing stability by introducing a super-discount class in its BMS. Several assumptions concerning policyholders and insurers behavior are tested. Diffusion theory is used to model the spread of the information concerning the new BMS among prospective

customers. A wide variety of market outcomes results: one company may take over the market or the two may survive with equal or unequal market shares, each specialising in a specific niche of the market. Before engaging in an aggressive competitive behavior, insurers should consequently be reasonably confident in their assumptions concerning the reactions of their policyholders to the new BMS.

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AUSTRALIAN ACTUARIAL JOURNAL

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GALE, A. C. *Leadership: the courage to commit: 2005 Presidential Address*. 1-143.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission to Australian Accounting Standards Board on liability adequacy test in AASB 1023 General Insurance Contracts*. 167-169.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission to the International Association of Insurance Supervisors (IAIS) on draft 'Cornerstones' paper*. 171-180.

INSTITUTE OF ACTUARIES OF AUSTRALIA. *Submission to Treasury on the Review of the Provision of pensions in Small Superannuation Funds*. 147-165.

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INSURANCE: MATHEMATICS & ECONOMICS

Volume **38** (1), 2006

BALLOTTA, L. & HABERMAN, S. *The fair valuation problem of guaranteed annuity options: the stochastic mortality environment case*. 195-214. In this paper, we extend the analysis of the behaviour of pension contracts with guaranteed annuity conversion options (as presented in Ballotta and Haberman [IME 33]) to the case in which mortality risk is incorporated via a stochastic model for the evolution over time of the underlying hazard rates. The pricing framework makes also use of a Black-Scholes/Heath-Jarrow-Morton economy in order to obtain an analytical solution to the fair valuation problem of the liabilities implied by these particular pension policies. The solution is not in closed form, and therefore, we resort to Monte Carlo simulation. Numerical results are investigated and the sensitivity of the price of the option to changes in the key parameters from the financial and mortality models is also analyzed.

BÄUERLE, N. & MÜLLER, A. *Stochastic orders and risk measures: consistency and bounds*. 132-148. We investigate the problem of consistency of risk measures with respect to usual stochastic order and convex order. It is shown that under weak regularity conditions risk measures preserve these stochastic orders. This result is used to derive bounds for risk measures of portfolios. As a by-product, we extend the characterization of coherent, law-invariant risk measures with the Fatou property to unbounded random variables.

CHEUNG, K. C. *Optimal portfolio problem with unknown dependency structure*. 167-175. This research studies a single-period expected utility-based optimal portfolio problem. Assets are divided into different groups. It is assumed that the actual dependency structure of the asset returns within each group is unknown, but assets belonging to different groups have independent returns. Instead of assuming any particular dependency structure within each group, we propose the maximin criterion as an alternative optimization criterion. The least favorable dependency structure is first identified, then we proceed to analyze the optimal portfolio problem as if this were the actual dependency structure. Sufficient conditions for ordering the aggregate amounts allocated to different groups, and for ordering the amounts allocated to different assets within each group are obtained.

EISELE, K.-T. *Recursions for compound phase distributions*. 149-156. In this paper, we study a simple recursion procedure for a compound phase-type distribution. This procedure is only based on the rationality of the characteristic function of the phase-type distribution of the claim number.

ESPINOSA, F. & VIVES, J. *A volatility-varying and jump-diffusion Merton type model of interest rate risk*. 157-166. According to many recent studies, Lévy processes with stochastic volatility seem to be the best candidates for replacing geometric Brownian motion (GBM) as a price process model. This means that the GBM model has to be generalised by introducing the possibility of jumps and allowing the volatility to be a stochastic process. In this paper, we present a generalisation of the traditional Lévy–Merton jump-diffusion model, allowing discrete stochastic volatility. In order to estimate jump instants and jump amplitudes, we use, and improve on, a method based on quadratic variation. We apply this method to two time series provided by the “Banco de España” comprising daily observations of interest rate for operations of 1 day and 1 year (from 4 January 1988 to 31 December 1998).

GELUK, J. L. & DE VRIES, C. G. *Weighted sums of subexponential random variables and asymptotic dependence between returns on reinsurance equities*. 39-56. Suppose X_1, X_2, \dots are independent subexponential random variables with partial sums S_n . We show that if the pairwise sums of the X_i 's are subexponential, then S_n is subexponential and $(S_n > x) \sim \sum_1^n P(X_i > x)$ ($x \rightarrow \infty$). The result is applied to give conditions under which $P(\sum_1^\infty c_i X_i > x) \sim \sum_1^\infty P(c_i X_i > x)$ as $x \rightarrow \infty$, where c_1, c_2, \dots are constants such that $\sum_1^\infty c_i X_i$ is a.s. convergent. Asymptotic tail probabilities for bivariate linear combinations of subexponential random variables are given. These results are applied to explain the joint movements of the stocks of reinsurers. Portfolio investment and retrocession practices in the reinsurance industry expose different reinsurers to the same subexponential risks on both sides of their balance sheets. This implies that reinsurer's equity returns can be asymptotically dependent, exposing the industry to systemic risk.

HIPP, C. *Speedy convolution algorithms and Panjer recursions for phase-type distributions*. 176-188. For phase-type claim size distributions, the Panjer algorithm can be simplified in the sense that the non local term in the recursion can be replaced by a local one, which yields a considerable reduction of computation time. This is done for the discrete case with arithmetic claim size, for the continuous case with a density for the claim size, and finally for the mixed case in which the claim size distribution is a mixture of two distributions, the first arithmetic,

the second continuous. In the last two cases, numerical integration can be replaced by the solution of an interacting system of differential equations.

HUANG, H.-C. & CAIRNS, A. J. G. *On the control of defined-benefit pension plans*. 113-131.

Conventionally, contribution rates for defined-benefit pension plans have been set with reference to funding levels without making allowance for current market interest rates: for example, on one-year bonds where rates of return on fund assets are not independent from one year to the next. We consider how to make use of market information to reduce contribution rate volatility. The purpose of this paper is to provide a model for determining an appropriate contribution rate for defined benefit pension plans under a model where interest rates are stochastic and rates of return are random. We extend previous work in two ways. First, we introduce a model for short-term interest rates, which can be used to help control contribution-rate volatility. Second, we model three assets rather than the usual one (cash, bonds and equities) to allow comparison of different asset strategies. We develop formulae for unconditional means and variances. We then discuss how variability can be controlled most efficiently by setting contribution rates with reference to current funding levels and interest rates.

KOISSI, M.-C., SHAPIRO, A. F. & HÖGNÄS, G. *Evaluating and extending the Lee–Carter model for mortality forecasting: Bootstrap confidence interval*. 1-20. This paper first studies the performance of the Lee–Carter [J. Am. Stat. Assoc. (1992)] model for mortality forecasting on the Nordic countries. Three approaches for computing the model parameters are compared: Singular Value Decomposition, Weighted Least Square and Maximum Likelihood Estimation.

Hypothetical projections are also made, based on variable period intervals. Secondly, the paper addresses an extension to the Lee–Carter method: a residual bootstrapped technique is used to construct confidence intervals for forecasted life expectancies. Uncertainties produced with this method incorporate the variability from all parameters in the model, while the original Lee–Carter method focuses on the variability in the time-varying parameter.

LIN, X. S. & PAVLOVA, K. P. *The compound Poisson risk model with a threshold dividend strategy*. 57-80. In this paper, we present the classical compound Poisson risk model with a threshold dividend strategy. Under such a strategy, no dividends are paid if the insurer's surplus is below a certain threshold level. When the surplus is above this threshold level, dividends are paid at a constant rate that does not exceed the premium rate. Two integro-differential equations for the Gerber–Shiu discounted penalty function are derived and solved.

The analytic results obtained are utilized to derive the probability of ultimate ruin, the time of ruin, the distribution of the first surplus drop below the initial level, and the joint distributions and moments of the surplus immediately before ruin and the deficit at ruin. The special cases where the claim size distribution is exponential and a combination of exponentials are considered in some detail.

MILEVSKY, M. A. & SALISBURY, T. S. *Financial valuation of guaranteed minimum withdrawal benefits*. 21-38. We develop a variety of methods for assessing the cost and value of a very popular 'rider' available to North American investors on variable annuity (VA) policies called a Guaranteed Minimum Withdrawal Benefit (GMWB). The GMWB promises to return the entire initial investment, albeit spread over an extended period of time, regardless of subsequent market performance. First, we take a static approach that assumes individuals behave passively and holds the product to maturity. We show how the product can be decomposed into a Quanto Asian Put plus a generic term-certain annuity. At the other extreme of consumer behavior, the dynamic approach leads to an optimal stopping problem akin to pricing an American put option, albeit complicated by the non-traditional payment structure. Our main result is that the No Arbitrage hedging cost of a GMWB ranges from 73 to 160 basis points of assets. In contrast, most products in the market only charge 30-45 basis

points. Although we suggest a number of behavioral reasons for the apparent under-pricing of this feature in a typically overpriced VA market, we conclude by arguing that current pricing is not sustainable and that GMWB fees will eventually have to increase or product design will have to change in order to avoid blatant arbitrage opportunities.

SCHRAGER, D. F. *Affine stochastic mortality*. 81-97. We propose a new model for stochastic mortality. The model is based on the literature on affine term structure models. It satisfies three important requirements for application in practice: analytical tractability, clear interpretation of the factors and compatibility with financial option pricing models. We test the model fit using data on Dutch mortality rates. Furthermore, we discuss the specification of a market price of mortality risk and apply the model to the pricing of a guaranteed annuity option and the calculation of required economic capital for mortality risk.

TANK, F., GEBIZLIOGLU, O. L. & APAYDIN, A. *Determination of dependency parameter in joint distribution of dependent risks by fuzzy approach*. 189-194. Frechet bounds for distribution of sum of dependent risks and Farlie–Gumbel–Morgenstein (FGM) distribution family are suggested for the analysis of dependent risks. Behaviour of the coefficient of dependency in FGM distribution is investigated by fuzzy approach. Deliberations are made for the dependency and uncertainty context.

YI, Z. & WENG, C. *An application of the alpha-power approximation in multiple life insurance*. 98-112. In this note, we combine the alpha-PAs and copula to approximate the survival probabilities of the general symmetric status. Two methods are constructed and the different approximations are compared with respect to stochastic ordering. As an application, these approximation methods are introduced to an endowment insurance contract and the effect of a value is discussed. Finally, two numeric examples are presented.

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BLEICHRODT, H. & EECKHOUDT, L. *Survival risks, intertemporal consumption, and insurance: the case of distorted probabilities*. 335-346. This paper explores how to evaluate changes in survival probabilities when people do not process probabilities linearly, as is commonly assumed in the literature, but distort probabilities. We show that the valuation of risks to life depends critically on two parameters: the elasticity of the probability weighting function and the elasticity of the utility function with respect to future consumption. Using estimates from the empirical literature we derive that the bias of erroneously ignoring probability distortion in general leads to cost–benefit ratios that are too high and that generate too much priority for programs that save young lives.

BURGERT, C. & RÜSCHENDORF, L. *Consistent risk measures for portfolio vectors*. 289-297. The main purpose to study risk measures for portfolio vectors $X = (X_1, \dots, X_d)$ is to measure not only the risk of the marginals X_i separately but to measure the joint risk of X caused by the variation of the components and their possible dependence. Thus, an important property of risk measures for portfolio vectors is consistency with respect to various classes of convex and dependence orderings. From this perspective, we introduce and study convex risk measures for portfolio vectors defined axiomatically and further introduce two natural and easy to interpret and calculate classes of examples of risk measures for portfolio vectors and investigate their consistency properties.

COLEMAN, T. F., LI, Y. & PATRON, M.-C. *Hedging guarantees in variable annuities under both*

equity and interest rate risks. 215-228. Effective hedging strategies for variable annuities are crucial for insurance companies in preventing potentially large losses. We consider discrete hedging of options embedded in guarantees with ratchet features, under both equity (including jump) risk and interest rate risk. Since discrete hedging and the underlying model considered lead to an incomplete market, we compute hedging strategies using local risk minimization. Our results suggest that risk minimization hedging, under a joint model for the underlying and interest rate, leads to effective risk reduction. Moreover, hedging with standard options is superior to hedging with the underlying when both equity and interest rate risks are appropriately modeled.

COSSETTE, H. E., LANDRIault, D. & MARCEAU, É. *Ruin probabilities in the discrete time renewal risk model.* 309-323. In this paper, we study the discrete time renewal risk model, an extension to Gerber's compound binomial model. Under the framework of this extension, we study the aggregate claim amount process and both finite-time and infinite-time ruin probabilities. For completeness, we derive an upper bound and an asymptotic expression for the infinite-time ruin probabilities in this risk model. Also, we demonstrate that the proposed extension can be used to approximate the continuous time renewal risk model (also known as the Sparre Andersen risk model) as Gerber's compound binomial model has been proposed as a discrete-time version of the classical compound Poisson risk model. This allows us to derive both numerical upper and lower bounds for the infinite-time ruin probabilities defined in the continuous time risk model from their equivalents under the discrete time renewal risk model. Finally, the numerical algorithm proposed to compute infinite-time ruin probabilities in the discrete time renewal risk model is also applied in some of its extensions.

FREES, E. W. & WANG, P. *Copula credibility for aggregate loss models.* 360-373. This paper develops credibility predictors of aggregate losses using a longitudinal data framework. For a model of aggregate losses, the interest is in predicting both the claims number process as well as the claims amount process. In a longitudinal data framework, one encounters data from a cross-section of risk classes with a history of insurance claims available for each risk class. Further, explanatory variables for each risk class over time are available to help explain and predict both the claims number and claims amount process. For the marginal claims distributions, this paper uses generalized linear models, an extension of linear regression, to describe cross-sectional characteristics. Elliptical copulas are used to model the dependencies over time, extending prior work that used multivariate t-copulas. The claims number process is represented using a Poisson regression model that is conditioned on a sequence of latent variables. These latent variables drive the serial dependencies among claims numbers; their joint distribution is represented using an elliptical copula. In this way, the paper provides a unified treatment of both the continuous claims amount and discrete claims number processes. The paper presents an illustrative example of Massachusetts automobile claims. Estimates of the latent claims process parameters are derived and simulated predictions are provided.

GUILLÉN, M., JØRGENSEN, P. L. & NIELSEN, J. P. *Return smoothing mechanisms in life and pension insurance: Path-dependent contingent claims.* 229-252. Traditional with-profits pension saving schemes have been criticized for their opacity, plagued by embedded options and guarantees, and have recently created enormous problems for the solvency of the life insurance and pension industry. This has fueled creativity in the industry's product development departments, and this paper analyzes a representative member of a family of new pension schemes that have been introduced in the new millennium to alleviate these problems. The complete transparency of the new scheme's smoothing mechanism means that it can be analyzed using contingent claims pricing theory. We explore the properties of this pension scheme in detail and find that in terms of market value, smoothing is an illusion, but also that the return smoothing mechanism implies a dynamic asset allocation strategy which

corresponds with traditional pension saving advice and the recommendations of state-of-the-art dynamic portfolio choice models.

HAU, A. *Production under uncertainty with insurance or hedging*. 347-359. This paper examines the output decision of a risk-averse producer facing profit risk in the presence of insurance or hedging. Conditions under which the producer's output increases upon the introduction of generic insurance are derived, giving rise to conditions for deductible insurance (commodity call options), coinsurance-type insurance (commodity futures), and restricted deductible insurance, respectively. This paper improves upon the literature by considering general profit risk, possibly revenue risk or cost risk, that may not be multiplicative. Moreover, unlike Machnes and Wong's [Geneva Pap. Risk Insurance Theory 28 (2003) 73-80] condition on the loading factor that may not lead to an explicit and unique value, the condition derived in this paper gives rise to a unique upper bound for the loading factor. Finally, their assumptions on the utility function, such as quadratic utility and constant absolute risk aversion for the case of restrictive deductible insurance and zero-loading are made substantially less restrictive.

HU, T., CHEN, J. & YAO, J. *Preservation of the location independent risk order under convolution*. 406-412. The location independent risk order has been used to compare different random assets in risk analysis without the requirement of equal means. Let (X_i, Y_i) , $i = 1, 2, \dots, n$, be independent pairs of random assets. It is shown that if X_i is less than Y_i in the location independent risk order for each i and the X_i and Y_i have log-concave density or probability functions, or if X_i is less than Y_i in the dispersive order and the X_i and Y_i have log-concave distribution functions, then $\sum_{i=1}^n X_i$ is less than $\sum_{i=1}^n Y_i$ in the location independent risk order. Similar results also hold for the excess wealth order.

JONES, B. L., PURI, M. L. & ZITIKIS, R. *Testing hypotheses about the equality of several risk measure values with applications in insurance*. 253-270. This paper explores statistical tests about the equality of risk measure values obtained using a distortion-based risk measure. We consider both the case in which the risk measure value is specified in the null hypothesis and the case in which it is not. In the former case, one- and two-sided alternatives are considered, and in the latter case, ordered and unordered alternatives are considered. Asymptotically most powerful tests are obtained, and asymptotic distributions of the test statistics are found using results about the asymptotic distributions of the risk measure values. Finally, we consider a numerical example and conclude the paper with notes on when the results of the paper could, or could not, be safely used.

KWON, H.-S. & JONES, B. L. *The impact of the determinants of mortality on life insurance and annuities*. 271-288. Extended risk classification has become an important issue recently in life insurance and annuity markets. Various risk factors have been explored and identified by past research. Using those risk factors, one can construct various risk classes. This enables insurers to provide more equitable life insurance and annuity benefits for individuals in different risk classes and to manage mortality/longevity risk more efficiently. The challenge of modeling mortality using various risk factors is to reflect complicated mortality dynamics in a model while maintaining statistical significance. This paper discusses the development of a mortality model that reflects the impact of various risk factors on mortality. Longitudinal survey data from the Canadian National Population Health Survey was used to determine the significant risk factors and quantify their effect on mortality. The model is used to illustrate how the various risk factors influence actuarial present values of life insurance and annuity benefits.

PAVLOVA, K. P., CAI, J. & WILLMOT, G. E. *The preservation of classes of discrete distributions under convolution and mixing*. 391-405. In this paper we consider some widely utilized classes

of discrete distributions and aim to provide a systematic overview about their preservation under convolution and mixing. Moreover, inclusion properties among these classes are discussed. This paper will serve as a detailed reference for the study and applications of the preservation of the classes of discrete distributions.

RAMASUBRAMANIAN, S. *An insurance network: Nash equilibrium*. 374-390. We considered insurance companies whose surplus processes are r.c.l.l. functions (like the sample paths of perturbed Levy processes). Suppose they have a treaty to diversify risk; accordingly, if one company needs a certain amount to prevent ruin, the other companies pitch in previously-agreed-upon fractions of the amount, and any shortfall is got from external sources. With each company trying to minimise its repayment liability, the situation is viewed upon as a d -person dynamic game with state space constraints and a Nash equilibrium is sought. Under certain natural conditions, it is shown that the Skorokhod problem of probability theory provides a (unique) Nash equilibrium. The thrust of the paper is entirely deterministic.

VERNIC, R. *Multivariate skew-normal distributions with applications in insurance*. 413-426. In this paper, we discuss the skew-normal distribution as an alternative to the classical normal one in the context of both risk measurement and capital allocation. As main risk measure, we consider the tail conditional expectation (TCE). Hence, we investigate an allocation formula based on the TCE, but we also consider Wang's [Wang, S, 2002. Working paper. SCOR Reinsurance Company] allocation formula. .

WU, X.-Y. & ZHOU, X. *A new characterization of distortion premiums via countable additivity for comonotonic risks*. 324-334. For premium calculation principles or risk measures, all existing works only consider the additivity for a finite number of comonotonic risks. As we all know, a limiting status of finite additivity is the additivity for countable risks. In this paper we investigate the countable additivity and generate new and elegant characterizations for Choquet pricing and distortion premium principles. We also study the countable exchangeability, as an extension to additivity. It leads to generalized Choquet pricing and generalized distortion premium principles.

YUEN, K. C., GUO, J. & WU, X. *On the first time of ruin in the bivariate compound Poisson model*. 298-308. This paper considers a bivariate compound Poisson model for a book of two dependent classes of insurance business. We focus on the ruin probability that at least one class of business will get ruined. As expected, general explicit expressions for this bivariate ruin probability is very difficult to obtain. In view of this, we introduce the so-called bivariate compound binomial model which can be used to approximate the finite-time survival probability of the assumed model. We then study some simple bounds for the infinite-time ruin probability via the association properties of the bivariate compound Poisson model. We also investigate the impact of dependence on the infinite-time ruin probability by means of multivariate stochastic orders.

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ANDRADE, E., SILVA, J. M. & CENTENO, M. DE L. *A note on bonus scales*. 601-607. We revisit the optimal bonus scales introduced by Norberg (SAJ 1976), Borgan, Hoem, and Norberg

(SAJ, 1981), and Gilde and Sundt (SAJ, 1989) and underline some potential problems of the linear scales. As a possible solution we propose the use of geometric scales.

BREUER, M. *Multiple losses, ex ante moral hazard, and the implications for umbrella policies.* 525-538. Under certain cost conditions the optimal insurance policy offers full coverage above a deductible, as Arrow and others have shown. However, many insurance policies currently provide coverage against several losses although the possibilities for the insured to affect the loss probabilities by several prevention activities (multiple moral hazard) are substantially different. This article shows that optimal contracts under multiple moral hazard generally call for complex reimbursement schedules. It also examines the conditions under which different types of risks can optimally be covered by a single insurance policy and argues that the case for umbrella policies under multiple moral hazard is limited in practice.

CAUDILL, S. B., AYUSO, M. & GUILLÉN, M. *Fraud detection using a multinomial logit model with missing information.* 539-550. Recently, Artís, Ayuso, and Guillén (2002, JRI 69; henceforth AAG) estimate a logit model using claims data. Some of the claims are categorized as "honest" and other claims are known to be fraudulent. Using the approach of Hausman, Abrevaya, and Scott-Morton (J. Econometrics 87), AAG estimate a modified logit model allowing for the possibility that some claims classified as honest might actually be fraudulent. Applying this model to data on Spanish automobile insurance claims, AAG find that 5 percent of the fraudulent claims go undetected. The purpose of this article is to estimate the model of AAG using a logit model with missing information. A constrained version of this model is used to reexamine the Spanish insurance claim data. The results indicate how to identify misclassified claims. We also show how misclassified claims can be identified using the AAG approach. We show that both approaches can be used to probabilistically identify misclassified claims.

CHOI, B. P. & WEISS, M. A. *An empirical investigation of market structure, efficiency, and performance in property-liability insurance.* 635-673. This study examines the relationships among market structure and performance in property-liability insurers over the period 1992-1998 using data at the company and group levels. Three specific hypotheses are tested: traditional structure-conduct-performance, relative market power, and efficient structure (ES). The results provide support for the ES hypothesis. The ES hypothesis posits that more efficient firms can charge lower prices than competitors, enabling them to capture larger market shares and economic rents, leading to increased concentration. Both revenue and cost efficiency are used in the analysis, and this is the first study to use revenue efficiency in this type of analysis. The results for the sample period as a whole and by year are consistent. The overall results suggest that cost-efficient firms charge lower prices and earn higher profits, in conformance with the ES hypothesis. On the other hand, prices and profits are found to be higher for revenue-efficient firms. Revenue X-efficiency is derived from activities such as cross-selling and may rely heavily on the use of detailed information from customer databases to identify potential customers. The implications of this research are that regulators should be more concerned with efficiency (both cost and revenue) rather than the market power that arises from the consolidation activity taking place in insurance.

DIONNE, G. & GHALI, O. *The (1992) bonus-malus system in Tunisia: an empirical evaluation.* 609-633. The objective of this study is to assess empirically what impact introduction of the bonus-malus system (BMS) has had on road safety in Tunisia. The results of the Tunisian experiment are of particular importance since, during the last decade, many European countries decided to eliminate their mandatory bonus-malus scheme. These results indicate that the BMS reduced the probability of reported accidents for good risks but had no effect on bad risks. Moreover, the reform's overall effect on reported accident rates is not statistically significant, but the exit variable is positive in explaining the number of reported accidents. To

avoid any potential selectivity bias, we also made a joint estimate of the reported accident and selection equations. The reform has a positive effect on the exit variable but still does not affect the accidents reported. This indicates that policyholders who switch companies are those attempting to skirt the imposed incentive effects of the new rating policy. Some of the control variables are statistically significant in explaining the number of reported accidents: the vehicle's horsepower, the policyholder's place of residence, and the coverages for which policyholders are underwritten.

GOSSNER, O. & PICARD, P. *On the consequences of behavioral adaptations in the cost-benefit analysis of road safety measures.* 577-599. It is sometimes argued that road safety measures or automobile safety standards fail to save lives because safer highways or safer cars induce more dangerous driving. A similar but less extreme view is that ignoring the behavioral adaptation of drivers would bias the cost-benefit rules of a traffic safety measure. This article derives cost-benefit rules for automobile safety regulation when drivers may adapt their risk-taking behavior in response to changes in the quality of the road network. The focus is on the financial externalities induced by accidents because of the insurance system as well as on the consequences of drivers' risk aversion. We establish that road safety measures are Pareto improving if their monetary cost is lower than the difference between their (adjusted for risk aversion) direct welfare gain with unchanged behavior and the induced variation in insured losses due to drivers' behavioral adaptation. The article also shows how this rule can be extended to take other accident external costs into account.

LOUGHRAN, D. S. *Deterring fraud: the role of general damage awards in automobile insurance settlements.* 551-575. Awards for pain and suffering and other noneconomic losses account for over half of all damages awarded under third-party auto insurance bodily injury settlements. This article hypothesizes that third-party insurers use general damage awards to reduce the incentive to submit exaggerated claims for specific damages for injuries and lost wages. Consistent with this hypothesis, the article finds evidence using data on over 17,000 closed bodily injury claims that special damage claims that exceed their expected value receive proportionally lower general damage awards than claims that do not. Among the implications of this research is the possibility that insurers will be less zealous in challenging fraudulent special damage claims under a third-party insurance regime than they will be under a first-party insurance regime in which access to general damages is limited.

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FREES, E. W. *Pension plan termination and retirement.* 1-27. Employee termination and retirement probabilities affect the valuation of employee benefit plans and thus are of concern to actuaries. To provide timely experience for the profession, the Society of Actuaries' Non-Mortality Decrement Task Force organized a data collection effort. Thirty-two contributors provided over 1.7 million life years of pension plan turnover data for years 1994-2000. This article summarizes the results of this data collection effort.

Traditionally, the most important determinants of termination and retirement are age, a proxy for attachment to the workforce, and service, a measure of attachment to a firm. This article documents the importance of these traditional quantities using current data and provides tables so that actuaries may quantitatively assess their importance when valuing pension plans.

For the middle working years, ages 25-55, we find female termination probabilities are higher than males, although the differences are smaller than has been true historically. The differences are insignificant for the younger working years or early service years. Moreover, for ages 55 and older, males have higher retirement probabilities than females.

We also document the effect of several plan characteristics: eligibility for postretirement health benefits, benefit formula, hourly/salary and union status, as well as plan size. To assess the effects of plan characteristics while controlling for age, service, and gender, we use multinomial logit analysis, a regression methodology suitable for categorical outcomes. We find that small plans have slightly higher termination probabilities compared to medium and large plans (plan size is our proxy for employer size). Union hourly plans have lower termination probabilities than salaried plans; in turn, salaried plans have lower termination probabilities than nonunion hourly plans. Firms that offer richer benefits enjoy lower turnover.

The data for this study were gathered using a traditional industry experience studies approach. To highlight the strengths and weaknesses of this approach, we compare this data set to several government sponsored probability samples on job turnover. In general, these samples are smaller and thus provide less credible estimates yet they allow the analyst to explore the complex interactions of the effects of several variables on turnover.

JIA, N. & TSUI, L. *Epidemic modelling using SARS as a case study*. 28-42. The recent Severe Acute Respiratory Syndrome (SARS) epidemic has highlighted a new dimension to the risks confronting insurance companies. Conventional approaches to insurance pricing take an almost exclusively retrospective view of future mortality experience, extrapolating past mortality trends into the future. Such an approach fails to take account of mortality shocks such as epidemics, which may arise spontaneously and that are not reflected in past experience. If actuaries are to maintain their position as risk experts in an ever-changing world, it is important for the actuarial profession to adopt a more comprehensive approach to assessing risks that goes beyond past experience.

This paper will take a look at the modelling of epidemics, using SARS as a case study, and will examine the potential impact of SARS and similar epidemics on insurance companies.

Joss, P. *The earnings implications of pension expense: a stochastic analysis of ten Canadian companies*. 43-55. Recent challenges to the actuarial pension model and a movement to harmonize international accounting standards both suggest that the current Canadian standards for pension accounting, CICA 3461, may see substantial revision during upcoming years. To understand better the implications of these possible accounting changes, this paper presents the results of a stochastic analysis that quantifies how the volatility of pension expense for a sample of ten Canadian companies sponsoring defined benefit plans will be increased by the adoption of immediate recognition accounting. For certain companies this increase is significant and is shown to have a material earnings impact. The implications of this earnings volatility for the future of defined benefit pension plans are also explored.

KIM, C. *Modeling surrender and lapse rates with economic variables*. 56-70. This paper presents surrender rate models with explanatory variables such as the difference between reference and crediting rates, policy age since issue, financial crises, unemployment and economy growth rates, and seasonal effects. The logit function and the complementary log-log function are used in modeling surrender rates.

This paper shows that the logit model and the complementary log-log model generally perform better than the existing surrender rate models such as the arctangent model. It also shows that the surrender rate models are different according to insurance policy types, and it finds proper surrender rate models for four insurance groups: protection plans, education plans, endowment, and annuities.

MILEVSKY, M. A. *Real longevity insurance with a deductible: introduction to advanced-life delayed annuities (ALDA)* 109-122. This paper explores the financial properties of a concept product called an advanced-life delayed annuity (ALDA). The ALDA is a variant of a pure deferred annuity contract that is acquired by installments, adjusted for consumer price inflation, and pays off toward the end of the human life cycle. The ALDA concept is aimed at the growing population of North Americans without access to a traditional defined benefit (DB) pension plan and the implicit longevity insurance that a DB plan contains. I show that under quite reasonable pricing assumptions, a consumer can invest or allocate \$1 per month, while saving for retirement, and receive between \$20 and \$40 per month in benefits, assuming the deductible in this insurance policy is set high enough. The ALDA concept might go a long way in mitigating the psychological barrier to voluntary lump-sum annuitization.

VANDUFFELL, S., HOEDEMAKERS, T. & DHAENE, J. *Comparing approximations for risk measures of sums of nonindependent lognormal random variables.* 71-82. In this paper we consider different approximations for computing the distribution function or risk measures related to a discrete sum of nonindependent lognormal random variables. Comonotonic upper and lower bound approximations for such sums have been proposed in Dhaene *et al.* (2002a,b). We introduce the comonotonic "maximal variance" lower bound approximation. We also compare the comonotonic approximations with two well-known moment-matching approximations: the lognormal and the reciprocal Gamma approximations. We find that for a wide range of parameter values the comonotonic "maximal variance" lower bound approximation outperforms the other approximations.

WONG, A. C. S. & CHAN, W.-S. *Mixture Gaussian time series modeling of long-term market returns.* 83-94. Stochastic modeling of investment returns is an important topic for actuaries who deal with variable annuity and segregated fund investment guarantees. The traditional lognormal stock return model is simple, but it is generally less appealing for longer-term problems. In recent years, the use of regime-switching lognormal (RSLN) processes for modeling maturity guarantees has been gaining popularity. In this paper we introduce the class of mixture Gaussian time series processes for modeling long-term stock market returns. It offers an alternative class of models to actuaries who may be experimenting with the RSLN process. We use monthly data from the Toronto Stock Exchange 300 and the Standard and Poor's 500 indices to illustrate the mixture time series modeling procedures, and we compare the fits of the mixture models to the lognormal and RSLN models. Finally, we give a numerical example comparing risk measures for a simple segregated fund contract under different stochastic return models.

ZHOU, X. *On a classical risk model with a constant dividend barrier.* 95-108. This paper considers a risk model with a constant dividend barrier. It first points out interesting connections between some previous results for this model and those for spectrally negative Lévy processes. An expression is then obtained for the joint distribution of the surplus immediately prior to ruin and the deficit at ruin, discounted from the time of ruin. Such an expression involves known results on the joint distribution at ruin for a classical risk model without barrier. Also discussed are the joint distributions related to the time periods when dividends are paid. In particular, this paper obtains the Laplace transform for the total dividend payments until ruin, and another expression for the expected present value of the total amount of dividend payments until ruin. The results do not require the positive loading condition.

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BIFFIS, E. & MILLOSovich, P. *The fair value of guaranteed annuity options*. 23-41. We discuss the fair valuation of Guaranteed Annuity Options, i.e. options providing the right to convert deferred survival benefits into annuities at fixed conversion rates. The use of doubly stochastic stopping times and of affine processes provides great computational and analytical tractability, while enabling to set up a very general valuation framework. For example, the valuation of options on traditional, unit-linked or indexed annuities is encompassed. Moreover, security and reference fund prices may feature stochastic volatility or discontinuous dynamics. The longevity risk is also taken into account, by letting the evolution of mortality present stochastic dynamics subject not only to random fluctuations but also to systematic deviations.

DENUIT, M. & FROSTIG, E. *Heterogeneity and the need for capital in the individual model*. 42-66. Loosely speaking, actuaries believe that the heterogeneity of the risks comprised in a given insurance portfolio tends to increase its dangerousness. This in turn leads to requiring more capital. This paper aims to formalize this intuitive idea in the individual model of risk theory. The impact of the heterogeneity (for claim occurrences and/or claim sizes) will be studied with the help of various stochastic orders. The concept of majorization, allowing for comparing the dispersion of the components of two vectors of real numbers, and the closely related Schur-increasingness, turn out to be the appropriate tools to deal with this problem. The method proposed in this paper distinguishes between diversification and dispersion: spreading maturities by dispersing claim occurrences lessens the need for capital, while increasing uncertainty in claim amounts increases the need for capital.

STEFFENSEN, M. *Surplus-linked life insurance*. 1-22. We link dividend and bonus payments of a general life and pension insurance contract to the actuarial notion of surplus generated in a Black-Scholes financial market. With this contract specification we obtain a general system of partial differential equations for the market value of future payments. We derive semi-explicit solutions in the cases of linear links. The specification of dividend and bonus payments as functions of surplus and corresponding valuation techniques are relevant both for valuation and management of existing life insurance products and for general product design.

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