43. ON THE DETERMINATION OF PLANETARY MASSES

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Abstract. A common method of determining the mass of a planet is to solve for the orbit of some other object several times with different trial values for the mass in question. It is pointed out that a parabola fitted to the values of the sums of the squares of the residuals may be used to obtain, not only the planetary mass, but also its probable error. Marsden's determination of the mass of Saturn from observations of Hidalgo is used as an example.

Undoubtedly the most preferable way to proceed in attempting to determine the mass of a planet from observations of a minor or some other major planet is to integrate simultaneously the trajectory and the variational equation, including partial differential coefficients with respect to this mass. On the other hand, it is a common practice to integrate only the trajectory with several different trial values of the mass in question, to make separate least squares solutions, often using two-body formulae for the partial differential coefficients, and finally to solve for the value of the mass from the minimum of the parabola which represents the sum of the squares of the residuals. Whenever this is done, the probable error of the resulting solution is equal to the probable error of unit weight multiplied by the square root of the latus rectum of this parabola.

The general form for the parabola is

$$(x - x_0)^2 = 4q(y - y_0),$$

or

$$A + Bx + Cx^2 = y, \tag{1}$$

where

$$A = \frac{x_0^2}{4q} + y_0$$

$$B = -\frac{2x_0}{4q}$$

$$C = \frac{1}{4q}.$$

To put x in dimensionless units and y in radians, let

$$x = \frac{M - M_0}{M_0}$$

$$y = \sum (v'')^2(\text{arc } 1'')^2,$$

where M is the reciprocal mass and v'' is a residual in seconds of arc. Equation (1) is formed for each orbit solution, and the coefficients (although only C is required,

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since it is the reciprocal of the latus rectum of the parabola) are determined by least squares.

We illustrate this proposition with the data published by Marsden (1972, Table I) for determining the mass of Saturn from the observations of Hidalgo. The symmetry of the data points is not typical of most applications of this kind. Let $M_0 = 3498.5$, and since there are 94 observations the sums of the squares of the residuals can be constructed from the mean residuals. The equations of condition are thus

$$A + 4.29(10^{-4}B) + 18.383(10^{-8}C) = 194.92(\text{arc } 1'')^2$$
$$A + 1.43(10^{-4}B) + 2.043(10^{-8}C) = 149.23(\text{arc } 1'')^2$$
$$A = 144.53(\text{arc } 1'')^2$$
$$A - 1.43(10^{-4}B) + 2.043(10^{-8}C) = 149.23(\text{arc } 1'')^2$$
$$A - 4.29(10^{-4}B) + 18.383(10^{-8}C) = 194.92(\text{arc } 1'')^2.$$

The normal equations are (B being completely separable)

$$5A + 40.852(10^{-8}C) = 832.84(\text{arc } 1'')^2$$
$$40.852(10^{-8}A) + 684.217(10^{-16}C) = 7776.18(\text{arc } 1'')^2(10^{-8})$$

and elimination of A gives

$$C = (1.665 \times \text{arc } 1'' \times 10^4)^2 = \left(\frac{1}{12.4}\right)^2.$$

The probable error of x is thus

$$0.6745 \times 1''\!.24 \times \text{arc } 1'' \times 12.4 = 0.000050,$$

and the probable error of M is

$$0.000050 M_0 = 0.17.$$

Reference

Marsden, B. G.: 1972, this Symposium, p. 239.

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