

## NOTES

# DOES INTRAFIRM BARGAINING MATTER IN THE LARGE FIRM'S MATCHING MODEL?

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We analyze the role of intrafirm bargaining in the large-firm matching model and underline the conditions under which the Pissarides equivalence between this model and the “single-worker” firm’s model holds.

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## 1. INTRODUCTION

In two recent fascinating papers, Stole and Zwiebel (1996a,b) presented a new mechanism, intrafirm bargaining, for studying the problem of the firm. Stole and Zwiebel show that employers, by exploiting diminishing returns in the productivity of labor, can manipulate the wage, thanks to hiring policies. More precisely, they show that there is overemployment compared to the standard wage-taker neoclassical firm. Moreover, all employees receive their reservation wage if employers adopt optimal hiring policies. Although their analysis is a partial equilibrium one, they argue that “many of the applications could be extended in an interesting way to a general equilibrium framework” (1996b, p. 201 n. 8). At first glance, these results cast doubts on the relevancy of most models of unemployment where workers are able to capture a part of quasi rents thanks to bargaining power. Accordingly, the sources of unemployment exhibited by the standard matching model of unemployment seem to be at odds with Stole and Zwiebel’s results [see, e.g., Pissarides (1990), Mortensen and Pissarides (1994), Caballero and Hammour (1996)].

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We incorporate the Stole–Zwiebel intrafirm bargaining mechanism into the standard matching model of unemployment. We use the large-firm version of the matching model of Pissarides (1990, Ch. 2), where the same ingredients as in Stole and Zwiebel (1996a,b) can be found (decreasing returns to labor and intrafirm bargaining over wages). It is shown that the assumptions of a perfect capital market (or no adjustment cost for capital) and constant returns to scale in all factors allow us to preserve the usual prediction of the matching model: Workers receive more than their reservation wage, and employment satisfies the same first-order condition as in another version of the Pissarides matching model, the “single-worker” firm. Thus, the equivalence between the two matching models, established in Pissarides, holds. We conclude that the standard matching model of unemployment is fully compatible with a rigorous treatment of intrafirm bargaining.

The paper is organized as follows: Section 2 is devoted to the analysis of hiring and investment. Intrafirm bargaining is presented in Section 3. The effects of intrafirm bargaining on unemployment are analyzed in Section 4. Concluding comments are in Section 5.

## 2. LABOR AND CAPITAL DEMAND

Let us consider a firm producing  $F(K, N)$  with  $K$  units of capital and  $N$  units of labor, where  $F$  is a standard production function with constant returns to scale. To determine the optimal factors demand for the large firm, it is very important to precisely define the timing of decisions in each small interval of time  $dt$ . We assume that the employer recruits, then simultaneously bargains wages with employees and chooses capital. In a such a framework, employment is a predetermined variable but neither wage nor capital is predetermined: They can be changed at any point of time without any delay. The discussion of the robustness of our conclusions with respect to the timing of decisions is postponed to the last section.

The problem of the firm is to maximize the discounted value of profits, given the law of motion for labor, knowing that the rental cost of capital is  $r + \delta$  (the interest rate plus the depreciation rate) per unit of time, and that the wage is continuously bargained. Therefore, capital and wage are a priori functions of employment,  $N$ , which are denoted, respectively, by  $w(N)$ ,  $K(N)$ . The capital stock being chosen so as to equalize the marginal productivity of capital to its rental costs,  $K(N)$  is defined by the following condition:

$$F_1[K(N), N] = r + \delta, \quad (1)$$

where  $F_i$ ,  $i = 1, 2$ , denotes henceforth the partial derivative of  $F$ .

Each firm takes the transition intensity  $q$  to fill a vacant position as given. The firm posts  $V$  vacancies at date  $t$  and jobs are destroyed at rate  $s$ . Therefore, denoting employment at date  $t$  by  $N$ , and employment at date  $t + dt$  by  $N'$ , the law of motion of jobs writes as

$$N' = N(1 - s dt) + Vq dt \quad (2)$$

If a vacant job costs  $\gamma$  per unit of time, and  $r$  is the exogenous discount rate, the value function for the problem of the firm solves the Bellman equation

$$\Pi(N) = \text{Max}_{\{V\}} \left( \frac{1}{1+r dt} \right) \{ [F[K(N), N] - w(N)N - \gamma V - (r + \delta)K(N)] dt + \Pi(N') \}, \tag{3}$$

subject to the law of motion of jobs (2).

Using (1), we can write the first-order and the envelope conditions for an optimal choice of  $V$  as

$$-\gamma + q \frac{d\Pi(N')}{dN'} = 0, \tag{4}$$

$$\left\{ F_2[K(N), N] - \frac{dw(N)}{dN}N - w(N) \right\} dt + (1 - s dt) \frac{d\Pi(N')}{dN'} = \frac{d\Pi(N)}{dN} (1 + r dt). \tag{5}$$

These two equations then imply that, at the optimum employment level  $N^*$ ,

$$F_2[K(N^*), N^*] - N^* \frac{dw}{dN}(N^*) - w(N^*) = \frac{\gamma}{q}(r + s). \tag{6}$$

### 3. INTRAFIRM WAGE BARGAINING

Let  $E$  be the expected present-discounted values of being employed at wage  $w(N)$  with discount factor  $r$ , exogenous Poisson rate of job destruction  $s$ , and  $U$  the life-time value of unemployment. Then, assuming that workers are risk-neutral with an infinite life, the surplus gotten by an employee paid a wage  $w(N)$  amounts to

$$E - U = \frac{w(N) - rU}{s + r}. \tag{7}$$

Let us denote by  $J$  the marginal value of a filled position for the firm. In the case of a firm with  $N$  employees, (1) and (3) imply that the value of a marginal job,  $J$ , destroyed at rate  $s$  satisfies

$$J = \frac{F_2[K(N), N] - w(N) - N dw(N)/dN}{r + s}, \tag{8}$$

and the last term represents the fact that hiring an extra worker will allow us to reduce all wages by  $dw(N)/dN$ . If wages solve the surplus sharing rule,  $\beta J = (1 - \beta)(E - U)$ , where  $\beta \in [0, 1]$  is an index of workers' bargaining power, then, using (7) and (8), they satisfy the following differential equation:

$$w(N) = (1 - \beta)rU + \beta \left\{ F_2[K(N), N] - N \frac{dw(N)}{dN} \right\}. \tag{9}$$

Solving for this differential equation in  $N$ , we obtain Stole and Zwiebel's solution<sup>1</sup> (generalized to  $\beta \geq 1/2$ ):

$$w(N) = N^{-1/\beta} \int_0^N x^{\frac{1-\beta}{\beta}} F_2[K(x), x] dx + (1 - \beta)rU. \quad (10)$$

This equation shows that the wage, negotiated at every date  $t$ , depends on employment, chosen before the wage.

#### 4. EFFECTS OF INTRAFIRM WAGE BARGAINING ON UNEMPLOYMENT

In the case of a CRS production function, the term  $F_2[K(x), x] = F_2(k, 1)$ , where  $k = K(x)/x$ , becomes a constant with respect to  $x$ , thanks to (1), and thus can be taken from the integral in equation (10). Accordingly, the wage simplifies as

$$w(N) = \beta F_2(k, 1) + (1 - \beta)rU, \quad (11)$$

which immediately implies that  $(dw/dN)(N) = 0$ . This property, together with (6) implies that the labor demand read as

$$F_2(k, 1) - w = \frac{\gamma}{q}(r + s), \quad (12)$$

In this case, (1) and (12) exactly correspond to the first-order conditions for capital and jobs of both Pissarides' single-worker matching model and Pissarides' large-firm model. Notably, they can be obtained from the following program:

$$\text{Max}_{\{K, N\}} \int_0^{\infty} e^{-rt} \left[ F(K, N) - wN - \frac{\gamma}{q}(\dot{N} + sN) - (\dot{K} + \delta K) \right] dt,$$

where  $w$  is considered as exogenous. In addition, the wage (11) is also exactly the same as in Pissarides' standard model. Therefore, usual results of the standard matching model are fully consistent with intrafirm wage bargaining in the large-firm matching model.

By contrast, out of a constant-returns-to-scale world, (6) rather than (12) holds, and the wage is a function of  $N$ , with  $dw/dN < 0$ . For an illustration, if  $F(K, N) = K^\alpha N^{1-\alpha-\phi}$ , one can show that the wage becomes, after solving the integral of equation (10),

$$w(N) = bN^{-\phi/(1-\alpha)} + (1 - \beta)rU,$$

where

$$b = \frac{1 - \alpha - \phi}{1/\beta - \phi/(1 - \alpha)} \left( \frac{\alpha}{r + \delta} \right)^{\frac{\alpha}{1-\alpha}} > 0.$$

Thus, the wage declines in  $N$ , all the more that the distance to CRS represented by  $\phi > 0$  is large. In such circumstances, the wage can be manipulated by the firm, and Stole and Zwiebel's result 1996a,b may apply, although not fully. In particular, it is

not necessarily the case that workers are just paid their reservation wage. Indeed, it is worth noting that combining (6) and (9) implies that

$$w(N^*) = rU + \frac{\beta}{1 - \beta}(r + s)\frac{\gamma}{q},$$

or in other words, that the existence of prematch costs (hiring costs, absent in Stole and Zwiebel's analysis) limits the possibility that employers have to pay workers their reservation wage. This only happens when  $\gamma \rightarrow 0$  (and in all cases in which  $\beta \rightarrow 0$ ).

## 5. CONCLUDING COMMENTS

Our conclusion is threefold: first, under constant returns to scale, the equivalence established by Pissarides between the large-firm matching model and the usual search model holds, if capital can be adjusted without any delay and any cost. This result also holds in applications of the large-firm model to two types of labor, as long as workers are perfect substitutes, as is the case in Wasmer (1999).

Second, this result would be true for timing of events other than the one that has been presented here: (a) if both capital investment and hiring decisions were taken after wage bargaining with incumbent workers, (b) if the firm first hired, then bargained wages and eventually chose the capital stock; or (c) if the firm first chose capital and labor, and then bargained over wages. In the latter case, capital investment made in  $t$  would not be consistent with the optimal investment in  $t + dt$ , and time inconsistency would make any commitment of the firm impossible, in which case, the firm would reinvest in  $t + dt$  and thus, given our assumption of costless capital adjustment, the Pissarides' equivalence again would hold.

Third, under assumptions of costly capital adjustment, predetermined capital stock, decreasing returns to scale, or imperfect substitute labor inputs, the new set of solutions exhibited by Stole and Zwiebel (1996a,b) has to be considered. Investigation of the matching model under these circumstances is a promising research direction. Our analysis nevertheless established that the existence of hiring costs may limit the possibilities the firm has to pay the reservation wage of workers.

### NOTE

1. One assumes, like in their paper, that the conditions for the existence of this integral are satisfied (these conditions are not very restrictive). This allows us to get rid of the constant term in the solution for the wage: In this case, this constant term can be shown to be zero, under the supplementary condition that the wage is finite when  $N \rightarrow 0$ .

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