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Excitation of ion Bernstein and ion cyclotron waves by a gyrating ion beam in a plasma column

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Abstract

Gyrating ion beams, produced by quick ionization of neutral beams, employed for plasma heating, are susceptible to ion Bernstein and ion cyclotron instabilities. The Bernstein wave, having large parallel phase velocity, is excited via cyclotron interaction whereas the ion cyclotron wave with lower parallel phase velocity could be driven by Cerenkov interaction as well. The maximally growing modes have transverse wave number of the order of inverse ion Larmor radius. The nonlocal effects cause reduction in the growth rate.

Keywords: Cerenkov interaction; Cyclotron interaction; Ion Bernstein wave; Ion cyclotron wave

1. INTRODUCTION

The ion Bernstein waves (IBWs) play an important role in radio frequency heating of tokamak in the ion cyclotron range of frequencies (ICRF), hence are a subject of continued attention (Bonoli *et al.*, 1997; Tripathi *et al.*, 1987; Paoletti *et al.*, 1999; Brizard & Kaufman, 1996; Sharma & Tripathi, 1988; Myra & D'lppolito, 1997; Zhao *et al.*, 2001; Li *et al.*, 2003; Sharma *et al.*, 1994). A large amplitude IBW can strongly couple with low frequency turbulence and influence plasma transport. It may also give rise to parametric instabilities and ponderomotive effects that have been suggested as cause for serious impurity release from the walls in experiment on IBW heating (Li *et al.*, 2001; Clark & Fisch, 2000; Kumar & Sharma, 1989). The IBW can penetrate the hot plasma core without strong attenuation until approaching the harmonic cyclotron layers where strong ion cyclotron damping occurs (Cardinali, 1993; Ono, 1993; Cardinali *et al.*, 1998). Plasma heating by directly launched ion Bernstein waves (IBWH) has been actively investigated in recent years (Sugaya, 1987; Ono *et al.*, 1988). As a result of their relatively short wavelength, the IBW can heat the bulk-ion distributions, and the wave polarization, and the relatively wide operating frequency range permit a flexible waveguide launcher design attractive for the compact ignition device. IBWH can also interact nonlinearly with subharmonics of the ion cyclotron frequencies, giving rise to new heating scenarios. Porkolab (1985) has shown that nonlinear ion Landau (cyclotron) damping efficiently absorbs the pump-wave power during ion Bernstein wave heating experiments in tokamak and tandem mirrors. The heating experiments on the Frascati Tokamak UP-grade (Cesario *et al.*, 2001) have reported efficient ion heating up to the fourth harmonic of hydrogen plasma. There is also a possibility that the IBW could be employed to enhance plasma confinement and drive poloidal current.

There is yet another potential source of IBW excitation, *viz*., the neutral particle beam injected into tokamak for auxiliary heating. The neutral beam quickly gets ionized to convert into a gyrating ion beam and latter can excite IBW. Saha *et al.* (1988) have experimentally observed the excitation of IBW by an ion beam in a beam created-plasma in an axial magnetic field. Lonnroth *et al.* (2002) have observed ion Bernstein mode excitation in their particle in cell simulations. Kuo *et al.* (1998) have studied parametric excitation of IBW by parallel-propagating Langmuir wave in a collisional magneto-plasma. Langmuir wave propagating along the geomagnetic field is considered as a pump for the parametric excitation of IBW and daughter Langmuir waves. Itoh *et al.* (1984) have obtained the nonlocal eigenmode of ICRF instability in the presence of parallel high energy beam component in toroidal plasma in an inhomogeneous magnetic field. The mode is the combination of the fast wave and IBW and is excited *via* kinetic interactions with beam particles. When the driving source of high energy particles overcomes the damping due to the bulk plasma and the

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resistive loss on the wall, the instability can occur. Mikhailenko *et al.* (2008) have developed linear theory of electrostatic ion cyclotron instabilities (Sperling & Perkins, 1976; Chen, 2000; Svidzinski & Swanson, 2000; Utsunomiya *et al.*, 2001; Brambilla, 1999) of the collisional magnetic field-aligned plasma shear flow, which is applicable to the ionospheric F-region. Chibisov *et al.* (2009) have investigated the electrostatic ion cyclotron instability of hydrogen plasma driven by an oxygen ion beam and resulting turbulent heating of both the ion species.

In this paper, we study the excitation of ion Bernstein and ion cyclotron instabilities by neutral beam turned gyrating ion beam in a plasma. We use Vlasov theory to obtain the response of gyrating ion beam to the field of the IBW. In Section 2, we study the problem in local approximation and deduce the growth rate. In Section 3, we develop the nonlocal theory using a slab model. The results are discussed in Section 4.

2. BEAM AND PLASMA RESPONSE

Consider plasma with static magnetic field $B_s\hat{z}$, ion density n_{0p}, ion mass m_i, and ion charge e. A gyrating ion beam of charge $Z_b e$ and mass m_b propagates through the plasma with equilibrium distribution function

$$f_0 = \frac{n_{0b}}{2\pi v_{0\perp}}\delta(v_\perp - v_{0\perp})\delta(v_{||} - v_{0||}). \tag{1}$$

We perturb this equilibrium by an electrostatic wave in the ion cyclotron range of frequency. In the local approximation, one may write the electrostatic potential as,

$$\phi = \phi_0 e^{-i(\omega t - \mathbf{k}.\mathbf{r})}. \tag{2}$$

The response of the gyrating ion beam is governed by the Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v}.\nabla f + \frac{Z_b e}{m_b}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right).\frac{\partial f}{\partial \mathbf{v}} = 0. \tag{3}$$

We express $f = f_0 + f_1$, linearize the Vlasov equation,

$$\frac{df_1}{dt} = \frac{Z_b e}{m_b}\nabla\phi.\frac{\partial f_0}{\partial \mathbf{v}}, \tag{4}$$

and solve it by following the usual procedure of integration along the unperturbed trajectories to obtain (Stix, 1962)

$$f_1 = -Z_b e\phi \sum_{s'}\sum_{s}\frac{J_{s'}(k_\perp\rho)J_s(k_\perp\rho)}{(\omega - k_{||}v_{||} - s\omega_{cb})} \times\left[s\frac{\partial f_0}{\partial\mu} + \frac{k_{||}}{m_b}\frac{\partial f_0}{\partial v_{||}}\right]\exp\left[-i\{\omega t - \mathbf{k}.\mathbf{r} - (s-s')\theta\}\right], \tag{5}$$

where s and s' are integers, $\rho = v_\perp/\omega_{cb}$, $\omega_{cb} = Z_b e B_s/mc$, $\mu = (1/2)m_b\, v_\perp^2/\omega_{cb}$ is the magnetic moment, and θ is the gyrophase angle ($\mathbf{v}_\perp$ makes with $\hat{x}$). The density perturbation turns out to be

$$n_{1b} = \int_0^\infty\int_0^{2\pi}\int_{-\infty}^{\infty} f_1 v_\perp dv_{||}d\theta dv_\perp = -(k^2/4\pi Z_b e)\chi_b\phi, \tag{6}$$

$$\chi_b = -\frac{\omega_{pb}^2}{\omega_{cb}^2}\sum_s\left[\frac{2sk_\perp}{v_{0\perp}k^2}\frac{J_s(k_\perp\rho_0)J_s'(k_\perp\rho_0)}{(\omega - k_{||}v_{0||} - s\omega_{cb})} + \frac{k_{||}^2}{k^2}\frac{\omega_{cb}^2 J_s^2(k_\perp\rho_0)}{(\omega - k_{||}v_{0||} - s\omega_{cb})^2} + \frac{k_{||}^2}{k^2}\frac{\omega_{cb}^2 J_{s-1}^2(k_\perp\rho_0)}{(\omega - k_{||}v_{0||})^2}\right]$$

where $\rho_0 = v_{0\perp}\omega_{\mathrm{cb}}$, $\omega_{\mathrm{pb}} = (4\pi Z_{\mathrm{b}}n_{\mathrm{b}}e^2/m_b)^{1/2}$.

The beam coupling to the wave is strong when $\omega - k_{||}v_{0||} \cong 0$ or $\omega - k_{||}v_{0||} \cong \omega_{cb}$. Thus, retaining only these three terms, we may write χ_b as (Kumar & Tripathi, 2004)

$$\chi_b = -\frac{\omega_{pb}^2}{\omega_{cb}^2}\left[\frac{k_\perp^2}{k^2}\frac{\omega_{cb}\{J_0^2(k_\perp\rho_0) - J_2^2(k_\perp\rho_0)\}}{2(\omega - k_{||}v_{0||} - \omega_{cb})} + \frac{k_{||}^2}{k^2}\frac{\omega_{cb}^2 J_1^2(k_\perp\rho_0)}{(\omega - k_{||}v_{0||} - \omega_{cb})^2} + \frac{k_{||}^2}{k^2}\frac{\omega_{cb}^2 J_0^2(k_\perp\rho_0)}{(\omega - k_{||}v_{0||})^2}\right]. \tag{7}$$

The density perturbation of Maxwellian plasma ions due to ϕ can be written as (Jain & Tripathi, 1987)$n_{1i} = -(k^2/4\pi e)\chi_i\,\phi$,

$$\chi_i = \frac{2\omega_{pi}^2}{k^2 v_{thi}^2}\left[1 - \sum_n \frac{\omega}{(\omega - n\omega_{ci})}I_n(b_i)\exp(-b_i)\right], \tag{8}$$

where $b_i = k_\perp^2 v_{thi}^2/2\omega_{ci}^2$, $\omega_{pi} = (4\pi n_{0pi}\; e^2/m_i)^{1/2}$, $I_n\;(b_i)$, and v_{thi} are the ion plasma frequency, modified Bessel function and thermal velocity of ions; and we have assumed $\omega - n\omega_{ci} \gg k_{||}\,v_{thi}$.

The density perturbation of plasma electrons due to the wave is

$$n_{1e} = (k^2/4\pi e)\chi_e\phi, \tag{9}$$

where

$$\chi_e = -\frac{\omega_p^2}{k^2}\left(\frac{k_{||}^2}{\omega^2} - \frac{k_\perp^2}{\omega_c^2}\right)\text{for } \omega \gg k_{||}v_{th}, \tag{10}$$

corresponding to the ion Bernstein wave,

$$\chi_e = \frac{2\omega_p^2}{k^2 v_{th}^2}\left(1 + i\sqrt{\pi}\frac{\omega}{k_z v_{th}}\right) = \frac{\omega_{pi}^2}{k^2 c_s^2}\left(1 + i\sqrt{\pi}\frac{\omega}{k_z v_{th}}\right)\text{for } \omega \ll k_{||}v_{th}, \tag{11}$$

corresponding to ion cyclotron wave, $\omega_p = (4\pi n_{0p}\; e^2/m)^{1/2}$, v_{th}, and c_s are the electron plasma frequency, thermal velocity of electrons and ion acoustic speed, respectively.

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3. INSTABILITY IN THE LOCAL APPROXIMATION

Using n_{1b}, n_{1p}, n_{1e} in the Poisson's equation $\nabla^2 \phi = 4\pi e(n_{1b} + n_{1i} + n_{1e})$, we obtain

$$\varepsilon\phi = 0, \tag{12}$$

Where $\varepsilon = 1 + \chi_b + \chi_i + \chi_e$. We consider two cases.

3.1. Ion Bernstein Mode Excitation ($\omega \gg k_z\, v_{the}$)

In this limit, the second and third terms in χ_b can be ignored and the dispersion relation Eq. (12) in the low beam density limit ($n_{0b} \ll n_{0p}$) can be written as

$$(\omega - \omega_R)(\omega - k_{||}v_{0||} - \omega_{cb}) = \Delta, \tag{13}$$

$$\omega_R = \omega_{ci}\left[1 + \frac{I_1(b_i)e^{-b_i}}{\left\{1 + \left(\frac{k^2 v_{thi}^2}{2\omega_{pi}^2}\right)\left(1 + \frac{\omega_p^2}{\omega_c^2}\right)\right\}}\right],$$

$$\Delta = \frac{\omega_{pb}^2}{\omega_{cb}^2}\left[\frac{\omega_{ci}\omega_{cb}I_1(b_i)e^{-b_i}\{J_0^2(k_\perp\rho_0) - J_2^2(k_\perp\rho_0)\}}{2\left\{1 + \frac{k^2 v_{thi}^2}{2\omega_{pi}^2}\left(1 + \frac{\omega_p^2}{\omega_c^2}\right)\right\}^2}\right].$$

Strong coupling between the beam and the wave occurs when $\omega_R = k_{\|}\, v_{0\|} = \omega_{cb}$ or

$$k_{||}v_{0||} = \omega_{ci} - \omega_{cb} + \omega_{ci}\delta', \tag{14}$$

where

$$\delta' = \frac{I_1(b_i)e^{-b_i}}{1 + \frac{k^2 v_{thi}^2}{2\omega_{pi}^2}(1 + \omega_p^2/\omega_c^2).}$$

The last term in Eq. (14) is smaller than ω_{ci}. For $\omega_{ci} > \omega_{cb}$, $k_{\|}\, v_{0\|} > 0$, i.e., the parallel phase velocity of the ion cyclotron wave is parallel to the parallel velocity of the beam, where as for $\omega_{ci} < \omega_{cb}$, they are antiparallel. Under Eq. (14), we write $\omega = \omega_R + i\gamma = k_{\|}v_{0\|} + \omega_{cb} + i\gamma$ in Eq. (13) and obtain the growth rate,

$$\gamma = (-\Delta)^{1/2}$$
$$= \frac{\omega_{pb}}{\omega_{cb}}\left[\frac{\omega_{ci}\omega_{cb}I_1(b_i)e^{-b_i}\{J_0^2(k_\perp\rho_0) - J_2^2(k_\perp\rho_0)\}}{2\left\{1 + \frac{k^2 v_{thi}^2}{2\omega_{pi}^2}\left(1 + \frac{\omega_p^2}{\omega_c^2}\right)\right\}^2}\right]^{1/2}. \tag{15}$$

The growth rate scales as half power of beam density. The parallel wave number for the maximally growing mode is given by Eq. (14). This $k_{\|}$ must also satisfy (1) $k_{\|} \ll (\omega - \omega_{ci})/v_{thi} = \omega_{ci}\ \ \delta'/v_{thi}$, i.e., $v_{0\|} \geq 2(\omega_{ci} - \omega_{cb})v_{thi}/\delta'$, and (2) $k_{\|} \ll \omega/v_{the}$ or $(\omega_{ci} - \omega_{cb})/\omega_{ci} \ll v_{0\|}/v_{the}$. The second condition is quite restrictive. It is satisfied when either the beam and plasma ions are the same species or the beam energy is in the MeV range when electron temperature is around 1 KeV. We have carried out the calculations of the growth rate for a hydrogen beam in a deuterium plasma with the following parameters: $m_i/m_b = 2$, $\rho_0/\rho_i = 2.5$, $\omega_{pb}/\omega_{cb} = 2$, $k^2v_{thi}^2/\omega_{pi}^2 = k^2\rho_0^2/1600$. In Figure 1, we plot the variation of growth rate as a function of $k_\perp\rho_i$. The growth rate is maximum for $k_\perp\rho_i \sim 1.8$ and falls off at larger $k_\perp\rho_i$.

3.2 ION CYCLOTRON INSTABILITY ($\omega \ll k_z\, v_{the}$)

In this case there are two possibilities, (a) $\omega \sim k_{\|}\, v_{0\|} + \omega_{cb}$ and (b) $\omega \sim k_{\|}\, v_{0\|}$. In the former case of cyclotron interaction, the second term in χ_b dominates and the dispersion relation takes the form

$$(\omega - \omega_R + i\Gamma)(\omega - k_{||}v_{0||} - \omega_{cb})^2 = \Delta', \tag{16}$$

$$\omega_R = \omega_{ci}\left[1 + \frac{I_1(b_i)e^{-b_i}}{\left\{1 + (T_i/T_e) + [(k^2v_{thi}^2)/(2\omega_{pi}^2)]\right\}}\right],$$

$$\Gamma = \sqrt{\pi}\frac{T_i}{T_e}\frac{\omega_{ci}^2 I_1(b_i)e^{-b_i}}{\left[k_z v_{the}\left(1 + \frac{T_i}{T_e} + \frac{k_z^2 v_{thi}^2}{2\omega_{pi}^2}\right)^2\right]},$$

$$\Delta' = \frac{\omega_{pb}^2}{\omega_{cb}^2}\left[\frac{k_{||}^2\omega_{ci}\omega_{cb}^2 I_1(b_i)e^{-b_i}J_1^2(k_\perp\rho_0)}{k^2\left\{1 + \frac{k^2 v_{thi}^2}{2\omega_{pi}^2}\left(1 + \frac{2\omega_p^2}{k^2 v_{th}^2}\right)\right\}^2}\right],$$

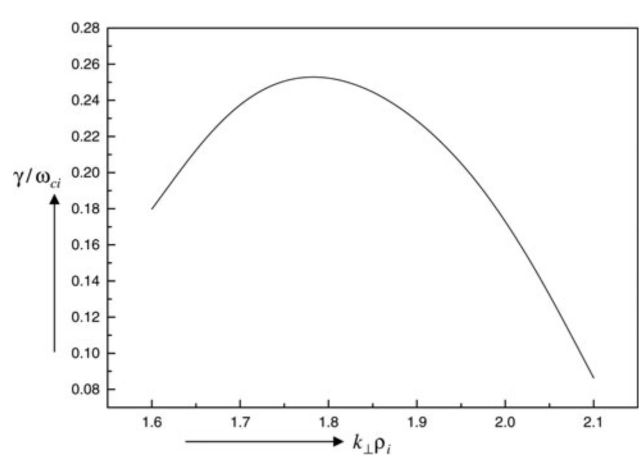

Fig. 1. Variation of local growth rate γ/ω_{ci} with $k_\perp\rho_i$ for ion Bernstein wave for $\omega_{cb}/\omega_{ci} = 2$, $\omega_p/\omega_c = 1.5$, $k_{\|}\rho_0 = 0.2$, $\omega_{pb}/\omega_{cb} = 2$, $\omega_p/\omega_c = 1.5$, $\rho_0/\rho_i = 2.5$, $k^2v_{thi}^2/\omega_{pi}^2 = k^2\,\rho_0^2\,/1600$.

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Γ is the damping rate of the ion cyclotron wave in the absence of the beam. Maximum growth occurs when $\omega_R = k_{\|}v_{0\|} + \omega_{cb}$. Expressing $\omega = \omega_R + \delta$, we obtain

$$\delta^3 + i\Gamma\delta^2 = \Delta'. \tag{17}$$

The roots of this equation are complex. Imaginary part of δ gives the growth rate. For $\Gamma \ll \delta$, Eq. (17) gives the growth rate

$$\gamma = \frac{\sqrt{3}}{2}\left[\frac{k_{\|}^2\omega_{pb}^2\omega_{ci}I_1(b_i)e^{-b_i}J_1^2(k_\perp\rho_0)}{k^2\left\{1+\frac{k^2v_{thi}^2}{2\omega_{pi}^2}\left(1+\frac{2\omega_p^2}{k^2v_{th}^2}\right)\right\}^2}\right]^{1/3}. \tag{18}$$

The growth rate scales as one-third power of the beam-density. In Figure 2, we have plotted the growth rate as a function of $k_\perp\rho_i$ for ion cyclotron wave for $\omega_{cb}/\omega_{ci} = 2$, $k_{\|}\ \rho_0 = 0.2$, $\omega_{pb}/\omega_{cb} = 2$, $\omega_p/\omega_c = 1.5$, $\omega_{pb}/\omega_{ci} = 2$, $\omega_p^2/k^2v_{th}^2 = 1600/k_\perp^2\rho_0^2$. The growth rate is maximum at $k_\perp\rho_i = 0.7$. For these parameters, the electron Landau damping term is weak.

In the other case of $\omega \sim k_{\|}\,v_{0\|}$, corresponding to Cerenkov interaction the last term in χ_b dominates, giving the dispersion relation

$$(\omega - \omega_R + i\Gamma)(\omega - k_{\|}v_{0\|})^2 = \Delta'', \tag{19}$$

where $\Delta'' = \Delta' J_0^2\,(k_\perp\rho_0)/J_1^2\,(k_\perp\rho_0)$. The maximum growth rate turns out to be

$$\gamma' = \gamma\left[\frac{J_0^2(k_\perp\rho_0)}{J_1^2(k_\perp\rho_0)}\right]^{1/3}. \tag{20}$$

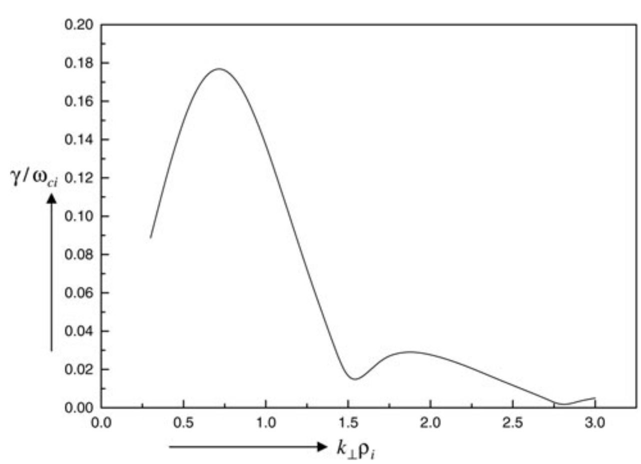

Fig. 2. Variation of local growth rate γ/ω_{ci} with $k_\perp\rho_i$ for ion cyclotron wave for $\omega_{cb}/\omega_{ci} = 2$, $\omega_p/\omega_c = 1.5$, $k_{\|}\rho_0 = 0.2$, $\omega_{pb}/\omega_{cb} = 2$, $\omega_{pb}/\omega_{ci} = 2$, $\rho_0/\rho_i = 2.5$, $k^2v_{thi}^2/\omega_{pi}^2 = k^2\,\rho_0^2\,/1600$, $\omega_p^2/k^2v_{th}^2 = 1600/k_\perp^2\,\rho_0^2$.

4. NONLOCAL EFFECTS ON ION BERNSTEIN MODE EXCITATION ($\omega \gg k_z\,v_{the}$)

Consider a plasma slab of x — width $2R$ and electron density n_{0p}. An ion beam of x— width $2R_b$, beam density n_{ob}, beam charge Z_be, and beam mass m_b propagates through it. Other plasma parameters are the same as before. The wave equation governing ϕ inside the plasma can be obtained from Eq. (12) by replacing $k_\perp^2$ by $k_y^2 - \partial^2/\partial x^2$ and expanding ε using Taylor expansion as $\varepsilon = \varepsilon(k_\perp^2 = k_y^2) - \frac{\partial\varepsilon}{\partial k_\perp^2}(\partial^2/\partial x^2)$,

$$\frac{\partial^2\phi}{\partial x^2} + \beta^2\phi = -\frac{\chi_b}{\beta_2}\phi, \tag{21}$$

Where $\beta = (\beta_1/\beta_2)^{1/2}$,

$$\beta_1 = \frac{\omega_{pi}^2}{\omega_{ci}^2b_i}\left[1 - \frac{\omega}{(\omega-\omega_{ci})}I_1(b_i)e^{-b_i} - \frac{m_i}{m}\frac{k_{\|}^2}{k_y^2}\frac{\omega_{ci}^2}{\omega^2}b_i\right],$$

$$\beta_2 = \frac{\omega_{pi}^2}{\omega_{ci}^2k_y^2}\left[-\frac{m}{m_i}\frac{k_{\|}^2}{k_y^2} + \frac{1}{b_i} + \frac{\omega b_i}{(\omega-\omega_{ci})}\frac{\partial}{\partial b_i}\left\{\frac{I_1(b_i)e^{-b_i}}{b_i}\right\}\right],$$

$I_1(b_i) \approx 1 - b_i = 1 - \left\{k_\perp^2v_{thi}^2/(2\omega_{ci}^2)\right\}$. We solve this equation iteratively. First, we ignore the beam term. Then Eq. (21) gives

$$\phi = A\cos\beta x. \tag{22}$$

The boundary condition, $\phi = 0$ at x = R, gives $\beta = \beta_l$, where

$$\beta_lR = (2l+1)\pi/2,\ l = 0, 1, 2, \tag{23}$$

In the first order perturbation theory, when beam term is non-zero (but small) the Eigen function may be treated to be unmodified, only the Eigen frequency is modified. Thus, using Eq. (22) in Eq. (21), multiplying the resulting equation by ϕ (where * denotes the complex conjugate) and integrating over x, we obtain

$$(\beta^2 - \beta_l^2)\int_{-R}^{R}\cos^2\beta_lx\,dx = -\frac{\chi_b}{\beta_2}\int_{-R_b}^{R_b}\cos^2\beta_lx\,dx. \tag{24}$$

Further simplifying Eq. (24) may be written as

$$(\omega - \omega_R)(\omega - k_{\|}v_{0\|} - \omega_{cb}) = \Delta_1,$$

$$\omega_R = \omega_{ci}\left[1 + \frac{I_1(b_i)e^{-b_i}}{\left(1 - b_i\frac{m_i}{m}\frac{k_{\|}^2}{k_y^2} - \beta_l^2\beta_2b_i\right)}\right],$$

$$\Delta_1 = p_1b_i\frac{\omega_{ci}^2}{\omega_{pi}^2}\frac{\omega_{pb}^2}{\omega_{cb}^2}\left[\frac{\omega_{cb}\omega_{ci}I_1(b_i)e^{-b_i}\{J_0^2(k_\perp\rho_0) - J_2^2(k_\perp\rho_0)\}}{2\left(1 - b_i\frac{m_i}{m}\frac{k_{\|}^2}{k_y^2} - \frac{\omega_{ci}^2}{\omega_{pi}^2}\beta_l^2\beta_2b_i\right)^2}\right],$$

$p_1 = (2\beta_l R_b + \sin 2\beta_l R_b)/(2\beta_l R)$. The factor p_1 characterizes the nonlocal effects.

The instability occurs with large growth rate when

$$\omega = k_{||} v_{0||} + \omega_{cb}.$$

and the maximum growth rate turns out to be

$$\gamma = (\Delta_1)^{1/2} = \left[p_1 b_i \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{\omega_{pb}^2}{\omega_{cb}^2} \left[\frac{\omega_{cb}\omega_{ci} I_1(b_i) e^{-b_i} \{J_0^2(k_\perp \rho_0) - J_2^2(k_\perp \rho_0)\}}{2\left(1 - b_i \frac{m_i}{m} \frac{k_{||}^2}{k_y^2} - \frac{\omega_{ci}^2}{\omega_{pi}^2} \beta_l^2 \beta_2 b_i\right)^2} \right] \right]^{1/2}. \tag{25}$$

In Figure 3, we have plotted the nonlocal growth rate γ/ω_{ci} with $k_\perp \rho_i$ for ion Bernstein wave for $\omega_{cb}/\omega_{ci} = 2$, $\omega_p/\omega_c = 1.5$, $k_{||}\rho_0 = 0.2$, $\omega_{pb}/\omega_{cb} = 2$, $\omega_p/\omega_c = 1.5$, $R_b = v_{thi}/\omega_{ci}$, $R = 2v_{thi}/\omega_{ci}$, $\rho_0/\rho_i + 2.5$, $k^2 v_{thi}^2/\omega_{pi}^2 = k^2\ \rho_0^2\ /1600$. The growth rate of IBW is maximum at $k_\perp \rho_i = 1..8$ and then falls sharply for larger $k_\perp \rho_i$.

5. NONLOCAL EFFECTS ON ION CYCLOTRON INSTABILITY ($\omega \ll k_z v_{the}$)

Expanding ε in a Taylor series around $k\perp^2 = k_y^2$ and replacing k_x by $-i(\partial/\partial x)$ in Eq. (12), we obtain

$$\frac{\partial^2 \phi}{\partial x^2} + \beta'^2 \phi = -\frac{\chi_b}{\beta'^2}\phi, \tag{26}$$

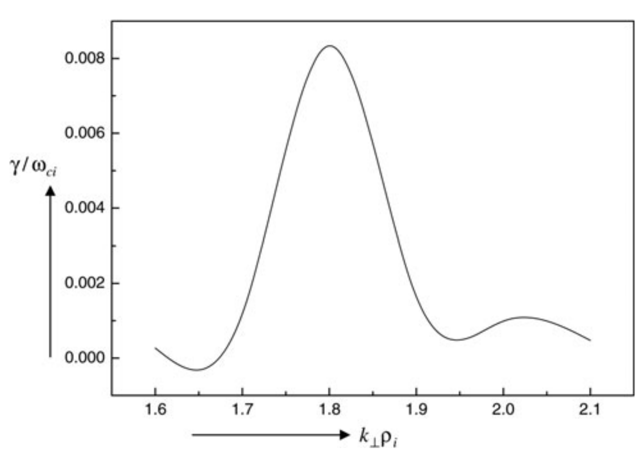

Fig. 3. Variation of nonlocal growth rate γ/ω_{ci} with $k_\perp \rho_i$ for ion Bernstein wave for $\omega_{cb}/\omega_{ci} = 2$, $\omega_p/\omega_c = 1.5$, $k_{||}\rho_0 = 0.2$, $\omega_{pb}/\omega_{cb} = 2$, $\omega_p/\omega_c = 1.5$, $m_i/m = 65$, $R_b = v_{thi}/\omega_{ci}$, $\omega_p/\omega_{pi} = 65$, $R = 2v_{thi}/\omega_{ci}$, $\rho_0/\rho_i = 2.5$, $k^2 v_{thi}^2/\omega_{pi}^2 = k^2\ \rho_0^2\ /1600$.

Where $\beta' - (\beta_1'/\beta_2')^{1/2}$,

$$\beta_1' = \frac{\omega_{pi}^2}{\omega_{ci}^2 b_i}\left[1 - \frac{\omega}{(\omega - \omega_{ci})} I_1(b_i) e^{-b_i} + \frac{2\omega_p^2}{\omega_{pi}^2}\frac{\omega_{ci}^2}{k^2 v_{th}^2} b_i\right],$$

$$\beta_2' = \frac{\omega_{pi}^2}{\omega_{ci}^2 k_y^2}\left[\frac{2m_i}{m}\frac{k_y^2 \omega_{ci}^2}{k^4 v_{th}^2} + \frac{1}{b_i} + \frac{\omega b_i}{(\omega - \omega_{ci})}\frac{\partial}{\partial b_i}\left\{\frac{I_1(b_i) e^{-b_i}}{b_i}\right\}\right].$$

Including the nonlocal effects as in Section 4, Eq. (26) gives

$$\phi = A' \cos \beta' x. \tag{27}$$

The boundary condition, $\phi = 0$ at $x = R$, gives $\beta' = \beta_l'$

$$\beta_l' R = (2l + 1)\pi/2,\ l = 0, 1, 2, \ldots \tag{28}$$

Thus, using Eq. (27) in Eq. (26), multiplying the resulting equation by ϕ^* and integrating over x, we obtain

$$(\beta'^2 - \beta_l'^2)\int_{-R}^{R} \cos^2 \beta_l' x dx = -\frac{\chi_b}{\beta_2}\int_{-R_b}^{R_b} \cos^2 \beta_l'\, x dx. \tag{29}$$

Now, there are two possibilities (c) $\omega \sim k_l\, v_{0||} + \omega_{cb}$ (d) $\omega \sim k_{||} v_{0||}$. In the former case, the second term in χ_b dominates and Eq. (29) takes the form

$$(\omega - \omega_R')(\omega - k_{||} v_{0||} - \omega_{cb})^2 = \Delta_1', \tag{30}$$

where

$$\omega_R' = \omega_{ci}\left[1 + \frac{I_1(b_i) e^{-b_i}}{\left(1 + b_i \frac{2m_i}{m}\frac{\omega_{ci}^2}{k^2 v_{th}^2} - \frac{\omega_{ci}^2}{\omega_{pi}^2}\beta_l'^2 \beta_2' b_i\right)}\right],$$

$$\Delta_1' = p_1' b_i \frac{\omega_{ci}^2}{\omega_{pi}^2}\frac{\omega_{pb}^2}{\omega_{cb}^2}\frac{k_{||}^2}{k^2}\frac{\omega_{cb}^2 \omega_{ci} I_1(b_i) e^{-b_i} J_1^2(k_\perp \rho_0)}{\left(1 + b_i \frac{2m_i}{m}\frac{\omega_{ci}^2}{k^2 v_{th}^2} - \frac{\omega_{ci}^2}{\omega_{pi}^2}\beta_l'^2 \beta_2' b_i\right)2},$$

$p_1' = (2\beta_l' R_b) + \sin 2\beta_l' R_b/(2\beta_l' R)$. Here $p_1{}'$ characterizes the nonlocal effects.

To obtain the growth rate we write, $\omega = \omega_R + i\gamma = \omega_{cb} + k_{||}\, v_{0||} + i\gamma$ and the growth rate turns out to be

$$\gamma = \frac{\sqrt{3}}{2}(\Delta_1')^{1/3} = \frac{\sqrt{3}}{2}\left[p_1' b_i \frac{\omega_{ci}^2}{\omega_{pi}^2}\frac{\omega_{pb}^2}{\omega_{cb}^2}\frac{k_{||}^2}{k^2}\frac{\omega_{cb}^2 \omega_{ci} I_1(b_i) e^{-b_i} J_1{}^2(k_\perp \rho_0)}{\left(1 + b_i \frac{2m_i}{m}\frac{\omega_{ci}^2}{k^2 v_{th}^2} - \frac{\omega_{ci}^2}{\omega_{pi}^2}\beta_l'^2 \beta_2 b_i\right)^2}\right]^{1/3}. \tag{31}$$

In the other case of $\omega \sim k_{||}\, v_{0||}$, the last term in χ_b dominates

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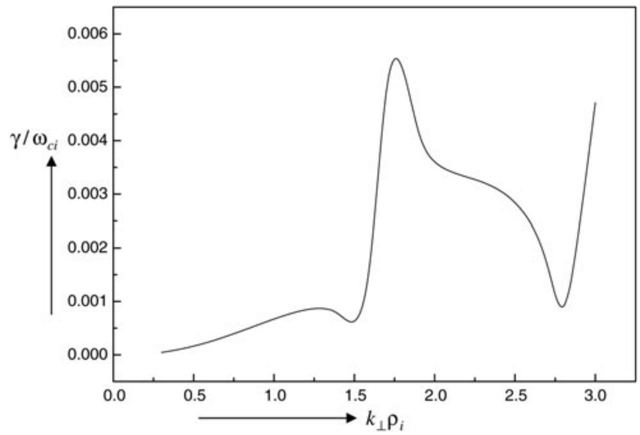

Fig. 4. Variation of nonlocal growth rate γ/ω_{ci} with $k_\perp\rho_i$ for ion cyclotron wave for $\omega_{cb}/\omega_{ci}=2$, $\omega_p/\omega_c=1.5$, $k_{\|}\rho_0=0.2$, $\omega_{pb}/\omega_{cb}=2$, $\omega_{pb}/\omega_{ci}=2$, $m_i/m=65$, $R_b=v_{thi}\ /\omega_{ci}$, $R=2v_{thi}\ /\omega_{ci}$, $\rho_0/\rho_i=2.5$, $k^2v_{thi}^2/\omega_{pi}^2=k^2\ \rho_0^2\ /1600$, $\omega_p^2/k^2v_{th}^2=1600/k_\perp^2\ \rho_0^2$.

and Eq. (29) takes the form

$$(\omega-\omega_R')(\omega-k_{||}v_{0||})^2=\Delta_1'', \tag{32}$$

where $\omega_R'=\omega_{ci}\left[1+\dfrac{I_1(b_i)e^{-b_i}}{\left(1+b_i\dfrac{2m_i}{m}\dfrac{\omega_{ci}^2}{k^2v_{th}^2}-\beta_l^{\prime 2}\beta_2'b_i\right)}\right]$,

$\Delta_1''=\Delta_1'J_0^2(k_\perp\rho_0)/J_1^2(k_\perp\rho_0)$.

The maximum growth rate turns out to be

$$\gamma'=\gamma\left[\frac{J_0^2(k_\perp\rho_0)}{J_1^2(k_\perp\rho_0)}\right]^{1/3}. \tag{33}$$

In Figure 4, we have plotted the nonlocal growth rate γ/ω_{ci} with $k_\perp\rho_i$ for ion cyclotron wave for $\omega_{cb}/\omega_{ci}=2$, $\omega_p/\omega_c=1.5$, $k_{\|}\rho_0=0.2$, $\omega_{pb}/\omega_{cb}=2$, $\omega_{pb}/\omega_{ci}=2$, $m_i/m=65$, $R_b=v_{thi}/\omega_{ci}$,$R=2v_{thi}/\omega_{ci}$, $\rho_0/\rho_i=2.5$, $k^2v_{thi}^2/\omega_{pi}^2=k^2\ \rho_0^2/1600$, $\omega_p^2/k^2v_{th}^2=1600/k_\perp^2\ \rho_0^2$. The growth rate of the ion cyclotron wave is small for smaller value of $k\perp\ \rho_i$ and is maximum at $k_\perp\rho_0=1.7$ and falls for larger $k_\perp\rho_i$.

6. DISCUSSION

A gyrating ion beam with delta function distributions in $v_\perp$ and $v_{\|}$, like the one produced by the ionization of neutral beam atoms during neutral beam heating of tokamak, can excite ion Bernstein and ion cyclotron waves of long and short parallel wavelengths. These waves propagate nearly perpendicular to the axial magnetic field. The growth rate of the IBW scales as one-third power of the ion beam density and is large for frequencies close to cyclotron harmonics $\omega\sim n\omega_{ci}$. It is maximum for $\omega\sim\omega_{ci}$ $(n=1)$ and decreases for higher values of n. When the region of destabilization is limited and x extent of the mode is relatively larger, then the growth rate of the instability is reduced. The total energy content of the mode is determined by its radial extent whereas the rate of energy supplied to its growth is determined by the size of the beam and also by the location of the beam with respect to the field structure of the mode.

In BWI, a beam of energetic neutral atoms is injected into the plasma. As the beam penetrates the plasma an increasing fraction of the beam atoms become ionized and trapped in the magnetic field of the tokamak. The deposition of the beam ions on available range of particle orbits is sensitive to the geometry of the beam injectors as well as the magnetic geometry. Ion cyclotron harmonic damping and the collisional energy exchange between the ions and electrons are main ion heating mechanisms for IBW heating. It can be used not only for ion heating, but also for electron heating *via* electron Landau damping. IBW experiments have been carried out in the HT-7 tokamak for several topics, such as heating, pressure profile and transport control and instability stabilization.

PIBW's are undamped plasma modes with $\mathbf{k}\perp\mathbf{B}_s$ and become Landau damped by electrons when the direction of propagation deviates by a small amount from exact perpendicularity. In an experiment for P (pressure of the interaction chamber) $=3-10\times10^{-5}$ torr, $n_e=2-15\times10^7$ cm^{-3}, $T_e\approx1.2$ eV, $T_i\sim0.5$ eV, $B_s=(600$–800 G$)$, $(\omega_{pi}^2+\omega_{pb}^2)/\omega_{ci}^2=14.0$, $n_{pi}/n_b=0.4$, Saha *et al.* (1988) found in most situations that $k_z=0$ and in some cases, the weak axial phase shift observed indicates a maximum value of 7.4×10^{-3} cm^{-1} for k_z. Taking $k_\perp=6.7$ cm^{-1}, the measured value, Saha *et al.* (1988) find that the propagation direction deviates from exact perpendicularity to B_s by 0.06° maximum. Also, the frequency of PIBW increases slightly with increase in n_{pi}/n_b, as observed experimentally. The value of the instability frequency found experimentally is 21% below the value of the instability frequency for the parameters:$B_s=720$ G, $n_{pi}=2.1\times10^7$ cm^{-3}, $n_b=5.3\times10^7$ cm^{-3}, $v_{thi}=0.49\times10^6$ cms^{-1}, $v_{0\perp}=1.45\times10^6$ cms^{-1}, and $(\omega_{pi}^2+\omega_{pb}^2)/\omega_{ci}^2=11.05$. The instability is a strong one as evidenced by the large growth rate of Im $(\omega/\omega_{ci})\approx0.2$. The value of $k\perp$ found theoretically for these parameters is 4.9, which is below the value found experimentally by 34%. Since the experimental value of $k_\perp$ is determined (for constant mode number) by the radius at which the wave amplitude is maximum, it is subjected to some uncertainty. For the parameters, $B_s=600$ G, $n_{pi}=2.1\times10^7$ cm^{-3}, $n_b=5.3\times10^7$ cm^{-3}, $v_{thi}=0.49\times10^6$ cms^{-1}, $v_{0\perp}=1.45\times10^6$ cms^{-1}, $(\omega_{pi}^2+\omega_{pb}^2)/\omega_{ci}^2=11.05$; the instability is stronger [Im $(\omega\ /\omega_{ci})\approx0.3$] than before.

IBW heating in tokomaks is based on the fact that the finite Larmor radius waves in the range of the ion cyclotron frequency excited from the low-field side of machine can penetrate into the hot plasma core without strong attenuation until the waves approach the harmonic cyclotron layers. The superconducting tokama (HT-7) has major radius 122 cm and minor radius 27.5 cm. The plasma current is about

120–170 kA. The toroidal magnetic field is in the range of $(1.5 - 5) \times 10^{13}$ cm^{-3}. The electron and ion temperatures are about 700 eV and 400 eV. The RF frequency is 24 – 30 MHz and the RF power of the generator can reach 300 kW. The IBW frequency is 24 – 30 MHz. For the plasma parameters of HT-7 with two species (hydrogen and deuterium) of working gas and assuming $k_{\perp 0}\,\rho_i \ll 1$, $k_{\perp 0}$ is the perpendicular wave number of the pump and ρ_i is the ion Larmor radius; the two parametric decay processes are: (1) decay into an IBW and ion cyclotron quasi mode where the quasi mode are characterized by $\omega = n\omega_{ci}$ and (2) decay into an IBW and a low-frequency electron Landau damped quasi-mode characterized by $\omega = k_{\parallel}\, v_{the}$.

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