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U.S. ELASTICITIES OF SUBSTITUTION AND FACTOR AUGMENTATION AT THE INDUSTRY LEVEL

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We provide industry-level estimates of the elasticity of substitution (σ) between capital and labor in the United States. We also estimate rates of factor augmentation. Aggregate estimates are produced. Our empirical model comes from the first-order conditions associated with a constant–elasticity of substitution production function. Our data represent 35 industries at roughly the 2-digit SIC level, 1960–2005. We find that aggregate U.S. σ is likely less than 0.620. σ is likely less than unity for a large majority of individual industries. Evidence also suggests that aggregate σ is less than the value-added share-weighted average of industry σ's. Aggregate technical change appears to be net labor–augmenting. This also appears to be true for the large majority of individual industries, but several industries may be characterized by net capital augmentation. When industry-level elasticity estimates are mapped to model sectors, the manufacturing sector σ is lower than that of services; the investment sector σ is lower than that of consumption.

Keywords: Elasticity of Substitution, Factor-Augmenting Technical Change, Labor-Augmenting Technical Change, Capital-Augmenting Technical Change, Corporate Taxation, Industry-Level Studies

1. INTRODUCTION

For the U.S. elasticity of substitution (σ) between capital and labor, there is no shortage of estimates. Recent examples include Caballero (1994), Jorgenson and Yun (2001), Antràs (2004), Chirinko et al. (2007), and Klump et al. (2007).[1] A consensus has emerged from this literature: "the evidence rather strongly rejects the Cobb–Douglas assumption of σ equal to one" [Chirinko (2008, p. 683)].

The pervasive use of Cobb–Douglas production functions in macroeconomics has focused many studies on whether σ is above or below unity.[2] However, variation in its value on *either* side of unity can have important policy implications. For example, in the case of a constant–elasticity of substitution (CES) technology, σ is the user cost elasticity of a firm's demand for capital. The larger σ, the larger

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the potential distortions associated with taxes on capital. Alternatively, the larger σ, the larger the potential gains to tax reforms [Chirinko (2002)].[3]

The effects of tax policy changes have been analyzed most commonly in relation to the economy's aggregate σ. However, if σ differs significantly across industries, then tax policy may be associated with interindustry distortions. For example, a tax-based increase in the user cost of capital will affect disproportionately the demand for capital in industries where σ is larger.

Such interindustry distortions have not been a focus of the literature. Chirinko (2002) surveys five computable general equilibrium (CGE) studies, all of which assume that one σ fits the entire economy.[4] Jorgenson and Yun (2001, p. 317) do report that the "largest [welfare] gains from capital tax reform are associated with equalizing tax burden on all assets and all sectors." However, they condition their exercises only on differences in σ between corporate versus noncorporate sectors.

How large are the differences across U.S. industry-level σ's? Surprisingly few studies provide an answer. Using plant-level data from 1972 to 1988, Caballero et al. (1995) estimate σ's for two-digit SIC industries. However, their data limit them to manufacturing industries only and their point estimates fall in the wide range from 0.01 to 2.00. Balisteri et al. (2003), using data from 1947 to 1999, estimate σ's for 28 industries that span the entire U.S. economy. However, they fail to use a framework that allows for biased technical change. Excluding biased technical change can bias estimates of σ toward unity (Antràs, 2004).[5] Also, their labor inputs are full-time equivalent employees rather than constant-quality indices [e.g., Ho and Jorgenson (1999)]. Because Balisteri et al.'s dependent variables are capital-to-labor ratios, this may also bias their estimates.

One aim of this paper is to provide industry-level σ estimates for the United States. We utilize Jorgenson's (2007) KLEM database.[6] This database contains annual data covering 35 industries at roughly the two-digit SIC level from 1960 to 2005. Spanning the entire U.S. economy, these data can also be used to estimate the aggregate U.S. σ.

As an empirical framework, we consider a CES production function [Arrow et al. (1961)] and the associated first-order conditions. These conditions yield relationships between factors of production and relative prices [Berndt (1976)] that can be derived while explicitly allowing for separate rates of labor- and capital-augmenting technical change [Antràs (2004)]. We estimate the first-order conditions separately using a generalized instrumental-variables (GIV) approach as well as OLS, using first differences to account for possible nonstationarity. We then also estimate a normalized system consisting of the first-order conditions and the production function, as suggested by León-Ledesma et al. (2010). The later approach improves efficiency and better incorporates the type of long-run information desirable for estimating σ.

Our findings suggest that U.S. σ is less than unity and likely less than 0.620. Furthermore, the same likely is true for the large majority of individual industries. Our findings do not support aggregate σ being greater than the value-added share-weighted average of industry σ's. Although the Cobb–Douglas hypothesis appears to be roundly rejected at both the aggregate and industry levels, there is still considerable variation across the industry σ point estimates.

To link our industry-level σ estimates more closely to economists' theoretical exercises, we follow Valentinyi and Herrendorf (2008) and use the estimates to compute σ's for sectors commonly found in general equilibrium models: (i) Agriculture, Manufacturing, and Services; (ii) Consumption and Investment; (iii) Tradables and Nontradables; and (iv) Agriculture and Nonagriculture. This is accomplished by utilizing the Bureau of Economic Analysis (BEA) benchmark input–output accounts to map industry outputs to final uses. The model sector σ's can be applied fruitfully in calibration exercises.

An additional result of our analysis is the estimation of rates of capital-augmenting and labor-augmenting technical change. Recent contributions to the macroeconomic growth literature have stressed misallocation of resources across sectors with different technologies [e.g., Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)]. Our industry-level estimates of factor augmentation may aid both in linking these theories to data and in extending them to richer specifications of technology. We find that factor augmentation is net labor–augmenting in the aggregate. However, at the industry level there is more uncertainty. Although a large majority of industries do seem to be characterized by net labor augmentation, this is by no means uniform across industries. For many industries, net capital augmentation may be the case.

This paper is organized as follows. Section 2 presents a CES production function permitting biased technical change and derives the first-order conditions that constitute our empirical model. The KLEM data are then characterized in Section 3. Section 4 presents and discusses the estimation results. The industry-level σ estimates are then mapped into general equilibrium model sector σ's in Section 5. The paper concludes in Section 6.

2. THE EMPIRICAL MODEL AND ITS ASSUMPTIONS

In estimating elasticities of substitution between labor and capital, we will assume that each industry's production possibilities can be represented by a CES production function [Arrow et al. (1961)] permitting factor-augmenting technical change [Antràs (2004)],

$$Y_{it} = \left[\alpha_i (B_{it} K_{it})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i)(A_{it} L_{it})^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{1}$$

where i and t index industries and time periods; Y_{it}, K_{it}, and L_{it} are value added, capital services, and labor services; and α_i and σ_i are constant parameters, the later being the elasticity of substitution. Technical change takes the form of factor augmentation and we assume that

$$B_{it} = B_{i0} e^{\lambda_{iK} t} \quad \text{and} \quad A_{it} = A_{i0} e^{\lambda_{iL} t}, \tag{2}$$

where λ_{iK} and λ_{iL} are respectively the rates of capital and labor augmentation. It is useful to introduce the terminology that if $(\lambda_{iL} - \lambda_{iK}) > 0$ there is *net labor augmentation*. Likewise, if $(\lambda_{iL} - \lambda_{iK}) < 0$ there is *net capital augmentation*.

It is well established in neoclassical growth theory that net labor augmentation is a necessary condition for balanced growth [Uzawa (1961); Jones and Scrimgeour (2008)]. Furthermore, Acemoglu (2003) explicitly models many firms that have a choice of pursuing either capital- or labor-augmenting technical change. When production of final output is characterized by $\sigma < 1$, on the balanced growth path no firms pursue capital-augmenting innovations; only the transition to balanced growth is characterized by capital augmentation. Empirically, recent aggregate studies support the conclusion of no positive capital augmentation (net or gross). Antràs (2004) estimates the US rate of capital augmentation to be about *negative* 1.5%. Klump et al. (2007) find that, for the United States, labor-augmenting technical change is exponential whereas capital augmentation is hyperbolic or logarithmic, in line with Acemoglu's (2003) predictions.

These empirical studies cannot, however, speak to whether aggregate patterns arise from labor-augmenting technical change characterizing (i) all industries or (ii) a predominant fraction of industries. The attempt to estimate λ_K and λ_L separately for each industry is a unique contribution of the present study.

Antràs (2004) has demonstrated the importance in practice of allowing for differential rates of capital-augmenting and labor-augmenting technical change (or, similarly, not constraining technical change to be Hicks-neutral) using aggregate data. Without this assumption, some of the observed variation in the data (e.g., output) is associated with factor variation (and therefore σ) rather than technical change. It is likely that differences between our industry-level results reported later and those of Balisteri et al. (2003) are due in part to our allowing different augmentation rates, whereas they do not. We make no claim that ours is the "correct" structure of the U.S. economy. Rather, we are able to capture (control for) more of the variation in our data that is due to technology. Put differently, Diamond et al.'s (1978) well-known impossibility theorem implies that one must specify the form of technology in advance to estimate the elasticity. Our specification is simply more general than Hicks neutrality (where $\lambda_K = \lambda_L$ by assumption).

Returning to the CES production function characterized by (1) and (2), the first-order conditions for profit maximization in competitive markets can be log-linearized into the following three relationships (where the i subscripts are suppressed):

$$yk_t = \delta_1 + \sigma rp_t + (1-\sigma)\,\lambda_K \cdot t + \varepsilon_{1t}; \quad \textbf{(3)}$$

$$yl_t = \delta_2 + \sigma wp_t + (1-\sigma)\,\lambda_L \cdot t + \varepsilon_{2t}; \quad \textbf{(4)}$$

$$kl_t = \delta_3 + \sigma wr_t + (1-\sigma)\,(\lambda_L - \lambda_K) \cdot t + \varepsilon_{3t}, \quad \textbf{(5)}$$

where *yk*, *yl*, and *kl* are $\ln(Y/K)$, $\ln(Y/L)$, and $\ln(K/L)$, respectively, whereas *rp*, *wp*, and *wr* are $\ln(R/P)$, $\ln(W/P)$, and $\ln(W/R)$. Based on marginal productivity conditions, W, R, and P are the prices of labor services, capital services, and

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output. The ε's are error terms. This framework is applied by Antràs (2004) to U.S. aggregate data. A similar framework (though omitting the distributional parameter α) is used by Behrman (1972) to estimate industry-level σ's for the Chilean economy.

Although we report results of estimating each of the preceding relationships, (3) through (5), separately, we also follow León-Ledesma et al. (2010) and consider a system composed of (3), (4), and the (log of the) production function (1). For estimation of a production function such as (1), the elasticity of substitution between capital and labor is a point elasticity. The production function itself is a solution to σ, defined as a function of the capital-to-labor ratio. The solution (including the other production function parameters) will be dependent on the "point" at which the elasticity is observed. Therefore, to bring such a function to data, one must fix benchmark values for the various variables, including factor inputs, factor income shares, and the level of production [de La Grandville (1989)].[7] León-Ledesma et al. (2010) follow Klump and Preissler (2000) and derive a normalization of the form

$$Y_t = Y_0 \left[\alpha_0 \left(\frac{e^{\lambda_K (t-t_0)} K_t}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{e^{\lambda_L (t-t_0)} L_t}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad \textbf{(6)}$$

where variables subscripted with 0 are benchmark values. The natural log of (3.6), along with the normalized first-order conditions, yields a normalized system for estimation,

$$yk_t - yk_0 = -\sigma\left[\ln(\alpha_0) + yk_0\right] + \sigma rp_t + (1-\sigma)\,\lambda_K \cdot (t - t_0) + \varepsilon_{1t}, \qquad \textbf{(7)}$$

$$yl_t - yl_0 = -\sigma\left[\ln(1-\alpha_0) + yl_0\right] + \sigma wp_t + (1-\sigma)\,\lambda_L \cdot (t - t_0) + \varepsilon_{2t}, \qquad \textbf{(8)}$$

$$y_t - y_0 = \frac{\sigma}{\sigma-1} \ln \left[\alpha_0 \left(\frac{e^{\lambda_K (t-t_0)} K_t}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0) \left(\frac{e^{\lambda_L (t-t_0)} L_t}{L_0} \right)^{\frac{\sigma-1}{\sigma}} \right] + \varepsilon_{3t}, \qquad \textbf{(9)}$$

where y is $\ln(Y)$. In practice, we will sample geometric averages from the data to compute yk_0, yl_0, and y_0; for t_0 we use the arithmetic average of time periods [León-Ledesma et al. (2010, p. 1342)]. Also, in practice, α_0 can be fixed using the arithmetic average of capital's share from the data prior to estimation.[8] Monte Carlo analysis by León-Ledesma et al. (2010) suggests that this normalized system estimation is considerably more effective than single equation approaches to estimating technological parameters.

3. THE DATA AND THEIR PROPERTIES

The Jorgenson (2007) KLEM database provides factor inputs and outputs, along with their prices, for 35 industries (corresponding approximately to the two-digit

SIC level) covering the entire U.S. economy. The constituent industries are listed in Table 1A (as well as 1B and 1C). The database is the product of Jorgenson et al. (1987)'s methodology, begins in 1960, and is updated through 2005 in the latest version.

The KLEM database combines industry data from the U.S. Bureau of Labor Statistics (BLS) and the U.S. Bureau of Economic Analysis (BEA). Variables include the quantity of output (Q) and the price of output (P); the quantity and price of capital services (K and R); and the quantity and price of labor inputs (L and W). Output is gross (including the value of energy and materials inputs). To conform to the empirical model in Section 2, we calculate real value added (Y), where value added ($R \cdot K + W \cdot L$) is deflated by the price of output (P).

The KLEM methodology has features desirable for a study of σ and technical change. First, it includes quality-adjusted labor services [see Ho and Jorgenson (1999)] that combine data on individuals from the U.S. Census and Current Population Survey (CPS) controlling for age, sex, educational attainment, and status as employee versus self-employed. Not controlling for different qualities of labor can result in those quality differences being captured, instead, in estimates of technical change.[9] Also, the price of capital services is constructed for each industry by accounting for the user cost of capital as a function of depreciation, rates of return, capital gains, and the tax structure [Jorgenson (1963), Hall and Jorgenson (1971), and Jorgenson and Yun (2001)].

For each industry, we construct the variables *yk*, *yl*, *kl*, *rp*, *wp*, and *rw*. For the system estimation we also construct y and the benchmark values, y_0, yk_0, yl_0, and α_0. We also use value-added shares to compute aggregate (weighted-average) values of these variables. Tables 1A through 1C present summary statistics for these variables. In the application of these time series to estimating (3)–(5) two potentially serious issues arise: *serial correlation* in the error terms and *nonstationarity and/or cointegration*.[10]

Durbin–Watson (DW) statistics based on OLS regressions of (3)–(5) using individual industry time series and aggregate time series indicate rejection of the null hypothesis of no serial correlation (most often at the 1% significance level) in all cases save one.[11] The obvious initial candidate to describe the serial correlation is a first-order autoregressive (AR(1)) process. When DW statistics are computed using the OLS residuals from AR(1) regressions, there are very few cases where the null is rejected.[12] Also, based on F-statistics from Breusch–Godfrey tests of serial correlation in the AR(1) residuals up to the fifth order, in the large majority of cases the null of no serial correlation is not rejected.

Based on serial correlation in the error terms of (3)–(5), the first estimation approach, for which we report results in Section 4, is a three-stage generalized instrumental variables (GIV) approach developed by Fair (1970) and applied by Antràs (2004) in the present context. We report GIV results for each equation, (3) through (5), separately. The stages are as follows. *First*, run a two-stage least squares (2SLS) regression where the instrumental variables (IVs) are lagged dependent and independent variables. *Second*, estimate an AR(1) regression of

TABLE 1A. Summary statistics of regression variables: *yk* and *rp*

Industry		*yk* Mean	*yk* Std. dev.	*yk* $\Delta_{1960-2005}$	*rp* Mean	*rp* Std. dev.	*rp* $\Delta_{1960-2005}$
1	Agriculture	0.733	0.277	0.708	0.029	0.322	1.030
2	Metal mining	0.675	0.348	−0.932	−0.065	0.313	−0.398
3	Coal mining	0.758	0.194	−0.116	−0.160	0.298	1.047
4	Oil and gas extraction	0.665	0.444	−0.927	0.373	0.433	−0.800
5	Nonmetallic mining	0.713	0.243	−0.573	−0.014	0.266	−0.213
6	Construction	2.180	0.242	−0.750	0.137	0.217	−0.054
7	Food and kindred products	0.714	0.124	0.364	−0.358	0.355	0.882
8	Tobacco	0.411	0.412	−1.417	−0.078	0.458	−1.532
9	Textile mill products	1.314	0.062	0.133	−0.092	0.164	0.678
10	Apparel	1.821	0.359	−1.193	0.051	0.322	−0.952
11	Lumber and wood	1.225	0.174	−0.253	0.006	0.163	0.289
12	Furniture and fixtures	1.635	0.068	0.171	0.038	0.196	0.177
13	Paper and allied	1.105	0.177	−0.303	0.029	0.183	−0.248
14	Printing, publishing, and allied	1.766	0.330	−1.057	0.401	0.309	−0.604
15	Chemicals	0.778	0.184	−0.277	0.026	0.189	0.029
16	Petroleum and coal products	0.650	0.326	0.101	0.099	0.326	0.692
17	Rubber and miscellaneous products	1.318	0.095	−0.074	−0.151	0.192	0.137
18	Leather	1.428	0.422	−1.345	0.006	0.241	−0.905
19	Stone, clay, and glass	1.292	0.188	−0.167	−0.035	0.323	0.024
20	Primary metal	1.074	0.241	−0.014	−0.073	0.313	0.586
21	Fabricated metal	1.231	0.235	−0.407	−0.016	0.207	0.462
22	Nonelectrical industry	1.402	0.187	0.260	−0.018	0.256	0.306
23	Electrical industry	0.661	0.226	0.490	−0.054	0.429	0.770
24	Motor vehicles	1.036	0.173	−0.051	−0.025	0.377	−0.319

TABLE 1A. Continued

Industry		*yk* Mean	*yk* Std. dev.	*yk* $\Delta_{1960-2005}$	*rp* Mean	*rp* Std. dev.	*rp* $\Delta_{1960-2005}$
25	Transportation equipment and ordnance	1.977	0.273	−0.528	−0.088	0.246	0.288
26	Instruments	2.615	0.385	−0.768	0.756	0.493	−0.623
27	Miscellaneous manufacturing	1.136	0.203	−0.295	−0.126	0.181	0.666
28	Transportation	1.117	0.109	0.198	−0.114	0.132	0.573
29	Communications	0.649	0.176	−0.410	0.020	0.111	−0.170
30	Electrical utilities	0.367	0.087	0.009	−0.023	0.107	0.099
31	Gas utilities	1.180	0.678	−1.335	0.752	0.634	−1.307
32	Trade	1.841	0.430	−1.262	0.359	0.338	−0.732
33	Finance, insurance, and real estate	0.359	0.093	0.335	−0.061	0.094	0.357
34	Services	1.983	0.333	−1.197	0.309	0.427	−1.440
35	Government enterprises	1.011	0.227	−0.688	0.073	0.311	−0.494
AG	Aggregate	0.545	0.031	0.132	0.112	0.131	−0.078

TABLE 1B. Summary statistics of regression variables: *yl* and *wp*

Industry		*yl*			*wp*		
		Mean	Std. dev.	$\Delta_{1960-2005}$	Mean	Std. dev.	$\Delta_{1960-2005}$
1	Agriculture	0.232	0.412	1.686	−0.481	0.400	1.378
2	Metal mining	0.380	0.406	0.779	−0.308	0.250	0.045
3	Coal mining	0.316	0.437	1.537	−0.237	0.303	0.632
4	Oil and gas extraction	1.462	0.241	0.291	0.080	0.292	−0.148
5	Nonmetallic mining	0.632	0.232	1.059	−0.041	0.149	0.574
6	Construction	0.221	0.060	0.136	0.078	0.060	−0.020
7	Food and kindred products	0.318	0.329	0.971	−0.123	0.214	0.696
8	Tobacco	0.994	0.173	−0.093	0.036	0.161	0.065
9	Textile mill products	0.006	0.378	1.427	−0.280	0.348	1.247
10	Apparel	0.089	0.285	1.178	−0.104	0.263	1.147
11	Lumber and wood	0.401	0.159	0.482	0.043	0.114	0.287
12	Furniture and fixtures	0.158	0.154	1.696	−0.073	0.170	0.695
13	Paper and allied	0.380	0.187	0.702	−0.042	0.175	0.671
14	Printing, publishing, and allied	0.387	0.091	0.443	0.089	0.069	0.254
15	Chemicals	0.528	0.327	1.141	−0.125	0.233	0.798
16	Petroleum and coal products	0.843	0.257	0.908	−0.059	0.218	−0.113
17	Rubber and miscellaneous products	0.157	0.200	0.696	−0.109	0.173	0.621
18	Leather	0.278	0.249	0.874	−0.028	0.174	0.765
19	Stone, clay, and glass	0.218	0.154	0.635	−0.099	0.139	0.537
20	Primary metal	0.324	0.188	0.754	−0.069	0.121	0.295
21	Fabricated metal	0.214	0.192	0.728	−0.083	0.124	0.429
22	Nonelectrical industry	−0.253	0.617	2.134	−0.533	0.636	2.119
23	Electrical industry	−0.290	0.883	2.885	−0.672	0.786	2.761
24	Motor vehicles	0.441	0.193	0.675	−0.009	0.217	0.825

TABLE 1B. Continued

Industry		*yl* Mean	*yl* Std. dev.	*yl* $\Delta_{1960-2005}$	*wp* Mean	*wp* Std. dev.	*wp* $\Delta_{1960-2005}$
25	Transportation equipment and ordnance	0.094	0.189	0.654	−0.046	0.174	0.542
26	Instruments	−0.029	0.271	1.090	−0.205	0.290	1.059
27	Miscellaneous manufacturing	0.292	0.280	1.038	−0.063	0.173	0.634
28	Transportation	0.318	0.167	0.666	−0.030	0.163	0.495
29	Communications	0.490	0.479	1.726	−0.285	0.399	1.429
30	Electrical utilities	1.076	0.194	0.693	−0.056	0.133	0.496
31	Gas utilities	0.993	0.187	0.389	−0.085	0.246	0.336
32	Trade	0.095	0.254	0.939	−0.165	0.220	0.769
33	Finance, insurance, and real estate	0.831	0.256	0.871	−0.243	0.256	0.830
34	Services	0.196	0.078	0.336	−0.014	0.096	0.406
35	Government enterprises	0.396	0.286	0.755	−0.123	0.207	0.618
AG	Aggregate	0.432	0.321	1.115	−0.179	0.224	0.771

TABLE 1C. Summary statistics of regression variables: *kl* and *wr*

		kl			*wr*		
Industry		Mean	Std. dev.	$\Delta_{1960-2005}$	Mean	Std. dev.	$\Delta_{1960-2005}$
1	Agriculture	−0.501	0.218	0.978	−0.509	0.248	0.349
2	Metal mining	−0.295	0.665	1.711	−0.243	0.429	0.443
3	Coal mining	−0.442	0.449	1.653	−0.077	0.300	−0.415
4	Oil and gas extraction	0.797	0.477	1.218	−0.293	0.471	0.652
5	Nonmetallic mining	−0.080	0.402	1.632	−0.027	0.350	0.787
6	Construction	−1.959	0.245	0.886	−0.059	0.215	0.035
7	Food and kindred products	−0.396	0.228	0.607	0.236	0.205	−0.186
8	Tobacco	0.584	0.336	1.324	0.114	0.447	1.597
9	Textile mill products	−1.308	0.362	1.339	−0.189	0.279	0.569
10	Apparel	−1.732	0.639	2.371	−0.155	0.553	2.100
11	Lumber and wood	−0.825	0.287	0.734	0.037	0.189	−0.002
12	Furniture and fixtures	−1.477	0.154	0.526	−0.111	0.300	0.518
13	Paper and allied	−0.725	0.349	1.005	−0.071	0.337	0.919
14	Printing, publishing, and allied	−1.379	0.355	1.501	−0.312	0.305	0.858
15	Chemicals	−0.250	0.462	1.418	−0.151	0.333	0.769
16	Petroleum and coal products	0.192	0.366	0.807	−0.159	0.346	−0.806
17	Rubber and miscellaneous products	−1.161	0.222	0.771	0.043	0.215	0.485
18	Leather	−1.150	0.662	2.219	−0.033	0.388	1.670
19	Stone, clay, and glass	−1.073	0.272	0.802	−0.063	0.392	0.513
20	Primary metal	−0.749	0.324	0.768	0.005	0.335	−0.291
21	Fabricated metal	−1.016	0.390	1.135	0.080	0.230	−0.033
22	Nonelectrical industry	−1.655	0.610	1.874	−0.514	0.715	1.814
23	Electrical industry	−0.951	0.817	2.395	−0.126	0.593	1.991
24	Motor vehicles	−0.595	0.246	0.726	0.016	0.505	1.144

TABLE 1C. Continued

Industry		*kl* Mean	*kl* Std. dev.	*kl* $\Delta_{1960-2005}$	*wr* Mean	*wr* Std. dev.	*wr* $\Delta_{1960-2005}$
25	Transportation equipment and ordnance	−1.883	0.429	1.183	0.042	0.359	0.254
26	Instruments	−2.645	0.603	1.859	−0.962	0.736	1.682
27	Miscellaneous manufacturing	−0.844	0.449	1.333	0.063	0.159	−0.032
28	Transportation	−0.800	0.155	0.468	0.084	0.168	−0.079
29	Communications	−0.159	0.638	2.136	−0.304	0.488	1.600
30	Electrical utilities	0.708	0.181	0.684	−0.033	0.123	0.398
31	Gas utilities	−0.187	0.789	1.724	−0.837	0.704	1.643
32	Trade	−1.745	0.680	2.201	−0.525	0.551	1.502
33	Finance, insurance, and real estate	0.472	0.183	0.536	−0.182	0.184	0.473
34	Services	−1.788	0.391	1.532	−0.323	0.507	1.846
35	Government enterprises	−0.616	0.463	1.443	−0.196	0.417	1.111
AG	Aggregate	−0.357	0.121	0.983	−0.127	0.143	0.851

the 2SLS residuals: $\hat{\varepsilon}_t = \rho\hat{\varepsilon}_{t-1} + \upsilon_t$. *Third*, use the estimated coefficient from the second stage to estimate (e.g., in the case of (3))

$$yk_t - \hat{\rho} yk_{t-1} = \delta_1 + \sigma\,(rp_t - \hat{\rho} rp_{t-1}) + (1-\sigma)\,\lambda_K \cdot t + \varepsilon_{1t}.$$

The third stage is, therefore, a feasible generalized least squares (FGLS) treatment assuming an AR(1) process for the errors. The 2SLS and FGLS components are to account, respectively, for endogeneity (which is almost inevitable in a production-function framework) and serial correlation in the OLS residuals.

Augmented Dickey–Fuller (ADF) (including time trends) rarely rejects the unit root null hypothesis (at the 10% level or better). Also, for 16 industries, a unit root cannot be rejected at conventional significance levels for any of the six variables. (The same is the case for the aggregate variables.) A unit root can be rejected for more than one of the six variables for only nine industries. Overall, there is compelling evidence that nonstationarity is a feature of the data.

Given nonstationarity, there arises the additional question of cointegration among the time series. Given our empirical model, there are six combinations of variables to consider: (*yk*, *rp*), (*yl*, *wp*), and (*kl*, *rw*). We perform maximum likelihood cointegration tests [Johansen and Juselius (1990) and Johansen (1995)] for these combinations. The results are mixed. For (*yk*, *rp*) the null of no cointegrating vectors can be rejected for 12 industries at the 10% level or better when one lag is considered. When two lags are included in the test, that null is rejected for either 7 (trace test) or 10 (max λ test) industries. Except for one industry (35), the null of at most one cointegrating vector is never rejected. The results are also mixed for the pairs (*yl*, *wp*), and (*kl*, *rw*), but the null of zero cointegrating vectors is less often rejected.[13] Overall, the evidence suggests that, considering any pair of variables, the pair is cointegrated of order (1, 1) in a minority of industries. Note, however, that the null of no cointegrating vectors is never rejected using the aggregate data.

If our regression variables are nonstationary but also not cointegrated (which seems to be the case for a majority of industries), then the OLS estimator of σ is inconsistent. We report results from OLS regressions of each equation, (3) through (5), using differenced data later. However, we do so with the caveat that information embodied in variable levels is discarded. This is particularly important information for estimating σ because the underlying theory assumes that "quantities entering the production function are long-run variables; that is, variables that have been adjusted to their optimal levels without incurring costly frictions" [Chirinko (2008, p. 673)].

Last, we report results from estimating the first-order conditions and production function jointly as a system. A nice property of the normalized system ((7), (8), and (9)) is that it incorporates cross-equation restrictions explicitly grounded in long-run relationships. Therefore, the approach stresses the long-run information contained in the variable levels. Furthermore, the Monte Carlo analysis of León-Ledesma et al. (2010) assumes that factor supplies are random walks with drifts

TABLE 2. Aggregate elasticity of substitution and technical bias estimates

Equation	OLS	GIV	OLS differenced		Normalized system GMM
σ_1	0.350	0.279	0.282	σ	0.177
95% C.I.	(0.182, 0.518)	(0.133, 0.425)	(0.138, 0.426)	95% C.I.	(0.097, 0.257)
λ_K	0.009	0.000	0.005	λ_K	0.035
R^2	.560	.260	.264	λ_L	0.061
σ_2	1.088	1.364	0.821	R^2 (equation (1))	.960
95% C.I.	(0.918, 1.258)	(1.282, 1.446)	(0.677, 0.965)	R^2 (equation (2))	.981
λ_L	−0.066	0.000	0.060	R^2 (equation (3))	.987
R^2	.997	.963	.752		
σ_3	0.206	0.416	0.249		
95% C.I.	(0.050, 0.362)	(0.258, 0.574)	(0.123, 0.375)		
$\lambda_L - \lambda_K$	0.020	0.000	0.023		
R^2	.975	.379	.264		

whose errors are uncorrelated. That analysis suggests the effectiveness of system estimation when the data are nonstationary. (We also report estimates based on the non-normalized system.)

System estimation is carried out by the generalized method of moments (GMM). As in our GIV estimations, we use lagged dependent and independent variables as instruments. Because the normalized system is nonlinear, we must choose initial parameter values. Our approach to this issue follows León-Ledesma et al. (2010, p. 1346): we estimate the first-order conditions, (7) and (8), as a two-equation system by OLS. The resulting coefficient estimates are then used as initial values in the three-equation system estimation.

4. ESTIMATION RESULTS

Because estimation of U.S. aggregate σ is prevalent in economics journals, whereas interindustry studies are rare, we begin in Table 2 by presenting our estimates of aggregate σ based on four approaches: (i) OLS (as a baseline), (ii) GIV, (iii) OLS differenced, and (iv) normalized system GMM.[14] Doing so places our work first in the context of existing studies before we move on to the more novel industry-level estimates. The columns corresponding to (i), (ii), and (iii) are single equation-by-equation approaches and therefore each contains three sets of results (σ_1, σ_2, and σ_3 for, respectively, (3), (4), and (5)). The column corresponding to (iv) is based on the three-equation system and therefore has one set of parameters.

4.1. Estimates of Aggregate σ, λ_L, and λ_K

The first thing to note in Table 2 is that the point estimates of σ from equation (2) (based on (yl, wp) are notably higher than those from equations (1) and (3). This is an empirical regularity reported in several U.S. studies [e.g., Eisner and Nadiri

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(1968), Lucas (1969), Berndt (1976), and Antràs (2004)]. Given this established regularity and greater agreement between the equation (1) and (3) estimates, our inclination is to discount (single) equation (4)-based estimates (though we report them throughout). Moreover, the industry-level results suggest that estimates based on (4) are not only generally higher than those based on (3) and (5), but also more fundamentally different. (See Section 4.3.)

If we focus on estimates of σ based on (3) and (5), the point estimates are always considerably below unity. Indeed, they range within the fairly narrow band from 0.206 to 0.416. Looking at the 95% confidence intervals, (i) the σ estimates are always statistically greater than zero and less than 0.574 and (ii) the confidence intervals are all overlapping. The normalized system GMM point estimate is notably lower: 0.177. Also, its 95% confidence interval upper bound is only 0.257. Although the normalized system σ estimate is low, the other parameter estimates are not at all unreasonable. Point estimates imply net labor augmentation at a rate of about 2.5% annually.

Klump et al. (2007) also use a three-equation normalized system approach using U.S. aggregate data and find σ to be between 0.5 and 0.6. Why is our estimate lower? Klump et al.'s (2007) data are not aggregated up from the industry-level but, instead, are drawn primarily from the U.S. NIPAs; they also cover a slightly different time period (1953 to 1998). Aside from their different data, Klump et al. use a nonlinear seemingly unrelated regression (SUR) estimator.[15]

Turning to the estimates of factor augmentation, both the OLS regressions in levels and the GIV regressions yield what may be viewed as unreasonable estimates of factor augmentation rates. Though OLS estimation of (5) yields a point estimate indicating net labor augmentation, the (4) estimate of λ_L is actually negative. The GIV point estimates for *all* factor augmentation rates (net and gross) are essentially zero. Given the importance of trend estimation, we suspect that nonstationarity of the data is to blame. The differenced OLS and normalized system GMM results are more reasonable.[16] Point estimates on λ_K are low relative to λ_K. For the differenced OLS regressions, (3) and (5) taken together imply a rather high rate of net labor augmentation: about 5.5% annually. However, the equation (4) point estimate is about 2.6%, whereas the GMM estimate is about 2.5% annually. These estimates are not only qualitatively but also quantitatively reasonable given trend growth in U.S. GDP.

4.2. Estimates of Industry-Level σ's

A novel contribution of this paper is industry-level estimates of σ and rates of factor augmentation. To preserve space (given the large number of estimates to report) we confine our reporting to results from GIV (Tables 3), OLS with differenced data (Table 4), and system GMM estimates (Table 5). These tables report σ estimates along with their 95% confidence intervals.

In Table 5, estimates based on both the normalized and non-normalized systems are reported. This is, first, for the sake of comparison—to understand how

TABLE 3. Estimates of the sector elasticities of substitution from GIV regressions

Industry	σ_1	σ_2	σ_3
1	0.605	0.839	−0.085
	(0.469, 0.741)	(0.673, 1.005)	(−0.287, 0.117)
2	0.624	1.312	0.754
	(0.404, 0.844)	(0.984, 1.640)	(0.380, 1.128)
3	0.281	0.804	0.086
	(0.137, 0.425)	(0.550, 1.058)	(−0.088, 0.260)
4	1.010	0.702	0.851
	(0.966, 1.054)	(0.574, 0.830)	(0.715, 0.987)
5	0.064	1.132	0.314
	(−0.070, 0.198)	(0.712, 1.552)	(0.040, 0.588)
6	0.382	1.008	0.290
	(0.234, 0.530)	(0.810, 1.206)	(0.118, 0.462)
7	0.293	1.432	−0.339
	(0.243, 0.343)	(1.250, 1.614)	(−0.551, −0.127)
8	0.864	0.581	0.568
	(0.818, 0.910)	(0.317, 0.845)	(0.468, 0.668)
9	0.142	1.146	0.167
	(0.026, 0.258)	(1.016, 1.276)	(−0.109, 0.443)
10	0.790	1.028	0.939
	(0.604, 0.976)	(0.970, 1.086)	(0.793, 1.085)
11	0.394	1.045	0.458
	(0.256, 0.532)	(0.857, 1.233)	(0.228, 0.688)
12	0.215	0.821	0.291
	(0.123, 0.307)	(0.699, 0.943)	(0.171, 0.411)
13	0.373	0.914	0.423
	(0.229, 0.517)	(0.774, 1.054)	(0.241, 0.605)
14	0.352	1.213	−0.013
	(0.204, 0.500)	(0.907, 1.519)	(−0.173, 0.147)
15	0.613	1.078	1.057
	(0.467, 0.759)	(0.832, 1.324)	(0.793, 1.321)
16	0.762	0.371	0.151
	0.614, 0.910)	(0.099, 0.643)	(−0.071, 0.373)
17	0.267	1.173	0.077
	(0.145, 0.389)	(1.021, 1.325)	(−0.117, 0.271)
18	0.417	1.186	0.577
	(0.185, 0.649)	(0.966, 1.406)	(0.285, 0.869)
19	0.366	1.043	0.283
	(0.254, 0.478)	(0.577, 1.509)	(0.115, 0.451)
20	0.574	0.982	0.451
	(0.436, 0.712)	(0.592, 1.372)	(0.217, 0.685)
21	0.441	1.260	0.476
	(0.245, 0.637)	(1.022, 1.498)	(0.186, 0.766)

TABLE 3. Continued

Industry	σ_1	σ_2	σ_3
22	0.597	0.965	0.816
	(0.489, 0.705)	(0.941, 0.989)	(0.744, 0.888)
23	0.292	1.030	0.819
	(0.180, 0.404)	(0.912, 1.148)	(0.563, 1.075)
24	0.377	0.672	0.281
	(0.291, 0.463)	(0.398, 0.946)	(0.157, 0.405)
25	0.671	1.004	0.798
	(0.409, 0.933)	(0.936, 1.072)	(0.584, 1.012
26	0.446	0.952	0.382
	(0.288, 0.604)	(0.824, 1.080)	(0.196, 0.568)
27	0.038	1.575	1.066
	(−0.278, 0.354)	(1.483, 1.667)	(0.426, 1.706)
28	0.303	0.827	0.110
	(0.167, 0.439)	(0.623, 1.031)	(−0.052, 0.272)
29	0.571	1.114	1.082
	(0.423, 0.719)	(0.950, 1.278)	(0.868, 1.296)
30	0.734	1.314	0.403
	(0.628, 0.840)	(1.138, 1.490)	(0.073, 0.733)
31	1.041	0.474	0.761
	(0.953, 1.129)	(0.264, 0.684)	(0.535, 0.987)
32	1.028	1.124	1.072
	(0.868, 1.188)	(1.004, 1.244)	(0.924, 1.220)
33	0.683	0.919	0.657
	(0.555, 0.811)	(0.805, 1.033)	(0.457, 0.857)
34	0.369	0.918	0.623
	(0.207, 0.531)	(0.766, 1.070)	(0.455, 0.791)
35	0.441	1.323	−0.017
	(0.315, 0.567)	(0.939, 1.707)	(−0.067, 0.033)

Note: The 95% confidence intervals are in parentheses.

normalization changes the results. Second, in Section 5, we will map industry σ estimates into σ's for sectors that commonly appear in general equilibrium models. A key argument for normalization in empirical work is that allowing σ to vary is "rather like moving from one [production] function $F(K,L)$ to another, $G(K,L)$" [Temple (2009, p. 3)]. Therefore we need a benchmark against which to evaluate the variation in the data (and the σ that it implies). However, this also suggests that normalized CES functions are most useful in facilitating the comparison of sectors that differ only in σ. This will not necessarily be the case in growth or CGE models. Rather, such models will often have sectoral production functions that have different and invariant structural parameters (in addition to σ). Non-normalized system estimates might be preferable in those contexts.[17]

TABLE 4. Estimates of the sector elasticities of substitution from OLS regressions of differenced variables

Industry	σ_1	σ_2	σ_3
1	0.341	0.407	0.025
	(0.249, 0.433)	(0.121, 0.693)	(−0.097, 0.147)
2	0.315	1.161	0.050
	(0.143, 0.487)	(0.735, 1.587)	(−0.292, 0.392)
3	0.220	0.326	0.052
	(0.092, 0.348)	(0.122, 0.530)	(−0.070, 0.174)
4	0.738	0.485	0.370
	(0.626, 0.850)	(0.285, 0.685)	(0.196, 0.544)
5	0.487	0.901	0.041
	(0.385, 0.589)	(0.501, 1.301)	(−0.153, 0.235)
6	0.398	0.992	0.313
	(0.258, 0.538)	(0.788, 1.196)	(0.147, 0.479)
7	0.160	1.012	−0.217
	(0.088, 0.232)	(0.608, 1.416)	(−0.331, −0.103)
8	0.615	0.237	0.178
	(0.555, 0.675)	(−0.039, 0.513)	(0.098, 0.258)
9	0.193	1.075	−0.162
	(0.067, 0.319)	(0.733, 1.417)	(−0.326, 0.002)
10	0.188	0.711	0.135
	(0.108, 0.268)	(0.499, 0.923)	(0.033, 0.237)
11	0.339	0.621	0.234
	0.229, 0.449)	(0.423, 0.819)	(0.112, 0.356)
12	0.165	0.569	0.055
	(0.057, 0.273)	(0.303, 0.835)	(−0.071, 0.181)
13	0.261	0.469	0.061
	(0.157, 0.365)	(0.279, 0.659)	(−0.739, 0.861)
14	0.196	1.126	−0.050
	(0.114, 0.278)	(0.814, 1.438)	(−0.178, 0.078)
15	0.577	0.900	0.123
	(0.483, 0.671)	(0.660, 1.140)	(−0.011, 0.257)
16	0.658	0.321	0.060
	(0.504, 0.812)	(0.079, 0.563)	(−0.128, 0.248)
17	0.267	1.139	0.005
	(0.147, 0.387)	(0.821, 1.457)	(−0.145, 0.155)
18	0.293	0.542	0.159
	(0.173, 0.413)	(0.254, 0.830)	(0.015, 0.303)
19	0.303	0.720	0.083
	(0.193, 0.413)	(0.240, 1.200)	(−0.055, 0.221)
20	0.296	0.576	0.071
	(0.158, 0.434)	(0.190, 0.962)	(−0.119, 0.261)
21	0.381	0.690	0.191
	(0.271, 0.491)	(0.442, 0.938)	(0.071, 0.311)

TABLE 4. Continued

Industry	σ_1	σ_2	σ_3
22	0.397	0.861	0.202
	(0.277, 0.517)	(0.613, 1.109)	(0.060, 0.344)
23	0.247	0.516	0.058
	(0.137, 0.357)	(0.212, 0.820)	(−0.066, 0.182)
24	0.289	0.786	−0.034
	(0.155, 0.423)	(0.284, 1.288)	(−0.242, 0.174)
25	0.365	0.941	0.484
	(0.193, 0.537)	(0.869, 1.013)	(0.316, 0.652)
26	0.427	1.001	0.297
	(0.271, 0.583)	(0.743, 1.259)	(0.113, 0.481)
27	0.222	1.060	−0.072
	(0.122, 0.322)	(0.664, 1.456)	(−0.208, 0.064)
28	0.316	0.747	0.100
	(0.184, 0.448)	(0.507, 0.987)	(−0.052, 0.252)
29	0.577	0.553	0.061
	(0.435, 0.719)	(0.265, 0.841)	(−0.131, 0.253)
30	0.658	0.791	0.244
	0.520, 0.796)	(0.475, 1.107)	(−0.068, 0.556)
31	0.981	0.487	0.663
	(0.843, 1.119)	(0.269, 0.705)	(0.423, 0.903)
32	0.332	0.875	0.203
	(0.230, 0.434)	(0.651, 1.099)	(0.073, 0.333)
33	0.423	0.476	0.083
	(0.311, 0.535)	(0.060, 0.892)	(−0.179, 0.345)
34	0.192	0.867	0.198
	(0.074, 0.310)	(0.689, 1.045)	(0.038, 0.358)
35	0.376	1.221	−0.017
	(0.344, 0.408)	(0.499, 1.943)	(−0.059, 0.025)

Note: The 95% confidence intervals are in parentheses.

The equation (3) GIV σ estimates are statistically significantly greater than zero (rejecting the Leontief case) for 33 out of 35 industries. In every one of those 33 cases, the confidence intervals lie entirely below unity (rejecting the Cobb–Douglas case). For only three industries (4, 31, and 32) is the point estimate greater than unity. The results based on equation (5) are similar. Statistical significance is found for 26 out of 35 industries; in 18 of those cases the confidence intervals lie entirely below unity. For only four industries (15, 27, 29, and 32) is the σ estimate statistically significant and greater than unity. (In one of those cases, however, the confidence interval reaches as low as 0.426.)

GIV estimates based on equation (4) fit the empirical regularity of being higher than those based on (3) or (5). Out of 35 statistically significant estimates, only 15 are below unity. Furthermore, confidence intervals lie entirely below unity for

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TABLE 5. Estimates of industry elasticities of substitution from GMM system estimation

Industry	Non-normalized	Normalized
1	0.675	−0.392
	(0.483, 0.867)	(−0.466, −0.318)
2	0.640	0.717
	(0.628, 0.652)	(0.703, 0.731)
3	0.779	0.615
	(0.775, 0.783)	(0.607, 0.623)
4	0.869	0.868
	(0.867, 0.871)	(0.866, 0.870)
5	0.630	0.629
	(0.626, 0.634)	(0.625, 0.633)
6	0.503	0.329
	(0.495, 0.511)	(0.311, 0.347)
7	0.389	0.382
	(0.385, 0.393)	(0.368, 0.396)
8	0.846	0.997
	(0.844, 0.848)	(0.995, 0.999)
9	1.092	0.210
	(1.080, 1.104)	(0.192, 0.228)
10	1.077	0.352
	(1.069, 1.085)	(0.338, 0.366)
11	0.821	1.050
	(0.809, 0.833)	(1.026, 1.074)
12	0.460	0.348
	(0.454, 0.466)	(0.338, 0.358)
13	0.431	0.426
	(0.415, 0.447)	(0.416, 0.436)
14	0.503	0.507
	(0.497, 0.509)	(0.501, 0.513)
15	0.521	0.522
	(0.517, 0.525)	(0.516, 0.528)
16	0.732	0.733
	(0.726, 0.738)	(0.729, 0.737)
17	0.387	0.383
	(0.377, 0.397)	(0.371, 0.395)
18	1.414	0.029
	(1.348, 1.480)	(0.005, 0.053)
19	0.458	0.456
	(0.454, 0.462)	(0.450, 0.462)
20	0.694	0.693
	(0.690, 0.698)	(0.689, 0.697)
21	0.681	0.668
	(0.675, 0.687)	(0.658, 0.678)

TABLE 5. Continued

Industry	Non-normalized	Normalized
22	0.823	0.822
	(0.821, 0.825)	(0.820, 0.824)
23	0.651	0.589
	(0.647, 0.655)	(0.585, 0.593)
24	0.493	0.490
	(0.481, 0.505)	(0.476, 0.504)
25	1.133	0.461
	(1.117, 1.149)	(0.451, 0.471)
26	0.727	0.602
	(0.732, 0.731)	(0.594, 0.610)
27	0.744	1.219
	(0.730, 0.758)	(1.177, 1.261)
28	0.595	0.598
	(0.461, 0.729)	(0.568, 0.628)
29	0.475	0.418
	(0.349, 0.601)	(0.410, 0.426)
30	1.002	1.001
	(1.002, 1.002)	(1.001, 1.001)
31	0.576	0.578
	(0.562, 0.590)	(0.572, 0.584)
32	0.428	0.425
	(0.426, 0.430)	(0.419, 0.431)
33	0.999	1.000
	(0.999, 0.999)	(1.000, 1.000)
34	0.686	0.600
	(0.682, 0.690)	(0.472, 0.728)
35	0.393	0.386
	(0.381, 0.405)	(0.376, 0.396)

Note: The 95% confidence intervals are in parentheses.

only five industries. On the other hand, for a full seven industries (7, 9, 17, 21, 27, 30, 32), the σ confidence intervals lie entirely above unity. Though we are inclined to discount the equation (4) estimates, combined with the GIV equation (5) results they suggest that at least two industries (27 and 32, *miscellaneous manufacturing* and *trade*) may have a substitution elasticity of at least unity. The former of these industries averaged less than 0.5% of value added over the period 1960 to 2005. *Trade*, on the other hand, had an average value-added share of over 16% (starting at about 18% and falling to about 15%.)

The OLS estimates based on differenced data are generally lower than the GIV estimates. We attribute this to the discarding of longer-run information in the levels of variables in favor of shorter-term information in the changes. Based on equation (3), all 35 industry σ estimates are statistically significant, and in 34 of those cases, the confidence intervals lie entirely below unity. Based on equation (4), estimates of σ are significantly different from zero for 34 industries. Based on only short-run variation, 16 of the confidence intervals now lie entirely below unity and only eight point estimates are greater than unity. (None of the confidence intervals lie entirely above unity.) Equation (5) estimates suffer from wide confidence intervals. Statistical significance holds for only 13 industries, though in all of those cases, the confidence intervals are entirely below unity.

The GMM three-equation system σ estimates (both normalized and non-normalized) are typically higher than both the GIV and differenced OLS estimates. However, they are still statistically significantly less than unity in a large majority of cases (30 industries for the non-normalized system; 31 industries for the normalized system). The confidence intervals are notably narrow. This is to be expected, given the structure imposed by the cross-equation restrictions. Rather troubling, though, is the estimate from the normalized system for industry 1 (*agriculture*), which is significantly *less* than zero. A negative σ is clearly not sensible in any economic sense. However, there are good theoretical reasons to prefer estimates based on the normalized system; we believe that it is unreasonable to discount them all based on one industry (which constitutes less than 3% of value added over our sample period).

4.3. Comparison of Industry-Level σ's across Estimators and with Aggregate Estimates

We believe that the overall picture portrayed in Tables 3, 4, and 5 is that of an economy where a large majority of industries are characterized by an elasticity of substitution between capital and labor below unity. In particular, this interpretation is consistent with our preferred industry σ estimates based on GMM estimation of the normalized production functions and first-order conditions.

However, there is undoubtedly substantial uncertainty associated with the industry-level σ estimates. Point estimates across estimation techniques often differ markedly whereas, on the other hand, confidence intervals overlap.[18] First, this represents the need for further research on estimating σ more precisely. Second, we would like to know if the industry-level estimates correlate across estimators. In other words, are all three estimation techniques (GIV, OLS with first differences, and GMM system) "leading us in the same direction" (hopefully toward the actual underlying technologies)?

Table 6 reports correlations between the industry-level σ estimates, by both equation and estimator. A striking feature of the correlations is that estimates based on equation (4) have either very low or negative correlations with estimates based on equations (3) and (5) or the three-equation system (normalized or otherwise). Indeed, only one of these correlations is positive (0.023 for GIV equation (2) and equation (3)). This one case is a common-estimator correlation. This brings new perspective to the empirical regularity that equation (4)-based σ estimates tend to be relatively high. Our results suggest that this is not merely an upward bias.

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TABLE 6. Correlations between different industry-level σ estimates

		GIV			Diff.				GMM
		1	2	3	1	2	3	GMM	(normalized)
GIV	1	1.000	−0.441	0.432	0.660	−0.365	0.614	0.182	0.118
	2		1.000	0.023	−0.460	0.687	−0.414	−0.034	0.006
	3			1.000	0.295	−0.107	0.443	0.258	0.354
Diff.	1				1.000	−0.342	0.633	0.062	0.293
	2					1.000	−0.238	−0.231	−0.059
	3						1.000	0.253	0.177
GMM								1.000	0.116
GMM (norm.)									1.000

Rather, estimation based on equation (4) results in fundamentally different results, often negatively correlated with those based on equations (3) and (5) alone or a system. This reinforces our inclination to discount equation (2)-based estimates. (It is also an insight unavailable from aggregate studies.)

Correlations between estimates based on equations (3) and (5) and the three-equation system are all positive. Seven of the correlations are greater than 0.400; eight are greater than 0.300. This suggests to us that our different estimates are finding some "rough" agreement with one another—again, hopefully detecting features of the underlying technologies.

Table 7 presents the aggregate σ estimates and confidence intervals from Table 2, along with the value-added share-weighted averages of industry-level σ estimates (denoted by σ_{WA}). They are again recorded by equation and estimator. We use the average value-added shares from 1960 to 2005 in the calculations.

TABLE 7. Aggregate elasticity of substitution estimates and weighted average of industry estimates

	GIV	Differenced		GMM	GMM (normalized)
$\sigma_{1,AG}$	0.279	0.282	$\sigma_{,AG}$	0.614	0.177
95% C.I.	(0.133, 0.425)	(0.138, 0.426)	95% C.I.	(0.608, 0.620)	(0.097, 0.257)
$\sigma_{1,WA}$	0.570	0.350	$\sigma_{,WA}$	0.688	0.621
$\sigma_{2,AG}$	1.364	0.821			
95% C.I.	(1.282, 1.446)	(0.677, 0.965)			
$\sigma_{2,WA}$	1.063	0.800			
$\sigma_{3,AG}$	0.416	0.249			
95% C.I.	(0.258, 0.574)	(0.123, 0.375)			
$\sigma_{3,WA}$	0.640	0.151			

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Solow (1964, p. 118) asserts that "[i]t seems plausible that, in general, elasticities of substitution should be smaller the more narrowly defined the industrial classification, and larger the degree of aggregation." If Solow is correct, then we would expect estimates of aggregate σ to be larger than the value-added share-weighted average of industry σ's.[19] However, the relationship between industry-level and aggregate σ's is less clear. Miyagiwa and Papageorgiou (2007) study a three-sector growth model with two sectors producing different intermediate goods that are inputs to production in a final goods sector.[20] The formula for the aggregate σ in their model is

$$\sigma = (\lambda_{1w}\theta_{1r} + \lambda_{1r}\theta_{1w})\sigma_1 + (\lambda_{2w}\theta_{2r} + \lambda_{2r}\theta_{2w})\sigma_2 + (\theta_{1w} - \theta_{2w})\,(\lambda_{1w} - \lambda_{1r})\,\phi.$$

This is a linear function of the sector-level substitution elasticities (σ_1 and σ_2 for the intermediate goods sectors; ϕ for the final goods sector) where the coefficients are functions of capital and labor income shares (θ_{ir} and θ_{iw}) and capital and labor endowment shares (λ_{ir} and λ_{iw}) in each of the intermediate goods sectors.[21]

For positive values of σ_1 and σ_2, the value of aggregate σ hinges on the coefficient weights and ϕ. Given an exogenous savings rate, ϕ stands in for substitution possibilities in consumption. To use Jones's (1965) terminology, the relationship between aggregate σ and industry substitution elasticities is dependent on the "intercommodity substitution by consumers" (ϕ) relative to the "intracommodity substitution between factors" (σ_1 and σ_2).

The evidence in Table 7 does not suggest that Solow's conjecture holds. Rather, for all cases where σ_{WA} lies outside of the estimated confidence intervals, it is *greater* than the estimated σ. Aggregate σ is *less than* the average of industry-level substitution elasticities. This is the case for both GMM system estimates—in particular our preferred normalized system estimate. Recalling the framework of Miyagiwa and Papageorgiou (2007), one interpretation of these results is that substitution possibilities for industry *outputs* are limited enough to offset the greater substitution possibilities for capital and labor *inputs* when they are considered economywide (rather than limited to within one industry).

4.4. Estimates of Industry-Level λ_L and λ_K

Tables 8A, 8B, and 8C report estimated rates of factor augmentation for GIV, OLS with first differences, and GMM system estimates, respectively. For the GIV and differenced OLS results, there are two ways to approach these estimates. First, one can look at the difference between λ_L (from equation (4)) and λ_K (from equation (3)). Second, one can look at the point estimate of $(\lambda_L - \lambda_K)$ from equation (5). Given our inclination to discount equation (4) estimates, we put more stock in the $(\lambda_L - \lambda_K)$ point estimate from equation (5). However, we comment on both perspectives later. In the case of GMM system estimation, both λ_K and λ_L are estimated separately. Therefore, the $(\lambda_L - \lambda_K)$ point estimate is always identical to the difference between the λ_L and λ_K estimates.

TABLE 8A. Estimates of factor augmentation from GIV regressions

Industry	λ_K	λ_L	$\lambda_L - \lambda_K$
1	0.001	0.002	0.000
2	0.001	−0.001	0.000
3	0.001	0.002	0.000
4	−0.010	0.002	0.003
5	0.000	−0.003	0.000
6	0.002	0.000	−0.001
7	0.001	0.000	0.000
8	0.001	0.001	0.001
9	0.001	−0.001	−0.001
10	0.004	0.000	−0.013
11	0.001	−0.004	−0.001
12	0.001	0.001	−0.001
13	0.001	0.002	−0.001
14	0.001	0.000	0.000
15	0.001	−0.005	0.000
16	0.001	0.001	0.000
17	0.001	−0.001	−0.001
18	0.001	−0.001	−0.001
19	0.001	−0.005	−0.001
20	0.001	0.011	−0.001
21	0.001	−0.001	−0.001
22	0.002	0.003	−0.003
23	0.001	−0.007	−0.002
24	0.001	0.001	0.000
25	0.003	0.000	−0.005
26	0.002	0.000	−0.002
27	0.001	0.000	0.000
28	0.001	0.001	0.000
29	0.001	−0.004	−0.001
30	0.001	−0.002	0.001
31	−0.005	0.001	0.001
32	−0.025	−0.001	0.008
33	0.001	0.006	0.001
34	0.001	0.001	−0.002
35	0.001	−0.001	0.000

In the case of GIV, looking at the separate λ_L and λ_K estimates, there is scant evidence that all (or even necessarily most) industry-level technical change is characterized as net labor-augmenting. The GIV estimates indicate that net labor augmentation is the case for 12 out of 35 industries. Similarly, looking at the GIV estimate of $(\lambda_L - \lambda_K)$, for only 18 industries is net labor augmentation indicated.

TABLE 8B. Estimates of factor augmentation from OLS regressions of differenced variables

Industry	λ_K	λ_L	$\lambda_L - \lambda_K$
1	0.012	0.042	0.023
2	−0.026	−0.100	0.040
3	−0.010	0.044	0.039
4	−0.027	0.016	0.034
5	−0.019	0.121	0.038
6	−0.027	0.438	0.028
7	0.006	−0.492	0.011
8	−0.029	−0.003	0.028
9	0.000	−0.039	0.028
10	−0.028	0.028	0.053
11	−0.012	0.018	0.021
12	0.004	0.016	0.012
13	−0.007	0.016	0.022
14	−0.026	−0.028	0.032
15	−0.015	0.094	0.033
16	−0.023	0.031	0.020
17	−0.003	0.002	0.017
18	−0.034	0.022	0.051
19	−0.006	0.020	0.019
20	−0.006	0.031	0.019
21	−0.021	0.031	0.031
22	0.005	0.049	0.041
23	0.009	0.066	0.054
24	0.001	0.003	0.016
25	−0.022	0.054	0.047
26	−0.019	−0.700	0.043
27	−0.013	−0.133	0.024
28	0.001	0.026	0.012
29	−0.016	0.047	0.048
30	−0.004	0.032	0.017
31	−0.063	0.010	0.042
32	−0.034	0.047	0.053
33	0.007	0.020	0.012
34	−0.025	−0.003	0.032
35	−0.018	0.000	0.031

The OLS with first-differences estimates contrasts starkly. Whether looking at separate λ_L and λ_K estimates (29 industries) or the $(\lambda_L - \lambda_K)$ estimate (all 35 industries), net labor augmentation is overwhelmingly indicated at the industry level. These results are consistent with a neoclassical economy where labor-augmenting technical change is not only necessary for balanced growth [Uzawa (1961) and Jones and Scrimgeour (2008)] but also endogenously the result of many

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TABLE 8C. Estimates of factor augmentation from GMM system estimation

	GMM			GMM (normalized)		
Industry	λ_K	λ_L	$\lambda_L - \lambda_K$	λ_K	λ_L	$\lambda_L - \lambda_K$
1	0.029	0.002	−0.027	0.021	0.030	0.009
2	−0.037	0.049	0.086	−0.040	0.049	0.089
3	−0.040	0.048	0.088	0.001	0.024	0.023
4	−0.041	0.022	0.063	−0.040	0.023	0.063
5	−0.023	0.023	0.046	−0.023	0.023	0.046
6	−0.029	0.000	0.029	−0.024	−0.001	0.023
7	−0.002	0.030	0.032	−0.001	0.030	0.031
8	−0.021	−0.021	0.000	0.584	−0.979	−1.563
9	0.060	0.009	−0.051	0.000	0.028	0.028
10	0.070	0.000	−0.070	−0.032	0.022	0.054
11	−0.052	0.030	0.082	0.260	−0.093	−0.353
12	0.005	0.010	0.005	0.004	0.011	0.007
13	−0.015	0.015	0.030	−0.016	0.015	0.031
14	−0.028	0.001	0.029	−0.030	0.002	0.032
15	−0.020	0.031	0.051	−0.019	0.031	0.050
16	−0.025	0.004	0.029	−0.023	0.038	0.061
17	−0.004	0.015	0.019	−0.004	0.016	0.020
18	0.039	−0.004	−0.043	−0.031	0.019	0.050
19	−0.010	0.011	0.021	−0.010	0.011	0.021
20	−0.023	0.020	0.043	−0.023	0.020	0.043
21	−0.043	0.002	0.045	−0.042	0.024	0.066
22	0.022	0.038	0.016	0.021	−0.002	−0.023
23	0.002	0.002	0.000	−0.015	0.073	0.088
24	0.005	0.006	0.001	0.004	0.005	0.001
25	0.071	−0.002	−0.073	−0.028	0.011	0.039
26	0.002	0.002	0.000	−0.008	0.016	0.024
27	−0.060	0.039	0.099	0.094	−0.025	−0.119
28	0.002	0.011	0.009	0.005	0.014	0.009
29	−0.018	0.042	0.060	−0.016	0.040	0.056
30	0.787	−1.656	−2.443	0.812	−1.696	−2.508
31	−0.054	0.011	0.065	−0.053	0.012	0.065
32	−0.038	0.020	0.058	−0.038	0.020	0.058
33	0.455	−0.852	−1.307	0.919	−1.736	−2.655
34	−0.011	−0.001	0.010	−0.014	0.001	0.015
35	−0.021	0.022	0.043	−0.021	0.023	0.044

firms in balanced growth choosing labor-augmenting technical change [Acemoglu (2003)]. However, factor-augmentation rates are estimated based on time trend coefficients. This would seem to be a case where excluding longer-run information on the levels of variables is particularly damaging.

The GMM system estimates are better suited to take into account the longer-run variation. Still, relative to GIV, the GMM results are less suggestive of qualitative heterogeneity in interindustry net factor-augmentation rates. For the non-normalized system, net labor augmentation holds (point estimate–wise) in 28 out of 35 industries. The result is similar when based on the normalized system: 29 out of 35 industries. There are only two industries where both normalized and non-normalized system GMM estimation suggests that net capital augmentation characterizes the data.

Comparing the GMM σ estimates from Table 5 with the factor augmentation rates from Table 8C, one notes that particularly implausible factor augmentation rates are reported for industries where σ is estimated to be almost exactly unity. For the Cobb–Douglas case, there is no distinction between labor-augmenting and capital-augmenting technical change. It is therefore not surprising that estimates of separate factor augmentation rates become meaningless in cases closely approximating the Cobb–Douglas case.

5. MODEL SECTOR σ ESTIMATES

All estimates reported so far are industry-level. However, the classification of industries in our data does not correspond to the choice of sectors in economists' general equilibrium models. Furthermore, because industries may produce more than one type of commodity (e.g., investment and consumption goods) and different industries may produce the same type of commodity (e.g., both *tobacco* and *leather* industries produce tradables), the mapping of industries to model sectors is not straightforward. In this section we define some sectors that are common in economists' models and then describe a method for providing just such a mapping.[22]

The method we follow is basically that of Valentinyi and Herrendorf (2008).[23] We define model sectors and then map our actual industry elasticity of substitution (σ) estimates to those sectors. The model sectors are agriculture (A), manufactured consumption (M), services (S), equipment investment (E), and construction investment (C). Weighted linear combinations of σ's for these basic sectors constitute various composite sector σ's of interest to economic theory:

$$\text{Agriculture } (A),\ \text{Manufacturing } (M;\ E;\ C),\ \text{and Services } (S); \tag{10}$$

$$\text{Consumption } (A;\ M;\ S) \text{ and Investment } (E;\ C); \tag{11}$$

$$\text{Tradables } (A;\ M;\ E) \text{ and Nontradables } (S;\ C); \tag{12}$$

$$\text{Agriculture } (A) \text{ and Nonagriculture } (M;\ S;\ E;\ C). \tag{13}$$

Our industry σ estimates are based on the 35-industry KLEM data, which roughly correspond to the older two-digit SIC classification. However, to identify the final uses of goods from various industries, we will use the BEA 1997 benchmark input–output accounts, which are based on the four-digit NAICS classification. For the basic model sectors, the NAICS industries and final uses of their output map as follows:

A is based on total expenditures on the production of NAICS industries 1110, 1120, and 1130; **(14)**

M is based on consumption expenditures on the production of NAICS industries 3110 through 3399; **(15)**

S is based on total expenditures on the production of NAICS industries 4200 through 8140; **(16)**

E is based on investment expenditures on the production of all NAICS industries *except* 2301 and 2302; **(17)**

C is based on total expenditures on the production of NAICS industries 2301 and 2302. **(18)**

Note that for construction investment (C) all expenditures are included because, by definition, all construction expenditures are investment expenditures.

Based on the "final uses" in terms of expenditures that are reported in Table 2 of the 1997 BEA benchmark accounts, investment expenditures are the sum of "Private fixed investment," the "Change in private inventories," and the investment share of net exports. Because the BEA does not provide this share, we follow Valentinyi and Herrendorf (2008) and assume that the investment share is equal to the investment share of not-exported output. In other words, define this share as the sum of "Private fixed investment" and the "Change in private inventories" divided by the sum of "Private fixed investment," the "Change in private inventories," and "Personal consumption expenditures." This share is then applied to net exports, i.e., the sum of "Exports of goods and services" and "Imports of goods and services." Likewise, consumption expenditures are the sum of "Personal consumption expenditures" and the consumption share of net exports.

For each of the set (A, M, S, E, C) we calculate a 1997 expenditure share in total U.S. GDP: $(y_A, y_M, y_S, y_E, y_C)$. These expenditure shares will be used as weights applied to KLEM industry data–based σ estimates to achieve to estimates for model composite sector groupings (10) through (13).

Turning to the KLEM industry classifications, recall that industry numbers and names are provided in Table 1A. We first categorize KLEM industries into agriculture (A'), manufacturing (M'), services (S'), equipment (E'), and

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construction (C'):

A' contains KLEM industry 1; **(19)**

M' contains KLEM industries 7 through 27; **(20)**

S' contains KLEM industries 32 through 34; **(21)**

E' contains KLEM industries 7 through 27 and 32 through 34; **(22)**

C' contains KLEM industry 6. **(23)**

For groupings (19) through (23) we use the average KLEM industry value-added shares from 1958 through 2005 ($v_1, \ldots, v_{35}$) and elasticity of substitution estimates ($\sigma_1, \ldots, \sigma_{35}$) to construct the following elasticities:

$$\sigma_{A'} = \sigma_1; \tag{24}$$

$$\sigma_{M'} = \frac{\sum_{i=7}^{27} v_i \sigma_i}{\sum_{i=7}^{27} v_i}; \tag{25}$$

$$\sigma_{S'} = \frac{\sum_{i=32}^{34} v_i \sigma_i}{\sum_{i=32}^{34} v_i}; \tag{26}$$

$$\sigma_{E'} = \frac{\sum_{i=7}^{27} v_i \sigma_i + \sum_{i=32}^{34} v_i \sigma_i}{\sum_{i=7}^{27} v_i + \sum_{i=32}^{34} v_i}; \tag{27}$$

$$\sigma_{C'} = \sigma_6. \tag{28}$$

Note that these value-added share-weighted elasticities do not account for the breakdown of industry production into investment and consumption. Input–output accounts are not available based on the KLEM (or older two-digit SIC) classification, so we rely on the input–output accounts that are available and the 1997 expenditure weights (y_A, y_M, y_S, y_E, y_C) defined earlier.

We now define model sector elasticity based on the groupings (10) through (13) and the components defined previously:

$$\sigma_{AGR} = \sigma_{A'} \quad \sigma_{SER} = \sigma_{S'} \quad \sigma_{MANU} = \sigma_{M'}; \tag{29}$$

$$\sigma_{CONS} = \frac{y_A \sigma_{A'} + y_M \sigma_{M'} + y_S \sigma_{S'}}{y_A + y_M + y_S} \quad \sigma_{INV} = \frac{y_E \sigma_{E'} + y_C \sigma_{C'}}{y_E + y_C}; \tag{30}$$

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TABLE 9. Estimates of model sector elasticities of substitution

Industry	GIV (σ_1)	GMM	GMM (normalized)
Agriculture	0.605	0.675	−0.392
Manufacturing	0.458	0.631	0.569
Services	0.676	0.691	0.657
Consumption	0.636	0.689	0.649
Investment	0.492	0.609	0.521
Tradables	0.489	0.678	0.631
Nontradables	0.643	0.670	0.621
Agriculture	0.605	0.675	−0.392
Nonagriculture	0.606	0.672	0.629

$$\sigma_{TRAD} = \frac{y_A\sigma_{A'} + y_M\sigma_{M'} + y_E\sigma_{E'}}{y_A + y_M + y_E} \quad \sigma_{NON-TRAD} = \frac{y_S\sigma_{S'} + y_C\sigma_{C'}}{y_S + y_C}; \qquad (31)$$

$$\sigma_{AGR} = \sigma_{A'} \quad \sigma_{NON-AGR} = \frac{y_M\sigma_{M'} + y_S\sigma_{S'} + y_E\sigma_{E'} + y_C\sigma_{C'}}{y_M + y_S + y_E + y_C}. \qquad (32)$$

Although some of these elasticities are straightforwardly based on value-added weighted KLEM industry-level σ estimates (e.g., σ_{AGR}), others have components that are weighted to account for only the share of the industries' output that is used for consumption or investment (e.g., σ_{CONS} and σ_{INV}).

The elasticities defined in (29) through (32) are reported in Table 9 based on our industry-level GMM system estimates (both normalized and not). We also include sector σ's based on equation (3) (yk) GIV estimates. Recall that the normalized system GMM estimated for KLEM industry 1 (*agriculture*) is negative. This anomaly, of course, carries through to the model sector σ_{AGR}. Because it is economically nonsensical, we have to discount it in our discussion.[24]

The most striking fact is how little difference there is across sector σ's based on any given estimation technique. Based on GIV, all sector σ's are between 0.458 and 0.676. For normalized system GMM (excluding σ_{AGR}), they range only between 0.527 and 0.544. For non-normalized system GMM, that range is 0.631 to 0.691. This is notable given that, for example, a typical CGE study of the effect of corporate taxation on investment typically compares calibrations using (aggregate) σ values of 0.5, 1.0, and 1.5.[25]

There are also some notable similarities across estimation techniques. The consumption sector's elasticity is always greater than the investment sector's. The services sector's elasticity is also always greater than that of the manufacturing sector.

For industry-level σ's, our preferred estimates are from the normalized system. However, as discussed in Section 4.2, general equilibrium models typically have sectoral production functions that have additional, invariant structural parameters besides σ. Therefore, our preferred model sector σ estimates are associated with the non-normalized system GMM estimates. These estimates also suggest that the tradables sector has a greater elasticity than the nontradables sector, though the difference is quite small (0.008).[26] Our preferred vector of model sector estimates is ($\sigma_{AGR}, \sigma_{MANU}, \sigma_{SER}, \sigma_{CON}, \sigma_{INV}, \sigma_{TRAD}, \sigma_{NON\text{-}TRAD}, \sigma_{NON\text{-}AGR}$)= (0.675, 0.631, 0.691, 0.689, 0.609, 0.678, 0.670, 0.672).

6. CONCLUSIONS

Is the most important concern in setting corporate tax policy potential distortions to the economy's aggregate capital stock, or are interindustry distortions in capital accumulation also an important subject for research? Are growth theorists to understand labor-augmenting technical change in the aggregate as the net result of various types of technical change at the firm and industry levels, or as the coordination of optimal technology choices across the economy? Are we to understand aggregate capital–labor substitution possibilities as basically a weighted average of industry-level substitution possibilities, or as more defined by consumers' willingness to substitute across different goods and services?

This paper attempts to shed light on these questions by providing industry-level estimates of the elasticity of substitution (σ) and rates of factor augmentation for the United States, along with aggregate estimates based on the same data. We exploit the first-order conditions associated with a CES production function to yield relationships between factors of production and relative prices. Additionally, time trends in these relationships imply separate rates of factor augmentation for labor and capital. We estimate these relationships with data covering 35 industries at roughly the two-digit SIC level from 1960 to 2005.

Our findings suggest that aggregate U.S. σ is less than unity and likely less than 0.620. The same likely is true for the large majority of individual industries. Our results suggest that aggregate σ is less than the value-added share-weighted average of industry σ's. This, in turn, suggests the importance of limited substitution possibilities for the uses of industry outputs. Although there is considerable heterogeneity in the industry-level estimates, the Cobb–Douglas hypothesis appears to be roundly rejected at both the aggregate and industry levels. The Cobb–Douglas hypothesis may be a reasonable starting point in static models that lack a specification of technology, but the evidence presented here suggests that it is inappropriate in dynamic contexts.

We estimate technical change in the aggregate to be net labor-augmenting. However, at the industry level there is much more uncertainty. There is little evidence that the type of technical change is uniform across industries, though the large majority of industries do appear to be characterized by net labor augmentation. For several individual industries, technical change may be characterized by net capital augmentation.

The estimates we report are industry-level. We attempt to link them to theory by mapping industry-level substitution estimates to sector classifications that are common in the general equilibrium models of economists. We find that σ's across model sectors lie in a fairly narrow range (relative to the range of values often explored in CGE models). Given that, we do find that manufacturing sector σ is likely lower than that of the services sector; also that the investment sector σ is likely lower than that of the consumption sector.

The aggregate and model σ estimates presented here are will hopefully be of use in calibration and policy experiments in CGE and growth models. However, it is important to note that the estimates themselves are model-dependent. In particular, we are assuming a particular specification of technology, which, in turn, determines how much and what type of variation in the data is the basis for estimating σ. Different assumptions on technology will generally yield different results. Despite this, we do believe that our specification allows us to capture more of the variation in technology than existing industry-level studies. We also think that the specification is reasonable in being consistent with the existence of long-run balanced growth in standard growth theory.

NOTES

1. See Chirinko (2008) for a review of the recent literature. Eisner and Nadiri (1968), Lucas (1969), and Berndt (1976) are pioneering studies in the literature.

2. An aggregate σ greater than unity is a sufficient condition for unbounded endogenous growth in a standard growth model with physical capital accumulation [e.g., Palivos and Karagiannis (2010)]. Nakamura and Nakamura (2008) develop a model where, even though final goods production using intermediate goods is Cobb–Douglas, endogenous substitution of capital for labor in intermediate goods production ("mechanization") can imply a CES production function with $\sigma > 1$. Also, Nishimura and Venditti (2004) shows that the value of σ can have implications for whether indeterminancy takes place in growth models.

3. Chirinko (2002) surveys CGE studies of corporate taxation and, for each, computes the "contribution [ξ] of a change in σ on the welfare increment following from a given tax reform" (p. 348). Across studies ξ lies in the range of 0.19 to 0.94 and is at least 0.50 in all but one study. The studies surveyed are Harberger (1959), Ballard et al. (1985), Engen et al. (1997), Roeger et al. (2002), and Altig et al. (2001). The last of these represents $\xi = 0.19$.

4. An exception is Fullerton and Rogers (1993), who carry out a CGE study with 19 industries, each with a separate σ. However, they ultimately base their calibration on "central-tendency" values based on various σ estimates predating 1976. They also do not report on interindustry effects. Fullerton and Rogers rely on the survey and compilation of estimates in Caddy (1976). For a comprehensive review of the first generation of empirical σ studies, Caddy is an excellent source.

5. Antràs (2004) and Klump et al. (2007, 2008) are examples of studies estimating aggregate σ that allow for biased technical change.

6. This database was originally developed by Jorgenson et al. (1987).

7. The importance of normalization was elaborated on subsequently by Klump and de La Grandeville (2000), Klump and Preissler (2000), and de La Grandville and Solow (2006).

8. This is particularly helpful because the nonlinearities makes convergence to a specific vector that a parameter estimates sometimes sensitive to the assumed vector of initial values. Alternatively, in estimating the non-normalized system, (2.3)–(2.5), we conduct estimation, leaving α a free parameter whenever possible. However, there are five industries (1, 10, 18, 25, and 33) for which convergence

is problematic while α is left free. In those few cases, the parameter is fixed at its arithmetic average from the data.

9. The aggregate studies of Antràs (2004) and Klump et al. (2007) utilize the Ho and Jorgenson (1999) labor input.

10. Diagnostics tests regarding (i) and (ii) are discussed later, but tables reporting the results in full were omitted for the sake of saving space. Those tables are available from the author upon request.

11. The exception is the DW statistics for industry 25; equation (1).

12. For only one industry (6) is the null rejected for each of the three specifications.

13. Either for seven (trace) or eight (max λ) industries in the case of (*yl*, *wp*); either for six (trace) or seven (max λ) industries in the case of (*kl*, *rw*).

14. For a thorough (though not exhaustive) list of aggregate σ estimates from the last 20 years, see Chirinko (2008, Table 1).

15. Klump et al. (2007) allow a more flexible specification of factor augmentation using a Box–Cox transformation. However, when they constrain factor augmentation to be exponential, their σ is estimate is still in the range 0.5 to 0.6.

16. "Reasonable" here refers specifically to the priors from neoclassical growth theory.

17. Temple (2009) provides a useful discussion on the (unsettled) issues concerning when and for what purposes normalization is appropriate. By providing both sets of industry and model sector estimates, we prefer to remain somewhat agnostic.

18. In writing this paper, purely in terms of exposition, the author notes that what may seem like "reasonable" variation across nine estimates of aggregate σ from three different equations and three different estimators (e.g., Table 2) may not "look" as "reasonable" when the estimates are multiplied by 35 industries (e.g., Tables 3, 4, and 5) and the variation in point estimates is not systematic across industries.

19. Solow's intuition is that, "one can imagine subindustries [*sic*] each of which has zero elasticity but which when aggregated exhibit highly variable factor proportions because of the mix of subindustries" [Solow (1964, p. 118)]. This certainly seems plausible. Moreover, Jones (1965) formally demonstrates in a two-sector general equilibrium model that "[aggregate] σ can be positive even if the elasticity of substitution in each industry is zero, for it incorporates the effect of intercommodity [*sic*] substitution by consumers as well as direct intracommodity [*sic*] substitution between factors" (p. 565).

20. Miyagiwa (2008) extends the model from a closed to a small open economy.

21. Endowment shares correspond to L_i/L and K_i/K, where $L$and K are total endowments of labor and capital in the economy.

22. An ideal mapping would also take into account intercommodity substitution possibilities for the outputs of different industries. [See Jones (1965), Miyagiwa and Papageorgiou (2007), and Section 2.] Unfortunately, doing so is not straightforward and is beyond the scope of this paper.

23. Valentinyi and Herrendorf's (2008) method is actually more complex than the one we apply here because they are concerned with mapping observed industry capital shares into model sector shares. Mapping income shares runs into an additional problem in that model sectors "typically use only capital and labor to produce final output [while] industries in the data use intermediate inputs, capital, and labor to produce intermediate inputs for other industries and final output" (p. 821). Therefore, model shares must accurately reflect the fact that some income generated by one industry's output is due to labor from other industries that produced its intermediate goods.

24. The negative KLEM industry estimate factors into other model sector σ's as well. However, we again note that agriculture is a very tiny part of U.S. value added over our sample (averaging 0.029); its effect on the other sector σ's is negligible.

25. See Chirinko (2002) for a survey. Altig et al. (2001) is an exception in reporting on a narrower range: σ equal to 0.8 and 1.0. Even this range is wide compared to those reported in Table 9 and Altig et al.'s results suggest that the elasticity of national income with respect to σ in that range is less than 0.2.

26. GMM estimation of the normalized system also suggests this; only the GIV estimates suggest otherwise.

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