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Derivation of an evolution equation for two-time correlation function

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Abstract. An expansion is developed of the two-point correlation function in the two-time correlation functions up to the expansion terms of third order. This expansion is necessary for the rigorous derivation of kinetic equations that objectively describe physical phenomena in a 'single' plasma with Langmuir turbulence. The correspondence of the expansion terms with various physical processes of the traditional weak plasma turbulence theory is discussed. Perspectives are outlined of application of the results to studies in the physics of non-ideal plasmas.

1. Introduction

In Erofeev (2000, 2002a, 2002b) we have highlighted two crucial shortcomings pertinent to traditional plasma physics. First, we pointed out the *senselessness* of the plasma ensemble considerations from the viewpoint of learning the physics of plasma evolution. Note that plasma theory was constructed from the beginning as a science oriented on the statistics of probabilistic plasma ensembles. Contrary to the most elementary common sense, it was accepted, after Gibbs (1902) with his success in erecting statistical mechanics, that we can always substitute real plasmas by probabilistic plasma ensembles and formulate conclusions about the physics of the plasma evolution on a basis of the evolution of the ensemble statistics. Factually, the idea of the ensemble method had exerted a key influence on the formulation of the most fundamental of the plasma physical theoretical notions†. Correspondingly, plasma theorists always freely substituted real plasmas by plasma ensembles, whether doing deliberately or unintentionally. We have demonstrated that some of the known plasma ensemble considerations lead to different final conclusions regarding the physics of plasma evolution. Hence, the ensemble substitution cannot help in gaining an objective picture of plasma physical evolution and generally obscures it.

The second shortcoming in traditional plasma theory is the absence of an adequate understanding of the essence and significance of the *asymptotic* character of convergency of *any* nonlinear perturbation expansion that we may invent for the justification of the semi-heuristic ideas of the physical phenomena in a plasma. Let

† Take, for instance, the notion *wave*. In tradition of physics, it was developed in an abstraction of a continuous medium. Real media consists of discrete elements: molecules, atoms, ions *etc.*, and the latter cannot be reduced to the former without ensemble averaging.

us expand this statement a bit more. For definiteness, we refer to the problems of plasma kinetics: plasma kinetic theory constitutes the most logical branch of plasma physical theory.

Undoubtedly, the fullest description of ideal classical plasma is given by simultaneous Maxwell and Klimontovich–Dupree equations (Dupree 1963; Klimontovich 1967), but their integration is technically impossible. (Really, it implies a simultaneous integration of motion equations for an immensely large total number of charged plasma particles.) This forces theorists to use more simplified approaches for the plasma description. None of the simplified approaches gives a picture of plasma evolution that does not diverge over time from the real plasma macroscopic evolution. Therefore, we can rely upon simplified descriptions of plasma evolution only on restricted time scales. With this, any trials to substantiate the recipes of corresponding descriptions by rigorous perturbation expansions will at best provide perturbations that first converge up to some optimal order of consideration, but then inevitably diverge. This gives rise to a problem of a *rational choice* of the leading order approximation in the perturbation expansion, since *different leading order approximations result in differing pictures of the plasma evolution, even within the converging series of the same perturbation technique*. Having never paid any attention to the asymptotical character of the perturbation convergency, the traditional plasma kinetic and plasma turbulence theories had also never paid proper attention to a sensible choice of basic objects for the theory to deal with†. Hence, the virtue of respective perturbation calculations is doubtful and most of the corresponding results should be reconsidered. In former papers we have shown how to develop the plasma kinetic description while refraining from the ensemble substitution and achieving a more adequate plasma description, as is only possible in view of the asymptotic convergency of the theory (Erofeev 1997, 1998a). Using the respective recipes, we have visualized substantial changes in the image of Langmuir turbulence. Namely, we discovered the intense decay of longwavelength Langmuir quanta that precludes the formation of the Langmuir condensate (see the appendix in Erofeev (2002a)) and prohibits Langmuir wave collapses (Erofeev 2002b)‡. In addition, we discovered that traditional results (Tsytovich 1966) undervalued the scattering of short Langmuir waves by plasma electrons (Erofeev 2000).

In view of the above observations, a problem arises of creating a logical theory of Langmuir turbulence in a 'single' evolving plasma. Developing a corresponding theory implies heightening the accuracy of the calculations reported in Erofeev (1997, 1998a). The reason for this is the necessity to derive the correct description of four-wave processes§, which assumes the calculation of an extra order in the expansion of the wave kinetic equation in powers of the turbulence energy density.

In this paper we derive the evolution equation for a *two-time* correlation function $\Phi_{ijkl}(\mathbf{R}, t, t')$,

$$\Phi_{ijkl}(\mathbf{R}, t, t') = \langle \delta F_{ij}(\mathbf{r} + \mathbf{R}, t) \delta F_{kl}(\mathbf{r}, t') \rangle_{\mathbf{r}}.$$

† Recall again the notion of the wave. The logic of Erofeev (1997) dictates the concept of a plasma wave that comprises the Lorentzian structure of the leading order of the wave frequency spectrum. Approaches of predecessors assumed a delta-functional frequency spectrum of the leading order of the wave correlation function.

‡ The idea of longwavelength Langmuir quanta conservation was always rendered as the most consistent of heuristic ideas of the traditional weak plasma turbulence theory.

§ Recall their role in images of Langmuir turbulence as explained in Erofeev (2002a, 2002b) after studies by Malkin (1982a, 1982b).

(Here $\delta\hat{F}$ describes the microstructure of the electromagnetic field tensor that is due to the plasma discreteness and the subscript $\mathbf{r}$ symbolizes the averaging over the 'physically large' plasma volume.) The corresponding derivation constitutes the first of two stages of the development of Langmuir wave kinetics. In the second stage, the evolution equation of the two-time correlation function is used for developing the final plasma kinetic description in terms of functions depending on only one time variable. In particular, it is used in the case of weakly turbulent collisionless plasma for developing equations that describe the evolution of the wave spectral density $n_{\mathbf{k}}(t)$ and the evolution of the electron and ion distribution functions $f_{\mathrm{e,i}}(\mathbf{p}, t)$. Note that the reported calculations are important not only for the problem of wave kinetics.

The final results of forthcoming calculations were formerly reported in Erofeev (1998b). This paper did not contain any descriptions of the calculations; the interpretations of results given in that paper have become a bit outdated.

This paper is organized as follows. In Sec. 2 we briefly describe the beginning of our calculation approach. In Sec. 3 we describe the logic of expansion of the *two-point* correlation function $\langle N_\alpha(\mathbf{r}+\mathbf{R}, \mathbf{p}, t)\delta F_{kl}(\mathbf{r}, t')\rangle$ in terms of the two-time correlation function: this expansion constitutes the basis of our title calculation. (Here N_α symbolizes the Klimontovich–Dupree distribution function.) For brevity we use a graphic means in the writing of intermediate formulae. We present the final form of the corresponding expansion in Sec. 4. In Sec. 5 we discuss its correspondence to the traditional conceptions of wave interactions in a turbulent plasma. The basic results of the study and possibilities of their application to plasma research are summarized in Sec. 6. Analytical expressions corresponding to the most important terms in the diagrammatic expansion of the two-point correlation function are left to the appendix.

2. Brief outline of the logic of kinetic calculations for 'single' plasmas

An expanded description of the basic ideas of our approach and the diverse aspects of the corresponding vision of problems of plasma kinetics can be found in Erofeev (1997, 1998a, 2000, 2002a, 2002b). In this paper we only highlight the conception of respective calculation logic, to a degree necessary for a better understanding of the study intended. The notation that we adopt throughout the paper is the same as was developed in Erofeev (1997).

The main intention of our calculations is to reduce the full plasma description to a simplified description of plasma physical evolution, which most adequately approximates the real plasma evolution. We should stress that the final kinetic plasma descriptions cannot bear any universality. The means of the plasma description and the corresponding interpretation of the final equations, depend substantially on the physical content of the problem under study. We were forced to substitute the ensemble averaging of the Klimontovich–Dupree distribution by an appropriate averaging in six-dimensional (6D) phase space of the particle spatial positions $\mathbf{r}$ and momentums $\mathbf{p}$; the design of the respective averaging correlates with the geometrical and other physical aspects of the problem. In addition, the very procedure of equation derivation depends on the physical situation: the last stage of kinetic calculations for turbulent collisionless plasma and that for quit plasma with Coulomb collisions should have a substantially different content. In this sense, our concept of plasma kinetic studies differs radically from traditional approaches that implied the existence of universal plasma kinetics (recall the common attitude to kinetics after Bogoliubov (1946), Born and Green (1949), Kirkwood (1946) and Yvon (1935) that is collectively known as the BBGKY kinetics).

The basic goal of our considerations is to develop an evolution equation for the well-defined statistic of the distribution function $f_{\mathrm{e,i}}(\mathbf{r}, \mathbf{p}, t)$, the phase space average of the Klimontovich–Dupree particle distribution†. The function f_α is advanced in time by the *two-point* correlation function, $\langle \delta F^{i\beta}(\mathbf{r} + \mathbf{R}, t') N_\alpha(\mathbf{r}, \mathbf{p}, t) \rangle_{\mathbf{r}}$; the two-point correlation function is advanced in time by the *three-point* correlation function, $\langle \delta F^{i\beta}(\mathbf{r} + \mathbf{R}', t'') \delta F^{j\gamma}(\mathbf{r} + \mathbf{R}, t') N_\alpha(\mathbf{r}, \mathbf{p}, t) \rangle_{\mathbf{r}}$; this is advanced by the four-point correlation function, etc. The corresponding evolution equations constitute some hierarchy. This equation hierarchy is insufficiently suitable for the constructive plasma description. Nevertheless, it can sometimes be used for the derivation of more useful kinetic equations that operate by functions depending on only one time variable. In particular, this is the case of a weakly turbulent collisionless plasma (and also the case of a plasma with a moderate level of Coulomb collisions which we discuss in Sec. 6). In this case, the plasma evolution can be described by simultaneous equations governing the evolution of the distribution functions f_α and the wave spectral density $n_{\mathbf{k}}$. These equations are obtained as a result of calculations falling into the two stages that were mentioned above. In the first stage we derive an approximate evolution equation for the two-point correlation function. To obtain this equation, we truncate the above equation hierarchy up to a necessary order and develop a respective perturbation calculations on the basis of the truncated equation hierarchy.

The two-point correlation function is always integrated in all the final equations. With an accuracy sufficient for integrations, it can be expressed in terms of a *two-time correlation function*. We introduce the two-time correlation function $\hat{\Phi}(\mathbf{r}', t', \mathbf{r}, t)$ as an averaged product of two terms $\delta \hat{F}$ that we take at points that are separated by a fixed spatial vector; the averaging is performed over the 'spatial projection' of the 6D parallelepiped-shaped neighborhood of the current point $(\mathbf{r}, \mathbf{p})$,

$$\Phi^{i\beta j\gamma}(\mathbf{r}', t', \mathbf{r}, t) = \langle \delta F^{i\beta}(\mathbf{r}', t') \delta F^{j\gamma}(\mathbf{r}, t) \rangle. \tag{1}$$

This function evolves according to the vacuum Maxwell equations when the external charge currents and densities are associated with the respective integrals of the two-point correlation functions. The expression of the two-point correlation function in terms of the two-time correlation function is obtained via the iterating of the evolution equation of the two-point correlation function. Obtaining this expression (and, consequently, the evolution equation of the two-time correlation function) closes the first stage of calculation of the final kinetic equations. It is the completing of this stage that constitutes the goal of this paper. Note that this stage has a common content both for a weakly turbulent collisionless plasma and for a quit plasma with a moderate level of Coulomb collisions.

In this way, we expand the two-point correlation function $\langle \delta N \delta \hat{F} \rangle$ in the two-time correlation functions $\hat{\Phi}$.

† Note that in this paper we do not specify the manner of phase space averaging. The final results of the study do not depend on it and only their further application raises the question of specific features of particular physical situations.

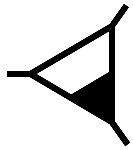

Figure 1. An asymmetrical vertex.

3. Expansion of the two-point correlation function in the two-time correlation functions

A straightforward calculation of the expansion of the two-point correlation function in the two-time correlation functions following the logic of Erofeev (1997) would have been very tedious. Fortunately, the body of intermediate manipulations can be substantially shortened due to some results and approaches developed in diagrammatic techniques.

Recall the graphic means of formulae writing that were introduced in Erofeev (1997).

The first of the theory objects is a bare Green function ${}^0G_\alpha(\mathbf{r}, \mathbf{p}, t, \mathbf{r}', \mathbf{p}', t')$, a solution to the equation

$$\left[\frac{\partial}{\partial t} + v^\beta \frac{\partial}{\partial r^\beta} + \frac{e_\alpha}{c} v_i {}^0F^{i\beta} \frac{\partial}{\partial p^\beta}\right] {}^0G_\alpha = \delta^3(\mathbf{p} - \mathbf{p}')\delta^3(\mathbf{r} - \mathbf{r}')\delta(t - t'). \tag{2}$$

We denote this function by a thin solid line. The second object is an operator of the electromagnetic Green function that can be associated with the well-known delayed potentials. We define this operator as a kernel for the integral expression of the EMF tensor in terms of the microdistribution N_α,

$$\begin{aligned} F_{ik}(\mathbf{r}, t) &= {}^0F_{ik}(\mathbf{r}) + \delta F_{ik}(\mathbf{r}, t), \\ \delta F_{ik}(\mathbf{r}, t) &= \sum_\alpha e_\alpha \int_{-\infty}^{t} \mathscr{F}_{ik}(\mathbf{r}, t, \mathbf{r}', \mathbf{v}', t') N_\alpha(\mathbf{r}', \mathbf{p}, t')\, d^3\mathbf{r}'\, d^3\mathbf{p}'\, dt'. \end{aligned} \tag{3}$$

(Here $\mathbf{v}'$ is the velocity of the particle (of kind 'α') with momentum $\mathbf{p}'$.) We denote this object by a dashed line. We also use a rectangle for the distribution function f_α and a wavy line for a *formal* function $\langle \delta N_\alpha(\mathbf{r}, \mathbf{p}, t)\delta N_{\alpha'}(\mathbf{r}', \mathbf{p}', t')\rangle$ (here $\delta N_\alpha = N_\alpha - f_\alpha$). As it is not a statistic in the strict sense of mathematics, it should always be connected with the dashed line of the electromagnetic Green function to form the statistically meaningful two-point or two-time correlation function. Nevertheless, at the intermediate stages of the forthcoming calculations we freely use the wavy lines that are not connected with dashed lines. We substantiate the appropriateness of this frivolity at the end of Sec. 4.

The next graphical object is an asymmetrical vertex as shown in Fig. 1. With this vertex, we associate the time moment t, the momentum $\mathbf{p}$, the space position $\mathbf{r}$ and the coefficient $-e_\alpha/c$. The vertex has one entry (on the left) and two exits. The vertex entry can be connected to the exit end of the line of the bare Green function, or to the exit of the line of the renormalized Green function that we define below.

The lower exit of the vertex is associated with differentiation with respect to the momentum $\mathbf{p}$ of the function connected to it. To make the analytical interpretation of diagrams easier, this exit is marked in black.

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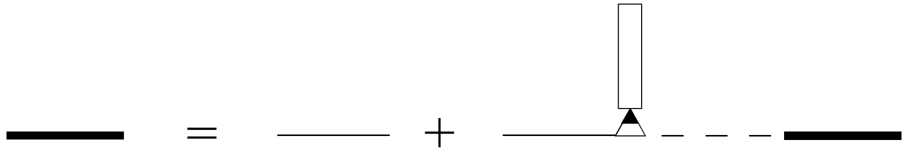

Figure 2. An equation of Dyson type for the renormalized Green function $\hat{G}_{\alpha\alpha'}$.

The upper exit is always connected to the dashed line of $\hat{\mathscr{F}}$. The related 'EMF-end', $\mathscr{F}^{i\beta}$, of the object is multiplied by v_i and by the momentum derivative $\partial/\partial p^\beta$ of the function connected to the lower exit.

The vertex entry is usually influenced by some operator, and when this is the case, integration over the corresponding variables $\mathbf{r}, \mathbf{p}$ and t takes place.

Finally, let us also introduce a *renormalized Green function* $G_{\alpha\alpha'}(\mathbf{r}, \mathbf{p}, t, \mathbf{r}', \mathbf{p}', t')$. We write it graphically as a thick solid line and define it as a solution to the Dyson-type equation shown in Fig. 2. Following this equation, the renormalized Green function can easily be expressed in terms of the operator $(\hat{\mathscr{F}}\hat{G})$. This operator is a solution to the vacuum Maxwell equations when the distributions of charges and charge currents are associated with the respective integrals of the very renormalized Green function. We present the corresponding equations at the end of the appendix.

The first stage of calculations consists of obtaining a sufficiently correct equation of evolution of the two-point correlation function. For this, we develop a formal perturbation expansion for 'correlators' $\langle \delta N_\alpha(\mathbf{r}, \mathbf{p}, t)\delta N_{\alpha'}(\mathbf{r}', \mathbf{p}', t')\rangle$. The principles for implementing such an expansion were first formulated in Erofeev (1996). The basic equation for developing the necessary perturbation technique is the nonlinear equation that governs the evolution of $\delta N_\alpha(\mathbf{r}, \mathbf{p}, t)$. This equation follows from the Klimontovich–Dupree equation,

$$\begin{aligned} &{}^0\hat{G}_\alpha^{-1}\delta N_\alpha(\mathbf{r}, \mathbf{p}, t) + \frac{e_\alpha}{c} v_i \frac{\partial f_\alpha}{\partial p^\beta} \sum_{\alpha'} e_{\alpha'} \int_{-\infty}^{t} \mathscr{F}^{i\beta}(\mathbf{r}, t, \mathbf{r}', \mathbf{v}', t')\delta N_{\alpha'}(\mathbf{r}', \mathbf{p}', t')\, d^3\mathbf{r}'\, d^3\mathbf{p}'\, dt' \\ &\quad = -\frac{e_\alpha}{c} v_i \frac{\partial}{\partial p^\beta}\delta N_\alpha(\mathbf{r}, \mathbf{p}, t) \sum_{\alpha'} e_{\alpha'} \int_{-\infty}^{t} \mathscr{F}^{i\beta}(\mathbf{r}, t, \mathbf{r}', \mathbf{v}', t')\delta N_{\alpha'}(\mathbf{r}', \mathbf{p}', t')\, d^3\mathbf{r}'\, d^3\mathbf{p}'\, dt'. \end{aligned} \tag{4}$$

Using this equation, we can iteratively develop successive approximations to an 'evolution equation' of our formal correlators $\langle \delta N_\alpha \delta N_{\alpha'}\rangle$. Conceptually, they generate a 'canonical' redaction of the Wyld diagram technique (Wyld 1961) first described by Zakharov and L'vov (1975). For our goal it suffices to start with an approximate relation from Erofeev (1996, Fig. 6). In view of (4), we should associate the symmetrical vertex of the respective diagram technique with the asymmetrical vertex from Fig. 1 by means of the relation shown in Fig. 3. With this, the role of the bare Green function of the auxiliary diagram technique goes to our renormalized Green function. For this reason we rewrite the equation from Fig. 6 of Erofeev (1996) in the form of the equation in Fig. 4. In this figure we substituted the thick lines of our renormalized Green function $G_{\alpha\alpha'}(\mathbf{r}, \mathbf{p}, t, \mathbf{r}', \mathbf{p}', t')$ for the thin lines of Fig. 6 from Erofeev (1996); the inverse operator of the bare Green function of the auxiliary diagram technique (i.e. the inverse operator of our renormalized Green function) is shown by the first two diagrams in the first square brackets in the figure.

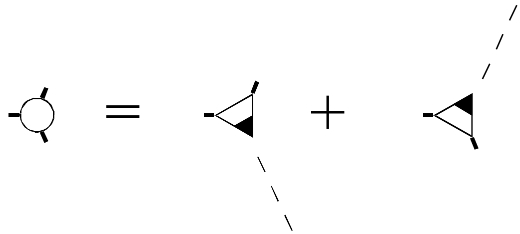

Figure 3. A symmetrical vertex of an auxiliary diagram technique.

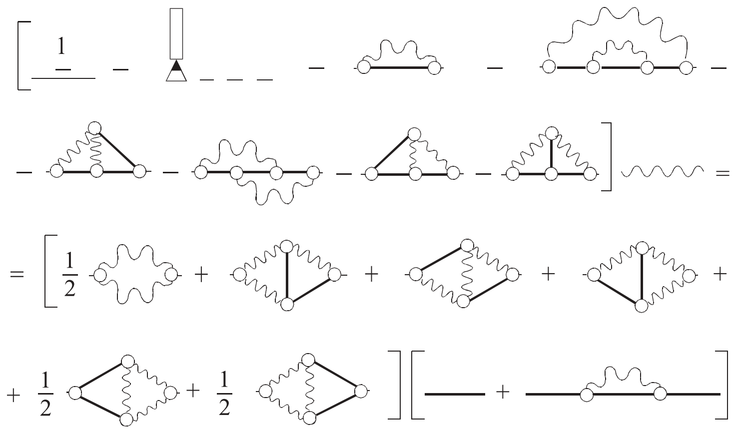

Figure 4. An auxiliary evolution equation for the *formal* function $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$.

We reiterate that the equation in Fig. 4 is a formal one, since the corresponding 'correlator' $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$ is not a consistent statistic. Still, if we attach the dashed lines of the electromagnetic Green function $\hat{\mathscr{F}}$ to the right ends of the diagrammatic expressions on the right-hand and left-hand sides of this equation, we get an actual evolution equation of a consistent statistic of the two-point correlation function $\langle \delta N_\alpha \delta \hat{F} \rangle$. It should be stressed that an absence of statistical sense in the inner lines of $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$ in terms of the respective equation does not lead to an absence of sense in the equation. The reason for this is that their analytical counterparts are integrated and under the sign of integration the contribution of these statistically empty functions is well reducible to integrals based on the consistent statistics only. (Ultimately, the evolution equation of the two-point correlation function only depends on the rough characteristics of the plasma macroscopic distributions and not on the tiny details of the microdistribution.)

Let us write the evolution equation of the two-point correlation function in the form given in Fig. 5 (compare with Fig. 16 in Erofeev (1997)). The first term on the right-hand side of the equation gives the lowest order in the expansion of the two-point correlation function in the two-time correlation functions. To obtain the second order in the expansion, we substitute the two-point correlation function, the multiplier in the second term of the right-hand side of the relation in Fig. 5, by all at the right-hand side. In the third iteration this recipe should be repeated, then we omit the diagrams that contain more than three wavy lines. As a result, we get the relation shown in Fig. 6. On the right-hand side of this relation, all the terms

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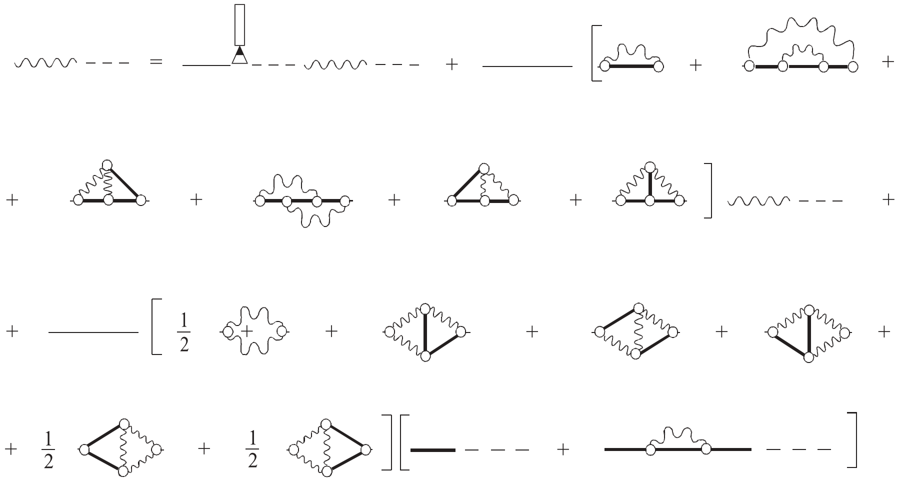

Figure 5. An expression for the two-point correlation function $\langle \delta N_\alpha \delta \hat{F} \rangle$: first iteration.

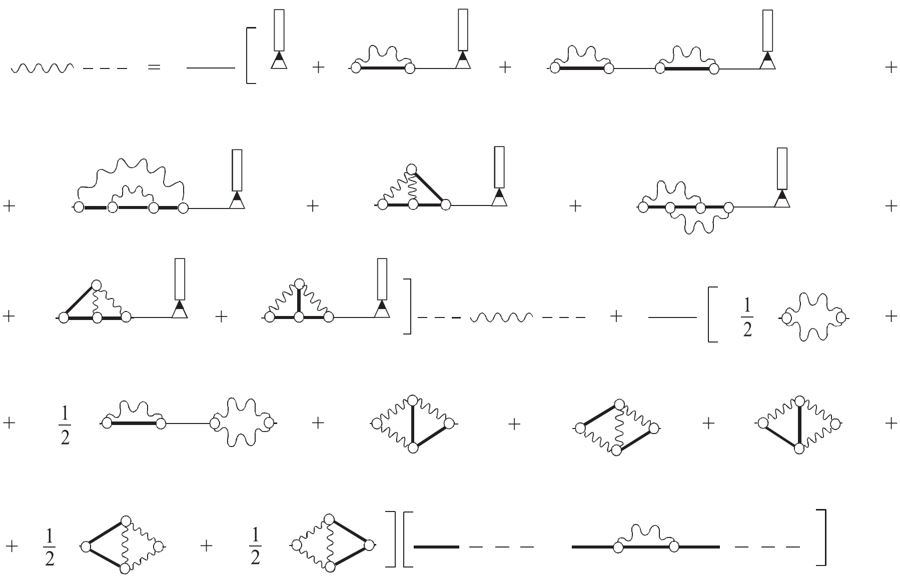

Figure 6. An expression for the two-point correlation function $\langle \delta N_\alpha \delta \hat{F} \rangle$: third iteration.

are retained that are necessary for expanding the two-point correlation function with a desired accuracy. That is, to obtain the expansion terms of the third order in powers of the turbulence energy density.

The rest of the calculations should consist of eliminating all of the bare wavy lines, i.e. formal functions $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$, from the diagrams of expansion of Fig. 6. Recall that in the corresponding analytical expressions the wavy lines are integrated and by this integral effect each of them can be substituted by an expression composed of the statistically meaningful functions only. Moreover, even the statistically consistent two-point correlation function (the wavy line with the attached dashed line) should be substituted in integrals (i.e. in diagrams) by the corresponding expansion

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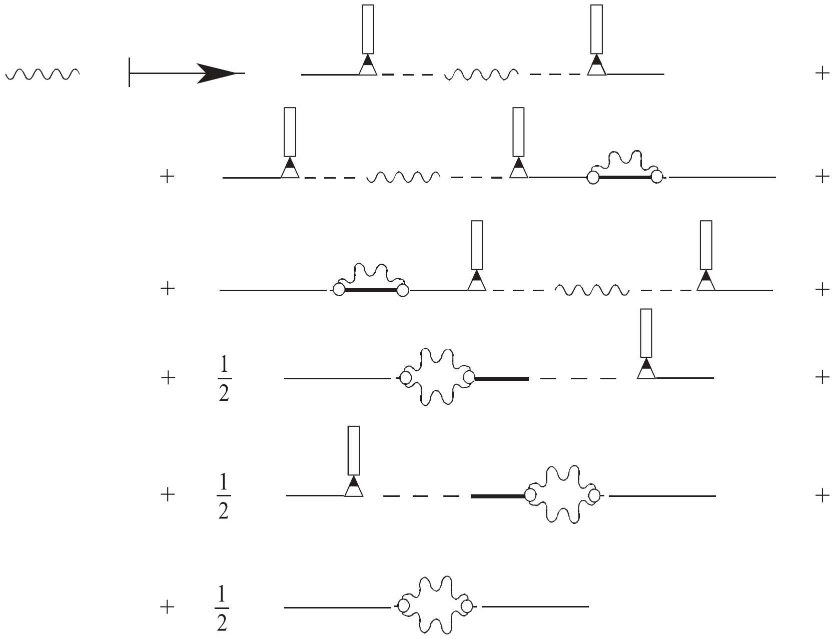

Figure 7. A substitution for the formal function $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$.

in the two-time correlation functions. To complete this stage, it suffices to substitute the wavy lines in the diagrams of Fig. 6 by sums of the graphic fragments given in Fig. 7. Note that the corresponding sum was generated according to the relation between the two-point and two-time correlation functions that is shown in Fig. 6. (Strictly speaking, in the above permutational fragments, the formal functions $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$ should also be eliminated. For this, the wavy lines should again be substituted by permutational fragments following Fig. 7, but this time it suffices to only keep the first term in the substitution.)

We have now finished the description of the expansion of the two-point correlation function in the two-time correlation functions. We present the corresponding final graphic relation in the next section.

4. Final expression for the two-point correlation function

An expansion of the two-point correlation function in the two-time correlation functions is given in Fig. 8. In this expression, the notation is modified for the sake of brevity. Namely, the wavy line stands for the two-time correlation function and a renormalized vertex is introduced following the recursive identity in Fig. 9.

The expression given in Fig. 8 needs some extra comments. All the diagrams are numbered in the figure. Each of them contains lines of the operator $\hat{\mathscr{F}}\hat{G}$ (i.e. dashed line attached to the bold solid line) except for diagrams 1 and 10. These diagrams should be regarded as collective images for sets of integral expressions. Namely, we should supplement the given diagrams by all diagrams, with a number of fragments structured as these on the left in Fig. 10 being substituted by the respective fragments on the right. The only exclusion is the left-most $\hat{\mathscr{F}}\hat{G}$-line of diagram 4: the

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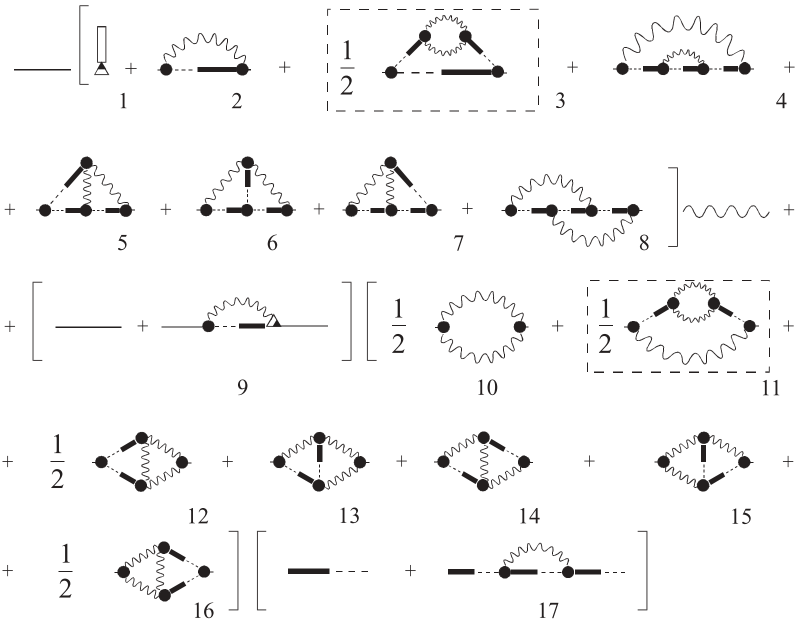

Figure 8. An expansion of the two-point correlation function $\langle \delta N_\alpha \delta \hat{F} \rangle$ in the two-time correlation functions $\langle \delta \hat{F} \delta \hat{F} \rangle$.

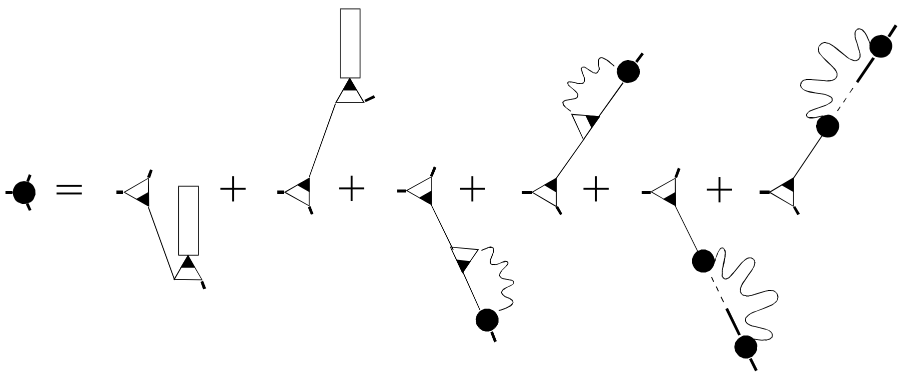

Figure 9. A recursive expression for the renormalized vertex $\hat{\mathscr{H}}$.

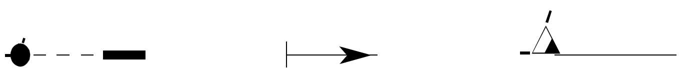

Figure 10. A substitution for the multiplication of diagrams in Fig. 8.

corresponding integral was used to organize the nonlinear renormalization of the left vertex in diagram 2.

Two of the diagrams, numbered as 3 and 11, are surrounded by dashed rectangles. We should keep the results of the multiplication of these diagrams with at least one of the $\hat{\mathscr{F}}\hat{G}$-lines in the upper branch of the diagram substituted. Note that the

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integrals corresponding to actually drawn diagrams 3 and 11 should be omitted, as well as the result of the multiplication of diagram 3 with a substitution for the lower branch only. In fact, they are enfolded into the integrals corresponding to diagrams 2 and 10, respectively.

We have completed the presentation of the expansion of the two-point correlation function in the two-time correlation functions. It only remains to translate the graphic formulae to the analytical formulae. This will be partially performed in the appendix.

Now let us discuss the degree of reliability of the results obtained. Consider the 'Vlasov plasma', i.e. an imaginary plasma consisting of electron and ion continuous fluids that fill the respective phase spaces $\mathbf{r}, \mathbf{p}_{\mathrm{e,i}}$ without gaps. Then the intermediate function $\langle \delta N_\alpha(\mathbf{r}, \mathbf{p}, t)\delta N_{\alpha'}(\mathbf{r}', \mathbf{p}', t')\rangle$ is a well defined statistic, unlike that in the Klimontovich–Dupree plasma. The evolution equation of this statistic is obtained by iterations following the principles of Erofeev (1996) and, with a sufficient accuracy, it is shown by Fig. 4. (Note that this graphic equation coincides with the corresponding approximation for the equation $\hat{G}^{-1}\hat{N} = \hat{\Phi}\hat{G}^{+}$ of the Wyld diagram technique. Here '+' stands for the Hermitian conjugate.) Starting from this evolution equation, we should derive the evolution equation for the two-time correlation function and the logic of the corresponding derivation repeats our calculation given in Figs 5–7. Due to this, our expansion of the two-point correlation function is repeated. That is, we have confirmed the above expansion at least for the Vlasov plasma case. However, there should not be any difference between the Vlasov and Klimontovich–Dupree plasmas with respect to the kinetics of collective phenomena, therefore the diagrams drawn represent the collective phenomena in the Klimontovich–Dupree plasma fairly well as well.

After the above analysis we may conclude that the substitution of the Vlasov plasma by the Klimontovich–Dupree plasma may lead to relative errors in the evolution equation that should be of the order of the ratio of the mean interparticle distance to the characteristic spatial scale of the plasma collective motions. In reality, our results have nothing to do with this ratio: the errors due to the plasma discreteness depend only on the volume of 6D parallelepipeds (that we use to define the distribution functions) and can essentially be lessened by enlarging this volume, when appropriate.

In the next section we discuss the correspondence of our expansion with the traditional plasma turbulence theory.

5. Correlation with the existing theory of Langmuir turbulence

In the previous section we presented the graphic image for the expansion of the two-point correlation function in the two-time correlation functions. We have derived it with sufficient accuracy for an adequate treating of the Langmuir turbulence. All the understanding of the Langmuir turbulence that was developed formerly in the plasma community has some correspondence with our results. In this section, we analyze the correspondence of our diagrams with the skeleton of the traditional weak Langmuir turbulence theory.

Let us consider first the diagrams of Fig. 8 as they are written, i.e. without the diagram multiplication following substitution from Fig. 10. (In this sense, diagrams 3 and 11 fall out of scope.) Then these diagrams represent all the wealth of the wave interaction in the turbulence. That is, had we substituted the corresponding

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expression to the evolution equation of the two-point correlation function, the latter equation would have properly accounted for the effects of the wave interaction in the plasma. To see this, let us recall the 'canonical' redaction of the Wyld diagram technique for turbulent wave fields (Zakharov and L'vov 1975). We should inform the reader that the Wyld diagram technique constitutes conceptually the most regular basis for studies of kinetics of the wave interaction in traditional classical weakly turbulent wave fields (see Zakharov 1974; Erofeev and Malkin 1989; Erofeev 1996).

When Fig. 8 is substituted into the vacuum Maxwell equations, then the piece in the square brackets in the two top lines in the figure generates the object that contains the inverse operator of the renormalized Green function of the respective canonical Wyld diagram technique. Correspondingly, the operator $\hat{\mathscr{F}}\hat{G}$ of our theory acquires the sense of the bare Green function of the canonical Wyld diagram technique that is multiplied by the bare Green function of our theory with the sign of integration of the latter over $\mathbf{p}$. The vertex of the canonical Wyld diagram technique we should associate with our vertex from Fig. 9, the entry of which is influenced by the operator in the second square brackets in Fig. 8 and then integrated over the momentums and summed over the particle kind with the weight $4\pi e_\alpha$. After this, the sum of diagrams 2 and 5–8 correlates with first expansion terms of the mass operator $\hat{\Sigma}$ of the Wyld's diagram technique and diagrams 10 and 12–16 with the expansion of a 'compact part of correlation function' $\hat{\Phi}$ of the latter. (The diagrams written in the last square brackets in Fig. 8 correlate with the first terms of expansion of the Hermitian conjugate of the renormalized Green function of the canonical Wyld diagram technique.)

Following the above, diagrams 2, 10 and 4 in Fig. 8 account for the three-wave interaction. (The analog of diagram 4 appears in a canonical diagram technique due to the iteration of the renormalized Green function in the analog of diagram 2.) That is, all the effects of interaction of the Langmuir waves and the ion sound waves can be described on the basis of these diagrams, particularly the piece of four-wave interaction of the Langmuir waves mediated by the ion sound. Note that this interaction exists even in the case of a strong damping of ion sound, i.e. in an isothermal plasma (Malkin 1982a). Apart from interaction of the Langmuir and the ion sound waves, the given diagrams depict the physics of interaction of the Langmuir waves with the electromagnetic waves (and all the other three-wave interaction phenomena).

The remaining part of the four-wave interactions can be developed from diagrams 5–8 and 14. Diagrams 12, 13, 15 and 16 can be rendered as comprising the additional renormalization of the 'three-wave interaction matrix element'. The necessary counterparts of corresponding renormalization in the diagrams of 'mass operator' are embedded in the same diagrams 5–8†.

From the above analysis only one important distinction is seen between our equations and those that one can develop constructing the Wyld's diagram technique from the dynamic equations of the three-wave interactions (the usual approach that can be exemplified by calculations in Zakharov and L'vov (1975), Malkin

† An idea of systematic division of the components of these diagrams in terms of the four-wave collision integral and corrections to the three-wave collision integral due to the renormalization of the 'matrix element' of the three-wave process can be learned from Zakharov and L'vov (1975) (see also Erofeev and Malkin (1989)). It can be characterized as a procedure of multiplication of the diagrams which we do not discuss here.

(1982a, 1982b), Erofeev and Malkin (1989)). The three-wave vertex of the canonical Wyld diagram technique is organized on the basis of the lowest order terms in the expression of our vertex from Fig. 9. (They are the first two in the right-hand side of the relation in the figure.) These terms are operated on by our bare Green function and then integrated over the momentums and summed over the particle kind with the weight $4\pi e_\alpha$. In our calculations, the analog of this vertex is a bit more complicated. First, we see that the above lowest order of vertex from Fig. 9 should be corrected according to the identity that is recursively written by the figure. Second, its entry is operated on by the nonlinear operator from second square brackets in Fig. 8 rather then by the single bare Green function. In this way, additional nonlinear corrections to the vertex have appeared that were absent in the canonical Wyld diagram technique. These corrections may be of some importance for the three-wave interactions. We may assume that they comprise the influence of the turbulence on the plasma dispersive properties that ultimately lead to modifications in the 'matrix element' of the three-wave phenomena. The usual Wyld diagram technique does not assume the presence of these modifications. We stress that they should manifest themselves only in the next after leading order of the turbulence kinetics.

Now let us disengage ourselves from the correspondence of our written diagrams and diagrams of the canonical Wyld diagram technique. Let us discuss the place of the Langmuir wave scattering induced by plasma particles within our diagrams. The leading order of this phenomenon is enfolded in diagram 2 itself and in a result of its multiplication following substitution from Fig. 10 (see Erofeev (2000)). What results from the diagram multiplications according to substitution in Fig. 10 is that they give either extra corrections to the collision integrals of three- and four-wave interactions or the corrections to the lowest order of kinetics of the wave scattering induced by plasma particles.

The above presented classification of diagrams and corresponding phenomena in a turbulent plasma do not have pretensions of the utmost rigorous one. We were concerned only with a rough scheme of correlation of our perturbation technique with the traditional weak plasma turbulence theory. Note that the latter itself does not possess sufficient rigorousness for a completed theory. For instance, the same term in the wave collision integral is equally attributed both to the three-wave interactions and to the wave scattering induced by plasma ions. (This term is imprinted in our diagram 2 from Fig. 8.) Still, we suppose that our diagrammatic expansion constitutes the most regular basis for studies of the plasma turbulent phenomena.

An additional idea regarding the rules of translation of our diagrammatic relations to the analytical writing can be perceived from the appendix.

6. Conclusions

In this paper, we derived an expansion of the two-point correlation function in the two-time correlation functions that is correct up to third order in powers of the turbulence energy density. This expansion permits the gained evolution equation to be written for the two-time correlation function that is necessary for a rigorous derivation of the kinetics of Langmuir turbulence.

Apart from the problems of Langmuir turbulence studies, our expansion is applicable to studies of weakly non-ideal (rarefied) plasmas. That is, for developing kinetics of single plasmas with finite small inverse numbers of particles in the Debye sphere. In the traditional paradigm of BBGKY plasma kinetics the given number is rendered as an expansion parameter. Note that only within the practice of the plasma ensemble studies the conceptions of BBGKY kinetics possessed some sense of constructive theory†. In view of the stated *uselessness* of the plasma ensemble studies (Erofeev 2000, 2002a, 2002b), inclusion of sequential equations of the BBGKY equation hierarchy to the plasma description do not result in approaching the objective picture of the plasma macrophysical evolution. In contrast, the expansion of the two-point correlation function in the two-time correlation functions permits the development of the picture of the plasma macrophysical evolution that is correct up to two first orders in the above inverse number of particles in the Debye sphere‡.

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Appendix. A truncated expression of the two-point correlation function in analytical form

Following traditional calculations by Malkin (1982a), the main channel of four-wave interaction in the weak Langmuir turbulence is the Langmuir wave scattering on the forced density inhomogeneities (or on intermediate ion sound). This statement is correct, although the adequate theory of the phenomenon is not yet developed. For this reason we deliberately skip the contributions of diagrams that correspond to the 'pure four-wave interactions'. That is, we omit diagrams 4–8 and 12–16 in Fig. 8. The remaining terms of our expansion depict the most important processes in the turbulence dynamics.

In the formulae presented we do not exploit the potentiality of Langmuir oscillations§.

† In particular, without the plasma ensemble averaging the notion of the two-particle distribution is void of any statistical sense, just like the formal object $\langle \delta N_\alpha \delta N_{\alpha'} \rangle$ that we used in this paper at the intermediate stages of our calculations. The same can be said about the three-particle and other multiparticle distributions.

‡ This inverse number coincides structurally with the ratio of Langmuir oscillation period, the typical time of electron flight through a field of charged particle, to a typical time of Coulomb collisions. Conceptually, the latter is an expansion parameter of the theory. Its analog in theory of weak plasma turbulence is a ratio of inverse width of turbulence spectrum in natural frequencies to typical times of the plasma and turbulence evolution.

§ The reason for this is that we would like to have an opportunity to not only study four-wave interactions in the Langmuir turbulence, but also in a general case of non-longitudinal turbulent wave fields. Besides, the general formulae writing seems to be desirable for studies of nonideal plasmas.

The analytical form of the recursive identity in Fig. 10 is

$$
\begin{aligned}
&{}^{\alpha}\mathscr{H}_{m'\beta'm''\beta''}(\boldsymbol{\xi},\boldsymbol{\xi}',\boldsymbol{\xi}'') = {}^{\alpha}\mathscr{M}_{m'\beta'm''\beta''}(\boldsymbol{\xi},\boldsymbol{\xi}')\delta^7(\boldsymbol{\xi}-\boldsymbol{\xi}'')\\
&\quad + {}^{\alpha}\mathscr{M}_{m''\beta''m'\beta'}(\boldsymbol{\xi},\boldsymbol{\xi}'')\delta^7(\boldsymbol{\xi}-\boldsymbol{\xi}'),\\
&{}^{\alpha}\mathscr{M}_{m'\beta'm''\beta''}(\boldsymbol{\xi},\boldsymbol{\xi}') = \frac{e_\alpha^2}{c^2}v_{m''}(\mathbf{p})\frac{\partial}{\partial p^{\beta''}}\left({}^0G_\alpha(\boldsymbol{\xi},\boldsymbol{\xi}')v_{m'}(\mathbf{p}')\frac{\partial f_\alpha(\boldsymbol{\xi}')}{\partial p'^{\beta'}}\right)\\
&\quad + \frac{e_\alpha^2}{c^2}v_{m''}(\mathbf{p})\frac{\partial}{\partial p^{\beta''}}\int {}^0G_\alpha(\boldsymbol{\xi},\boldsymbol{\xi}_1)\,d^7\boldsymbol{\xi}_1\,v_{m_1}(\mathbf{p}_1)\frac{\partial}{\partial p_1^{\beta_1}}({}^0G_\alpha(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)\,d^7\boldsymbol{\xi}_2\\
&\quad \times {}^{\alpha}\mathscr{H}_{m_3\beta_3m'\beta'}(\boldsymbol{\xi}_2,\boldsymbol{\xi}_3,\boldsymbol{\xi}')\,d^7\boldsymbol{\xi}_3\,\Phi^{m_1\beta_1m_3\beta_3}(\mathbf{r}_1,t_1,\mathbf{r}_3,t_3))\\
&\quad - \frac{e_\alpha}{c}v_{m''}(\mathbf{p})\frac{\partial}{\partial p^{\beta''}}\int {}^0G_\alpha(\boldsymbol{\xi},\boldsymbol{\xi}_1)\,d^7\boldsymbol{\xi}_1\,{}^{\alpha}\mathscr{H}_{m_4\beta_4m_2\beta_2}(\boldsymbol{\xi}_1,\boldsymbol{\xi}_4,\boldsymbol{\xi}_2)\,d^7\boldsymbol{\xi}_2\,d^7\boldsymbol{\xi}_4\\
&\quad \times \sum_{\alpha'}\int(\hat{\mathscr{F}}\hat{G})^{m_4\beta_4}_{\alpha'}(\mathbf{r}_4,t_4,\boldsymbol{\xi}_5)\,d^7\boldsymbol{\xi}_5\\
&\quad \times {}^{\alpha'}\mathscr{H}_{m_3\beta_3m'\beta'}(\boldsymbol{\xi}_5,\boldsymbol{\xi}_3,\boldsymbol{\xi}')\,d^7\boldsymbol{\xi}_3\,\Phi^{m_2\beta_2m_3\beta_3}(\mathbf{r}_2,t_2,\mathbf{r}_3,t_3).
\end{aligned}
\tag{A 1}
$$

In this formula the notation $\boldsymbol{\xi}_i$ introduces the seven-dimensional 'vector' composed of time t_i, position $\mathbf{r}_i$ and momentum $\mathbf{p}_i$; the feature ${}^{\alpha}\mathscr{H}_{m'\beta'm''\beta''}(\boldsymbol{\xi},\boldsymbol{\xi}',\boldsymbol{\xi}'')$ is the renormalized vertex.

Structurally, the expression in Fig. 8 consists of two terms. The first term can be regarded as a graphical expression connected to the wavy line. Formally, this expression describes a linear *response* of the two-point function to a change in the two-time function. As its analytical counterpart, let us introduce $\mathscr{R}_{\alpha m\gamma}(\mathbf{r},\mathbf{p},t,\mathbf{r}_1,t_1)$.

The analytical counterpart of the other term in Fig. 8 we denote by $\mathscr{P}^{kl}_\alpha(\boldsymbol{\xi},\mathbf{r}',t')$. Then the expression in Fig. 8 for the two-point correlation function yields the relation

$$
\langle\delta N_\alpha(\boldsymbol{\xi})\delta F^{kl}(\mathbf{r}',t')\rangle = \int \mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi},\mathbf{r}_1,t_1)\,d^3\mathbf{r}_1\,dt_1\,\Phi^{m\gamma kl}(\mathbf{r}_1,t_1,\mathbf{r}',t') + \mathscr{P}^{kl}_\alpha(\boldsymbol{\xi},\mathbf{r}',t').
\tag{A 2}
$$

The expansion of the response $\hat{\mathscr{R}}$ consists of three terms, $\hat{\mathscr{R}} = {}^0\hat{\mathscr{R}} + {}^3\hat{\mathscr{R}} + {}^3\widehat{\delta\mathscr{R}}$. The first describes the linear dispersive properties of the given species of the plasma particles,

$$
{}^0\mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi},\mathbf{r}_1,t_1) = -\frac{e_\alpha}{c}\int {}^0G_\alpha(\boldsymbol{\xi},\boldsymbol{\xi}_1)\,d^3\mathbf{p}_1\,v_m(\mathbf{p}_1)\frac{\partial}{\partial p_1^\gamma}f_\alpha(\boldsymbol{\xi}_1).
\tag{A 3}
$$

The second term ${}^3\hat{\mathscr{R}}$ accounts for the three-wave interactions and the effect of the wave scattering induced by plasma particles,

$$
\begin{aligned}
{}^3\mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi},\mathbf{r}_1,t_1) &= \int {}^0G_\alpha(\boldsymbol{\xi},\boldsymbol{\xi}_2)\,d^7\boldsymbol{\xi}_2\,{}^{\alpha}\mathscr{H}_{m_5\varepsilon m_6\beta}(\boldsymbol{\xi}_2,\boldsymbol{\xi}_5,\boldsymbol{\xi}_6)\,d^7\boldsymbol{\xi}_5\,d^7\boldsymbol{\xi}_6\\
&\quad \times \sum_{\alpha'}\int(\hat{\mathscr{F}}\hat{G})^{m_5\varepsilon}_{\alpha'}(\mathbf{r}_5,t_5,\boldsymbol{\xi}_3)\,d^7\boldsymbol{\xi}_3\,{}^{\alpha'}\mathscr{H}_{m_4\delta m\gamma}(\boldsymbol{\xi}_3,\boldsymbol{\xi}_4,\boldsymbol{\xi}_1)\,d^7\boldsymbol{\xi}_4
\end{aligned}
$$

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$$\times d^3\mathbf{p}_1 \Phi^{m_6\beta m_4\delta}(\mathbf{r}_6, t_6, \mathbf{r}_4, t_4) - \frac{e_\alpha}{c}\int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2\, v_{m_2}(\mathbf{p}_2)$$

$$\times \frac{\partial}{\partial p_2^\beta}\left(\int {}^0G_\alpha(\boldsymbol{\xi}_2, \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, {}^\alpha\mathscr{H}_{m_4\delta m\gamma}(\boldsymbol{\xi}_3, \boldsymbol{\xi}_4, \boldsymbol{\xi}_1)\right) d^7\boldsymbol{\xi}_4$$

$$\times d^3\mathbf{p}_1 \Phi^{m_2\beta m_4\delta}(\mathbf{r}_2, t_2, \mathbf{r}_4, t_4). \tag{A 4}$$

The third term ${}^3\widehat{\mathscr{R}}$ describes corrections that are due to diagram 3 in Fig. 8. Its expression is

$$ {}^3\widehat{\delta\mathscr{R}} = {}^{31}\widehat{\delta\mathscr{R}} + {}^{32}\widehat{\delta\mathscr{R}} + {}^{33}\widehat{\delta\mathscr{R}} + {}^{34}\widehat{\delta\mathscr{R}}, \tag{A 5}$$

$${}^{31}\delta\mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi}, \mathbf{r}_1, t_1) = \sum_{\alpha'} \frac{e_\alpha e_{\alpha'}}{2c^2}\int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2) v_{m_2}(\mathbf{p}_2)\, d^7\boldsymbol{\xi}_2 \frac{\partial}{\partial p_2^{\beta_2}}$$

$$\times\left(\int {}^0G_\alpha(\boldsymbol{\xi}_2, \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, {}^\alpha\mathscr{H}^{m_5\beta_5 m_6\beta_6}(\boldsymbol{\xi}_3, \boldsymbol{\xi}_5, \boldsymbol{\xi}_6)\right) d^7\boldsymbol{\xi}_5\, d^7\boldsymbol{\xi}_6$$

$$\times \int (\hat{\mathscr{F}}\hat{G})^{m_2\beta_2}_{\alpha'}(\mathbf{r}_2, t_2, \boldsymbol{\xi}_1)\, d^3\mathbf{p}_1\, v_m(\mathbf{p}_1)$$

$$\times \frac{\partial}{\partial p_1^\gamma}\left(\int {}^0G_{\alpha'}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_4)\, d^7\boldsymbol{\xi}_4\, {}^{\alpha'}\mathscr{H}^{m_7\beta_7 m_8\beta_8}(\boldsymbol{\xi}_4, \boldsymbol{\xi}_7, \boldsymbol{\xi}_8)\right) d^7\boldsymbol{\xi}_7\, d^7\boldsymbol{\xi}_8$$

$$\times \Phi_{m_5\beta_5 m_7\beta_7}(\mathbf{r}_5, t_5, \mathbf{r}_7, t_7)\Phi_{m_6\beta_6 m_8\beta_8}(\mathbf{r}_6, t_6, \mathbf{r}_8, t_8), \tag{A 6}$$

$${}^{32}\delta\mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi}, \mathbf{r}_1, t_1) = \frac{e_\alpha^2}{2c^2}\int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2\, v_{m_2}(\mathbf{p}_2)\frac{\partial}{\partial p_2^{\beta_2}}$$

$$\times\left(\int {}^0G_\alpha(\boldsymbol{\xi}_2, \boldsymbol{\xi}_1)\, d^3\mathbf{p}_1\, v_m(\mathbf{p}_1)\frac{\partial}{\partial p_1^\gamma}\right.$$

$$\times\left.\left(\int {}^0G_\alpha(\boldsymbol{\xi}_1, \boldsymbol{\xi}_4)\, d^7\boldsymbol{\xi}_4\, {}^\alpha\mathscr{H}^{m_7\beta_7 m_8\beta_8}(\boldsymbol{\xi}_4, \boldsymbol{\xi}_7, \boldsymbol{\xi}_8)\right)\right) d^7\boldsymbol{\xi}_7\, d^7\boldsymbol{\xi}_8$$

$$\times \sum_{\alpha'}\int (\hat{\mathscr{F}}\hat{G})^{m_2\beta_2}_{\alpha'}(\mathbf{r}_2, t_2, \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, {}^{\alpha'}\mathscr{H}^{m_5\beta_5 m_6\beta_6}(\boldsymbol{\xi}_3, \boldsymbol{\xi}_5, \boldsymbol{\xi}_6)\, d^7\boldsymbol{\xi}_5\, d^7\boldsymbol{\xi}_6$$

$$\times \Phi_{m_5\beta_5 m_7\beta_7}(\mathbf{r}_5, t_5, \mathbf{r}_7, t_7)\Phi_{m_6\beta_6 m_8\beta_8}(\mathbf{r}_6, t_6, \mathbf{r}_8, t_8), \tag{A 7}$$

$${}^{33}\delta\mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi}, \mathbf{r}_1, t_1) = -\frac{e_\alpha}{2c}\int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2\, v_{m_2}(\mathbf{p}_2)$$

$$\times \frac{\partial}{\partial p_2^{\beta_2}}\left(\int {}^0G_\alpha(\boldsymbol{\xi}_2, \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, {}^\alpha\mathscr{H}^{m_6\beta_6 m_7\beta_7}(\boldsymbol{\xi}_3, \boldsymbol{\xi}_6, \boldsymbol{\xi}_7)\right) d^7\boldsymbol{\xi}_6\, d^7\boldsymbol{\xi}_7$$

$$\times \sum_{\alpha'}\int (\hat{\mathscr{F}}\hat{G})^{m_2\beta_2}_{\alpha'}(\mathbf{r}_2, t_2, \boldsymbol{\xi}_5)\, d^7\boldsymbol{\xi}_5\, {}^{\alpha'}\mathscr{H}_{m_4\beta_4 m\gamma}(\boldsymbol{\xi}_5, \boldsymbol{\xi}_4, \boldsymbol{\xi}_1)\, d^7\boldsymbol{\xi}_4\, d^3\mathbf{p}_1$$

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$$\times \sum_{\alpha''} \int (\hat{\mathscr{F}} \hat{G})^{m_4\beta_4}_{\alpha''}(\mathbf{r}_4, t_4, \boldsymbol{\xi}_8)\, d^7\boldsymbol{\xi}_8 \,{}^{\alpha''}\mathscr{H}^{m_9\beta_9 m_{10}\beta_{10}}(\boldsymbol{\xi}_8, \boldsymbol{\xi}_9, \boldsymbol{\xi}_{10})\, d^7\boldsymbol{\xi}_9\, d^7\boldsymbol{\xi}_{10}$$

$$\times \Phi_{m_6\beta_6 m_9\beta_9}(\mathbf{r}_6, t_6, \mathbf{r}_9, t_9)\Phi_{m_7\beta_7 m_{10}\beta_{10}}(\mathbf{r}_7, t_7, \mathbf{r}_{10}, t_{10}),$$

$$^{34}\delta\mathscr{R}_{\alpha m\gamma}(\boldsymbol{\xi}, \mathbf{r}_1, t_1) = -\sum_{\alpha'} \frac{e_{\alpha'}}{2c} \int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2 \,{}^{\alpha}\mathscr{H}^{m_4\beta_4 m_5\beta_5}(\boldsymbol{\xi}_2, \boldsymbol{\xi}_4, \boldsymbol{\xi}_5) d^7\boldsymbol{\xi}_4$$

$$\times d^7\boldsymbol{\xi}_5 (\hat{\mathscr{F}} \hat{G})_{\alpha' m_4\beta_4}(\mathbf{r}_4, t_4, \boldsymbol{\xi}_1)\, d^3\mathbf{p}_1\, v_m(\mathbf{p}_1)$$

$$\times \frac{\partial}{\partial p_1^\gamma}\left(\int {}^0G_{\alpha'}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_8)\, d^7\boldsymbol{\xi}_8 \,{}^{\alpha'}\mathscr{H}^{m_9\beta_9 m_{10}\beta_{10}}(\boldsymbol{\xi}_8, \boldsymbol{\xi}_9, \boldsymbol{\xi}_{10})\right) d^7\boldsymbol{\xi}_9\, d^7\boldsymbol{\xi}_{10}$$

$$\times \sum_{\alpha''} \int (\hat{\mathscr{F}} \hat{G})_{\alpha'' m_5\beta_5}(\mathbf{r}_5, t_5, \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3 \,{}^{\alpha''}\mathscr{H}^{m_6\beta_6 m_7\beta_7}(\boldsymbol{\xi}_3, \boldsymbol{\xi}_6, \boldsymbol{\xi}_7)\, d^7\boldsymbol{\xi}_6\, d^7\boldsymbol{\xi}_7$$

$$\times \Phi_{m_6\beta_6 m_9\beta_9}(\mathbf{r}_6, t_6, \mathbf{r}_9, t_9)\Phi_{m_7\beta_7 m_{10}\beta_{10}}(\mathbf{r}_7, t_7, \mathbf{r}_{10}, t_{10}). \tag{A 8}$$

The term $\mathscr{P}^{kl}_\alpha(\boldsymbol{\xi}, \mathbf{r}', t')$ can be presented as follows:

$$\mathscr{P}^{kl}_\alpha(\boldsymbol{\xi}, \mathbf{r}', t') = \sum_{\alpha'\alpha''} \int ({}^0\hat{G} + {}^0\widehat{\delta G})_{\alpha\alpha'}(\boldsymbol{\xi}, \boldsymbol{\xi}_1)\, d^7\boldsymbol{\xi}_1 ({}^3\hat{\Phi} + {}^3\widehat{\delta\Phi})_{\alpha'\alpha''}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2$$

$$\times ((\hat{\mathscr{F}} \hat{G}) + \delta(\hat{\mathscr{F}} \hat{G}))^{kl}_{\alpha''}(\mathbf{r}', t', \boldsymbol{\xi}_2). \tag{A 9}$$

In this formula, ${}^0\widehat{\delta G}$ introduces the correction to the bare Green function that is due to diagram 9 in Fig. 8,

$$({}^0\hat{G} + {}^0\widehat{\delta G})_{\alpha\alpha'}(\boldsymbol{\xi}, \boldsymbol{\xi}_1) = \delta_{\alpha\alpha'}\left[{}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_1) + \frac{e_\alpha^2}{c^2} \int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2\, v_{m_2}(\mathbf{p}_2) \frac{\partial}{\partial p_2^{\beta_2}} \right.$$

$$\left. \times \left(\int {}^0G_\alpha(\boldsymbol{\xi}_2, \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, v_{m_3}(\mathbf{p}_3) \frac{\partial}{\partial p_3^{\beta_3}} {}^0G_\alpha(\boldsymbol{\xi}_3, \boldsymbol{\xi}_1) \right) \Phi^{m_2\beta_2 m_3\beta_3}(\mathbf{r}_2, t_2, \mathbf{r}_3, t_3) \right]$$

$$- \frac{e_{\alpha'}}{c} \int {}^0G_\alpha(\boldsymbol{\xi}, \boldsymbol{\xi}_2)\, d^7\boldsymbol{\xi}_2 \,{}^{\alpha}\mathscr{H}_{m_3\beta_3 m_4\beta_4}(\boldsymbol{\xi}_2, \boldsymbol{\xi}_3, \boldsymbol{\xi}_4)\, d^7\boldsymbol{\xi}_3\, d^7\boldsymbol{\xi}_4$$

$$\times (\hat{\mathscr{F}} \hat{G})^{m_4\beta_4}_{\alpha'}(\mathbf{r}_4, t_4, \boldsymbol{\xi}_5)\, d^7\boldsymbol{\xi}_5 v_{m_5}(\mathbf{p}_5)$$

$$\times \frac{\partial}{\partial p_5^{\beta_5}} {}^0G_{\alpha'}(\boldsymbol{\xi}_5, \boldsymbol{\xi}_1) \Phi^{m_3\beta_3 m_5\beta_5}(\mathbf{r}_3, t_3, \mathbf{r}_5, t_5). \tag{A 10}$$

The function $({}^3\hat{\Phi} + {}^3\widehat{\delta\Phi})_{\alpha\alpha'}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$ represents some analog of the 'compact part of the correlation function' of the canonical Wyld's diagram technique; '3' in this notation hints on the three-wave interactions. The expression of this function is:

$$({}^3\hat{\Phi} + {}^3\widehat{\delta\Phi})_{\alpha\alpha'}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2) = \frac{1}{2} \int {}^{\alpha}\mathscr{H}^{m_3\beta_3 m_4\beta_4}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_3, \boldsymbol{\xi}_4)\, d^7\boldsymbol{\xi}_3\, d^7\boldsymbol{\xi}_4$$

$$\times {}^{\alpha'}\mathscr{H}^{m_5\beta_5 m_6\beta_6}(\boldsymbol{\xi}_2, \boldsymbol{\xi}_5, \boldsymbol{\xi}_6)\, d^7\boldsymbol{\xi}_5\, d^7\boldsymbol{\xi}_6\, \Phi_{m_3\beta_3 m_5\beta_5}(\mathbf{r}_3, t_3, \mathbf{r}_5, t_5)$$

$$\times \Phi_{m_4\beta_4 m_6\beta_6}(\mathbf{r}_4, t_4, \mathbf{r}_6, t_6)$$

$$-\frac{e_\alpha}{2c}v_{m_1}(\mathbf{p}_1)\frac{\partial}{\partial p_1^{\beta_1}}\left(\int {}^0G_\alpha(\boldsymbol{\xi}_1,\boldsymbol{\xi}_3)\,d^7\boldsymbol{\xi}_3\,{}^\alpha\mathscr{H}^{m_4\beta_4m_5\beta_5}(\boldsymbol{\xi}_3,\boldsymbol{\xi}_4,\boldsymbol{\xi}_5)\right)d^7\boldsymbol{\xi}_4\,d^7\boldsymbol{\xi}_5$$

$$\times\,{}^{\alpha'}\mathscr{H}_{m_6\beta_6m_7\beta_7}(\boldsymbol{\xi}_2,\boldsymbol{\xi}_6,\boldsymbol{\xi}_7)\,d^7\boldsymbol{\xi}_6\,d^7\boldsymbol{\xi}_7\sum_{\alpha''}\int(\hat{\mathscr{F}}\hat{G})^{m_7\beta_7}_{\alpha''}(\mathbf{r}_7,t_7,\boldsymbol{\xi}_8)\,d^7\boldsymbol{\xi}_8$$

$$\times\,{}^{\alpha''}\mathscr{H}^{m_9\beta_9m_{10}\beta_{10}}(\boldsymbol{\xi}_8,\boldsymbol{\xi}_9,\boldsymbol{\xi}_{10})\,d^7\boldsymbol{\xi}_9\,d^7\boldsymbol{\xi}_{10}\,\Phi_{m_4\beta_4m_9\beta_9}(\mathbf{r}_4,t_4,\mathbf{r}_9,t_9)$$

$$\times\,\Phi_{m_5\beta_5m_{10}\beta_{10}}(\mathbf{r}_5,t_5,\mathbf{r}_{10},t_{10})\Phi^{m_1\beta_1m_6\beta_6}(\mathbf{r}_1,t_1,\mathbf{r}_6,t_6)-\frac{e_{\alpha'}}{2c}v_{m_2}(\mathbf{p}_2)\frac{\partial}{\partial p_2^{\beta_2}}$$

$$\times\left(\int {}^0G_{\alpha'}(\boldsymbol{\xi}_2,\boldsymbol{\xi}_8)\,d^7\boldsymbol{\xi}_8\,{}^{\alpha'}\mathscr{H}^{m_9\beta_9m_{10}\beta_{10}}(\boldsymbol{\xi}_8,\boldsymbol{\xi}_9,\boldsymbol{\xi}_{10})\right)d^7\boldsymbol{\xi}_9\,d^7\boldsymbol{\xi}_{10}$$

$$\times\,{}^{\alpha}\mathscr{H}_{m_3\beta_3m_4\beta_4}(\boldsymbol{\xi}_1,\boldsymbol{\xi}_3,\boldsymbol{\xi}_4)\,d^7\boldsymbol{\xi}_3\,d^7\boldsymbol{\xi}_4\sum_{\alpha''}\int(\hat{\mathscr{F}}\hat{G})^{m_3\beta_3}_{\alpha''}(\mathbf{r}_3,t_3,\boldsymbol{\xi}_5)\,d^7\boldsymbol{\xi}_5$$

$$\times\,{}^{\alpha''}\mathscr{H}^{m_6\beta_6m_7\beta_7}(\boldsymbol{\xi}_5,\boldsymbol{\xi}_6,\boldsymbol{\xi}_7)\,d^7\boldsymbol{\xi}_6\,d^7\boldsymbol{\xi}_7\,\Phi_{m_6\beta_6m_9\beta_9}(\mathbf{r}_6,t_6,\mathbf{r}_9,t_9)$$

$$\times\,\Phi_{m_7\beta_7m_{10}\beta_{10}}(\mathbf{r}_7,t_7,\mathbf{r}_{10},t_{10})\Phi^{m_4\beta_4m_2\beta_2}(\mathbf{r}_4,t_4,\mathbf{r}_2,t_2)$$

$$+\frac{1}{2}\frac{e_\alpha e_{\alpha'}}{c^2}\int v_{m_1}(\mathbf{p}_1)\frac{\partial}{\partial p_1^{\beta_1}}\left(\int {}^0G_\alpha(\boldsymbol{\xi}_1,\boldsymbol{\xi}_3)\,d^7\boldsymbol{\xi}_3\,{}^\alpha\mathscr{H}^{m_5\beta_5m_6\beta_6}(\boldsymbol{\xi}_3,\boldsymbol{\xi}_5,\boldsymbol{\xi}_6)\right)d^7\boldsymbol{\xi}_5d^7\boldsymbol{\xi}_6$$

$$\times\,v_{m_2}(\mathbf{p}_2)\frac{\partial}{\partial p_2^{\beta_2}}({}^0G_{\alpha'}(\boldsymbol{\xi}_2,\boldsymbol{\xi}_4)\,d^7\boldsymbol{\xi}_4\,{}^{\alpha'}\mathscr{H}^{m_7\beta_7m_8\beta_8}(\boldsymbol{\xi}_4,\boldsymbol{\xi}_7,\boldsymbol{\xi}_8))\,d^7\boldsymbol{\xi}_7\,d^7\boldsymbol{\xi}_8$$

$$\times\,\Phi^{m_1\beta_1m_2\beta_2}(\mathbf{r}_1,t_1,\mathbf{r}_2,t_2)\Phi_{m_5\beta_5m_7\beta_7}(\mathbf{r}_5,t_5,\mathbf{r}_7,t_7)\Phi_{m_6\beta_6m_8\beta_8}(\mathbf{r}_6,t_6,\mathbf{r}_8,t_8).\quad\text{(A 11)}$$

The term $((\hat{\mathscr{F}}\hat{G})+\delta(\hat{\mathscr{F}}\hat{G}))^{kl}_{\alpha''}(\mathbf{r}',t',\boldsymbol{\xi}_2)$ introduces the correction to the operator $(\hat{\mathscr{F}}\hat{G})$. We reiterate that the latter operator correlates with the bare Green function of the canonical Wyld's diagram technique; the correction $\delta(\hat{\mathscr{F}}\hat{G})$ introduces the 'turbulent' renormalization of this bare Green function. The corresponding formula is

$$((\hat{\mathscr{F}}\hat{G})+\delta(\hat{\mathscr{F}}\hat{G}))^{kl}_{\alpha}(\mathbf{r}',t',\boldsymbol{\xi}_2)=(\hat{\mathscr{F}}\hat{G})^{kl}_{\alpha}(\mathbf{r}',t',\boldsymbol{\xi}_2)+\sum_{\alpha'\alpha''}\int(\hat{\mathscr{F}}\hat{G})^{kl}_{\alpha'}(\mathbf{r}',t',\boldsymbol{\xi}_3)\,d^7\boldsymbol{\xi}_3$$

$$\times\,{}^{\alpha'}\mathscr{H}^{m_4\beta_4m_5\beta_5}(\boldsymbol{\xi}_3,\boldsymbol{\xi}_4,\boldsymbol{\xi}_5)\,d^7\boldsymbol{\xi}_4\,d^7\boldsymbol{\xi}_5\,(\hat{\mathscr{F}}\hat{G})_{\alpha''m_4\beta_4}(\mathbf{r}_4,t_4,\boldsymbol{\xi}_6)d^7\boldsymbol{\xi}_6$$

$$\times\,{}^{\alpha''}\mathscr{H}^{m_7\beta_7m_8\beta_8}(\boldsymbol{\xi}_6,\boldsymbol{\xi}_7,\boldsymbol{\xi}_8)\,d^7\boldsymbol{\xi}_7\,d^7\boldsymbol{\xi}_8\,(\hat{\mathscr{F}}\hat{G})_{\alpha m_7\beta_7}(\mathbf{r}_7,t_7,\boldsymbol{\xi}_2)$$

$$\times\,\Phi_{m_5\beta_5m_8\beta_8}(\mathbf{r}_5,t_5,\mathbf{r}_8,t_8)$$

$$-\sum_{\alpha'}\frac{e_{\alpha'}}{c}\int(\hat{\mathscr{F}}\hat{G})^{kl}_{\alpha'}(\mathbf{r}',t',\boldsymbol{\xi}_3)\,d^7\boldsymbol{\xi}_3v_{m_3}(\mathbf{p}_3)$$

$$\times\frac{\partial}{\partial p_3^{\beta_3}}\left(\int {}^0G_{\alpha'}(\boldsymbol{\xi}_3,\boldsymbol{\xi}_4)\,d^7\boldsymbol{\xi}_4\,{}^{\alpha'}\mathscr{H}_{m_5\beta_5m_6\beta_6}(\boldsymbol{\xi}_4,\boldsymbol{\xi}_5,\boldsymbol{\xi}_6)\right)d^7\boldsymbol{\xi}_5\,d^7\boldsymbol{\xi}_6$$

$$\times (\hat{\mathscr{F}}\hat{G})_{\alpha}^{m_6\beta_6}(\mathbf{r}_6, t_6, \boldsymbol{\xi}_2)\Phi^{m_3\beta_3 m_5\beta_5}(\mathbf{r}_3, t_3, \mathbf{r}_5, t_5)$$

$$- \frac{e_\alpha}{c}\sum_{\alpha'}\int (\hat{\mathscr{F}}\hat{G})_{\alpha'}^{kl}(\mathbf{r}', t', \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, {}^{\alpha'}\mathscr{H}_{m_4\beta_4 m_5\beta_5}(\boldsymbol{\xi}_3, \boldsymbol{\xi}_4, \boldsymbol{\xi}_5)\, d^7\boldsymbol{\xi}_4\, d^7\boldsymbol{\xi}_5$$

$$\times (\hat{\mathscr{F}}\hat{G})_{\alpha}^{m_4\beta_4}(\mathbf{r}_4, t_4, \boldsymbol{\xi}_6)\, d^7\boldsymbol{\xi}_6 v_{m_6}(\mathbf{p}_6)\frac{\partial}{\partial p_6^{\beta_6}}{}^0G_\alpha(\boldsymbol{\xi}_6, \boldsymbol{\xi}_2)\Phi^{m_5\beta_5 m_6\beta_6}(\mathbf{r}_5, t_5, \mathbf{r}_6, t_6)$$

$$+ \frac{e_\alpha^2}{c^2}\int (\hat{\mathscr{F}}\hat{G})_{\alpha}^{kl}(\mathbf{r}', t', \boldsymbol{\xi}_3)\, d^7\boldsymbol{\xi}_3\, v_{m_3}(\mathbf{p}_3)$$

$$\times \frac{\partial}{\partial p_3^{\beta_3}}\left(\int {}^0G_\alpha(\boldsymbol{\xi}_3, \boldsymbol{\xi}_4)\, d^7\boldsymbol{\xi}_4 v_{m_4}(\mathbf{p}_4)\frac{\partial}{\partial p_4^{\beta_4}}{}^0G_\alpha(\boldsymbol{\xi}_4, \boldsymbol{\xi}_2)\right)$$

$$\times \Phi^{m_3\beta_3 m_4\beta_4}(\mathbf{r}_3, t_3, \mathbf{r}_4, t_4). \tag{A 12}$$

To complete our presentation, it remains to formulate the recipe for the calculation of the function $(\hat{\mathscr{F}}\hat{G})_{\alpha}^{kl}(\mathbf{r}, t, \boldsymbol{\xi})$. Conceptually, this function is the solution to Maxwell equations when the 'charge density' is associated with the bare Green function. The corresponding equations were written in the appendix of Erofeev (1997) and we repeat them here:

$$\frac{1}{c}\frac{\partial}{\partial t}(\hat{\mathscr{F}}\hat{G})_{\alpha\beta\gamma} = -\frac{\partial}{\partial r^\beta}(\hat{\mathscr{F}}\hat{G})_{\alpha\gamma 0} + \frac{\partial}{\partial r^\gamma}(\hat{\mathscr{F}}\hat{G})_{\alpha\beta 0}, \tag{A 13}$$

$$\frac{1}{c}\frac{\partial}{\partial t}(\hat{\mathscr{F}}\hat{G})_{\alpha}{}^{\beta 0} = -\frac{\partial}{\partial r^\gamma}(\hat{\mathscr{F}}\hat{G})_{\alpha}{}^{\beta\gamma} - \frac{4\pi}{c}\int d^3\mathbf{r}_1\, dt_1 \sigma^{\beta m \cdot}_{\cdot\cdot\gamma}(\mathbf{r}, t, \mathbf{r}_1, t_1)$$

$$\times (\hat{\mathscr{F}}\hat{G})_{\alpha m}{}^{\cdot\gamma}_{\cdot}(\mathbf{r}_1, t_1, \mathbf{r}', \mathbf{p}', t') - \frac{4\pi}{c}e_\alpha\int d^3\mathbf{p}\, v^{\beta 0}G_\alpha(\mathbf{r}, \mathbf{p}, t, \mathbf{r}', \mathbf{p}', t'). \tag{A 14}$$

In the last equation, the notation $\sigma^{\beta m \cdot}_{\cdot\cdot\gamma}(\mathbf{r}, t, \mathbf{r}_1, t_1)$ is for a conductivity tensor that is given by

$$\sum_\alpha e_\alpha \int d^3\mathbf{p}\, v^\beta \mathscr{R}^{m\ \cdot}_{\alpha\ \cdot\ \gamma}(\mathbf{r}, \mathbf{p}, t, \mathbf{r}_1, t_1).$$

When developing the lowest order kinetic equations for the weak turbulence in Erofeev (1997, 1998a), the value of $\sigma^{\beta m \cdot}_{\cdot\cdot\gamma}(\mathbf{r}, t, \mathbf{r}_1, t_1)$ was calculated neglecting the time evolution of the plasma parameters, which corresponded to the order of consideration. In addition, it was appropriate to omit nonlinear corrections in $\hat{\mathscr{R}}$. Due to this, it was possible to get solution to equations (A 13) and (A 14) using techniques of the Fourier–Laplace transformation. The non-zero matrix elements of the Fourier–Laplace transform of the operator $(\hat{\mathscr{F}}\hat{G})$ was then

$$(\hat{\mathscr{F}}\hat{G})_{\alpha\mathbf{k}\omega}^{\beta 0}(\mathbf{p}') = \frac{1}{-i\omega + 4\pi\sigma_{\mathbf{k}\omega}}4\pi e_\alpha\int d^3\mathbf{p}\frac{k^\beta k_\gamma v^\gamma}{k^2}{}^0G_{\alpha\mathbf{k}\omega}(\mathbf{p}, \mathbf{p}'). \tag{A 15}$$

For the problems of Langmuir kinetics the neglections mentioned are incorrect and we should include into the scope the terms with the first-order time derivatives of the linear part of the response ${}^0\hat{\mathscr{R}}$ and also corrections to ${}^0\hat{\mathscr{R}}$ of the first order in the turbulence energy density. This does not lead to an essential complication in the calculation of the matrix elements of the $(\hat{\mathscr{F}}\hat{G})$-operator.

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