Ultimately seamanship, like airmanship, is a matter of maintaining the ability to remain in the medium; that is, the ability to remain safely afloat. In addressing this perpetual requirement the seaman navigator has constantly to satisfy himself that in proceeding he is not standing into danger. Many dangers are locus-related and publicised, so that the modern voyager does not sally forth into the great unknown, as was often the case in the time of Captain Cook. Yet the number of maritime calamities remains constant, despite the continuing improvement of navigational aids.

Danger, hazard and peril are those things which frustrate the safe and timely conclusion of a voyage. They encompass, for example, volcanic islands rising out of the sea or, more usually, storms and other inclement weather which cause cargo to shift or cause dynamic failure and the ingress of water or, more usually still, the waywardness of other navigators. Of these categories, the first may be deemed an act of God in response to which man must act as best he can. The second calls for improvements in design, the better to cope with the onslaught of the elements. All call for prescience and seamanship, the better to cope with whatever situation arises.

The Rules of the Road provide statutory guidelines for both awareness and action but, in the last resort, the navigator must get and keep his ship out of harm's way as best he may. The essence of the matter is controlled motion, in the environment pertaining and in the domain of the ship. Particular force vectors have to be resolved or controlled if the ship and the persons onboard are not to become prisoners of circumstance, such as collision or being engulfed.

A moving floating machine, her entrained water and the water in her future path comprise the domain over which control is vital, and it follows that success must lie in the prescient control of the ship, which can only be achieved by knowledge of the particular characteristics of the particular ship, in the particular circumstances. Such is the essence of good seamanship and the foundation of safe navigation.

KEY WORDS

1. Risk analysis 2. Collisions

'Trans-oceanic Passages by Rhumbline Sailing'

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1. INTRODUCTION. In the paper 'Trans-oceanic passages by rhumbline sailing' contributed by Captain Ivica Tijardović,[1] the following formulae were used to find the rhumbline course and derive the minimum distance by a differential method:

$$D = \delta\phi \sec\theta + (\delta\lambda - M\tan\theta)\cos\phi_2, \quad (1)$$

$$\sin\theta = M\cos\phi_2/\delta\phi. \quad (2)$$

The terms in these equations are as defined in Captain Tijardović's Fig. 1. The equations offer the mariner a simple and rational method of saving distance as compared with a direct rhumbline track between two positions, but some questions arise which are worthy of discussion.

2. ANALYSIS. This paper provides further insight into several controversial points. These are:

(i) In the navigational context, the term 'middle latitude' seems ambiguous.

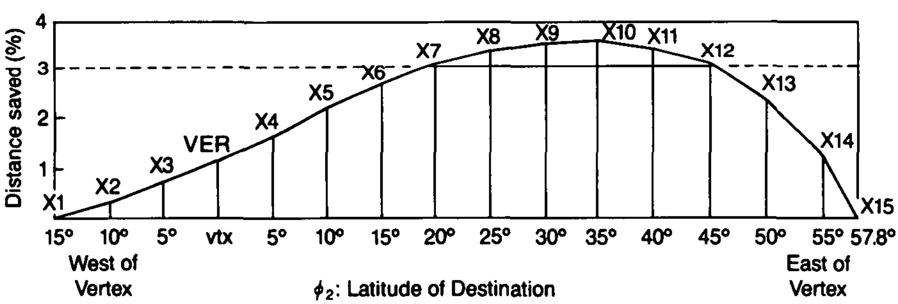
(ii) Is it necessary that the vertex of a great circle must be between the positions of departure and destination?

(iii) Is the method easy to apply in practice?

Demonstration. Starting from the original Tijardović example, the departure position is kept fixed but the destination position X_i ($i = 1, 2, 3, \ldots, 15$) is taken at difference of longitude steps of 5° each side of the vertex. The percentage of distance saved compared with the direct rhumbline (mercator) track is then calculated for each of the destination positions X_i and the results are shown in Table 1 and Fig. 1.

TABLE 1. COMPARISON OF RESULTS BASED ON THE TIJARDOVIĆ METHOD AND EXAMPLE

	ϕ_2	λ_2	θ_r	D_r	θ	D	Diff. $D_r - D$	Saving (%)
X_1	51−46′·8	176−40·6	61°·5	2110·5	60°·0	2108·6	1′·9	0·09
X_2	52−19′·2	171−40·6	63°·2	2307·7	58°·3	2299·3	8′·4	0·36
X_3	52−38′·4	166−40·6	64°·9	2503·4	57°·9	2484·0	19′·4	0·77
V_{tx}	52−44′·8	161−40·6	66°·7	2699·6	57°·8	2688·3	31′·3	1·16
X_4	52−38′·4	156−40·6	68°·5	2898·3	57°·9	2849·0	49′·3	1·70
X_5	52−19′·2	151−40·6	70°·4	3101·5	58°·3	3032·9	68′·6	2·21
X_6	51−46′·8	146−40·6	72°·3	3311·5	58°·9	3222·2	89′·3	2·70
X_7	51−00′·0	141−40·6	74°·2	3529·5	59°·9	3419·2	110′·3	3·13
X_8	50−00′·0	136−40·6	76°·6	3758·1	61°·1	3629·4	128′·7	3·42
X_9	48−42′·6	131−40·6	78°·1	4000·3	62°·6	3855·2	145′·1	3·62
X_{10}	47−07′·8	126−40·6	80°·1	4257·9	64°·6	4101·2	156′·7	3·68
X_{11}	45−12′·0	121−40·6	80°·2	4534·3	56°·9	4375·0	158′·3	3·51
X_{12}	42−54′·9	116−40·6	84°·3	4831·4	69°·9	4681·1	150′·3	3·11
X_{13}	40−12′·0	111−40·6	86°·5	5152·3	73°·8	5028·0	124′·9	2·42
X_{14}	37−01′·4	106−40·6	88°·7	5500·8	78°·6	5429·4	71′·2	1·29
X_{15}	35° N	103−51·2	90°·0	5683·9	90°·0	5683·9	0	0



ϕ_2: Latitude of Destination
λ_2: Longitude of Destination
θ_r: Course of Mercator track
D_r: Distance of Mercator track

Fig. 1. Illustration of the results from Table 1

There are two findings from the results presented in Table 1 and Fig. 1.

(i) The maximum percentage saving falls at the position X_{10} (47° 07′ N, 126° 40·6′ W). This is not the 'middle latitude' between the positions of departure and destination, although it is in what are loosely termed 'middle latitudes'. It would be less confusing if the term 'middle latitude' were replaced by the term 'moderate latitude' in the Tijardović paper.

(ii) Even when the vertex of the great circle is not located between the departure and destination positions, the method still saves distance compared with the direct rhumbline track, but the percentage saving is relatively small, as for the positions X_1, X_2 and X_3.

3. ALTERNATIVE METHOD. We consider the form of equation (2). The difference of meridional parts M depends on the latitudes ϕ_1 and ϕ_2 of the departure and destination positions, respectively. The term $\cos\phi_2$ depends on the latitude of the destination. Clearly, we can derive a value for the initial course θ as long as we know the departure and destination positions. Thus a turnpoint on the parallel of ϕ_2 can be found whatever relationship exists between turnpoint and destination.

We now consider the alternative strategy of finding the longitude (λ_P) of the turnpoint P instead of finding the initial course θ.

From Fig. 2, taking the departure position as A and the destination position as B, we have:

$$AP = \delta\phi \sec\theta$$

and

$$\theta = \tan^{-1}\left(\frac{\lambda_P-\lambda_1}{M}\right)$$

so that

$$AP = \delta\phi \sec\tan^{-1}\left(\frac{\lambda_P-\lambda_1}{M}\right), \tag{3}$$

$$PB = (\lambda_2-\lambda_P)\cos\phi_2. \tag{4}$$

From (3) and (4)

$$D = \delta\phi \operatorname{sectan}^{-1}\left(\frac{\lambda_P-\lambda_1}{M}\right)+(\lambda_2-\lambda_P)\cos\phi_2,$$

where $D = AP+PB$

$$D = \delta\phi\sqrt{\left(\frac{(\lambda_P-\lambda_1)^2+M^2}{M^2}\right)}+[(\lambda_2-\lambda_1)-(\lambda_P-\lambda_1)]\cos\phi_2. \tag{5}$$

To find the condition for the distance D to be a minimum, we differentiate the right-hand side with respect to the longitude difference $(\lambda_P-\lambda_1)$ and equate to zero. Thus:

$$0 = \frac{\delta\phi(\lambda_P-\lambda_1)}{M\sqrt{[(\lambda_P-\lambda_1)^2+M^2]}}-\cos\phi_2,$$

$$\delta\phi^2(\lambda_P-\lambda_1)^2 = M^2\cos^2\phi_2(\lambda_P-\lambda_1)^2+M^4\cos^2\phi_2,$$

$$(\delta\phi^2-M^2\cos^2\phi_2)(\lambda_P-\lambda_1)^2 = M^4\cos^2\phi_2,$$

$$(\lambda_P-\lambda_1) = \pm\frac{M^2\cos\phi_2}{\sqrt{(\delta\phi^2-M^2\cos^2\phi_2)}}. \tag{6}$$

Discussion. Referring to Fig. 2(*a*), we can see that, if the destination B is located between the departure position A and the turnpoint P, we must alter course to the west, which will increase the total distance D. Therefore the turnpoint P must be located between A and B. This means that there are limits of the longitude of B such that the

TABLE 2. COMPARISON OF REPRESENTATIVE EXAMPLES

	A, departure		B, destination		P, turnpoint		θ_r, rhumb-line course	θ_g, great course	θ, course	D_r, rhumb-line distance	D_g, great distance	D, distance	D_r-D, difference of D_r, D	D_g-D difference of D_g, D	Saving (%) (D_r-D) 100 / D_r
	ϕ_1	λ_1	ϕ_2	λ_2	ϕ_P	λ_P									
1	0·0	0·0	15·0	30·0	15·0	60·8	63·3	61·8	76·1	2004·6	1993·6	5530·0	−3525·4	−3536·47	−175·86%
2	0·0	0·0	15·0	60·0	15·0	60·8	75·9	72·8	76·1	3693·8	3667·3	3791·4	−97·6	−124·10	−2·64%
3	0·0	0·0	15·0	90·0	15·0	60·8	80·5	75·0	76·1	5448·5	5400·0	5432·4	16·1	−32·43	0·30%
4	0·0	0·0	15·0	120·0	15·0	60·8	82·8	72·8	76·1	7221·2	7132·7	7171·1	50·1	−38·35	0·69%
5	15·0	0·0	30·0	30·0	30·0	42·9	61·6	56·3	69·3	1893·7	1882·0	3220·3	−1326·5	−1338·26	−70·05%
6	15·0	0·0	30·0	60·0	30·0	42·9	74·9	63·7	69·3	3451·8	3407·6	3435·5	16·3	−27·96	0·47%
7	15·0	0·0	30·0	90·0	30·0	42·9	79·8	60·9	69·3	5079·0	4953·9	4994·4	84·6	−40·50	1·67%
8	15·0	0·0	30·0	120·0	30·0	42·9	82·3	51·6	69·3	6725·3	6407·3	6553·2	172·1	−145·88	2·56%
9	30·0	0·0	45·0	30·0	45·0	37·6	57·7	49·1	63·3	1685·6	1673·1	2323·2	−637·7	−650·11	−37·83%
10	30·0	0·0	45·0	60·0	45·0	37·6	72·5	54·6	63·3	2989·1	2923·2	2950·5	38·6	−27·29	1·29%
11	30·0	0·0	45·0	90·0	45·0	37·6	78·1	49·1	63·3	4369·2	4157·7	4223·3	145·9	−65·57	3·34%
12	30·0	0·0	45·0	120·0	45·0	37·6	81·0	37·8	63·3	5771·3	5237·1	5496·1	275·2	−258·97	4·77%
13	45·0	0·0	60·0	30·0	60·0	37·0	50·3	39·2	56·1	1409·3	1397·0	1823·5	−414·2	−426·48	−29·39%
14	45·0	0·0	60·0	60·0	60·0	37·0	67·5	44·8	56·1	2348·4	2273·6	2302·2	46·2	−28·58	1·97%
15	45·0	0·0	60·0	90·0	60·0	37·0	74·5	39·2	56·1	3375·9	3134·3	3202·2	173·6	−67·89	5·14%
16	45·0	0·0	60·0	120·0	60·0	37·0	78·3	28·8	56·1	4430·6	3850·6	4102·2	328·4	−251·61	7·41%

Key: θ_r: the course of rhumbline sailing. θ_g: the initial course of great circle sailing. θ: the course of the aforenamed method. D_r: the distance of rhumbline sailing. D_g: the distance of great circle sailing. D: the distance of this method. D_r-D: the difference between D_r and D. D_g-D: the difference between D_g and D. Save percentage: the ratio of $(D_r-D)\,100/D_r$.

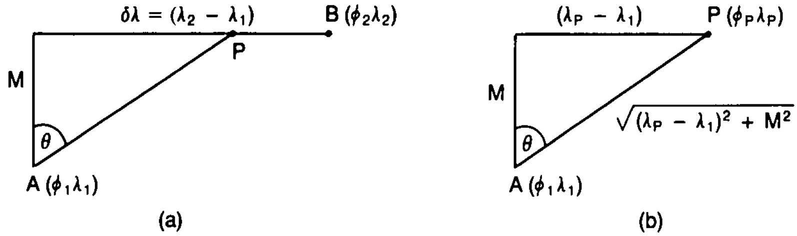


Fig. 2. Mercator chart plots

difference of longitude between A and B must be greater than that between A and P. Symbolically:

$$|\lambda_2-\lambda_1| \geqslant |\lambda_P-\lambda_1| = \frac{M^2\cos\phi_2}{\sqrt{(\delta\phi^2-M^2\cos^2\phi_2)}}.$$

This represents a judgement as to whether the method will save on the total distance or not.

To calculate the total distance D, even if the initial course is unknown, a version of equation (5) may be used:

$$D = (1/M)\sqrt{[(\lambda_P-\lambda_1)^2+M^2]}+(\lambda_2-\lambda_P)\cos\phi_2. \tag{7}$$

4. COMPARISON OF CASES. Using the rapid calculating capabilities of the computer, it has been possible to find comparative data for rhumbline sailing, great-circle sailing, and the above method in many cases. Initial courses, distances, differences of distances and distance-saved percentages have been calculated. The results for some representative examples are illustrated in Table 2.

The results presented in Table 2 may be summarized as follows:

(i) In cases 1, 2, 5, 9 and 13, $\lambda_2 < \lambda_P$ and $|\lambda_2-\lambda_1| \leqslant |\lambda_P-\lambda_1|$, which implies an alteration of course to the west on arrival at the turnpoint so that the distance-saved percentage is negative.

(ii) In cases 4, 8, 12 and 16, the difference of latitude $\delta\phi$ between A and B is fixed. If the latitudes of both A and B are increased then the initial course decreases, the distance difference D_r-d increases and the distance saved percentage increases.

(iii) In cases 3, 4, 7 and 8, the departure position A(ϕ_1 λ_1) is fixed and the latitude (ϕ_2) of the destination position B is also fixed. As the longitude (λ_2) of the destination position increases, the initial course θ remains unchanged. The distance difference D_r-D increases and the distance saved percentage increases.

5. CONCLUSIONS. There are a number of findings from this study.

(i) Theoretically, the method can be used in any latitude but, the higher the latitude, the more distance will be saved.

(ii) The position of the vertex of the great circle joining the departure and destination points is irrelevant.

(iii) The prevailing environmental conditions should be considered when using the method, such as tidal streams, current, wind, sea state and weather. For example, it may be more efficient to take a course off the wind, or so as to take advantage of a favourable current. Or it may be prudent to seek shelter from a cyclone.

(iv) The method finds a turnpoint on the parallel of latitude which passes through the destination such that the total distance is a minimum. We could perhaps infer that there must exist another turnpoint on some other parallel of latitude which gives a further reduction in the overall distance. Would this be a simple matter to establish, and is it worth further discussion?

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KEY WORDS

1. Marine navigation. 2. Voyage planning.

Astro Without Azimuth

Charles Brown

Comments have been made on recent papers[1,2,3] which discussed a method of deriving an OP (observed position) from celestial observations, which does not depend upon the use of azimuth and intercept and which does not require a CP (chosen position) or DR position to be input into the calculation. Using the same principle, a method is given for calculating an OP where there has been observer movement between sights.

1. A DISCUSSION OF THE BASIC METHOD. The writer became interested in this topic in 1980 when applying similar thinking to radio bearings; subsequently the general principles evolved were applied to the determination of an OP from celestial observations.

A computer program using the Epson HX20 PC was evolved and has been in use since 1982/83. The method used is similar to that described by Spencer[2] and is based on the concept of great circles (GCs) pivoting about the GPs (geographical positions) and intersecting at distances equal to the ZD (zenith distance).

The solution for position is mathematically simple but laborious and depends almost exclusively upon the use of the spherical cosine equation. Resolution of the ambiguity in determined position(s) can be either automatic (e.g. using computed and observed ZDs for the third sight), manually or by reference to the retained DR. Any number of observations can be handled (depending on memory available) – the program automatically selecting pairs of sights from those available, together with the discriminating third sight.

The method used by the writer additionally employs automatic date/time recordings[5] at each observation; this enables the declination (DEC) and Greenwich Hour Angle (GHA) to be derived for each body from the internal almanac[4] (see Appendix 1).

The following comments refer to the basic concept outlined above and in references 1 and 2.

(a) The use of a third body to resolve ambiguity in position fails when all three bodies lie in the same GC, – an unlikely occurrence but it can exist, for example, when taking successive sights of the Sun when the zenith of that body lies near the equator; safeguards are built into the program.

(b) Narrow angle intersections (or virtual reciprocal bearings) can be a problem

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