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Altitude Aiding for GPS Systems using Elevation Map Datasets

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This paper reports the development of a DGPS navigation system integrated with altitude aiding. In this system, a digital height dataset is used for altitude augmentation. A two-dimensional (2-D) positioning algorithm is discussed and modified based on previous publications. The developed algorithm was implemented on the Brunel Inverse DGPS system. The performance of the new developed system is experimentally verified and compared with three-dimensional (3-D) GPS and DGPS systems. The experimental results showed 86% availability of positioning services; whereas for the 3-D GPS system alone, the availability was only 63% of time. In addition, the accuracy of the system was improved from 7·1 to 6·1 m (RMS) for GPS and from 6·0 to 5·1 m (RMS) for DGPS when compared to standalone 3-D modes.

KEY WORDS

1. GPS. 2. Land Navigation. 3. Augmentation.

1. INTRODUCTION. In urban canyon areas, the availability and accuracy of GPS-based systems can be degraded due to blocking of the satellite signals. If another sensor can provide altitude information, two-dimensional (2-D) GPS positioning can be a remedy for some applications, providing an improvement in system performance. During normal GPS receiver operations, pseudo-ranges from four GPS satellites are needed to solve the equations for the receiver's antenna position because of the four unknowns: three position unknowns and the receiver's clock bias (x, y, z, Δt). When less than four satellites are available, or the current geometry of satellites is poor, a GPS receiver (system) can operate in an augmented mode using additional positioning information from an external sensor.

The concept of 2-D positioning systems is well known (DoD, 1996; Parkinson and Spilker, 1996; Brown and Struza, 1993; McBurnley and Braisted, 2000). The more detailed descriptions have been patented in several applications using different sensors for altitude augmentation. Nevertheless, most of these systems have only been described theoretically without creating a real working system that could be tested in order to assess its performance and usability. This paper presents and discusses a 2-D positioning algorithm to which changes are proposed. This proposed algorithm, using a digital height dataset for altitude augmentation, has been implemented on the Brunel University 2-D inverse DGPS positioning system. The performance of the newly developed system is experimentally verified and compared with 3-D GPS and DGPS systems.

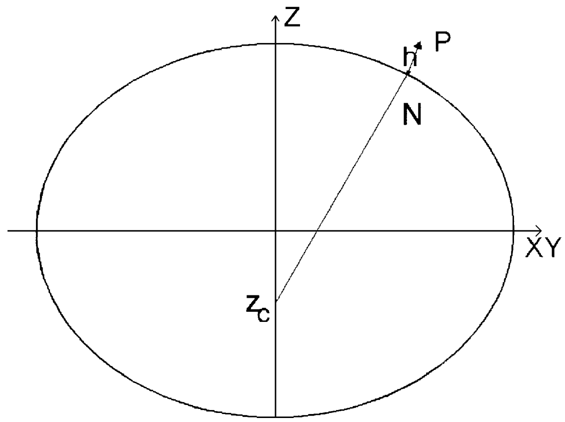

Figure 1. WGS-84 ellipsoid and normal N in the direction to point P with altitude *h* over the ellipsoid.

2. THEORETICAL BACKGROUND.

2.1. *2-D Position Calculation.* If less than four pseudo-range measurements to different satellites are made, then the normal system of positioning equations is under-determined, and the position unknowns cannot be solved. Hence, the complete 3-D GPS navigation solution is unavailable. In performing 2-D positioning using altitude aiding, it is assumed that an additional range measurement is available that is exactly in the vertical direction relative to the Earth's surface; this is equivalent to having a satellite at the centre of the Earth (Parkinson and Spilker, 1996; McBurnley and Braisted, 2000). This range measurement is effectively a distance from the centre of the Earth to the user antenna based on the assumption that the user altitude above the reference ellipsoid is constant. WGS-84 is the ellipsoidal model of the Earth used in GPS positioning (DoD, 1996). Because of the non-spherical shape of the Earth the altitude can be only considered as a range using spherical approximation if the approximate location of the calculated position is initially known.

As can be seen from Figure 1, another problem, which results from the non-spherical shape of the Earth, is that the normal N, which is exactly in the direction to point P, does not go through the centre point of the ellipsoid. The description given in the literature (Parkinson and Spilker, 1996; and McBurnley and Braisted, 2000) uses the centre of the ellipsoid as the centre of the sphere representing the constant altitude.

A simulation has been carried out to determine the inaccuracy of locating a pseudo-satellite at the centre of the Earth (see Figure 2). Assuming that a given sphere should be exactly over the provided area of the reference ellipsoid, we placed the centre of the sphere at the centre of the ellipsoid. The sphere radius was the distance between the centre and the predicted point location. Changing the position of the point on the sphere within a distance of ± 100 m, the maximum deviation of the distance from the sphere over the reference WGS-84 ellipsoid was observed. The selected point had co-ordinates: latitude 45° N, longitude 0° and ellipsoidal height 100 m. These values were chosen based on the fact that, for latitude of 45°, the approximation would be the least accurate, longitude value would not affect it, and 100 m was taken as a hypothetical ellipsoidal height for land applications. The maximum change from the

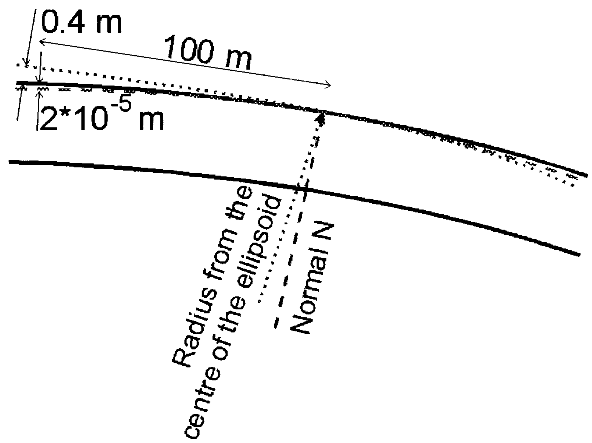

Figure 2. Inaccuracies due to different locations of pseudo-satellites.

assumed distance (altitude) between the ellipsoid and sphere was 0·4 m. The same simulation was performed with the centre of the sphere placed at the point (0, 0, z_c), which lays exactly on the normal N. The sphere radius was the distance between this point and the predicted point location. In this case, the maximum change of the distance between the reference ellipsoid and a given sphere was less than 2×10^{-5} m. Usually the altitude error is in a range of a few to tens of metres. In this case, both of the approximations are reasonably accurate. However, using the second proposed method would give more accurate results.

Adding an altitude-aiding equation to the set of 3 equations representing pseudo-range measurements to satellites (Kaplan, 1996) would lead to the following set of positioning equations:

$$\begin{aligned} \rho_1 &= |\boldsymbol{s_1} - \boldsymbol{u}| + ct_\mathrm{u} + \epsilon_{\rho_1} \\ \rho_2 &= |\boldsymbol{s_2} - \boldsymbol{u}| + ct_\mathrm{u} + \epsilon_{\rho_2} \\ \rho_3 &= |\boldsymbol{s_3} - \boldsymbol{u}| + ct_\mathrm{u} + \epsilon_{\rho_3} \\ |\hat{\boldsymbol{u}} - \boldsymbol{c}_\mathrm{s}| &= |\boldsymbol{c}_\mathrm{s} - \boldsymbol{u}| + \epsilon_\mathrm{alt}, \end{aligned} \tag{1}$$

where ρ_i is the pseudo-range observation between a user and satellite i, corrected by known biases, $\boldsymbol{s}_j = (s_{xj}, s_{yj}, s_{zj})$ is the position of j-th satellite at the time when the pseudo-range measurement were taken, $\boldsymbol{u} = (u_x, u_y, u_z)$ current position solution, t_u the user's GPS receiver clock offset, c the speed of light, $\epsilon_{\rho j}$ the pseudo-range error, $\hat{\boldsymbol{u}} = (\hat{u}_x, \hat{u}_y, \hat{u}_z)$ the estimated initial user position, and c_s the centre of the sphere representing the altitude aiding equation.

This non-linear set of equations can be solved by techniques based on linearisation (Kaplan, 1996). The linearised result of these equations, by expanding in a Taylor series about the approximate position, is given by the following:

$$\begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta A \end{bmatrix} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ a_{x3} & a_{y3} & a_{z3} & 1 \\ a_{x\mathrm{alt}} & a_{y\mathrm{alt}} & a_{z\mathrm{alt}} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ -c\Delta t \end{bmatrix} + \begin{bmatrix} \epsilon_{\mathrm{sv}1} \\ \epsilon_{\mathrm{sv}2} \\ \epsilon_{\mathrm{sv}3} \\ \epsilon_\mathrm{alt} \end{bmatrix}, \tag{2}$$

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where:

$$a_{xj} = \frac{s_{xj} - \hat{u}_x}{r}, \qquad a_{yj} = \frac{s_{yj} - \hat{u}_y}{r}, \qquad a_{zj} = \frac{s_{zj} - u_z}{r}, \tag{3}$$

$$r_j = |\mathbf{s_j} - \hat{\mathbf{u}}| = \sqrt{(s_{xj} - \hat{u}_x)^2 + (s_{yj} - u_y)^2 + (s_{yj} - u_z)^2}, \tag{4}$$

$$a_{x\text{alt}} = \frac{-\hat{u}_x}{\sqrt{\hat{u}_x^2 + \hat{u}_y^2 + \hat{u}_z^2}}, \qquad a_{y\text{alt}} = \frac{-\hat{u}_y}{\sqrt{\hat{u}_x^2 + \hat{u}_y^2 + \hat{u}_z^2}}, \qquad a_{z\text{alt}} = \frac{-u_z}{\sqrt{\hat{u}_x^2 + \hat{u}_y^2 + \hat{u}_z^2}}. \tag{5}$$

$\Delta\rho_j = \hat{\rho}_j - \rho_j$, and $\hat{\rho}_j$ is a pseudo-range from the predicted estimated initial position of the *j*-th satellite, and ΔA the difference between the given-required altitude and the initial position altitude.

Altitude aiding can be also used when a receiver tracks more than 4 satellites. Then the positioning processor can select only the three satellites that will yield the best possible navigation solution as it is proposed in Barry (2001) or can use 'all satellites in view' with the over-determined set of equations. Using the second method, the resulting altitude solution will be a compromise between the supplied altitude and the altitude solution that would be available from the 3-D GPS position solution. The most common method to solve an over-determined set of equations is least squares (Kaplan, 1996). The altitude measurement equation should be appropriately weighted, as the standard deviation of supplied altitude error is usually different from the standard deviation of pseudo-range measurements from satellites.

The scaled form of the over-determined set of positioning equations is:

$$\begin{bmatrix} \Delta\rho_1/\sigma_{\text{SV}1} \\ \cdots \\ \Delta\rho_3/\sigma_{\text{SV}n} \\ \Delta A/\sigma_{\text{alt}} \end{bmatrix} = \begin{bmatrix} a_{x1}/\sigma_{\text{SV}1} & a_{y1}/\sigma_{\text{SV}1} & a_{z1}/\sigma_{\text{SV}1} & 1/\sigma_{\text{SV}1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{x3}/\sigma_{\text{SV}n} & a_{y3}/\sigma_{\text{SV}n} & a_{z3}/\sigma_{\text{SV}n} & 1/\sigma_{\text{SV}n} \\ a_{x\text{alt}}/\sigma_{\text{alt}} & a_{y\text{alt}}/\sigma_{\text{alt}} & a_{z\text{alt}}/\sigma_{\text{alt}} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ -c\Delta t \end{bmatrix} + \begin{bmatrix} \epsilon_{\text{sv}1}/\sigma_{\text{SV}1} \\ \cdots \\ \epsilon_{\text{sv}3}/\sigma_{\text{SV}n} \\ \epsilon_{\text{alt}}/\sigma_{\text{alt}} \end{bmatrix}, \tag{6}$$

where σ_{alt} is the standard deviation of the error in altitude measurement, and $\sigma_{\text{SV}i}$ is the standard deviation of satellite range measurement errors.

3. 2-D NAVIGATION ALGORITHM.

The following navigation algorithm has been proposed for altitude augmentation using digital maps integrated with a GPS or DGPS system. Referring to the flowchart of Figure 3, the navigation process for determining 2-D GPS or DGPS position from pseudo-range measurements consists of the following stages:

(a) Collect pseudo-range measurements and satellite navigation data from all tracked satellites discarding information from satellites with unhealthy status or elevation angle below a given threshold.
(b) Apply atmospheric corrections using differential data from a DGPS reference receiver or calculated from ionospheric and tropospheric models.
(c) Determine if there is sufficient information to calculate a 2-D position. Three valid pseudo-range measurements with relevant satellite ephemeris data are required. In a cold start when the previous position is unavailable, in order to

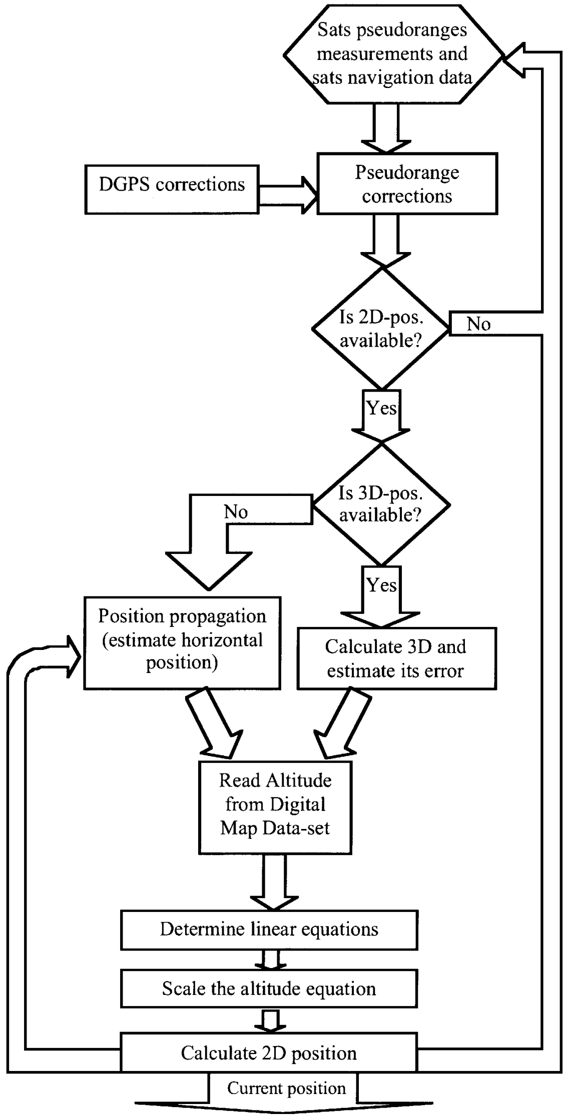

Figure 3. Flow chart of the process to determine 2-D position with altitude augmentation using digital map dataset.

read the altitude value from the map four pseudo-range measurements are required.

(d) In order to read the altitude from the elevation map, an initial horizontal position is required. Determine if there is sufficient information to calculate 3-D fix:
   (i) If yes, calculate 3D position.
   (ii) If no, estimate the current horizontal position based on the preceding position information. Assuming that the user location has not changed significantly to affect the altitude, the latest position fix can be used or the position can be extrapolated using Kalman filtering.

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(e) Read the altitude information from the digital map.
(f) Determine linear equations using pseudo-range measurements to all satellites plus altitude aiding equation.
(g) If the total number of equations is greater than four, then scale the altitude equation in order to calculate a least square solution with an over-determined set of equations.

4. EXPERIMENTAL VERIFICATION.

4.1. *Experimental set-up – Brunel Integrated Inverse DGPS system.* One form of the DGPS implementation is an Inverse DGPS (Blackwell, 1985), which can be used successfully in many applications with centralised architecture, where the remote units report their location to the service centre. Using inverse DGPS releases the user from carrying a large map dataset and a differential receiver for correction data, which significantly simplifies the Mobile Unit.

The proposed 2-D positioning algorithm was integrated with the Brunel Inverse DGPS system. The block diagram of Brunel Inverse DGPS system is presented in Figure 4. The Mobile Unit is based on a standard GSM mobile telephone and GPS

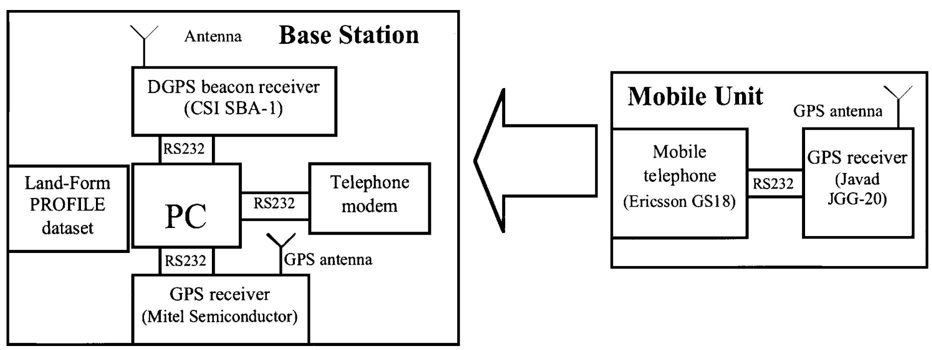

Figure 4. Brunel Integrated Inverse DGPS system – Block Diagram.

receiver. Currently, circuit data switched channel is used for positioning data transmission. SMS data communication or packet data communication using GPRS terminals can be used as either of these techniques would permit simultaneous data and voice transmission. The Base Station is implemented using a PC. Pseudo-range and positioning data are received via a modem from the Mobile Unit. In order to calculate DGPS position, up-to-date ephemeris and pseudo-range correction data are required. The beacon receiver provides DGPS correction data from the General Lighthouse Authorities (GLAs) marine beacon receivers within the UK. Ephemeris data are provided from a separate GPS receiver at the base station.

4.2. *Altitude Augmentation Datasets.* Land-Form PROFILE maps from Ordnance Survey were used as the database for altitude information for GPS and DGPS position augmentation. Land-Form PROFILE is the 1:10,000 scale digital height dataset covering the whole of Great Britain (Ordnance Survey, 1999). The height accuracy of any point derived from a contour map depends upon the nature of the ground. It is typically better than one half of the vertical interval of the source contour data, which is either 5 or 10 m.

4.3. *Configuration of the Experiments.* At Brunel University campus, we conducted experiments to validate our 2-D positioning systems and its performance compared with 3-D GPS and DGPS systems. Knowing that 2-D positioning would be especially beneficial in a situation when a GPS receiver would be able to track only a small number of satellites due to signal blockage and, having in mind different navigation and tracking applications that would be expected to work in a built-up city area, we choose a pedestrian walk adjacent to buildings. The mobile unit was mounted on a person, with the GPS antenna placed on the shoulder of the walker (Garaj *et al*., 2000).

For the purpose of the accuracy measurements, the person walked exactly along a marked route. Along this route, some benchmark points were marked, which were accurately surveyed as the reference points. Whenever the person passed a known benchmark point, an accurate time synchronized with the GPS time was logged. Assuming a constant velocity of the walker between the benchmarks, the reference truth for each position fix was achieved with an accuracy of about 1 m during the whole trail.

The experiment was repeated four times at different times of the day and so with different constellations of satellites as GPS system concept relies upon the dynamics of satellites, which constantly change position with reference to positions on the Earth. For the performance assessment, two parameters were considered: the availability of the positioning service and the accuracy of the system. The positioning service was available when the GPS receiver tracked at least 4 satellites for 3-D type of position solution or when using at least 3 satellites for 2-D type of solution. Additional constraints were placed upon satellite visibility in terms of elevation mask angle and geometry, to minimise the possibility of a positioning service generating a marginal position solution. The elevation mask was set up to 5° as a commonly used value for performance assessments (DoD and DoT, 1995). The threshold parameter related to the dilution of precision can always vary and be assigned a value depending upon expected accuracy of a system. Increasing the value of this parameter, we can improve the availability at the cost of degrading the overall accuracy of the system. In our experiments, the threshold for the dilution parameter was set up to 5 for HDOP (Horizontal Dilution of Precision). Selecting this value should maintain the horizontal accuracy of 30 and 20 m for GPS and DGPS systems through 95% of the time when the HDOP value is 5. A position solution within 20 m of true position would always allow a user to determine on which street he or she is currently located, so that the system can always differentiate between closely separated roads.

4.4. *The Results*. Positioning data received from the Mobile Unit were processed at the Base Station and generated four types of position solution:

(a) 3-D GPS,
(b) 2-D GPS,
(c) 3-D DGPS, and
(d) 2-D DGPS.

The accuracy was expressed using the most commonly used accuracy measures:

RMS (root mean square) – the square root of the average of the squared errors,
CEP (circular error probable) – a circle's radius, centred at the true position, containing 50% of the points in the horizontal scatter plot, and
R95 (horizontal 95% accuracy) – a circle's radius, centred at the true position, containing 95% of the points in the horizontal scatter plot.

Table 4. The comparison of the accuracy when both 3-D and 2-D positioning available.

	Accuracy (m)			Availability %
	RMS	CEP	R95	
3-D GPS	7·1	5·1	13·0	63
2-D GPS	6·1	4·5	11·5	
3-D DGPS	6·0	3·4	12·7	
2-D DGPS	5·1	3·2	9·7	

Table 5. The accuracy and availability of 2-D positioning when 3-D was unavailable.

	Accuracy (m)			Availability %
	RMS	CEP	R95	
2-D GPS	7·8	4·6	16·8	23
2-D DGPS	6·0	3·7	11·3	

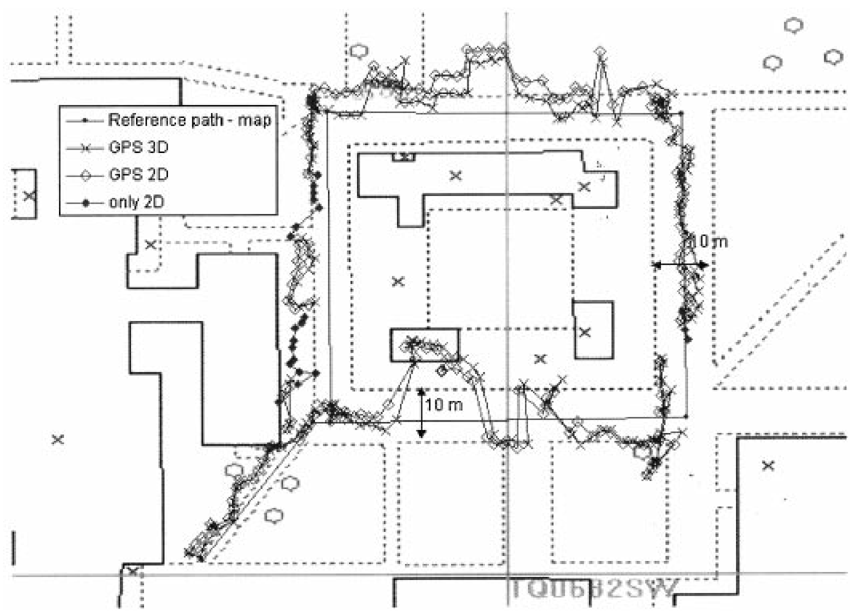

Figure 5. The results plotted on the map: first trail GPS.

Table 4 includes the accuracy assessment based only on records when both 2-D and 3-D positions were available. It can be seen from the results that using 2-D always gives better accuracy for both GPS and DGPS modes. However, the difference may not be significant for many applications.

Table 5 shows the accuracy of 2-D positioning when 3-D position was unavailable. The accuracy maintained the level of the accuracy of that when 3-D was available.

Tables 4 and 5 also show the availability of 3-D and 2-D as a percentage of total time. The difference in 3-D and 2-D availability is significant, and it can be seen that 2-D improved the availability by 23% from 63% to 86% when the system was in use.

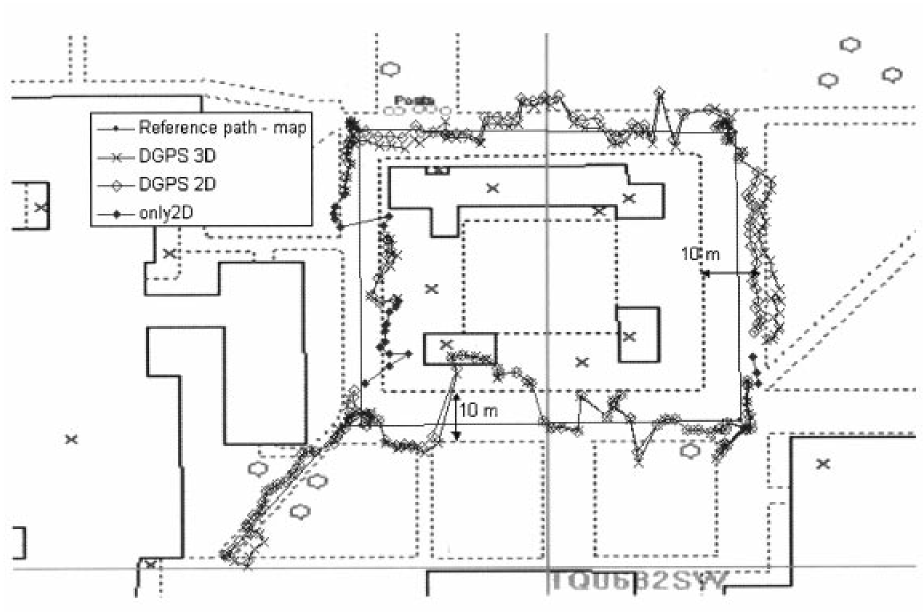

Figure 6. The results plotted on the map: first trail DGPS.

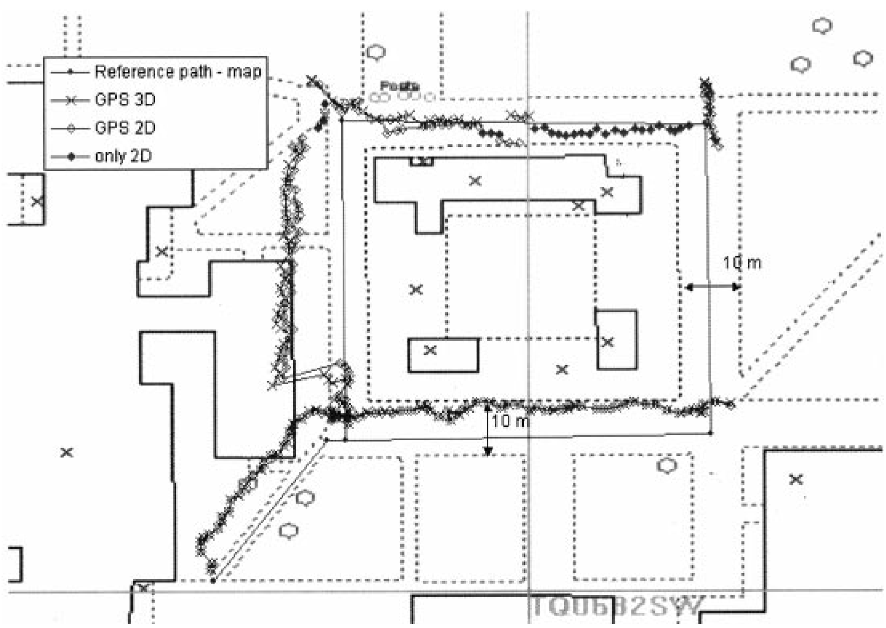

Figure 7. The results plotted on the map: second trail GPS.

The graphical representation of the experimental results for first 3 trails are plotted on the corresponding Ordnance Survey Land-Line map. The reference path obtained from surveying points coincides very well with the true walking path on the map. The differences between surveyed position and position read from the map where no greater than 0·5 m. Figures 5, 7 and 9 present the results comparing 3-D and 2-D for

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GPS mode and Figures 6, 8 and 10 for DGPS mode. In Figures 5 and 6, it can be seen that the plotted 2-D and 3-D path are very close to each other. There are short sections where 3-D positioning was unavailable. 2-D positioning was available most of the time. Comparing GPS and DGPS results we can see smaller deviations of obtained results from the reference path for DGPS. If we assume that the user is walking on a pedestrian walk, we can always correctly identify where he or she is, but we cannot say on which exact side of a pavement the user is located. In Figures 7 and 8 there is a long section when neither 3-D nor 2-D was not available. In Figures 9 and

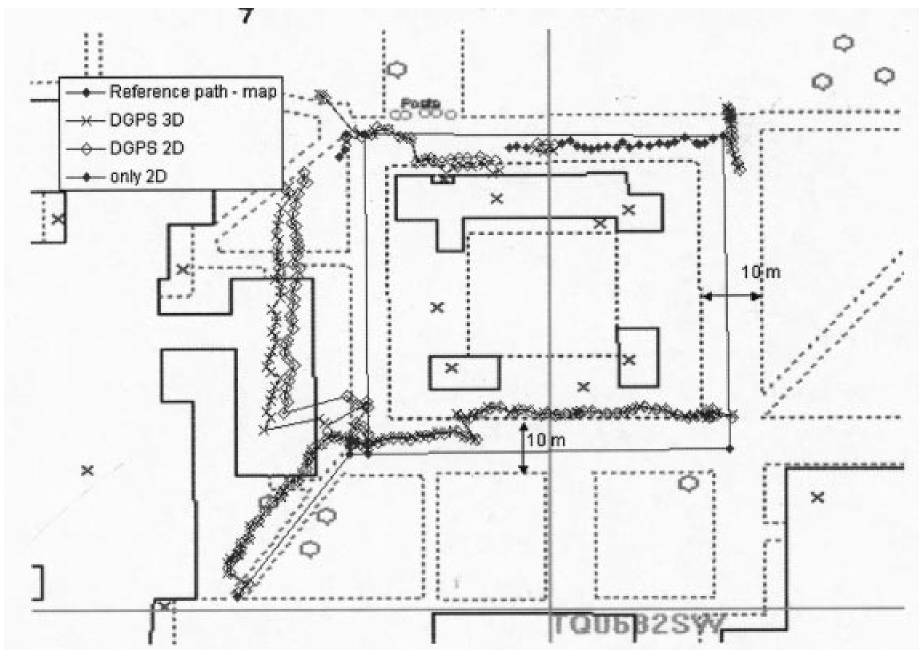

Figure 8. The results plotted on the map: second trail DGPS.

10, at the north section of the path, it can be noticed that 3-D path deviates more than 2-D path.

5. CONCLUSIONS.

In urban areas, GPS and DGPS performance can be significantly degraded due to the satellite signal blockage. Altitude augmentation can be an efficient way of improving GPS and DGPS performance in such land applications. Existing digital height datasets can be used and integrated with the existing systems. At Brunel University, an Inverse DGPS Positioning augmented with digital altitude map datasets has been developed using very accurate Ordnance Survey maps. The system was tested and experimentally verified in an urban canyon area. The augmented system is shown to improve availability in urban canyons substantially. When standalone GPS availability was only 63 %, it provided an improvement to 86 %. The accuracy of the system was also improved from 7·1 to

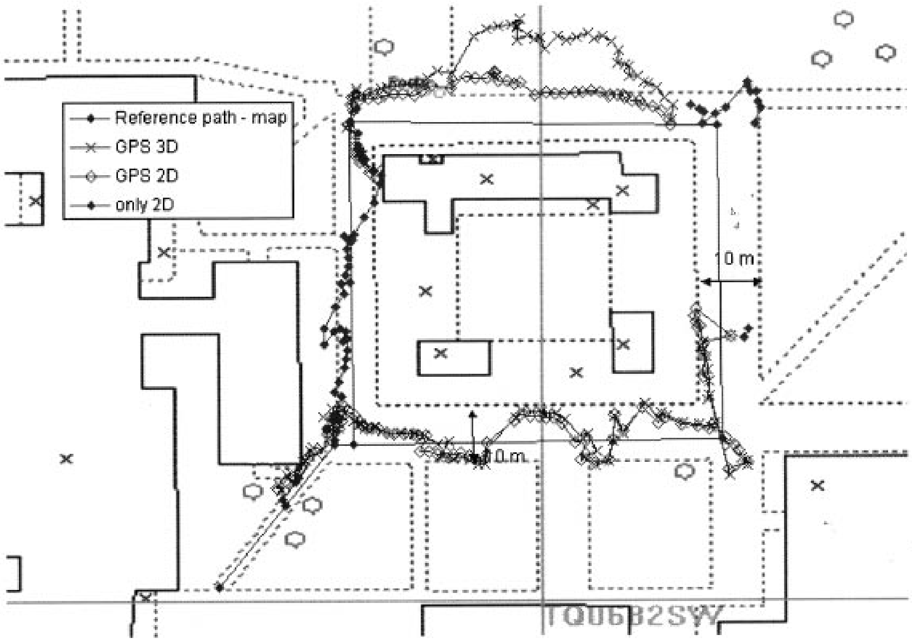

Figure 9. The results plotted on the map: third trail GPS.

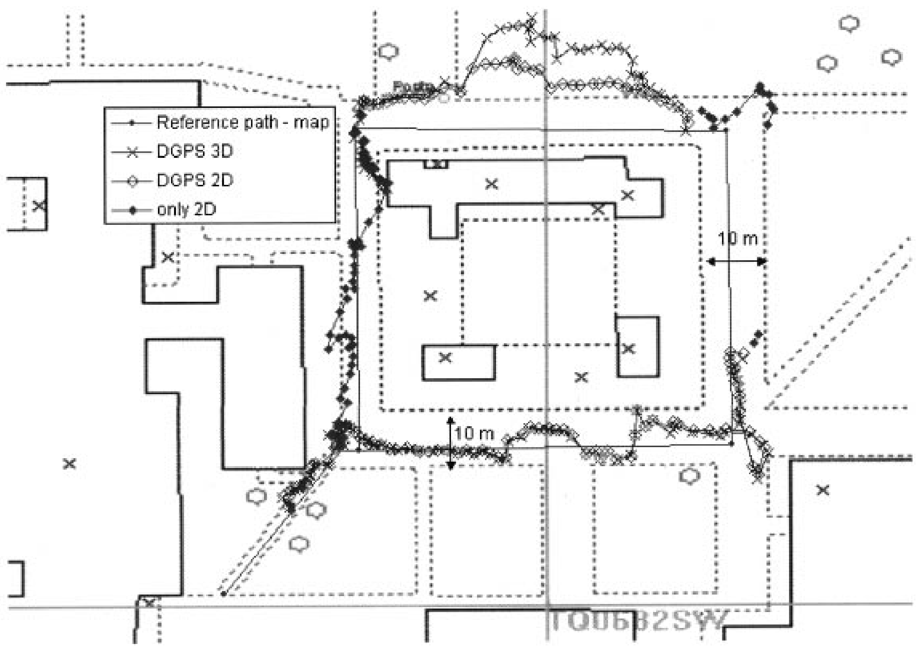

Figure 10. The results plotted on the map: third trail DGPS.

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6·1 m (RMS) for GPS and from 6·0 to 5·1 m (RMS) for DGPS. This system could be used as a positioning module for most vehicle and personal location and navigation systems, providing significant performance improvement in urban areas in terms of positioning service availability and accuracy.

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