Correspondence

DEAR EDITOR,

2 + 2 = 5

Note 106.20 quotes these words of George Orwell from an essay of 1939:-

> ' It is quite possible that we are descending into an age in which two plus two will make five when the Leader says so.'

Here is a proof of this fact, taken from my book *Comic Sections Plus*.

Addition can be performed with any objects. For example, you can add a number of spoons to a number of spoons and wind up with a number of spoons.

What happens if you add plus signs together? What, for example, is two plus signs added to two plus signs? Perhaps it should be

$$(+\ +) + (+\ +) = +\ +\ +\ +\ +,$$

or, in other words, $2 + 2 = 5$?

I would be interested to see how readers would refute this demonstration if a pupil were to offer it.

10.1017/mag.2023.31

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Feedback

On 106.12: Martin Lukarevski writes: Tran Quang Hung gives a proof of the Pythagorean theorem in n-dimensions. The three-dimensional case for a tetrahedron is of course the most interesting and is known as de Gua's theorem. It was published by J. P. de Gua de Malves in 1783, in [1] but it was already known to Descartes 1619-1621.

Recently de Gua's theorem was used for a derivation of Heron's formula [2], so a direct proof is desirable and we give one here. Let $OABC$ be a tetrahedron with right corner at vertex O and let $l = OA$, $m = OB$ and $n = OC$. Let the perpendicular from A to B meet BC at L. Then OL is also perpendicular to BC. From the right triangle OBC it follows that $CL = \frac{OC^2}{CB}$.

Hence

$$\begin{aligned}\text{Area}\,(ABC)^2 &= \tfrac{1}{4}BC^2AL^2 = \tfrac{1}{4}\left(OB^2 + OC^2\right)\left(AC^2 - CL^2\right)\\ &= \tfrac{1}{4}\left(m^2 + n^2\right)\left(l^2 + n^2 - \frac{n^4}{m^2 + n^2}\right)\\ &= \tfrac{1}{4}\left(l^2m^2 + m^2n^2 + n^2l^2\right)\\ &= \text{Area}\,(ABO)^2 + \text{Area}\,(BCO)^2 + \text{Area}\,(CAO)^2.\end{aligned}$$

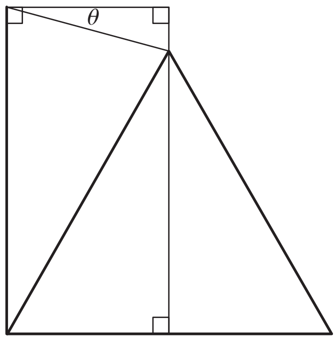
https://doi.org/10.1017/mag.2023.32 Published online by Cambridge University Press

References

1. J. P. de Gua de Malves, *Histoire*, Acad. Sc. Paris 1783 (1786), p. 375
2. J. M. Levy-Leblond, A symmetric 3D proof of Heron's Formula, *Mathematical Intelligencer*, **43**(2) (2021) pp. 37-39.3

10.1017/mag.2023.32

Paul Stephenson writes: In his July 2022 Feedback to 'What makes a good Proof without Words', Martin Lukarevski asks readers for such a demonstration of the fact that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. The diagram is one suggestion. The bold line segments are equal. We infer that (a) $\theta = \frac{\pi}{12}$ and (b) $\tan\theta = 2 - \sqrt{3}$.



10.1017/mag.2023.33

Amendment to Feedback: On 106.06 in November 2022

In the statement of the Lemma, the word 'negative' ought to have been 'non-negative'. The correct version is:

Lemma: If f is strictly increasing on the open interval (A, B), where A is non-negative, then the function $g(x) = xf(x)$ is *strictly superadditive* on that interval, namely,

$$g(x) + g(y) < g(x + y)$$

whenever x, y, $x + y$ belong to (A, B).

This was pointed out to us as a typo by Robert M Young and Jack Calcut and we apologise to them for failing to correct it.

Reference

R. Young, J. Calcut, On 106.06. *Math. Gaz.*, **106** (November 2022) pp. 549-550. doi:10.1017/mag.2022.144

10.1017/mag.2023.47