PARTICIPATING PAYOUT LIFE ANNUITIES: LESSONS FROM GERMANY

BY

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ABSTRACT

This paper analyzes the framework of German participating payout life annuities (PLAs), which offer guaranteed minimum benefits as well as participation in insurers' surpluses. We show that the process of sharing surpluses between shareholders and policyholders follows transparent and consistent rules. Subsequently, we develop an asset-liability model for a stylized German life insurer that offers PLAs to evaluate benefit variability and insurer stability given stochastic mortality and capital market developments. Our results suggest that guaranteed benefits can be provided with high credibility via PLAs, while, at the same time, annuitants receive attractive money's worth ratios. Moreover, we show that it might be difficult to offer a fixed benefit annuity providing the same lifetime utility as a PLA for the same premium and a comparably low insolvency risk. Overall, PLA schemes may be an efficient way to deal with risk factors that are highly unpredictable and difficult to hedge over the long run, such as systematic longevity and investment risks.

KEYWORDS

Surplus determination, surplus distribution, asset-liability management, investment risk, systematic longevity risk.

1. INTRODUCTION

Reaching retirement, individuals face the challenge of drawing down assets they accumulated during their working lives. One traditional approach is to purchase an immediate payout annuity from an insurance company, which entitles the annuitant to periodic and lifelong payments. This transfers the individual's longevity risk to the insurance company, which organizes risk pools across a sufficiently large number of annuitants. Risk pooling allows the insurer to hedge the idiosyncratic part of longevity risk, i.e. the (uncorrelated) uncertainty of individual lifetimes. It is well documented in the literature, however, that risk pooling is not effective in managing systematic mortality risk and investment

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risk.[1] The former refers to the stochastic variation of mortality rates over time, while the latter to the uncertainty associated with fluctuating capital markets and interest rates. Both risk components affect the overall pool of annuitants.

Annuity benefits may be fixed in nominal terms, vary over time through indexation, or depend on the insurance company's overall experience regarding asset returns and mortality (*participating* or *with-profits annuity*). In many countries, fixed annuities are the most popular annuity product.[2] These allow the annuitant to transfer both investment risk and longevity risk to the insurance company. Yet the currently low interest rate environment reduces the lifelong benefits and, hence, the attractiveness of fixed nominal annuities. At the same time, systematic longevity risk and the growing volatility of capital markets make it increasingly difficult to price the long-term cash flow streams of fixed annuities in a prudent manner. Consequently, annuitants are exposed to credit risk, as insurance companies may default on their obligations.[3] Participating payout life annuities (PLAs) allow sharing of longevity and investment risks between annuitants and an insurance company, which may provide a means of overcoming the disadvantages of fixed annuities.[4]

In the German market, which is the focus of this paper, PLAs are the standard annuity product (see Bohnert and Gatzert, 2012).[5] Typically, they offer guaranteed minimum benefits for the remaining lifetime and an additional non-guaranteed surplus. The guaranteed benefits are calculated using conservative actuarial assumptions on investment returns and the cohort's mortality experience. Therefore, life insurance companies can expect to earn a systematic surplus. A large proportion of the surplus generated by the insurance company has to be shared with and distributed to policyholders, whereby the mechanics of surplus allocation are regulated by the supervising authority. As pointed out by Albrecht and Maurer (2002), life insurance companies use special smoothing techniques in an effort to stabilize surplus rates over time.[6]

This paper explores the basic features of immediate PLA products in the German life insurance market.[7] We describe the process of surplus determination, the regulatory framework of sharing surpluses between shareholders and policyholders, and the smoothing mechanism used to stabilize the distribution of surpluses over time. Moreover, we develop an asset-liability model of a PLA provider that accounts for uncertain investment returns and mortality developments. This model allows us to study the risk and return profile of annuitant payout streams as well as the profit/loss exposure of the insurance company.

The idea of sharing longevity and investment risk using with-profits life insurance policies is not new. Early work by Ogborn and Wallas (1955) explores the possible characteristics of various profit-participation schemes for life annuities. Several studies discuss the valuation of the liabilities associated with participating life insurance policies as well as the pricing of the inherent explicit and implicit options (see Briys and de Varenne, 1997; Grosen and Jørgensen, 2000; Miltersen and Persson, 2003; Tanskanen and Lukkarinen, 2003; Gatzert and Kling, 2007; Zemp, 2011). Another strand of the literature focuses on the asset-liability management of with-profit life insurance contracts (see Kling *et al.*, 2007; Gatzert, 2008; Gerstner *et al.*, 2008; Bohnert *et al.*, 2012). Yet these papers primarily concentrate on the accumulation phase of with-profit life policies, while we study the postretirement phase. We also analyze the welfare implications of participating annuity contracts for annuitants from an expected utility perspective. Finally, we concentrate our analysis explicitly on the situation in the German annuity market.

The remainder of this paper is organized as follows. Section 2 describes the general characteristics of German PLAs and the mechanism for determining and distributing surpluses. In Section 3, we develop our realistically calibrated asset-liability framework and present simulation results for annuity benefits, insurer profitability and ruin probabilities. Finally, Section 4 provides the conclusion.

2. Participating life annuities

2.1. General characteristics

The payout stream of German PLAs consists of two parts: *guaranteed benefits* and *distributed surpluses*. Guaranteed benefits have to be paid for the remaining lifetime of the annuitant. Hence, they have to be calculated "on the safe side" (see §11 Act on the Supervision of Insurance Undertakings (VAG)) to ensure the long-term ability of insurers to honor the obligations from the annuity contracts. To this end, calculation of premiums and reserves for guaranteed lifetime benefits is based on conservative first-order actuarial assumptions. The first-order actuarial assumptions are specified when the contract is signed and cannot be changed during the lifetime of the annuitant. The main parameters are low guaranteed interest rates, conservative mortality tables and prudent cost rates.

Since premiums are calculated in a conservative way, life insurance companies can expect to earn a systematic surplus. The basis for calculating surpluses is the distance between first-order and second-order actuarial assumptions. The second-order assumptions are determined by the insurer at the end of every financial year and depend on investment returns, mortality and cost experiences, as well as other sources of return, such as reinsurance. As surpluses result not only from the annuity provider's entrepreneurial skills or management abilities but also to a substantial extent from the legally prescribed prudent calculation, insurance companies are obliged to share the positive return from every source with the policyholders (see §153 VAG). Sharing profits with the annuitants means paying a not guaranteed amount in addition to the guaranteed benefits. However, losses are not shared.

Usually annuities are offered by life insurance companies as a part of their overall product portfolios. Other important product lines include term life insurance and endowment policies. Changes in the second-order actuarial assumptions have different impact on the return of each product line. For example, a reduction of the actual life expectancy increases mortality returns for annuity products, but lowers mortality returns for pure life products. To share profits fairly and to prevent uncontrolled cross subsidies, surpluses have to be calculated separately for each product group. Furthermore, the set of policies per product has to be split into subsets with matching first-order assumptions, so-called *profit series*. Surpluses have to be calculated separately for each profit series.

When signing the contract, the annuitant can choose between two participation schemes: *surplus annuitization* and *lump-sum surplus distribution*. If the policyholder chooses the former, surpluses are annuitized based on the same actuarial assumptions that were used to calculate premiums. In this case, the annuitized surpluses raise benefits and also become part of the guaranteed benefits in subsequent years. If the lump-sum option is chosen, the annuitant receives surpluses year by year as one-time payments that do not become part of the guaranteed benefits.

To protect guaranteed payments promised to annuitants, German life insurance companies are subject to a comprehensive regulatory framework codified in the Act on the Supervision of Insurance Undertakings (VAG) and supervised by the German Federal Financial Supervisory Authority (BaFin). Besides solvency requirements and building sufficient actuarial reserves, life insurance companies also have to account for quantitative restrictions on their investments (e.g. maximum exposures to equities, real estate and alternative investments) and the use of financial derivatives. Moreover, each life insurance company must nominate an appointed actuary that supervises the calculation of premiums and reserves for guaranteed benefits, and who is also involved in supervising the determination, allocation and distribution of surpluses to policyholders. Finally, policyholders are protected through a mandatory solvency fund for German life insurers (Protektor Lebensversicherungs AG).[8] In the case of insurer insolvency, this institution takes over the policies as well as the remaining assets and pays the guaranteed future benefits.

2.2. PLA return sources

Based on data provided by the BaFin, Table 1 presents aggregated surpluses of all German life insurers from 2007 to 2010, itemized by return sources. Legislation stipulates that insurers must determine and distribute surpluses from mortality, assets and costs, as well as performance in reinsurance and other sources. The two main sources of return are assets and mortality. Over the period 2007 to 2010, insurers generated annual profits of more than 6,000 million Euros from mortality, a number that has been rather stable over time. Asset returns, on the other hand, exhibited high volatility. In 2007, asset returns contributed 62% of overall surpluses. This number decreased to only 13% in 2008, and increased again to 46% (54%) in 2009 (2010).

Cost returns are generated due to safety margins calculated for acquisitions of new contracts and running expenses. Other returns include profits generated

TABLE 1

SURPLUS ANALYSIS BY SOURCE OF RETURN.

	2007		2008		2009		2010	
Source of Return	In Million €	In % of Surplus	In Million €	In % of Surplus	In Million €	In % of Surplus	In Million €	In % of Surplus
Mortality	6,352	46.2	6,489	95.3	6,464	54.7	6,459	53.1
Assets	8,530	62.0	892	13.1	5,485	46.4	6,569	54.0
Costs	913	6.6	771	11.3	1,147	9.7	1,179	9.6
Others	−2,041	−14.8	−1,346	−19.8	−1,277	−10.8	−2,080	−16.8
Distributed Surplus	13,754		6,815		11,819		12,158	

Notes: Aggregated values over all product groups of all 101 (100/99/97) German life insurers in 2007 (2008/2009/2010). Source: BaFin, Statistics for Direct Insurers, 2009 (2010).

by reinsurance and premium reductions. Table 1 shows that cost and other returns are low compared to asset and mortality returns.

Asset returns are calculated as the difference between the net investment returns and the interest rate used to calculate guaranteed benefits (GIR). The net investment return contains coupon payments received on fixed income investments, dividends from stocks and rents from property investments. Gains and losses resulting from sales, acquisitions, or revaluations of assets are also included. As shown in Figure 1, asset surpluses are generated because the GIR is significantly below the net investment returns. The maximum GIR, annuity providers can choose, is set by German Ministry of Finance, and it usually amounts to 60% of the average yield of government securities over the last 10 years. As illustrated in Figure 1, the GIR decreased successively from 4% in 1994 to 2.25% in 2010. In January 2012, the maximum GIR was again reduced to 1.75%.

Given that German insurance companies have to earn at least the guaranteed interest every year, their investment policies favor allocation to bonds (see Table 2). In 2010, 66.5% of all assets in the German life insurance industry were directly invested in bonds. With a share of 24.6%, (institutional) investment funds constituted the second largest asset class. The lion's share of these assets was again held in fixed income funds. For example, the biggest German life insurer reported that 88% of the investment funds held in 2011 were bond funds, and the remaining 12% were equity funds (Allianz, 2012). These numbers are comparable to those of other major companies in the German life insurance market. Hence, we can conclude that approximately 90% of insurers' assets are allocated to bonds or bond-like investments. The remaining 10% of assets are invested in direct/indirect holdings of stocks, equity-like assets and properties.

The mortality return is calculated as the product of current actuarial reserve and the difference of expected and actual mortality. Actual mortality is observed by the insurer at the end of every financial year. Expected mortality, by contrast,

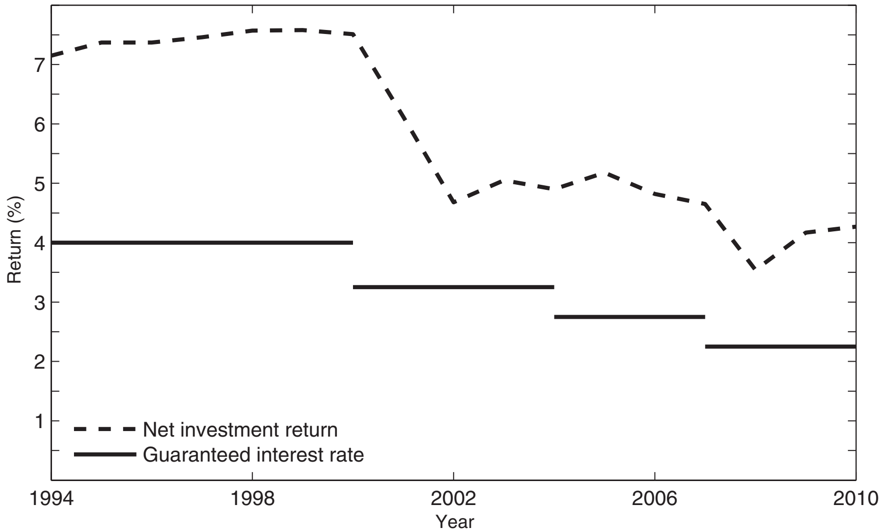

FIGURE 1: Realized net investment returns and guaranteed interest rates (2004–2010).

Notes: Average net investment return over all German life insurers. Maximum possible guaranteed interest rate according to premium refund order. Source: GDV (2011).

TABLE 2

AVERAGE ASSET ALLOCATION.

Asset Class	Weight (%)
Bonds	66.5
Investment Funds	24.6
Assets of Affiliated Companies	2.9
Properties	1.5
Direct Stocks Holdings	0.6
Other	3.9

Notes: Equally weighted asset allocation over all German life insurers in 2010. Source: BaFin, Statistics for Direct Insurers, 2010.

is taken from the mortality table used to calculate the annuity premium. For pricing annuities, German life insurers currently apply mortality tables recommended by the German Association of Actuaries (DAV), called "DAV 2004 R".[9] These cohort life tables are available since 2004 and depend on sex, age and year of birth. Prior to that, life tables called "DAV 1994 R" were used, which only considered sex and age.

For men born in 1947, Figure 2 presents annuitants' mortality rates as well as actual population mortalities. The former are based on the "DAV 2004 R" table and the corresponding trend adjustment factors provided by DAV. The latter are based on a period mortality table provided by the Human Mortality Database,[10]

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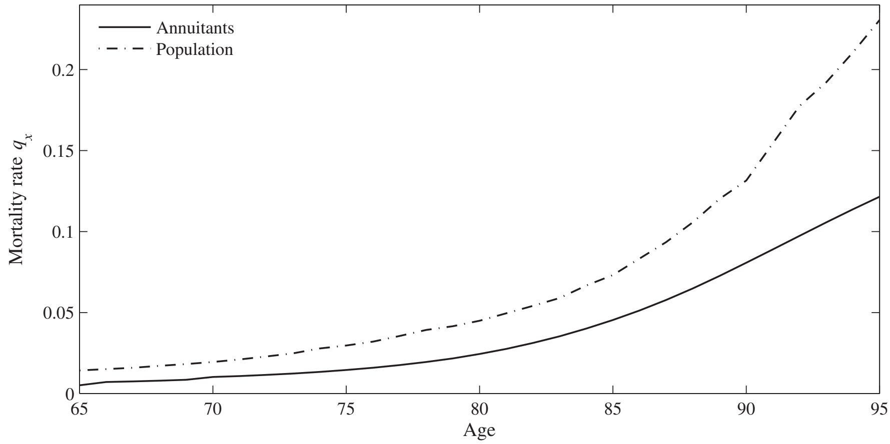

FIGURE 2: Expected and actual mortality rates: German males.

Notes: Mortality rates of a male born in 1947. German population as provided by the Human Mortality Database, cohort table forecast by the LC model. Annuitants' mortality and forecast as in mortality tables "DAV 2004 R". Source: German Actuarial Society, Human Mortality Database.

which we transform into a cohort table by projecting future mortality rates using the Lee and Carter (LC, 1992) model. The figure shows a substantial deviation for pricing guaranteed benefits of PLAs.[11] These highly conservative mortality assumptions used in the German life annuity market will result in systematic mortality returns in expectation.

2.3. Mechanics of surplus determination, allocation and distribution

Figure 3 summarizes the process of determination, allocation and distribution of surpluses, regulated by the BaFin. A life insurance company's overall surplus first must be determined by policy category, e.g. annuities, term life insurance or other insurance lines. Next, surpluses determined for each policy category must be broken down by profit series, and itemized by source of return. In the following step, the determined surplus by product must be allocated among policyholders and shareholders according to prespecified sharing rules. Finally, the *allocated surplus* must be distributed among policyholders. Typically, allocated surpluses are not fully paid out to policyholders in any given year. Instead they are partially stored in a special reserve fund, which enables the insurer to smooth surplus payouts to policyholders over time.

Surplus Determination: Surpluses are determined according to the contribution formula in (1) on a single contract basis (see Wolfsdorf, 1997). Profits $g_{x,t}$ due to an x-year-old male in year t of a contract can be broken down into a mortality return $g_{x,t,q}$, an asset return $g_{x,t,i}$ and a cost return $g_{x,t,K}$.

$$g_{x,t} = g_{x,t,q} + g_{x,t,i} + g_{x,t,K}. \tag{1}$$

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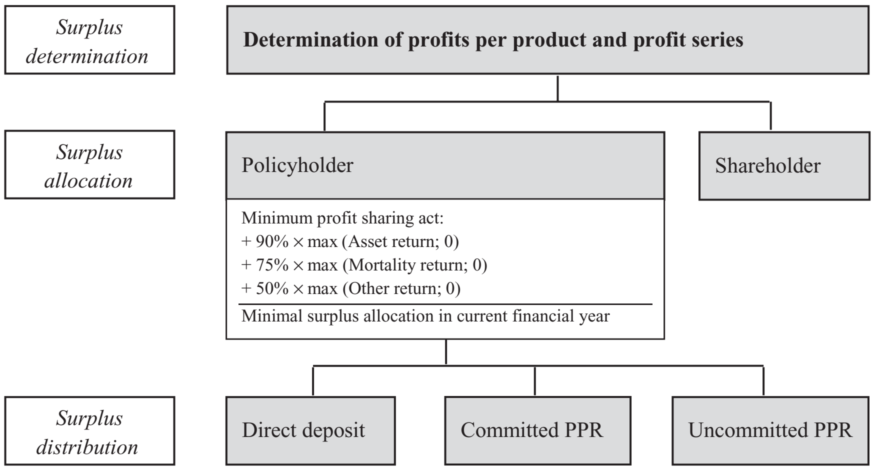

FIGURE 3: Process of surplus determination, allocation and distribution.

Source: Authors' illustration.

Here, the mortality return $g_{x,t,q}$ is the deviation between actual mortality q^{II}_{x+t} and expected mortality q^{I}_{x+t} multiplied by the actuarial reserve ${}_{t+1}V_x$. Hence, mortality returns become positive if the mortality observed at the end of the financial year is higher than that used in calculating the PLA.

$$g_{x,t,q} = {}_{t+1}V_x(q^{II}_{x+t} - q^{I}_{x+t}). \tag{2}$$

The asset return $g_{x,t,i}$ is the actuarial reserve of the previous year ${}_tV_x$ less guaranteed annuity payments L^x_t, running expenses σ^I_t and other costs γ^I_t based on the sum insured S multiplied by the difference of actual net investment return[12] i^{II}_t and $GIR\, i^I_t$.

$$g_{x,t,i} = ({}_tV_x - L^x_t - \gamma^I_t S - \sigma^I_t)(i^{II}_t - i^I_t). \tag{3}$$

The cost return $g_{x,t,K}$ refers to the difference between the expected and actual costs for managing an insurance contract compounded with the actual interest rate.

$$g_{x,t,K} = ((\gamma^I_t - \gamma^{II}_t)S + (\sigma^I_t - \sigma^{II}_t))(1 + i^{II}_t). \tag{4}$$

Surplus Allocation: Policyholders are entitled to participate in every source of return. Regulation requires that the minimum amount to be shared with the annuitants is at least 90% of asset returns, at least 75% of mortality returns, and at least 50% of other return sources. Annuitants only participate in positive return categories and cross charging between categories is prohibited. Therefore, a negative return in any category will directly reduce the equity capital of the insurance company.

TABLE 3

REALIZED SURPLUSES AND SURPLUS ALLOCATION (2005–2010).

	2005	2006	2007	2008	2009	2010
Surplus (In Billion €)	14.2	14.1	13.5	6.6	11.6	11.8
Surplus Allocation (%)	92.9	92.6	92.6	86.9	90.0	90.0
In Percent of the PPR	2.6	2.5	2.3	1.1	1.9	1.9

Notes: Aggregated values over all German life insurers. Source: BaFin, Annual Report, 2009 (2010).

While the above-mentioned percentages are minimum requirements, it is customary that more than 90% of surpluses from all return sources are allocated to policyholders. Based on data provided by the BaFin, Table 3 shows actual surplus allocations from 2005 to 2010, aggregated across all German life insurers. Over this period, on average, about 92% of all surpluses were allocated to the policyholders.

Surplus allocation and distribution must be approved by the insurer's board, taking into account the recommendations of the appointed actuary. The supervisory authority monitors that the minimum surplus distribution requirements are met, and it has the right to intervene if surpluses are distributed inappropriately. Moreover, the insurer must disclose detailed information about surplus allocation and distribution in its annual report.

Surplus Distribution and Smoothing: The annuity provider can distribute the allocated surplus among three accounts: an uncommitted provision for premium refunds (uncommitted PPR), a committed provision for premium refunds (committed PPR), and direct deposits. The PPR positions are special items in the life insurer's balance sheet and play a key role in distributing and smoothing surpluses. Their sum is the second largest item on the liabilities side of the balance sheet, exceeded only by the actuarial reserve. Surpluses to be paid to the beneficiaries within the next 2 years are assigned to the committed PPR. Within the committed PPR account, assigned distributed surpluses are recorded on a single contract basis. The uncommitted PPR is a collective buffer account belonging to all insured that is used to smooth fluctuations of the distributed surpluses over time. Here, the insurer can set aside reserves in good times and withdraw them when needed. Surpluses to be immediately paid to the beneficiaries are assigned to direct deposits.

The committed PPR as well as direct deposits are tied reserves to which policyholders have legal claims. Hence, these reserves require Solvency Capital. Funds in the uncommitted PPR, on the other hand, are untied reserves and they do not require Solvency Capital. Consequently, the insurer is interested in a well-filled uncommitted PPR account. Allocated surpluses, however, cannot be assigned to the uncommitted PPR arbitrarily. The regulator stipulates that the sum of the committed and uncommitted PPRs is limited to the sum of all

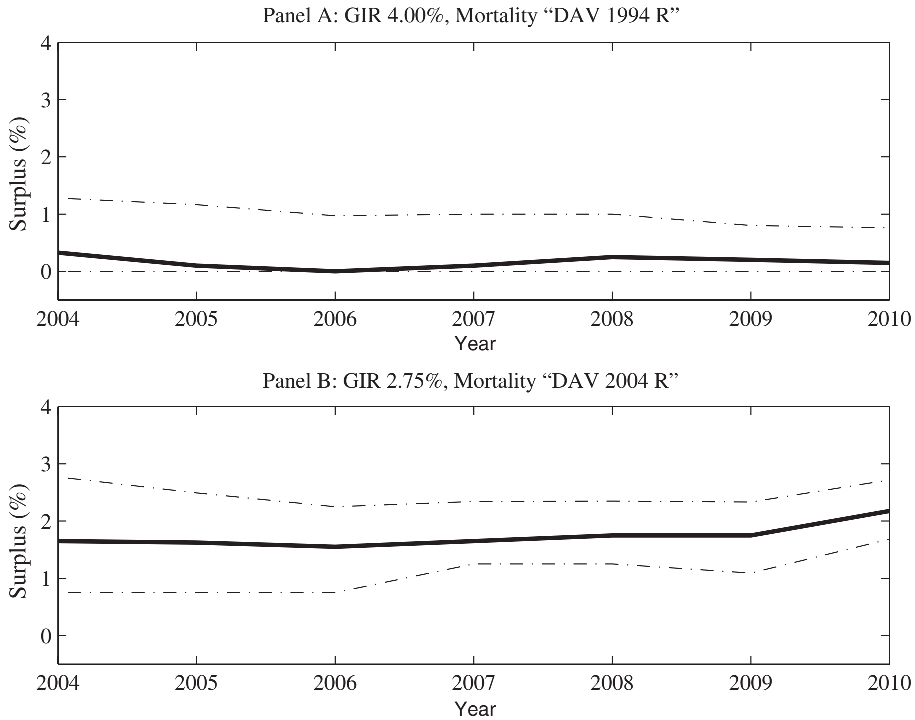

FIGURE 4: Development of distributed surpluses (2004–2010): German life insurers.

Notes: Range of distributed surpluses for different first-order actuarial assumptions in percent of the actuarial reserve. Lower (upper) dash–dotted line represents the 5% (95%) quantile; solid line represents the average. Source: Assecurata Profit Sharing Studies (2004 to 2010), Authors' illustration.

allocations to the PPR over the three previous years. Hence, the uncommitted PPR is indirectly limited. In addition, annuity providers do business in a competitive market environment, where the level of surplus distributed to annuitants is the dominant factor by which potential clients measure the performance of an insurer.

2.4. Historical distributed surpluses and implied benefit variations

Drawing on data from the Surplus Sharing Studies of Assecurata — a leading insurance broker — for the years 2004 to 2010, Figure 4 illustrates the distributed surpluses in the German life insurance industry, presenting averages across companies as well as the 5% and 95% quantiles. Here, the distributed surplus is defined as the increase in the annuity benefit as a percentage of the actuarial reserve. Panel A depicts the distributed surpluses for the profit series based on a GIR of 4% and mortality from the "DAV 1994 R" table, while Panel B shows results for the profit series based on a GIR of 2.75% and the "DAV

2004 R" table. The former profit series represents the market environment in the mid-1990s, when guaranteed interest rates were high and assumed mortality rates were higher than today. By contrast, the second profit series corresponds to a more current market situation with both lower guaranteed interest and lower mortality rates. Naturally, the resulting guaranteed benefits in Panel A are higher than those in Panel B.

The level of distributed surpluses obviously depends on the actuarial assumptions underlying the respective profit series. While average annual distributed surpluses for the product with high guaranteed benefits only amount to around 0.25% of actuarial reserves (Panel A) those for the profit series with lower guaranteed benefits amount to over 1.5% (Panel B). Looking at the 5% and 95% quantiles of the range of surplus as well as their fluctuation over time, it can be seen that the German life insurance industry as a whole was able to maintain rather stable distributed surpluses within each profit series.

3. Modeling PLA payouts and insurer stability within an Asset-Liability Management framework under investment and longevity risk

Having discussed the key characteristics determining German PLAs, we now investigate whether the parameters stipulated by regulation and/or adopted by the insurance industry result in sustainable guaranteed benefits, distributed surpluses and company stability. To this end, we develop a stochastic asset and liability model for a stylized German life insurance company that sells only one product, a single-premium PLA, to a specific cohort of similar individuals exposed to capital market risk as well as systematic and idiosyncratic longevity risk. For our stylized insurance company, we assume a run-off scenario, no new contracts are signed.

3.1. Model and calibration

3.1.1. *Capital market model.* The portfolio of our life insurance company includes two assets: stocks and bonds. The stochastic dynamics of bond prices are described by a Cox–Ingersoll–Ross (CIR) model, which posits that the short rate r_t^{CIR} evolves according to:[13]

$$dr_t^{CIR} = \alpha(\mu^{CIR} - r_t^{CIR})d_t + \sigma^{CIR}\sqrt{r_t^{CIR}}dW_t^1, r_0^{CIR} > 0, \tag{5}$$

where α and μ^{CIR} are positive scalars, σ^{CIR} is the volatility parameter and W_t^1 is a standard Wiener process (see, e.g., Fischer *et al.*, 2003). Due to the affine structure of the CIR model, the entire term structure of interest rates is determined

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by the short rate r_t^{CIR} according to:

$$R(t,\tau) = -\frac{1}{\tau}A(\tau) + \frac{1}{\tau}H(\tau)r_t^{CIR}, \quad (6)$$

where $R(t,\tau)$ represents the τ-period spot rate at time t, and $A(\tau)$ and $H(\tau)$ are given by[14]

$$\mathrm{A}(\tau) = \frac{2\alpha\mu^{CIR}}{(\sigma^{CIR})^2}\ln\left[\frac{2\gamma e^{(\alpha+\lambda+\gamma)\tau/2}}{(\alpha+\lambda+\gamma)(e^{\gamma\tau}-1)+2\gamma}\right], \quad (7a)$$

$$\mathrm{H}(\tau) = \frac{2(e^{\gamma\tau}-1)}{(\alpha+\lambda+\gamma)(e^{\gamma\tau}-1)+2\gamma}, \quad (7b)$$

$$\gamma = \sqrt{(\alpha+\lambda)^2 + 2(\sigma^{CIR})^2}. \quad (7c)$$

With the spot rates derived from the CIR model, the price B_t^T of the coupon-paying bond at time t with a constant coupon rate C^T, a face value of N and maturity at T is given by

$$B_t^T = N\left[\sum_{i=t+1}^{T} C^T e^{-R(t,i-t)} + e^{-R(t,T-t)}\right]. \quad (8)$$

Each year, the company must earn at least the GIR and therefore, it is interested in a stable income stream from bonds. Consequently, we assume that the company only invests in coupon paying par bonds with fixed initial maturity T. With $B_0^T = B_T^T = N$ at purchase time t, the par yield is given by

$$C^T = \frac{1-e^{-R(t,T-t)}}{\sum_{i=t+1}^{T} e^{-R(t,i-t)}}. \quad (9)$$

Stock prices S_t evolve according to the following stochastic process:

$$S_t = S_{t-1}\cdot e^{r_t^{CIR}+r_t^{RP}} = S_{t-1}\cdot e^{r_t^{CIR}+\mu^{RP}+\sigma^{RP}W_t^2}. \quad (10)$$

Here, r_t^{CIR} is the short rate described by the CIR model. $r_t^{RP} = \mu^{RP} + \sigma^{RP}W_t^2$ is the (stochastic) equity risk premium (net of non-stochastic dividends) with constants μ^{RP} and σ^{RP} and W_t^2 is a standard Wiener process uncorrelated with W_t^1.[15] As the company relies on a regular asset income stream to finance periodic annuity payments, we explicitly model dividends D_t paid on stock holdings. In particular, we assume that dividend payments evolve according to

$$D_t = S_{t-1}\cdot(e^{\mu^D}-1), \quad (11)$$

where μ^D is a constant.

To calibrate the CIR model, we rely on historical data provided by the Deutsche Bundesbank.[16] Using the martingale approach by Bibby and

Sørensen (1995), we calibrate the short-rate process in (5) to the 1-month interest rate over the period January 1976 to December 2011. For estimating the market price of risk, we fit (6) to historical German government bond spot rates with maturities of 1–10 years over the period January 1988 to December 2011 using an MSE approach. This produces the following parameter estimates: $\mu^{CIR} = 0.02612$, $\alpha = 0.03495$, $\sigma^{CIR} = 0.04651$ and $\lambda = -0.06799$. The initial short rate is set to $r_0^{CIR} = 1\%$.

The development of the stock prices and dividend rates is calibrated using the DAX Total Return Index and DAX Price Index over the same period (January 1988 to December 2011). This produces the following parameter estimates: $\mu^{RP} = 0.2\%$, $\sigma^{RP} = 25\%$ and $\mu^{D} = 2.3\%$. The insurer's asset allocation follows a constant mix strategy: the portfolio is rebalanced annually toward the targeted allocation when assets are sold to pay benefits to the annuitants. In case the stock exposure exceeds the target exposure, the insurance company sells a higher percentage of stocks to pay the benefits.

3.1.2. *Mortality model.* Drawing on the approach of Lee and Carter (1992), the stochastic dynamics of the annuitants' actual mortality rates $q_{x,c}^{II}$ for age x and calendar year c are described by

$$\log q_{x,c}^{II} = a_x + b_x k_c + \epsilon_{x,c}, \tag{12}$$

where a_x and b_x are age-specific constants, k_c is a single time-varying factor and $\epsilon_{x,c}$ is an error term capturing the remaining variation. To estimate future mortality rates, the time-dependent component k_c is forecasted by using a random walk with drift:

$$k_c = k_{c-1} + \mu^{LC} + \varepsilon_c. \tag{13}$$

Here, μ^{LC} is the drift of k_c and ε_c is normally distributed ($\varepsilon_c \sim \tilde{N}(0, \sigma^{LC})$). For notational convenience, we will subsequently drop the index c and denote $q_{x,c}^{II}$ as q_x^{II}, for it is understood that actual mortality rates are time varying.

As surpluses from mortality are determined by the difference between anticipated and observed mortality in the collective pool, we must track the actual (stochastic) mortality experience in the pool of annuitants. To this end, we assume that the PLA is initially sold to a cohort of n equal-aged individuals of the same sex. For each individual $i \, (i = 1, \ldots, n)$, the indicator variable I_t^i takes the value 1 if i is alive at time t and 0 if the annuitant is dead. The (stochastic) number NI_t of living annuitants at time t is then given by

$$NI_t = \sum_{i=1}^{n} I_t^i, \tag{14}$$

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with $NI_0 = n$. Over time, the sequence of indicator variables I_t^i for each individual i forms a Markov chain with

$$\begin{aligned} P(I_{t+1}^i = 1 | I_t^i = 1) &= 1 - q_{x+t}^{II} = p_{x+t}^{II}, \\ P(I_{t+1}^i = 0 | I_t^i = 1) &= q_{x+t}^{II}, \\ P(I_{t+1}^i = 0 | I_t^i = 0) &= 1. \end{aligned} \tag{15}$$

We calibrate the LC model to German mortality data from the Human Mortality Database.[17] This produces parameter estimates: $a_{65} = -3.9$, $b_{65} = 0.01$, $k_{2010} = -27.8$, $\mu^{LC} = -2.7$ and $\sigma^{LC} = 1.9$.

3.1.3. *Company model.* In order to distribute surpluses to the policyholders, we must determine the companies' profits (surplus determination). Subsequently, surpluses are allocated to policyholders and shareholders. Finally, to smooth the annual surplus payout, surpluses are distributed to the committed and uncommitted PPRs. In our company model, we only account for asset and mortality returns and do not include costs. In this case, the contribution formula to determine the profits for each contract in year t (1) reduces to

$$g_{x,t} = g_{x,t,q} + g_{x,t,i}. \tag{16}$$

The mortality return $g_{x,t,q}$ is given by

$$g_{x,t,q} = {}_{t+1}V_x(\tilde{q}_{x+t}^{II} - q_{x+t}^{I}), \tag{17}$$

with actuarial reserve ${}_{t+1}V_x$, actually observed mortality $\tilde{q}_{x+t}^{II} = 1 - NI_{t+1}/NI_t$ and expected mortality q_{x+t}^{I} taken from the mortality table "DAV 2004 R". The asset return $g_{x,t,i}$ is given by

$$g_{x,t,i} = ({}_tV_x - L_t^x)(i_t^{II} - i_t^{I}). \tag{18}$$

Here, L_t^x are the payments to the annuitants, and i_t^I is the GIR used to price the annuity. i_t^{II} is the realized investment return from the bond–stock portfolio.

$$i_t^{II} = \frac{\alpha_{S,t} D_t + \alpha_{B,t} C^T(t) + \beta_{S,t}(S_t - S_0) + \beta_{B,t}(B_t^T - N)}{NI_t({}_tV_x - L_t^x)}, \tag{19}$$

where $\alpha_{S,t}(\alpha_{B,t})$ is the number of stocks (bonds) held in year t, and D_t ($C^T(t)$) is the dividend (coupon) payment received on each stock (bond). $\beta_{S,t}$ denotes the number of stocks sold at market price S_t and, hence, $\beta_{S,t}(S_t - S_0)$ represents the realized gain/loss with respect to the purchase price S_0. The realized gain/loss from selling $\beta_{B,t}$ units of bonds at market price B_t^T relative to the initial value of N is given by $\beta_{B,t}(B_t^T - N)$, and $NI_t({}_tV_x - L_t^x)$ is the total actuarial reserve (after benefit payouts). If the company holds stocks/bonds with different initial values, assets are sold according to the FIFO rule.

Next, profits must be allocated to policyholders and shareholders, taking into account the regulatory minimum requirements presented in Section 2.3. Recall that policyholders are eligible to receive at least 75% of positive mortality returns and 90% of positive asset returns, while they do not participate in negative returns. To facilitate the calibration of our model to market data, we allow the company to allocate a fixed percentage ap of total surpluses to policyholders, as long as the allocated amount exceeds the regulatory minimum. In stress situations, insurers will typically reduce the allocation of surplus to annuitants to the regulatory minimum. We take this into account and posit that the insurer allocates the regulatory minimum once the insurer's equity capital has dropped to or below 75% of its initial value. Consequently, the total allocated surplus AS_t to policyholders is calculated according to

$$AS_t = \begin{cases} NI_t \cdot \max[0.75 \cdot \max(g_{x,t,q}, 0) \\ \qquad + 0.9 \cdot \max(g_{x,t,i}, 0), ap \cdot g_{x,t}], & E_t > 0.75 E_0 \\ NI_t \cdot [0.75 \cdot \max(g_{x,t,q}, 0) \\ \qquad + 0.9 \cdot \max(g_{x,t,i}, 0)], & E_t \leq 0.75 E_0. \end{cases} \tag{20}$$

The remaining profits are distributed to shareholders. Hence, the equity capital E_t develops according to

$$E_t = (1 + R(t-1, 1)) E_{t-1} + g_{x,t} \cdot NI_t - AS_t, \tag{21}$$

where $R(t-1, 1)$ is the 1-year spot rate at time $t-1$.

After the allocated surplus has been determined, it has to be distributed to the committed and the uncommitted PPR. Here, the insurer will trade off keeping distributed surpluses stable over time, while maintaining a sufficient collective buffer account. We solve this trade-off by determining the distributed surplus that maximizes the sum of two parabolic objective functions, one for the uncommitted PPR and the other for the committed PPR. The first objective function reaches its maximum when the uncommitted PPR is equal to a prespecified percentage of the actuarial reserve. The second objective function takes a maximum value when the distributed surplus is the same as the previous year. Here, as long as an economically feasible solution can be found, we restrict the deviation of the distributed surplus from its value in the previous year, i.e. $DistributedSurplus_t / DistributedSurplus_{t-1} \in [lb; ub]$.

The liability side of our hypothetical company's balance sheet has four items: the actuarial reserve, the committed PPR, the uncommitted PPR and the company's equity capital. The actuarial reserve consists of premiums collected from annuitants to finance the guaranteed benefits. The committed and uncommitted PPRs are initially endowed with 2% and 3% of the actuarial reserve, respectively. This allows the insurer to distribute surpluses to the annuitants already in the first year, a common policy of German insurance companies. In 2012, for example, German life insurers offered PLAs with an average initial surplus of approximately 2.15%.[18] These funds are provided by the insurer as an interest-free loan and are repaid by the cohort of annuitants over time.[19]

In line with market observations,[20] the insurer's equity capital is set to 1.5% of the balance sheet total. As long as the equity capital exceeds 75% of its initial value, shareholders receive dividends of 2.3% of the current (book) equity value at the end of each financial year. We set the target value of the uncommitted PPR to 6% of the actuarial reserve, which is the market average total PPR of 8% less the initial committed PPR of 2%.[21] Moreover, we set lb to 0.8 and ub to 1.25.

3.1.4. *Money's worth ratio and the utility-equivalent fixed life annuity.* To determine the value that PLAs deliver to the annuitants, we follow two approaches. First, we draw on Mitchell *et al.* (1999) and estimate the money's worth ratio (MWR) of the PLAs. The MWR is calculated as the present value of (expected) PLA payouts relative to the initial annuity premium P:

$$MWR = \frac{1}{P}\left[L_0^x + \sum_{t=1}^{\omega-x} \frac{{}_tp_x^{II} L_t^x}{\prod_{i=0}^{t-1}(1 + E(f_{i,i+1}))}\right]. \tag{22}$$

Here, ${}_tp_x^{II} = \prod_{i=0}^{t-1}(1 - q_{x+i}^{II})$ is the probability that an x-year-old male will survive the next t years, which evolves stochastically as described in Section 3.1.2. L_t^x is the (uncertain) annuity benefit in year t, and ω is the terminal age of the mortality table. $\mathrm{E}(f_{i,i+1})$ are the expected one-period forward rates for investments from time i to time $i+1$ generated by the CIR model. Based on the actuarial equivalence principle, the initial annuity premium is calculated according to

$$P = GB \cdot \sum_{t=0}^{\omega-x} \frac{{}_tp_x^{I}}{(1 + GIR)^t}, \tag{23}$$

where ${}_tp_x^{I} = \prod_{i=0}^{t-1}(1 - q_{x+i}^{I})$ is the t-period survival probability at age x under first-order actuarial assumptions, GIR is the guaranteed interest rate and GB is the guaranteed annual annuity benefit.[22]

If the MWR is equal to 1, the German policyholder can expect an annuity benefit of one Euro in present value terms for every Euro paid in premiums. A MWR below (above) 1 implies that the premium charged by the insurance company exceeds (falls short of) the present value of the PLA. $MWRs$ below 1 are common, but this does not imply that rational individuals will not buy such annuities. For example, Mitchell *et al.* (1999) report that individuals without a bequest motive still prefer buying an annuity with a MWR of 0.8 versus following an optimal consumption and investment strategy. In our case, the annuitant receives a stochastic path of benefit payments L_t^x for each simulation run. We can derive the simulated distribution of $MWRs$ by calculating the MWR over each path.

We also assess PLAs within an expected utility framework, which allows us to determine how individuals with different risk aversion and time preferences value the stochastic PLA income stream. Specifically, we calculate the utility-equivalent fixed annuity for policyholders with time additive constant relative risk aversion (CRRA) utility. An individual's expected lifetime utility is given by

$$U = E\left(\sum_{t=0}^{\omega-x} \beta^t \cdot {}_t p_x^{II} \frac{L_t^{x(1-\gamma)}}{1-\gamma}\right). \tag{24}$$

Here, γ denotes the coefficient of relative risk aversion and the discount factor $\beta < 1$ represents the individual's subjective time preference. The expected lifetime utility U for the simulated benefits of a PLA is then transformed into a utility-equivalent fixed life annuity EA:

$$EA = \left[\frac{U(1-\gamma)}{\sum_{t=0}^{\omega-x} \beta^t \cdot {}_t p_x^{II}}\right]^{\frac{1}{(1-\gamma)}}. \tag{25}$$

The EA can be interpreted as the constant guaranteed lifelong income stream that the annuitant will require to give up the upside potential of a PLA with stochastic surpluses.

3.2. Base case simulation results

Next, we evaluate benefit payout streams as well as the insurance company's stability prospects implied by the model described above. To this end, we simulate 50,000 independent sample paths for a cohort of 10,000 males age 65 in 2012 who purchase a PLA with initial guaranteed benefits of €10,000 per year. Insurance premiums for all annuitants are calculated using the same first-order actuarial assumptions: a guaranteed interest rate of 1.75% per year and annuitant mortality tables "DAV 2004 R". To focus the analysis on the impact of the various participation schemes, we abstract from expense loadings. This results in a premium of €201,640. In line with empirical evidence, the fixed asset allocation is a 10%/90% stock/bond mix, with bonds having an initial maturity of 10 years.[23] The surplus allocation parameter, which specifies the distribution of profits between annuitants and shareholders, is set to $ap = 92\%$. In the base case scenario, the distributed surpluses are used to raise the annual guaranteed payments (surplus annuitization). Moreover, we explore the effects of distributing surpluses through not-guaranteed yearly lump-sum payments (surplus lump-sum).

3.2.1. *The annuitant's perspective.* From the beneficiaries' point of view, the main focus of the analysis is on the development of uncertain annuity benefits. To this end, Figure 5 presents fancharts of the distributions of annual distributed surpluses (as a percent of the actuarial reserve) as well as the distributions of resulting annual payments paid to a representative annuitant from age

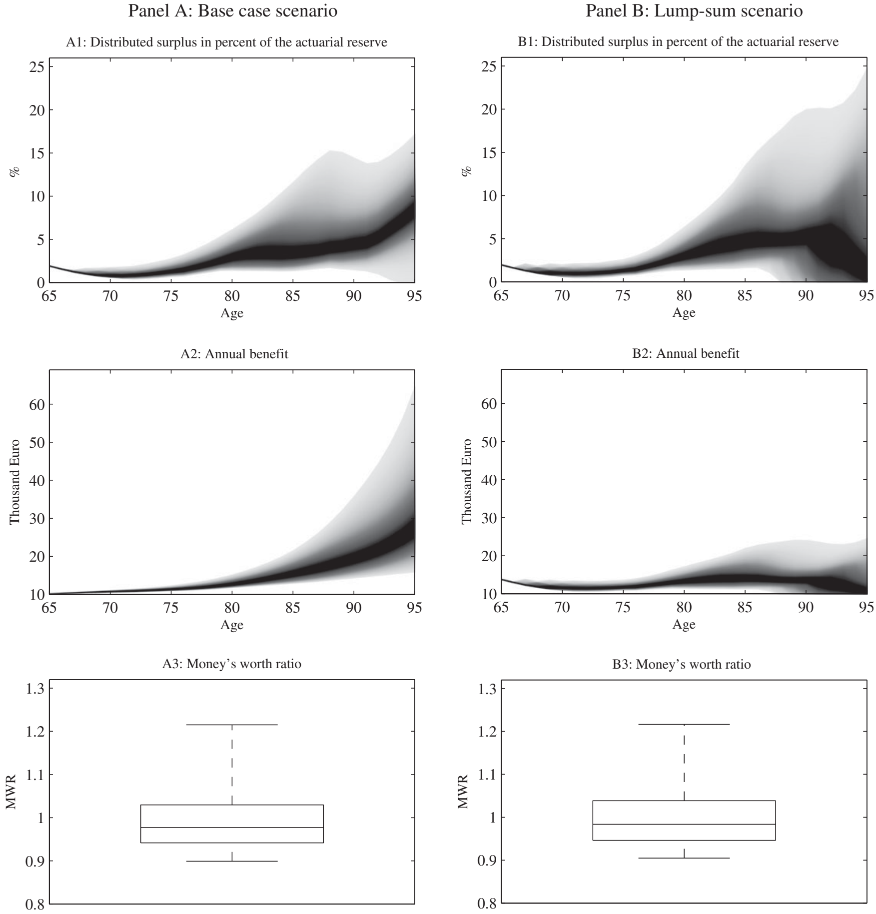

FIGURE 5: Simulated distributed surplus, annual benefits and money's worth ratio.

Notes: Simulated distribution of distributed surpluses and annual benefit payments (5%–95% quantiles), and range of the MWR (50,000 simulations). Male age 65 in 2012; initial guaranteed PLA benefits: €10,000 (present value: €201,640); GIR: 1.75%; mortality: "DAV 2004 R"; asset allocation: 10% stocks/90% bonds (with 10 years maturity); surplus allocation to annuitants: 92%. Panel A: distributed surpluses are used to raise the annual benefits (base case). Panel B: surpluses distributed through lump-sum payments (lump-sum). Fancharts: darker areas represent higher probability mass. Boxplots: lower (upper) whisker represents the 5% (95%) MWR quantile; 25%/50%/75% quantiles represented by the box. Source: Authors' calculations.

65 to 95. Moreover, Figure 5 shows boxplots of the distributions of the simulated MRWs. The left-hand side of this figure, Panel A, shows the results for our baseline participation scheme surplus annuitization, while the right-hand side, Panel B, focuses on the alternative surplus lump-sum participation approach.

Starting from an initial level of 2% of the actuarial reserve, the distributed surpluses decrease to about 1% over the first 10 years for both payout schemes

(Panels A1 and B1). During that time, the uncommitted PPR must be built up to the target level, which limits funds available for distribution to annuitants. Due to the smoothing mechanism, however, distributed surpluses are not reduced to zero and, in fact, the insurance company does not even cut the surpluses by the allowed 20% per year. When the uncommitted PPR reaches an adequate funding level, average distributed surpluses increase over the following 5 years. Around age 80, distributed surpluses amount to about 3% p.a. of the actuarial reserve in both cases and continue to grow to 4.5% in the surplus annuitization case (Panel A1) and to about 5.3% in the surplus lump-sum case (Panel B1) until age 90. Subsequently, the distributed surplus increases at an even higher rate in the base case setting, while it is sharply decreasing in the lump-sum scenario. If surpluses are annuitized, cash outflows to annuitants in excess of the guaranteed benefits are low early in retirement (Panel A2). Yet accumulated surpluses increase the actuarial reserve and more surplus-generating assets are kept by the insurance company early on. By contrast, under the lump-sum distribution scheme, surpluses are paid out immediately (Panel B2), which reduces the company's potential to generate additional surpluses over time.

With an initial short rate of 1%, the par bonds purchased at contract initiation pay the same annual coupon of only 1.63% on each simulation path. After 10 years, when the bonds mature and the principal must be reinvested, the interest rate level has significantly increased in expectation, as the long-term mean of the short rate is 2.6%. Consequently, newly purchased bonds pay a higher coupon rate, 2.39% on average, which supports the stronger upward trend in distributed surpluses from age 75. At the same time, there is a substantial spread in coupon rates due to the stochastic term structure, which drives up the dispersion of distributed surpluses.

Turning to the distributions of MWRs, we find that the median MWR in the base case is 97.7% (Panel A3), compared to 98.4% in the surplus lump-sum scenario (Panel B3). Moreover, the spread between the 5% quantile (lower whisker) and the 95% quantile (upper whisker) of the $MWRs$ is marginally wider in the base case. As can be seen in Panels A2 and B2, however, benefit payments in the base case are substantially lower than those in the lump-sum case, until around age 85. Hence, $MWRs$ are slightly higher in the surplus lump-sum scenario, as higher payments in the early years outweigh the lower annuity benefits at very advanced ages due to discounting.

It should again be noted that cost loadings charged by the insurance company to cover expenses for acquisition, administration and management are excluded from the analysis. Such expenses increase annuity premiums and reduce $MWRs$. For immediate PLAs, the German Insurance Association reports average costs loadings of 6.6% on top of the actuarially fair initial premium (GDV, 2011). When considering expense loadings of this magnitude, the median MWR in our base case scenario decreases from around 98% to around 91%. This compares to average $MWRs$ of around 85% in the German annuity market over

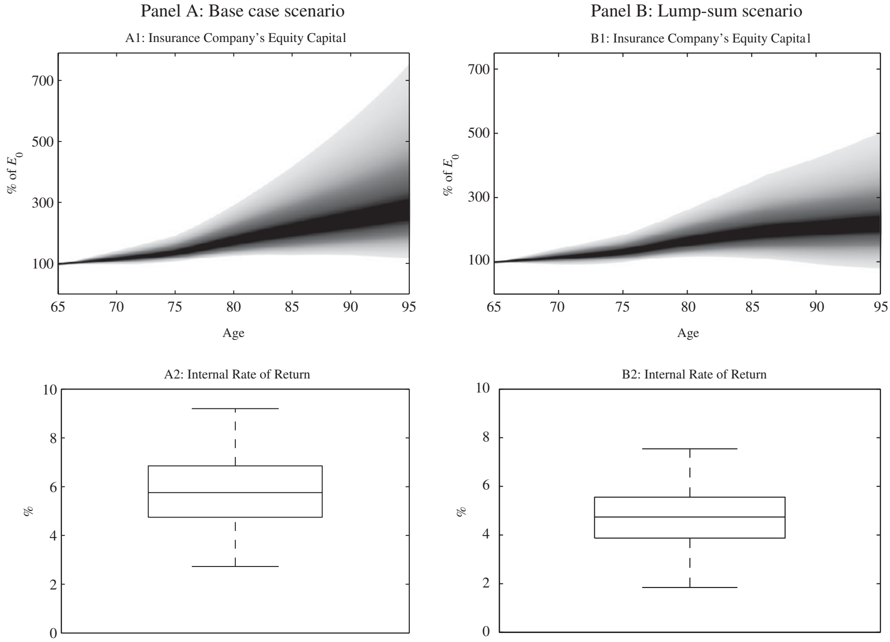

FIGURE 6: Simulated equity capital and internal rates of return.

Notes: Simulated distribution of the insurance company's equity capital (5%–95% quantiles), and range of the IRR (50,000 simulations). Male age 65 in 2012; initial guaranteed PLA annual benefits: €10,000 (present value: €201,640); GIR: 1.75%; mortality: "DAV 2004 R"; asset allocation: 10% stocks/90% bonds (with 10 years maturity); surplus allocation to annuitants: 92%. E_0 is the initial equity capital. Panel A: distributed surpluses are used to raise the annual benefits (base case). Panel B: surpluses distributed through lump-sum payments (lump-sum). Fancharts: darker areas represent higher probability mass. Boxplots: lower (upper) whisker represents the 5% (95%) IRR quantile; 25%/50%/75% quantiles represented by the box. Source: Authors' calculations.

the period 1997–2006, as estimated by Kaschützke and Maurer (2011) based on observed annuity quotes.

3.2.2. *The insurer's perspective.* Besides looking at the development of annuity benefits, it is of interest to study the viability of the annuity provider. From an insurer's point of view, it is essential that sufficient equity capital is available to ensure the payments of the guaranteed benefits also in the case of adverse capital market and mortality developments. Shareholders, on the other hand, are interested in receiving an adequate return on their investment. Figure 6 presents fancharts of the distributions of the insurer's equity capital (in percent of its initial value E_0) over a 30-year horizon (from the annuitants' age 65 to 95). Moreover, Figure 6 presents boxplots of the distributions of the simulated internal rates of return. The left-hand side of this figure, Panel A, presents the results for our baseline participation scheme surplus annuitization, while the right-hand side, Panel B, focuses on the alternative surplus lump-sum participation approach.

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Looking at the base case (lump-sum) scenario, we see that the insurer's equity capital on average increases to around 270% (220%) of its initial value over our 30-year period, despite the annual dividend payouts (Panels A1 and B1). Overall, the insurance company is rather stable. At the end of our 30-year horizon, insurer equity in the base case (lump-sum) scenario exceeds 120% (86%) of its initial value in 95% of our simulations and will be fully exhausted in only 0.49% (1.42%) of the cases.

Next, we turn to shareholders return on investment. To this end, we calculate the internal rate of return (IRR; Arrow and Levhari, 1969) for each simulation run, accounting for the initial investment, the periodic dividend payments, and the values of equity, the committed PPR, and the uncommitted PPR that remain after the last annuitant has died. The median IRR in the base case scenario is around 5.8%, with a range of 2.7% to 9.2% for the 5% to the 95% quantiles. In the lump-sum scenario, the median value comes to 4.7% and the 5% (95%) quantile amounts to 1.8% (7.6%). This spectrum of IRRs indicates that the shareholders of the life insurer can anticipate moderate but stable returns. Therefore, we conclude that the insurer does not unduly withhold surpluses from annuitants in an effort to increase shareholder value.

3.3. Sensitivity analysis

Next, we explore the robustness of our results with respect to the variation of central model parameters. In particular, we vary the bond fraction and maturity as well as the level of surplus allocation to the policyholders. Moreover, we study the impact of selling the PLAs to female annuitants. Table 4 presents for alternative calibrations the development of average distributed surpluses over time and the insurance company's ruin probability (calculated as percentage of simulation runs when the insurer's equity capital is fully exhausted).

As one would expect, average distributed surpluses rise with the stock fraction, and this effect is particularly pronounced at higher ages. Increasing stock fractions, however, result in higher risk exposure for the annuitants, as the company's ruin probability rises accordingly. While in the no stocks case, only 0.01% of the simulation runs results in negative equity capital, the insurer would be ruined in almost 17% of our simulations if the maximum stock fraction (35%) was chosen. As the term structure of interest rates is typically increasing, bonds with shorter maturities generate lower returns. Consequently, insurers that decide to only purchase bonds with a maturity of 5 years distribute marginally lower surpluses and exhibit a substantially higher ruin probability of approximately 1.8%. Restricting the allocation of surpluses to annuitants to its regulatory minimum, on the other hand, reduces the ruin probability by one-third compared to the base case, while there is little impact on the average level of distributed surpluses. Average distributed surpluses for female annuitants are considerably lower than those of males. Only females above age 90 will receive substantially higher surpluses than their male counterparts. For younger (older) females, the spread between first-order and second-order mortality rates is lower (higher) than for

TABLE 4

AVERAGE DISTRIBUTED SURPLUS AND INSURER'S RUIN PROBABILITY FOR ALTERNATIVE CALIBRATIONS.

	Age							Ruin Probability (in bps)
	65	70	75	80	85	90	95	
Base Case	2.0	0.9	1.3	3.0	3.5	4.5	8.5	49
Lump-Sum	2.0	1.1	1.4	3.0	4.7	5.3	0.8	142
No Stocks	2.0	0.7	1.0	2.2	2.8	4.1	7.6	1
Max Stock Fraction	2.0	1.1	1.8	3.6	4.5	5.7	10.3	1,670
Short Bond Maturity	2.0	0.7	1.3	2.7	3.5	4.3	8.4	177
Regulatory Min	2.0	0.8	1.3	2.8	3.2	4.2	8.1	32
Female	2.0	0.7	0.4	1.0	2.4	6.1	14.4	28

Notes: Average distributed surplus in percent of the actuarial reserve at specified age, and ruin probability of the annuity provider in Basis Points (50,000 simulations). Base case assumptions: male age 65 in 2012; initial guaranteed PLA annual benefits: €10,000 (present value: €201,640); GIR: 1.75%; mortality: "DAV 2004 R"; surplus annuitization; asset allocation: 10% stocks/90% bonds (with 10 years maturity); surplus allocation to annuitant: 92%. Lump-Sum: lump-sum annuitization. No Stocks: asset allocation: 0% stocks/100% bonds. Max Stock Fraction: asset allocation: 35% stocks/65% bonds. Short Bond Maturity: maturity of bonds 5 years. Regulatory Min: surplus allocation to annuitant: 90% asset returns, 75% mortality returns. Female: female age 65 in 2012. Source: Authors' calculations.

males, and so are the surpluses from mortality. Consequently, annuity benefits at younger ages are lower than in the base case, which supports the insurance company to overcome the initially low interest rate environment and reduces the probability of early ruins.

Table 5 presents the distributions of the IRR and the MWR for the seven scenarios under scrutiny. With rising stock fractions, the dispersion of both IRRs and MWRs increases. Annuitants benefit from higher risk-taking, since the guaranteed part of the annuity essentially resembles a put option. Annuitants participate in the increased upside potential, while they are protected from downside risk. Consequently, higher stock fractions have little impact on the 5% quantile of the MWR, whereas the median and particularly the higher quantiles increase sharply. This, however, only holds as long as the insurance company is able to pay the guaranteed annuity benefit, which is less likely the higher the exposure to stocks, as indicated by the ruin probabilities in Table 4. From the perspective of shareholders, this results in limited upside potential in terms of their IRR, while they are exposed to substantial downside risk. The median (95% quantile) of the IRR increases by 1.6% (3.3%) when moving from the no stocks allocation to the regulatory maximum stock allocation. Yet the 5% quantile drops from 2.7% to −100%, that means the shareholder lost its entire investment in this worst-case scenario. Allocating only the regulatory minimum of surpluses to the annuitants increases (reduces) the median IRR (MWR) by 1.3% (1.4%) compared to the base case. IRRs from selling PLAs to females are almost equal to those in the base case. This also holds for the center of the MWR distribution, whereas the 5% (95%) quantile is lower (higher) than

TABLE 5

DISTRIBUTION OF INTERNAL RATES OF RETURN AND MONEY'S WORTH RATIOS.

	IRR Quantile (%)					MWR Quantile (%)				
	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%
Base Case	2.7	4.6	5.8	7.0	9.2	89.9	93.8	97.7	103.8	121.5
Lump-Sum	1.8	3.8	4.7	5.7	7.6	90.5	94.2	98.4	104.7	121.6
No Stocks	2.7	4.2	5.1	6.2	8.4	90.1	91.7	93.4	96.4	107.0
Max. Stock Fraction	−100.0	4.1	6.7	8.7	11.7	87.0	92.8	105.5	127.9	180.2
Short Bond Maturity	1.7	4.0	5.4	6.8	9.4	88.1	92.3	96.8	104.0	124.5
Regulatory Min	3.5	5.7	7.1	8.5	11.1	89.7	93.1	96.3	100.9	113.2
Female	2.7	4.6	5.8	7.1	9.9	85.4	92.5	98.0	105.5	127.9

Notes: Internal rates of return and money's worth ratios in percent (50,000 simulations). Base case assumptions: male age 65 in 2012; initial guaranteed PLA annual benefits: €10,000 (present value: €201,640); GIR: 1.75%; mortality: "DAV 2004 R"; surplus annuitization; asset allocation: 10% stocks/90% bonds (with 10 years maturity); surplus allocation to annuitant: 92%. Lump-Sum: lump-sum annuitization. No Stocks: asset allocation: 0% stocks/100% bonds. Max Stock Fraction: asset allocation: 35% stocks/65% bonds. Short Bond Maturity: maturity of bonds 5 years. Regulatory Min: surplus allocation to annuitant: 90% asset returns, 75% mortality returns. Female: female age 65 in 2012. Source: Authors' calculations.

in the base case. As annuity benefits are substantially back-loaded for females, later payments have a higher impact on the *MWR*. Consequently, *MWRs* for two simulation paths exhibit substantial spread even for small differences in the number of years annuity benefits must be paid.

3.4. Utility analysis

Finally, we evaluate the utility of annuitants using PLAs with alternative payout schemes, surplus allocation rules and asset allocation policies. To this end, we transform the simulated PLA payout streams into a utility-equivalent fixed life annuity by inverting a time-additive CRRA utility function (24) and (25). Table 6 presents the results for alternative rates of time preference and risk aversion. We classify annuitants with a coefficient of relative risk aversion of $\gamma = 2/5/10$ as having low/medium/high-risk aversion, and those with a subjective discount factor of $\beta = 0.98/0.96/0.94$ as patient/normal/impatient individuals.

In the base case scenario, the equivalent yearly fixed life annuity for patient annuitants with a low-level risk aversion is €11,859. This means, the annuitant requires 18.59% higher lifelong fixed benefits compared with a PLA with €10,000 guaranteed payments plus uncertain surplus. The utility drawn from PLAs decreases with increasing risk aversion and impatience. Naturally, individuals with higher risk aversion dislike the inherent volatility in PLA benefits. Hence, the utility-equivalent fixed life annuity benefit is only €11,330 for a highly risk averse but patient individual. At the same time, PLA benefits are comparably low in the early years and increase measurably only late in life, which is of

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TABLE 6

UTILITY EQUIVALENT FIXED LIFE ANNUITY.

Time Preference	Patient			Normal			Impatient		
Risk Aversion	Low	Medium	High	Low	Medium	High	Low	Medium	High
Base Case	11.9	11.6	11.3	11.4	11.2	11.1	11.1	11.0	10.9
Lump-Sum	12.5	12.4	12.4	12.2	12.2	12.1	11.8	11.8	11.7
No Stocks	11.5	11.3	11.1	11.2	11.1	10.9	11.0	10.9	10.8
Max Stock Fraction	12.2	11.7	11.6	11.5	11.3	11.2	11.1	11.1	10.9
Short Bond Maturity	11.8	11.5	11.3	11.4	11.2	11.0	11.1	10.9	10.8
Regulatory Min	11.8	11.5	11.3	11.4	11.2	11.0	11.1	11.0	10.8
Female	11.4	11.2	11.0	11.1	11.0	10.9	10.9	10.8	10.8

Notes: Equivalent fixed life annuity (in €thousands) that generates the same utility as a PLA with a guaranteed initial lifelong benefits of yearly €10,000 for alternative scenarios based on a time-additive CRRA utility function. Calibrations of time preference: $\beta = 0.98$ (patient), $\beta = 0.96$ (normal), $\beta = 0.94$ (impatient); calibration of risk aversion: $\gamma = 2$ (low), $\gamma = 5$ (medium), $\gamma = 10$ (high). Base case assumptions: male age 65 in 2012; initial guaranteed PLA benefits: €10,000; GIR: 1.75%; mortality: "DAV 2004 R" (PLA present value: €201,640); surplus annuitization; asset allocation: 10% stocks/90% bonds (with 10 years maturity); surplus allocation to annuitant: 92%. Lump-Sum: lump-sum annuitization. No Stocks: asset allocation: 0% stocks/100% bonds. Max Stock Fraction: asset allocation: 35% stocks/65% bonds. Short Bond Maturity: maturity of bonds 5 years. Regulatory Min: surplus allocation to annuitant: 90% asset returns, 75% mortality returns. Female: female age 65 in 2012. Source: Authors' calculations.

less appeal the more impatient the individual is. Consequently, an impatient annuitant with low risk aversion is indifferent between a PLA and a fixed annuity of €11,092. In general, this pattern can be observed for all scenarios analyzed here. The only setting that exhibits noteworthy differences is the lump-sum scenario. Independent of risk aversion and impatience, PLAs with lump-sum surplus distribution generate significantly higher utility. This is an intuitive result, as surpluses are paid out earlier than under the surplus annuitization scheme. At the same time, the level of risk aversion has little impact on the utility-equivalent life annuity, as the total variation of benefits is much smaller in the lump-sum case (see Figure 5).

To put these numbers into perspective, we use the base case and conduct the following thought experiment. Let us assume that a life insurance company offers a non-participating life annuity with fixed benefits of €11,859 per annum, i.e. the utility-equivalent fixed annuity for a patient individual with low risk aversion. To offer such an annuity for the same premium as the PLA, the insurer must calculate the fixed annuity using an interest rate of 3.19%, when relying on the same mortality table. This is 1.44% higher than the GIR of 1.75% used in calculating the guaranteed benefits (€10,000 p.a.) of the PLA. Since the initial coupon rate for long-term bonds is only 1.63% (given our capital market assumptions), guaranteeing an interest of 3.19% results in substantial insolvency risk. To quantify this risk, we redo our simulation for a cohort of 10,000 individuals that purchase a guaranteed fixed annuity of €11,859 instead of a PLA with an initially guaranteed €10,000 plus surplus participation, relying on the

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same assumptions with respect to premiums paid to the insurance company, capital market and mortality developments, as well as asset allocation. We then evaluate how many of our simulations result in negative equity capital in at least 1 year, i.e. in how many cases, the insurer is ruined: we find this number to be 78%. This compares to the ruin probability of only 0.5% in our PLA base case, which provides the same lifetime utility. These results suggest that, for our (stylized) market situation, insurers will face substantial difficulties to offer a fixed life annuity for the same premium as a PLA having comparable insolvency risk and lifetime utility.

4. Discussion and conclusions

This paper analyzes participating payout life annuities (PLAs), which are the dominating product in the German market. PLAs offer relatively low guaranteed lifetime benefits in combination with access to parts of the surplus generated by the insurer. In contrast to traditional life annuities with fixed benefits, PLA benefit payments can fluctuate over time. At the same time, the surplus does not depend on the performance of a specific asset portfolio chosen by the annuitant, as, e.g. in the case of an investment-linked variable payout annuity, but it depends on the insurance company's overall experience regarding mortality and investments. A distinct feature of German PLAs is the possibility to annuitize distributed surpluses. In fact, this is the predominant surplus appropriation scheme in Germany. Here, distributed surpluses increase guaranteed benefits and, hence, annuity payments are increasing monotonically.

The key question with respect to PLAs is how surpluses are determined and allocated among policyholders and shareholders. We show that in Germany the process of surplus determination, allocation and distribution mostly follows transparent and clear rules, and is strictly monitored by the appointed actuaries and the Federal Financial Supervisory Authority. Hence, an insurance company's management has limited leeway with respect to discretionary decision-making. Yet, despite its transparency, the mechanics are complex and not easily understandable even for financially literate individuals.

Our analysis of the German market also shows that insurance companies smooth surplus distribution over time. To this end, insurers have two instruments at hand. First, surpluses are not fully distributed to the individual policyholders in the year they are generated. Instead, insurance companies retain a limited fraction of surpluses in a buffer account, which can be distributed in case returns are low. Second, investment returns on assets held by the insurance companies are determined on the basis of book rather than market values.

From our simulation analysis, we learn that insurance companies offering PLAs based on the German regulatory framework are able to provide guaranteed minimum benefits with high credibility. This is due to the fact that minimum benefits are calculated using conservative assumptions regarding mortality experience and investment performance. At the same time, simulated MWRs come to around 98%, on average. This indicates that annuity providers do not unduly take advantage of the conservative assumptions, as the participation scheme provides a way to transfer realized profits back to the policyholders.

In a further analysis, we study the impact on annuitant utility provided by the payout stream of PLAs for individuals with different levels of risk aversion and impatience. Our calculations show that it might be difficult to offer a fixed benefit annuity providing the same lifetime utility as a PLA for the same premium and a comparably low insolvency risk. Overall, PLA schemes may be an efficient way to deal with risk factors that are highly unpredictable and difficult to hedge over the long run, such as systematic mortality and investment risks.

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Notes

1. For implications of independent risks on insurance pricing and risk management, see Albrecht (1981) and Cummins (1991), and more specifically for life insurers, see Gründl *et al*. (2006).
2. Boardman (2006) reports that about 80% of the private life annuities sold in the United Kingdom pay fixed nominal benefits. According to IRI Fact Book (2011), one-third of annuity asset in the United States are fixed and two-thirds are variable annuities. Yet most variable annuities offer the possibility to transfer accumulated assets into fixed nominal payout annuities in the decumulation phase.
3. See Goldsticker (2007) for this point. Work by Grosen and Jørgensen (2002) explores the impact of interest rate risk on life insurance liabilities and insolvency risk.
4. Related concepts that link life annuity benefits to the experienced mortality within a risk pool are described in Piggott *et al.* (2005), Denuit *et al.* (2011), Richter and Weber (2011), and Maurer *et al.* (2013). For an overview of alternative annuity designs, see Rocha *et al.* (2011).
5. In the United States, TIAA-CREF issues life annuities where benefit payments evolve according to investment returns and the mortality experience of annuitants. For more information on the TIAA-CREF product, see Weil and Fisher (1974) and Brown *et al.* (2001).
6. Guillén *et al.* (2006) study various return smoothing mechanisms in life and pension insurance.
7. Life insurance is the largest sector in the German private insurance market with earned premiums of 156.1 billion Euro in 2009 and total reserves of about 30% of the German GDP. Within the life insurance business, annuity products have a market share of about 20%.
8. Protektor only covers policies sold by insurers domiciled in Germany and regulated by the German supervisor, but not policies sold in Germany by insurers from other EU member states.

9. Almost all insurers calculate premiums based on the DAV mortality tables. Only the biggest German insurance company, Allianz AG, develops its own mortality tables for calculating private annuities, as only their portfolio of policies is large enough to support viable mortality estimates. The "DAV 2004 R" will also be the base for calculating products with unisex tables, which will be mandatory from December 21, 2012.

10. We use the German Life tables (period 1×1) for males; see http://www.mortality.org.

11. A formal comparison between the annuitant-specific mortality table "DAV 2004 R" and the population table using the A/E ratio (see McCarthy and Mitchell, 2010) shows that the annuitant table assumes a mortality structure which on average is about 40% lower than the population table.

12. According to the Act on the Supervision of Insurance Undertakings, surplus determination has to follow the German commercial code, which stipulates that investment returns have to be calculated based of the book values of the assets.

13. Here and throughout the paper, we work under the physical probability measure P.

14. The parameter λ is a function of the market price of risk $q(t, r^{CIR})$. Specifically, $\lambda = q(t, r^{CIR})\sigma/\sqrt{r^{CIR}}$.

15. Nonetheless, stocks and bonds are correlated, since the short rate influences the stock return.

16. Specifically, we use the following time series: SU0104, WZ9808, WZ9810, WZ9812, WZ9814, WZ9816, WZ9818, WZ9820, WZ9822, WZ9824, WZ9826. Available at: http://www.bundesbank.de.

17. Specifically, we use the German Life tables (period 1×1) for Males and Females; last modified: October 26, 2012, version MPv5 for the period 1990–2010. See http://www.mortality.org.

18. See http://www.assekurata.de/content.php?baseID=130&dataSetID=614; accessed December 3, 2012.

19. In practice, a going concern life insurance company with various profit series will redistribute funds from the already existing uncommitted PPR to the committed PPR of a new cohort, and the new cohort will repay this "interest-free loan" through their contributions to the uncommitted PPR.

20. BaFin, Statistics for Direct Insurers, 2010.

21. As stated in the Annual Report 2011 of the BaFin, insurers have an average total PPR (uncommitted and uncommitted) of 8%.

22. Here and throughout the analysis, we disregard explicit costs in terms of loadings.

23. We abstract from modeling additional asset classes like real estate and private equity.

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