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MINIMUM CONSUMPTION REQUIREMENTS: THEORETICAL AND QUANTITATIVE IMPLICATIONS FOR GROWTH AND DISTRIBUTION

SATYAJIT CHATTERJEE
Federal Reserve Bank of Philadelphia

B. RAVIKUMAR
University of Iowa

We study the impact of a minimum consumption requirement on the rate of economic growth and the evolution of wealth distribution. The requirement introduces a positive dependence between the intertemporal elasticity of substitution and household wealth. This dependence implies a transition phase during which the growth rate of per-capita quantities rise toward their steady-state values and the distributions of wealth, consumption, and permanent income become more unequal. We calibrate the minimum consumption requirement to match estimates available for a sample of Indian villagers and find that these transitional effects are quantitatively significant and depend importantly on the economy's steady-state growth rate.

Keywords: Growth, Inequality, Wealth Distribution, Minimum Consumption

1. INTRODUCTION

This paper explores the evolution of per-capita consumption and wealth and the evolution of the household distribution of consumption and wealth in a poor economy during the initial stages of economic growth. A key assumption is that a household's consumption expenditure cannot fall below a positive level each period. The presence of a minimum consumption requirement implies that a poor household's elasticity of substitution between consumption at different dates (i.e., its intertemporal elasticity of substitution [IES]) may be low compared to that of a rich household. The main objective of this paper is to study, qualitatively and quantitatively, the implications of such wealth-induced differences in IES

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for economic growth and for the evolution of consumption and wealth distributions.

The motivation for our work stems from two sources. First, there is evidence for minimum consumption requirement. Using panel data on Indian villagers, Atkeson and Ogaki (1996, 1997) and Rosenzweig and Wolpin (1993) estimate minimum consumption requirements that amount to a significant fraction of total consumption expenditures of the average household. In addition, Rebelo (1992) and Ogaki et al. (1996) point to the low savings rates and low interest elasticity of savings in poor countries as indirect evidence of minimum consumption requirement. Given the central role of savings and capital accumulation in economic development, it seems appropriate to incorporate this feature into models of growth.

Our second motivation is that the dependence of IES on household wealth implied by minimum consumption requirement opens a potentially important link between the macroeconomic performance of a country (i.e., its aggregate dynamics) and the evolution of its consumption and wealth distributions. The recent resurgence of interest in growth economics has led researchers to reexamine the links between inequality and economic growth. Benabou (1996a) surveys a large and growing literature on the different ways in which income inequality affects economic growth. Channels explored in this literature include the role of politics [e.g., Persson and Tabellini (1992)], the role of educational policies [e.g., Glomm and Ravikumar (1992)], the role of markets and specialization [e.g., Tamura (1992, 1996)], and the role of "endogenous sorting" [e.g., Benabou (1996b), Durlauf (1996)]. In contrast to these studies, inequality does not affect growth in our model. Instead, we focus on the possibility that the rate of economic growth might affect the evolution of income inequality because rich and poor households respond in different ways to the same growth opportunity.

We accomplish two tasks in this paper. First, we present theoretical results on the impact of a minimum consumption requirement on the rate of economic growth and the evolution of the household distribution of consumption and wealth for an economy with a linear production technology. We show that such a requirement implies a transition phase during which the growth rates of per-capita quantities rise toward their steady-state values and the distributions of consumption, permanent income, and wealth become progressively more unequal. We also explore how the nature of this transition phase is affected by different initial conditions. For instance, economies that start out poorer exhibit slower growth and a more unequal distribution of wealth over time.

Second, we calibrate this model to match consumption patterns and estimates of minimum consumption requirements available for Indian villages and show that the effect of minimum consumption requirement may be quantitatively important, especially if the underlying rate of return on capital permits only modest growth. We also find that the relationship between (wealth and consumption) inequality and rate of economic growth depends on the length of time economies have spent on their sustained growth paths. For instance, among economies that have spent only a short time on their sustained growth path, the relationship between rate of economic growth and wealth inequality is positive; that is, faster-growing economies show bigger increases in wealth inequality. On the other hand, among economies that have spent a long time on their sustained growth paths, the relationship between inequality and rate of economic growth is shaped like an inverted U.

Finally, this study adds to a growing literature that emphasizes the role of wealth-induced differences in IES in understanding growth and distribution. Rebelo (1992), in the study mentioned above, used differences in IES resulting from minimum consumption requirement to suggest an explanation for the puzzling differences in the rates of growth of economies linked by world capital markets. Chatterjee (1994) explored the role of wealth-induced differences in IES (arising from minimum consumption requirement and other sources) in shaping the evolution of wealth distribution in the standard neoclassical growth model. Easterly (1994), in a model with fixed factors and minimum consumption requirement, shows how economic policies determine whether countries stagnate or not. Most recently, Caselli and Ventura (1996) have proposed a model of income distribution in which wealth-induced differences in IES play a prominent role and have applied it to U.S. data.

2. THE MODEL

2.1. Preferences

Households have preferences over consumption sequences. These preferences, common to all households, are of the form

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - \alpha)^{1-\sigma} - 1}{1 - \sigma} \right],$$

where $c_t \geq \alpha$ is consumption in period t, $\alpha > 0$ is the minimum required consumption in each period, $\beta \in (0, 1)$ is the discount factor, and $\sigma > 0$ is a parameter that controls the curvature of the momentary utility function. For $\sigma = 1$, the momentary utility function is interpreted to be $\ell\text{n}(c_t - \alpha)$.

When $\alpha = 0$, the curvature parameter σ is the inverse of the intertemporal elasticity of substitution. When $\alpha > 0$, the IES in period t is no longer a constant. The new expression for the IES can be derived by partially differentiating the logarithm of the intertemporal marginal rate of substitution between periods t and $t + 1$ with respect to the logarithm of (c_{t+1}/c_t) and taking its inverse. This yields

$$\text{IES}_t = \frac{1}{\sigma} \left(\frac{c_{t+1} - \alpha}{c_{t+1}} \right). \tag{1}$$

Thus, as consumption increases beyond α, the IES increases from 0 to $1/\sigma$. The positive dependence of IES on the level of consumption is the key feature of these preferences.

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2.2. Technology

All households have access to a technology for making the good. Input for the technology is the stock of capital $K_t \geq 0$ and the output is $Y_t \geq 0$. The technology is linear and of the form

$$Y_t = aK_t,$$

where $a > 0$. Households also have access to a technology for accumulating capital,

$$K_{t+1} = (1-\delta)K_t + X_t,$$

where $\delta \in (0, 1)$ is the rate of depreciation of capital and X_t is gross investment in period t. We assume that $a > \delta$.

2.3. Endowments

Each household has enough initial capital to ensure itself the minimum required level of consumption each period; that is, $k_0^i > \alpha/(a-\delta)\ \forall i$.

2.4. Markets

There are really only two distinct goods in each period: the beginning of period capital stock and the single good that it can be used to produce. Let the equilibrium rental rate for one unit of beginning-of-period capital in terms of the single good in period t be r_t. Obviously, $r_t = a - \delta \equiv r$.

2.5. Optimization

Household i chooses sequences $\{c_t^i\}$ and $\{x_t^i\}$ to

$$\max \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^i - \alpha\right)^{1-\sigma} - 1}{1-\sigma} \right],$$

subject to, $\forall t \geq 0$,

$$c_t^i + x_t^i = ak_t^i$$

$$k_{t+1}^i = (1-\delta)k_t^i + x_t^i$$

$$c_t^i \geq \alpha.$$

Define discretionary consumption to be $\tilde{c}_t^i = c_t^i - \alpha$ and discretionary capital stock to be $\tilde{k}_t^i = k_t^i - \alpha/r$. The optimization problem is then equivalent to one where the

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household chooses $\{\tilde{c}_t^i\}$ and $\{\tilde{k}_{t+1}^i\}$ to

$$\max \sum_{t=0}^{\infty} \beta^t \left[\frac{(\tilde{c}_t^i)^{1-\sigma} - 1}{1-\sigma} \right],$$

$$\text{subject to, } \forall\, t \geq 0,$$

$$\tilde{c}_t^i + \tilde{k}_{t+1}^i = (1+r)\tilde{k}_t^i$$

$$\tilde{c}_t^i \geq 0.$$

This problem can be given the following recursive formulation:

$$v(\tilde{k}) = \max \left\{ \frac{\tilde{c}^{1-\sigma} - 1}{1-\sigma} + v(\tilde{k}') \right\},$$

$$\text{subject to}$$

$$\tilde{c} + \tilde{k}' \leq (1+r)\tilde{k}$$

$$\tilde{c} \geq 0.$$

PROPOSITION 1 (Decision rules). *If* $\beta(1+r)^{1-\sigma} < 1$, *the optimal decision rules for discretionary consumption and discretionary capital stock are*

$$c_t^i - \alpha = (1+r)\left\{1 - [\beta(1+r)^{1-\sigma}]^{\frac{1}{\sigma}}\right\}\left(k_t^i - \alpha/r\right)$$

$$k_{t+1}^i - \alpha/r = [\beta(1+r)]^{\frac{1}{\sigma}}\left(k_t^i - \alpha/r\right).$$

For ease of exposition, the formal proofs of all propositions are collected in the Appendix.

2.6. Equilibrium

Although households have different levels of beginning-of-period capital stocks, with the market interest rate being $(a - \delta)$ and all households having access to the common technology, the equilibrium allocation is the same as that achieved by each household operating in isolation. Thus, we may view these household decision rules as equilibrium decision rules as well.

The equilibrium per-capita quantities of capital and consumption then are $\bar{k}_t \equiv (\sum_i k_t^i)/N$ and $\bar{c}_t \equiv (\sum_i c_t^i)/N$.

2.7. The Interpretation of Capital Stock

Because the model's capital stock is not subject to diminishing returns, it is best to view its empirical counterpart to be a broad measure of capital. What might this measure be? Consider an economy in which households obtain income from labor effort and accumulated assets. Let the beginning-of-period stock of assets

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of household i be a_t^i and its labor earnings for the period be e_t^i. Let the period t return on assets be r_t. Define ρ_{t+s} to be $\prod_{i=0}^{s}(1+r_{t+i})^{-1}$ for $s = 0, 1, 2, \ldots$. Then, household i's present-value budget constraint is

$$\left(\sum_{s=0}^{\infty} \rho_{t+s} c_{t+s}^i\right)(1+r_t) = \left(a_t^i + \sum_{s=0}^{\infty} \rho_{t+s} e_{t+s}^i\right)(1+r_t).$$

Define $w_t^i \equiv a_t^i + \sum_{s=0}^{\infty} \rho_{t+s} e_{t+s}^i$. The present-value budget constraint then can be written as $c_t^i + w_{t+1}^i = (1+r_t) \cdot w_t^i$. Furthermore, if $z_t \equiv w_{t+1}^i - w_t^i$, the constraint can be expressed as

$$c_t^i + z_t^i = r_t \cdot w_t^i,$$

$$w_{t+1}^i = w_t^i + z_t^i.$$

If r_t is set to $(a - \delta)$ and w_t^i to k_t^i, this environment becomes isomorphic to our model. Thus, the model capital stock can be reasonably interpreted as the value of human and nonhuman wealth. This interpretation is exact if interest rates are constant over time.

3. THEORETICAL ANALYSIS

3.1. Evolution of Per-Capita Capital Stock

Multiplying both sides of the decision rule for discretionary capital stock by $1/N$ and summing over i yields an expression for the growth rate of per-capita capital stock

$$\frac{\bar{k}_{t+1} - \bar{k}_t}{k_t} = \left\{[(1+r)\beta]^{\frac{1}{\sigma}} - 1\right\}\left(1 - \frac{\alpha}{r k_t}\right). \tag{2}$$

We begin with a simple proposition about the evolution of per-capita capital stock.

PROPOSITION 2 (Existence of representative household). *The evolution of per-capita capital stock is the same as the evolution of capital stock of a household endowed with the per-capita stock of capital.*

Thus, for the purpose of studying aggregate or per-capita dynamics, there is no loss of generality in restricting attention to a representative household. That is, the inequality in wealth does not affect the evolution of per-capita income and growth. The existence of a representative household is a consequence of the type of utility function assumed and of the assumption that all households have access to the same linear technology. The next proposition brings out the role played by the minimum consumption requirement in the evolution of per-capita capital stock.

PROPOSITION 3 (Minimum consumption requirement and transition dynamics). *Consider a representative household economy with* $\beta(1+r) > 1$. *The rate of growth of per-capita capital stock increases over time and converges to*

$\{[(1+r)\beta]^{\frac{1}{\sigma}} - 1\}$ *as* $t \to \infty$. *For transition paths with different* α *but identical* $\underline{k}_0$, *the growth rate of per-capita capital stock at any point in time is lower for economies with higher* α. *For transition paths with identical* α *but different* $\underline{k}_0$, *the growth rate of per-capita capital stock at any point in time is lower for economies with lower* $\underline{k}_0$.

As in the economy without minimum consumption, our economy grows over time if $(1+r)\beta > 1$ and its steady-state growth is $\{(1+r)\beta\}^{\frac{1}{\sigma}} - 1$. Thus, the minimum consumption requirement does not influence steady-state growth but its presence introduces transition dynamics; that is, the steady-state growth rate is achieved asymptotically rather than immediately.[1] Furthermore, an economy's growth rate of per-capita capital stock always lags behind that of a similar economy with a lower minimum consumption requirement or that of a similar economy with higher initial per-capita capital stock. This is true for the level of per-capita capital stock as well.

Whereas β, σ, and r pin down the steady-state growth rate, the initial growth rates depend on the initial capital stock and the level of the required minimum consumption. An economy that starts out with per-capita capital stock close to the level needed to maintain minimum consumption, i.e., close to α/r, will have low initial growth rates.

As in the standard linear growth model, the growth rate of consumption and capital stock depend positively on the intertemporal elasticity of consumption. The reason that transition dynamics exist in this model (and not in the standard one) is that the IES is time-varying. In particular, the IES in period t depends positively on the level of consumption in period t with the dependence becoming negligible as consumption moves farther away from the minimum required level.

To see this formally, note that Proposition 1 implies that

$$(\bar{c}_{t+1} - \bar{c}_t)/\bar{c}_t = \left\{[(1+r)\beta]^{\frac{1}{\sigma}} - 1\right\}\left(1 - \frac{\alpha}{\bar{c}_t}\right).$$

Approximating $\{[(1+r)\beta]^{\frac{1}{\sigma}} - 1\}$ by a first-order Taylor expansion around $r^* = 1/\beta - 1$ yields

$$(\bar{c}_{t+1} - \bar{c}_t)/\bar{c}_t \approx \beta(r - r^*)\left(1 - \frac{\alpha}{\bar{c}_t}\right)\left(\frac{1}{\sigma}\right).$$

If the time interval is small, $\bar{c}_{t+1}$ will be very close to $\bar{c}_t$ and the right-hand side of the above equation can be written as $\beta(r - r^*) \cdot \mathrm{IES}_t^{\mathrm{REP}}$, where $\mathrm{IES}_t^{\mathrm{REP}}$ is the intertemporal elasticity of substitution of the representative household. Thus,

$$(\bar{c}_{t+1} - \bar{c}_t)/\bar{c}_t \approx \beta(r - r^*) \cdot \mathrm{IES}_t^{\mathrm{REP}}. \quad \textbf{(3)}$$

Hence, the transition dynamics of consumption may be understood in terms of how the IES of the representative household evolves with economic growth. When the economy is poor, $\mathrm{IES}_t^{\mathrm{REP}}$ is close to zero and the growth rate of consumption is very low. As the standard of living of the average household improves, $\text{IES}_t^{\text{REP}}$ increases toward $1/\sigma$ and the growth rate of consumption converges toward $\beta(r-r^*)/\sigma$.[2]

3.2. Evolution of Wealth Inequality

This section explains the role of the minimum consumption requirement in shaping the distribution of capital stock across households. We start with the definition of Lorenz ordering, a concept that will be used repeatedly in this section. Let the share of total capital held by household i in period t be $s_t^i \equiv k_t^i/N\bar{k}_t$. The household distribution of capital shares in period t is the N-vector $\{s_t^1, s_t^2, \ldots s_t^N\} \equiv \mathbf{s}_t$. Then:

DEFINITION 1. *Arrange households in order of increasing stock of capital. Let* $\mathbf{s}$ *and* $\mathbf{s}'$ *be two different household distributions of capital shares. Then,* $\mathbf{s}$ *is Lorenz superior to* $\mathbf{s}'$ *if* $\sum_{i=1}^{m} s'^{i} \leq \sum_{i=1}^{m} s^{i}$ *for all* $1 \leq m \leq N$ *with the inequality holding strictly for at least one* m.

The concept of Lorenz superiority captures the essence of what it means for a distribution to have less inequality than another. Any useful scalar measure of inequality should assign a lower inequality index to a distribution that is Lorenz superior in comparison to another. Commonly used inequality measures (e.g., the Gini coefficient and the coefficient of variation) indeed do so. For this reason, we present the model's implications about the evolution of wealth inequality in terms of Lorenz orderings.

Note from Proposition 1 that the distribution of capital shares will obviously remain constant over time if $\beta(1+r)=1$. The following proposition describes how the household distribution of capital shares evolves when there is growth.

PROPOSITION 4 (Growth with increasing inequality). *If* $\beta(1+r)>1$, *then for all* $t \geq 0$, *the per-capita capital stock in period* $t+1$ *is greater than that in period* t, *but the household distribution of capital shares in period* t *is Lorenz superior to the household distribution of capital shares in period* $t+1$. *However, as* $t \to \infty$ *there is no change in the household distribution of capital shares.*

This result may be intuitively explained as follows: For household i, the analog of equation (3) is

$$\left(c_{t+1}^i - c_t^i\right)/c_t^i \approx \beta(r-r^*) \cdot \text{IES}_t^i. \tag{4}$$

Because wealthier households will consume more in every period (consumption in each period is a normal good), we know from the definition of IES that the IES of rich households will be higher than that of poor households. Thus, the consumption of rich households will grow faster than that of poor households. To support their steeper consumption profile, rich households save a higher fraction of their income relative to poor households; that is, they accumulate capital faster. Consequently, the distribution of capital shares becomes more unequal over time. Furthermore, as the level of consumption of all households increases, their IES's converge to $1/\sigma$. Thus, in the limit, all households accumulate wealth at the same rate, and there is no change in the distribution of capital shares.

Because household i's period t consumption is a linear function of k_t^i with positive intercept and slope terms, statements concerning the evolution of household capital shares have their analogues in household consumption as well. Thus, it is also the case that in the presence of minimum consumption requirements, each household's consumption and the inequality in household consumption grows over time. Because income is proportional to k_t^i, the same is true of income as well.[3]

Our results on the evolution of the distribution of wealth and consumption differ from a similar model with no minimum consumption. It follows from Proposition 1 [or from equations (4) and (1)] that without a minimum consumption requirement, the inequality in household wealth and consumption remains constant over time. Furthermore, if we compare two economies initially identical in all respects except that one has a minimum consumption requirement and the other does not, the long-run distribution in the minimum consumption economy is more unequal.

The next proposition compares two economies with different initial distributions of capital shares.

PROPOSITION 5. *Consider two economies*, $h = 1, 2$, *identical in all respects in the initial period except that the initial distribution of capital shares in economy* 1, $\mathbf{s}_0^1$, *is Lorenz superior to* $\mathbf{s}_0^2$, *the initial distribution of capital shares in economy* 2. *Then* $\mathbf{s}_t^1$ *is Lorenz superior to* $\mathbf{s}_t^2$ *for all* $t > 0$.

Again, the result is intuitive. An economy that has a more dispersed distribution of capital stock also will have a more dispersed distribution of IES across households. Thus, the potential for the distribution of wealth to get more unequal is greater for such an economy. Therefore, all else remaining the same, an economy that starts out with more inequality will never catch up (in terms of inequality) with one that starts out with less inequality.

The following proposition compares two economies with different initial per-capita capital stocks.

PROPOSITION 6. *Consider two economies*, $h = 1, 2$, *identical in all respects in the initial period except that* $\bar{k}_0^2 > \bar{k}_0^1$. *Then* $\bar{k}_t^2 > \bar{k}_t^1$ *and* $\mathbf{s}_t^2$ *is Lorenz superior to* $\mathbf{s}_t^1$ *for all* $t > 0$.

A lower initial per-capita capital stock means that every household in economy 1 has proportionately less capital than every household in economy 2. Because the IES is much more sensitive to changes in wealth when wealth is low, the growth rate of household capital stock will be lower for poor households than for rich. Consequently, the distribution of capital shares in period 1 will be more unequal for economy 1 than for economy 2. Then, in all future periods, there will be two reasons for the distribution of capital shares to be more unequal in economy 1 relative to economy 2: First, the per-capita capital stock will continue to be lower in economy 1 than in economy 2 and, second, the distribution of wealth in economy 1 will be more unequal than in economy 2 (see Proposition 5).

Recall from equation (2) that an economy that starts out with a lower per-capita capital stock will see slower growth. Proposition 6 tells us that this economy also will see a bigger increase in inequality over any length of time. Therefore, we can conclude that initial poverty retards growth and leads to a more unequal distribution of wealth. The next proposition compares two economies with different minimum consumption requirements.

PROPOSITION 7. *Consider two economies, $h = 1, 2$, identical in all respects in the initial period except that $\alpha^1 > \alpha^2$. Then $\overline{k}_t^2 > \overline{k}_t^1$ and $\mathbf{s}_t^2$ is Lorenz superior to $\mathbf{s}_t^1$ for all $t > 0$.*

The intuition here is similar to that underlying Proposition 6, but with one difference. In period 1 and in all future periods, there is an additional reason for the distribution of capital shares in economy 2 to be Lorenz superior to that in economy 1, namely, the minimum consumption requirement is higher in economy 1 than in economy 2. This lowers the IES of poor households more than that of rich households and is, therefore, another force contributing toward greater inequality of capital shares in economy 1.

3.3. Steady-State Growth and Evolution of Inequality

Because economic development typically is viewed as a shift from a zero- or low-growth path to a high-growth path, it may be important to understand the connection between the evolution of inequality and the rate of steady-state growth. As it turns out, this connection is complex enough that little can be said about the Lorenz orderings of distributions across economies with different steady-state growth rates. The purpose of this section is to explain why this is so.

Three parameters affect the steady-state growth rate in our economy: β, r, and σ. However, changes in the rate of economic growth usually are associated with changes in r, and so, we will concentrate on this case here and hold fixed the preference parameters β and σ.

Assume then that economy 1 has a higher rate of return than economy 2, i.e., $r_1 > r_2$ and that all other initial conditions are exactly the same. Suppose, for the moment, that the distribution of IES's is the same across the two economies in the initial period. Equation (4) suggests that economy 1 will experience a greater increase in the wealth inequality than economy 2. For instance, the variance of the anticipated growth rate of household consumption (and, by implication, of household capital stock) in the initial period will be higher in economy 1 than in economy 2.[4] Then, in period 1, economy 1 will have a higher per-capita capital stock [as implied by equation (2)] and a more unequal distribution of capital shares. Now note that because economy 1 has a higher per-capita capital stock, Proposition 6 suggests that it should have a more equal distribution of capital shares in period 2 than economy 2. On the other hand, because economy 1 has a more unequal distribution of wealth than economy 2, Proposition 5 suggests that it should have a more equal distribution of capital shares in period 2 than economy

2. These opposing tendencies make statements about Lorenz ordering difficult. Indeed, there is no guarantee that the two distributions can, in fact, be ranked by the Lorenz criterion.[5]

4. QUANTITATIVE ANALYSIS

4.1. Parameter Selection and Calibration

The parameters of our model are α, σ, β, r, and the initial distribution of the capital stock. We use available estimates of the first three parameters. The data pertain to a sample of households in three Indian villages (the so-called ICRISAT data) for the period 1975–76 to 1985–86. Townsend (1994, p. 588, Table A.1) describes some of the key features of this data set. We calibrate r to match the observed average growth rate of consumption in these villages and we calibrate the distribution of initial capital stock so that the implied mean and standard deviation of consumption match the observed mean and standard deviation of consumption per adult over this period.

We consider two values of minimum consumption: 177 rupees per year per adult and 245 rupees per year per adult. The first estimate is from Atkeson and Ogaki (1996, Table 2) and the second is from Rosenzweig and Wolpin (1993, Table 2). As a percentage of average consumption, these estimates correspond to 58% and 80%, respectively.[6]

The estimate of the curvature parameter σ, reported by Rosenzweig and Wolpin, is 0.964. The Atkeson–Ogaki estimation strategy does not provide an estimate of σ, but their estimate of minimum consumption is entirely consistent with a σ value of 0.964. Therefore, we associate the Rosenzweig–Wolpin estimate of σ with the minimum consumption requirement of 177 rupees as well.

Neither the Atkeson–Ogaki nor the Rosenzweig–Wolpin estimation strategy yields estimates of β. However, in doing their estimation, Rosenzweig and Wolpin assume that the annual discount factor β is 0.95. To be consistent, we assume that $\beta = 0.95$ as well, which fixes the model period as 1 year.

We calibrate r to match the observed growth rate of consumption in these village economies. As noted by Townsend (1994), the average consumption level appears to remain constant over the sample period in these villages. Therefore, equation (3) implies that r must be $1/\beta - 1 = 0.0526$.

Given the values of α, σ, β, and r, we calibrate the initial-period distribution of capital stock to match the observed mean and the standard deviation of consumption across households. The procedure is as follows: For each of the two estimates of α, we use the observed mean and standard deviation of consumption to calculate the mean and standard deviation of *discretionary* consumption. We then use the decision rule for discretionary consumption (Proposition 1) to infer the mean and standard deviation of *discretionary capital stock*. We assume that discretionary capital stock is lognormally distributed, and so, the mean and standard deviation of discretionary capital stock are used to infer the *distribution* of discretionary

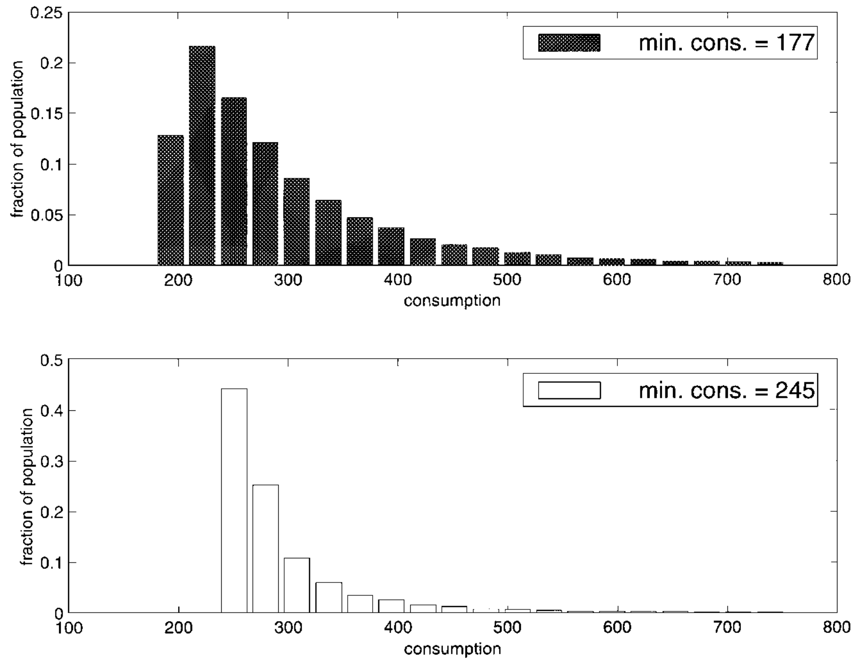

FIGURE 1. Initial-consumption distributions.

capital stock. The initial distribution of capital stock is then just a translation of this distribution of discretionary capital stock by the quantity $+\alpha/r$.[7]

An implication of this procedure is that the degree of inequality in the initial period's capital stock and consumption is *less* for the economy with $\alpha = 245$ as compared to the economy with $\alpha = 177$. The reason for this is as follows: When α is set to the higher value, our procedure requires us to keep the mean and standard deviation of observed consumption unchanged. If the consumption by each household is shifted up by the difference between 245 and 177, the standard deviation would remain unchanged but the mean would be higher. To bring about a *spread-preserving* decrease in the mean, it is necessary to shift the mass of the distribution toward the minimum consumption of 245. As shown in Figure 1, this results in less inequality in the initial period's consumption distribution as compared to the case where $\alpha = 177$. The same is true for the initial distribution of capital stock.

4.2. Evolution of Per-Capita Consumption and Wealth

In this section, we explore the quantitative impact of minimum consumption requirement on aggregate dynamics. These findings show how our calibrated economy would behave if its steady-state growth rate was positive instead of zero.

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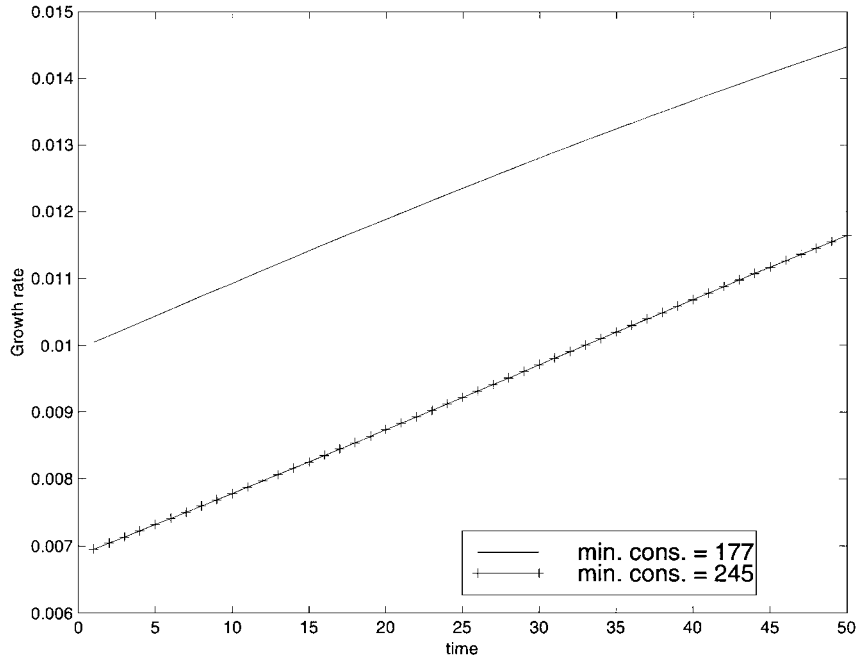


FIGURE 2. Growth rate of per-capita consumption when long-run growth rate is 2%.

Consider first the case where the rate of return on capital is such that it generates a steady-state growth rate of 2%. Figure 2 shows the path of per-capita consumption over a 50-year period for two economies: $\alpha = 177$ and 245. The existence of minimum consumption slows down the growth rate of the per-capita consumption substantially. For the $\alpha = 177$ economy, the initial growth rate is only 1%. Even at the end of 50 years (roughly two generations), the growth rate is 1.45%. The pace of growth is even slower for the economy with $\alpha = 245$: The initial growth rate of consumption is 0.7%, and at the end of 50 years it is only 1.16%. The results for growth rate of per-capita capital stock, shown in Figure 3, are very similar.

The behavior of these two economies can be understood by examining the behavior of IES for the representative household along the transition path. For the $\alpha = 177$ economy, the IES in the initial period is 0.53, and at the end of 50 years, it is 0.76. For the $\alpha = 245$ economy, the corresponding values are lower: 0.37 in the initial period and 0.61 at the end of 50 years.

The extent to which the minimum consumption requirement retards economic growth depends on the economy's steady-state growth rate. Economies on a high-growth path overcome the effects of minimum consumption relatively quickly. Figure 4A shows the evolution of the growth rate of consumption for economies with the same initial capital stock and $\alpha = 177$ but with steady-state growth rates

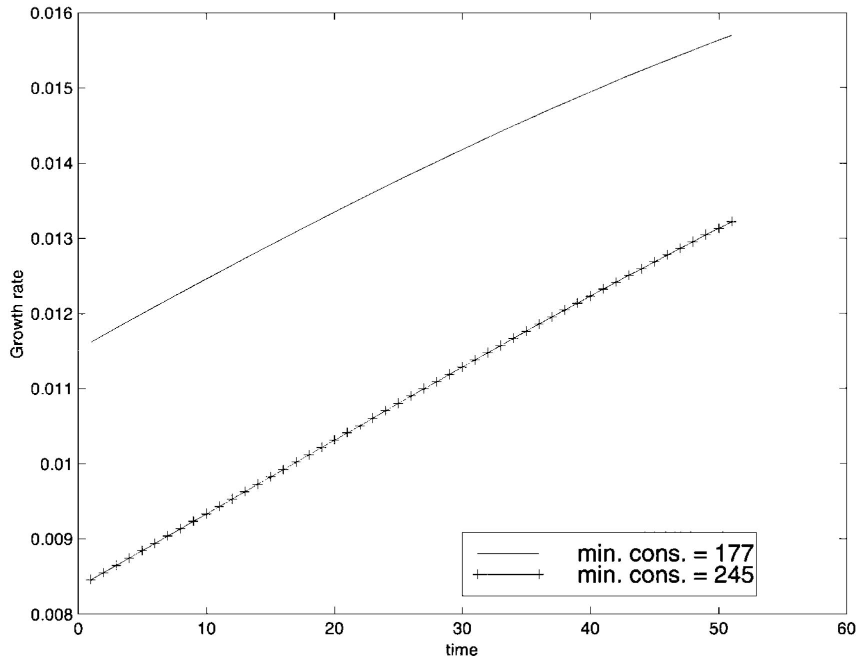
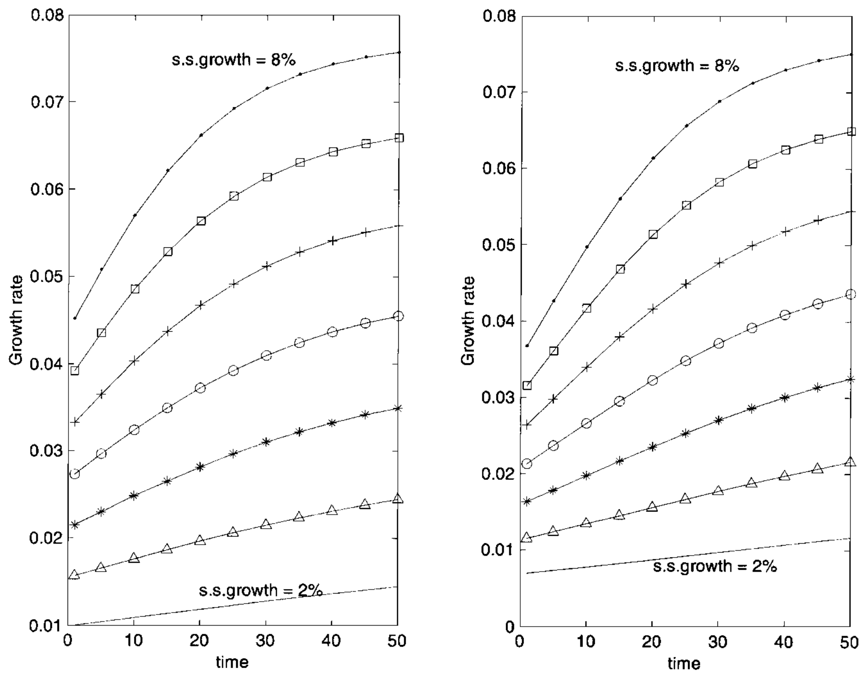


FIGURE 3. Growth rate of per-capita wealth when long-run growth rate is 2%.

2%, 3%, . . . , 8%. (As before, the differences in steady-state growth rates are due to different rates of return to capital.) The effect of minimum consumption does not last very long in the high-growth economy: For an economy with a steady-state growth rate of 8%, the initial growth rate is 4.51%, but after 50 years the growth rate is 7.57%. A similar result is shown in Figure 4B, where $\alpha = 245$: For an economy with a steady-state growth rate of 8%, the initial growth rate is only 3.67%, but after 50 years it is 7.51%, almost the same as in the $\alpha = 177$ economy.

The contrast between the low- and high-growth economies also can be understood by comparing the IES's. For instance, when $\alpha = 177$, the IES of the representative household in the initial period for the high-growth economy is 0.62 and at the end of 50 years it is 1.02. Notice that the IES at the end of 50 years is very close to the $1/\sigma$ value of 1.04.

These simulations suggest that wealth-dependent IES's may be important for understanding the cross-country evidence on economic growth. For instance, consider the *convergence* literature. Barro and Sala-i-Martin (1992) show that the standard neoclassical growth model with constant IES implies too rapid a convergence in national incomes relative to the data. Lower IES slows down the rate of convergence and, thus, a model with minimum consumption requirement has the potential for matching the observed rates of income convergence.



A) Minimum Consumption = 177 **B)** Minimum Consumption = 245

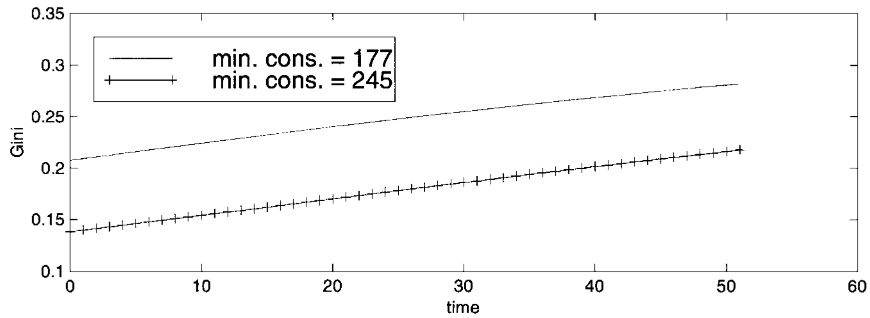
FIGURE 4. Per-capita consumption growth across economies.

4.3. Evolution of Inequality

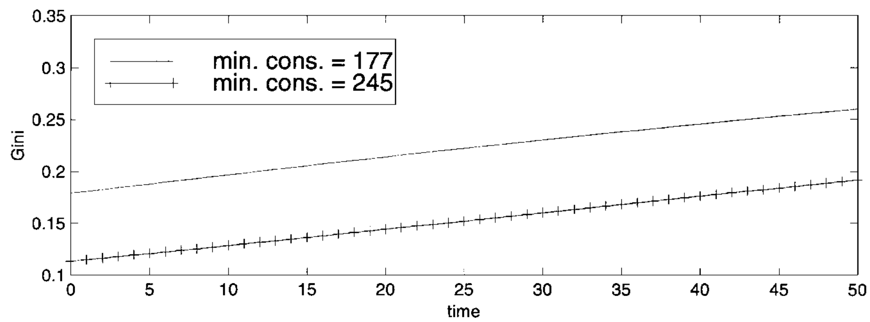
We measure inequality in each period by the Gini coefficient. The qualitative results of the preceding section tell us that inequality in consumption and wealth increases over time. The Gini coefficient quantifies this increase.

Figure 5A illustrates the evolution of wealth inequality for our two economies when the steady-state growth rate is 2%. The response of wealth inequality to this modest rate of economic growth is substantial. For instance, for the $\alpha = 177$ economy, the Gini coefficient rises from 0.207 to 0.246 in 25 years, a rise of about 19% in one generation.[8] Figure 5B shows corresponding evolution in consumption inequality. For the $\alpha = 177$ economy, the Gini coefficient of wealth inequality rises from 0.180 to 0.223 in 25 years, a rise of about 29% in (roughly) one generation.

The differences in the IES of rich and poor households underlie this increase. In the initial period, the IES of the bottom decile is 0.290 whereas that of the top decile is 0.790. Even after 25 years, these elasticities are 0.398 and 0.868, respectively, implying a strong potential for further increases in inequality. Indeed, if we roll the economy forward for another 25 years, the Gini coefficient of wealth inequality increases to 0.279.[9]



A) Wealth Inequality



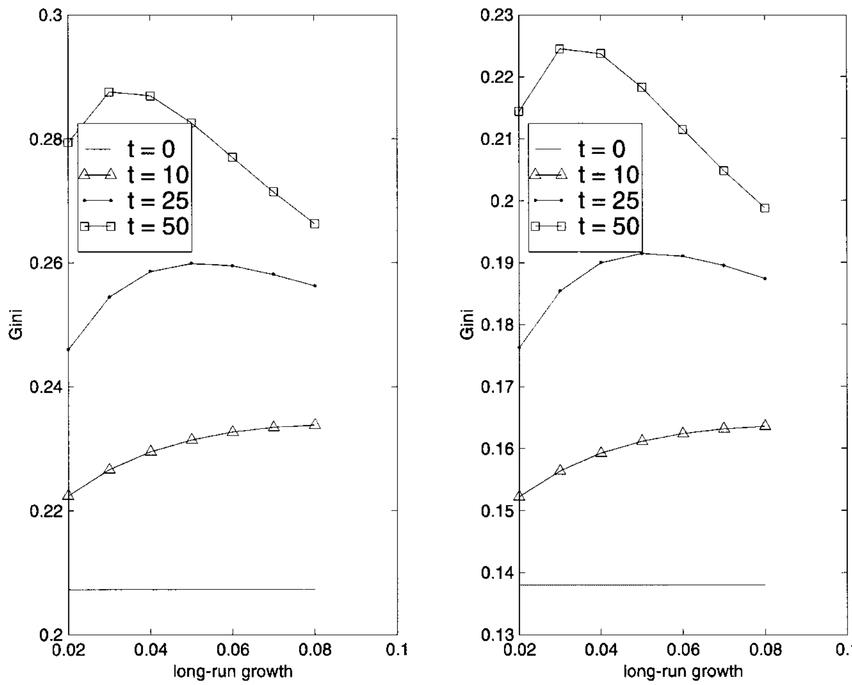
B) Consumption Inequality

FIGURE 5. Evolution of inequality when long-run growth rate is 2%.

As one would expect, the impetus for increases in inequality is greater for the economy with $\alpha = 245$. For instance, in Figure 5B the Gini coefficient of consumption inequality rises from 0.116 to 0.155 over a period of 25 years, an increase of about 34%.[10]

4.4. Steady-State Growth and Evolution of Inequality

Figures 6A and 6B plot the evolution of wealth inequality for our two economies for steady-state growth rates ranging from 2 to 8%. (Again, the growth rates differ because of differences in the underlying rate of return on capital.) The striking finding is that the relationship between rate of steady-state growth and wealth inequality at some future point in time need not be monotonic. As explained in the theoretical section of the paper, the nonmonotonicity reflects the interplay of two opposing forces. On the one hand, a higher rate of return serves to magnify the differences in the growth rates of household capital stocks and thereby contributes to increasing inequality (the "rate-of-return effect"). On the other hand, the consumption by each household at each point in time is further away from the minimum required level in the fast-growing economy. As a result, differences in IES



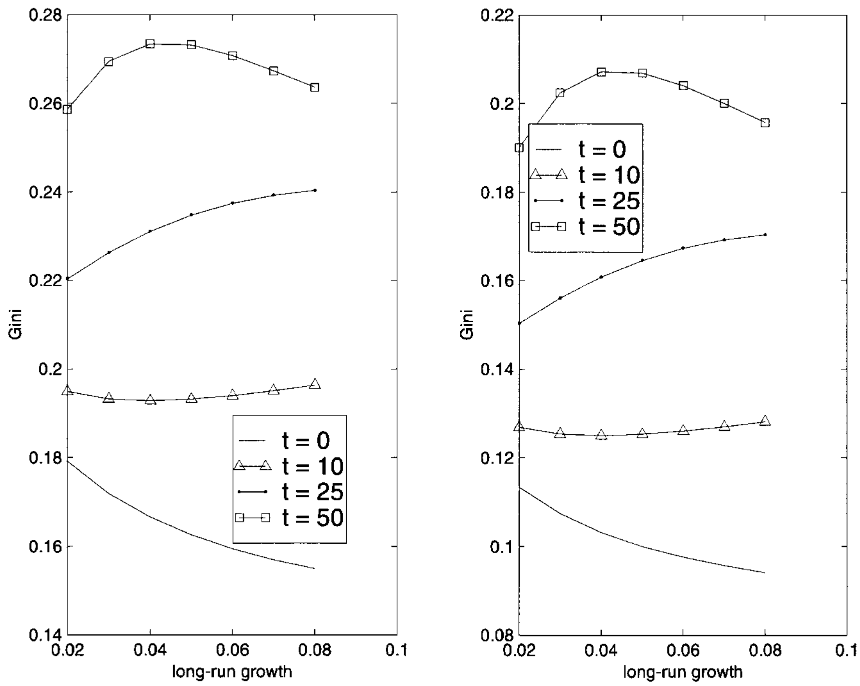
A) Minimum Consumption = 177 **B)** Minimum Consumption = 245

FIGURE 6. Wealth inequality across economies.

decline more rapidly for the fast-growing economy, and the potential for inequality to increase drops faster as well (the "declining-differences-in-IES effect").

Figure 6A shows which of these two forces dominates at different horizons for the $\alpha = 177$ economy. At the end of 10 years, the relationship between the degree of wealth inequality and steady-state growth is positive: The faster the growth, the more unequal the distribution of wealth. Over this period, the rate-of-return effect dominates the declining-differences-in-IES effect. However, at the end of 25 years, the relationship is nonmonotonic with a peak in inequality at about the 5% growth rate. Evidently, the additional 20 years allow the relatively slow-growing economies, for which the declining-differences-in-IES effect is weak, to catch up and surpass the wealth inequality of the fast-growing economies. A similar pattern is evident in Figure 6B for the economy with $\alpha = 245$.

These simulations suggest the a group of countries that have spent relatively more time on their sustained growth path may display a different relationship between wealth inequality and rates of economic growth than a group of countries that have spent relatively less time on their sustained growth paths. Thus, it might be important in cross-country studies of wealth distribution to control for time elapsed on the sustained growth path.



A) Minimum Consumption = 177 **B)** Minimum Consumption = 245

FIGURE 7. Consumption inequality across economies.

Figures 7A and 7B show the evolution of consumption inequality in these simulations. The relationship between consumption inequality and steady-state growth is more complex. Note that, in the initial period, the relationship between steady-state growth and consumption inequality is *negative*: Consumption inequality decreases with faster steady-state growth. Then, at the end of 10 years, the relationship is U-shaped: Inequality first declines and then rises with steady-state growth. At the end of 25 years, the relationship is monotonically increasing, and, finally, at the end of 50 years, the relationship resembles an inverted-U: There is a distinct peak in consumption inequality at around the 4% growth rate.

The reason consumption inequality behaves differently is that an increase in r causes the *initial* distribution of consumption to change in favor of poor households. To see why, note that, because σ is calibrated to a value of less than one, the substitution effect of an increase in r on *discretionary* consumption dominates the income effect. Therefore, an increase in r causes all households to substitute future consumption for initial-period consumption. Because the rich do more of this substitution than the poor, the distribution of initial-period consumption shifts in favor of poor households. The complex temporal behavior of the relationship between consumption inequality and steady-state growth is the result of interaction between this initial redistribution effect and the other effects in operation (namely, the rate-of-return effect and the declining-difference-in-IES effect).

The difference between the behavior of wealth and consumption inequality holds an important lesson, namely, that trends in wealth inequality (or inequality in permanent income) need not match the trend in consumption inequality. Because data on household consumption are often more readily available than data on wealth or permanent income, researchers have used the former as a proxy for the latter. However, such proxies may be quite misleading. To see why, suppose that, for some country, researchers have one observation on consumption inequality from a period when the economy was stagnant and another observation from a period during which the economy was on its sustained growth path. Depending on how long the economy had spent on the sustained growth, consumption inequality could be lower or higher than the consumption inequality recorded during the stagnant era. This is because consumption distribution improves at the start of sustained growth. Of course, after this initial improvement, consumption inequality worsens over time, but consumption inequality at the time of the second observation could well be lower than at the time of the first observation. In contrast, inequality in wealth would be higher at the time of the second observation. Thus, one might erroneously conclude from the consumption inequality data that permanent income or wealth inequality improved over this period when actually it did not.

5. CONCLUSIONS

Several streams of work suggest that understanding the relationship between economic growth and distribution of income and wealth may be important. The political economy literature has stressed the importance of changes in the distribution of income in determining how collective decisions that have an enormous impact on economic growth are affected [see Persson and Tabellini (1992)]. In a similar vein, sociologists have documented that periods of social and political upheaval (like revolutions) tend to occur when economic conditions are improving and have suggested that this timing of revolutions might be explained if periods of economic growth coincide with periods of increasing *relative* deprivation [see Runciman (1966) and Gurr (1970)].

We show that if households have to satisfy a common minimum consumption requirement, periods of economic growth will be accompanied by increasing relative deprivation. This happens because poor households have a lower intertemporal elasticity of substitution in consumption and therefore accumulate wealth at a slower rate relative to richer households.

We find that when the minimum consumption requirement is set to the available estimates, its effects on the rate of economic growth and the distribution of consumption and wealth are significant. For instance, an economy that has a steady-state growth rate of 2% will have attained only half its steady-state growth rate at the end of 25 years and would see its Gini coefficient of wealth inequality increase about 25% over this period.

We also find that the relationship between the rate of economic growth and consumption and wealth inequality depends on how long a time the economies in question have spent on their sustained growth paths. Among the economies that have been on their sustained growth path for a long period of time, the medium-growth economies tend to experience the greatest increases in wealth inequality. For economies that have experienced sustained growth for only a short period of time, the increase in wealth inequality is likely to be greatest for the fast-growing economies.

Finally, we note that, although we focused this article on the role of minimum consumption requirements in poor countries, our results are clearly applicable to poverty-stricken groups in more affluent countries as well. Even for the United States, there is systematic evidence that the rate of growth of consumption is lower for poor households than for middle- and high-income households [Lawrance (1991)]. If these differences are symptomatic of minimum consumption requirements, poor households as a group would exhibit the type of relative deprivation discussed in this paper.

NOTES

1. Note that, from equation (2), the growth rate is zero if $\beta(1+r)=1$. In this case there are no transitional dynamics even with a minimum consumption requirement.

2. Because equation (3) is an approximation, we do not use it (or its analog for an individual) to prove any results in this paper. However, we do use it to provide intuition for some of our results.

3. Because the model abstracts from idiosyncratic income uncertainty, and because the capital stock is best interpreted as the sum of human and nonhuman wealth, the concept of income for which the model makes predictions is permanent income.

4. Of course, the distribution of initial-period IES will not be the same in the two economies. The IES's depend on the level of consumption, and even with identical initial distributions of capital stocks, these consumption levels will depend on the rate of return to capital. However, it is easy to show that despite this complication the distribution of capital shares in period 1 will be worse in economy 1 than in economy 2.

5. In situations where distributions cannot be ranked by the Lorenz criterion, researchers have suggested weaker alternatives. For instance, Shorrocks (1983) suggests replacing the Lorenz criterion of superiority by the generalized Lorenz criterion, "$\bar{k}\sum_{i=1}^{m} s^i \leq \bar{k}' \sum_{i=1}^{m} s'^i$ for all $1 \leq m \leq N$ and inequality holding strictly for some m." By this criterion, an economy with a higher r will, *ceteris paribus*, also have a superior distribution of wealth in all periods beyond the initial one.

6. The average consumption level was computed by combining the average consumption levels reported by Townsend (1994, Table A.1) for each of the three villages.

7. The key property of the lognormal distribution that makes it suitable for modeling distributions is that it is negatively skewed with its median being less than its mean. For a detailed discussion of lognormality in the context of income distributions, see Aitchison and Brown (1969, Ch. 11).

8. For an $\alpha = 0$ economy that starts out with the same conditions as the $\alpha = 177$ economy, the Gini coefficient would have remained at 0.207 at the end of 25 years and its per-capita wealth would have been higher.

9. The ratio of the consumption of the top decile to consumption of the bottom decile rises from 2.99 to 3.74 at the end of 25 years and to 4.62 at the end of 50 years.

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10. As explained earlier, the fact that the $\alpha = 245$ economy has a smaller Gini coefficient is a consequence of our calibration procedure, which assigns a more equal initial-period distribution of consumption and wealth to this economy relative to the $\alpha = 177$ economy.

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APPENDIX: PROOFS OF PROPOSITIONS

We first present two useful results that bring out the connection between household growth rates of capital and the evolution of household distribution of capital shares. These results are purely logical in nature and hold independently of any particular model of economic growth. We use these results in the proofs of various propositions. Throughout this section, $\gamma_{t+1}^i \equiv (k_{t+1}^i - k_t^i)/k_t^i$ and $\bar{\gamma}_{t+1} \equiv (\bar{k}_{t+1} - \bar{k}_t)/\bar{k}_t$.

The first result gives a sufficient condition under which the household distribution of capital shares in different periods in a given economy can be Lorenz ranked.

LEMMA A.1. *Suppose $k_t^j > k_t^i$ implies that $\gamma_{t+1}^j > \gamma_{t+1}^i$. Then $\boldsymbol{s}_t$ is Lorenz superior to $\boldsymbol{s}_{t+1}$.*

Proof. Arrange households in period t in order of increasing capital stocks. Because $k_t^j > k_t^i$ implies that $k_{t+1}^j > k_{t+1}^i$, this ordering also ranks households in order of increasing capital stocks in period $t+1$. Next, observe that $s_{t+1}^i > s_t^i$ if and only if $\gamma_{t+1}^i > \bar{\gamma}_{t+1}$. Let M be such that $k_t^i \le \bar{k}_t\ \forall i \le M$ and $k_t^i > \bar{k}_t\ \forall i > M$. Then, $\sum_{i=1}^m s_{t+1}^i < \sum_{i=1}^m s_t^i$ $\forall m \le M$ and $\sum_{i=m}^N s_{t+1}^i > \sum_{i=m}^N s_t^i\ \forall m > M$. The second inequality implies that $1 - \sum_{i=m}^N s_{t+1}^i < 1 - \sum_{i=m}^N s_t^i\ \forall m > M$. Because $\sum_{i=1}^{m-1} s_t^i + \sum_{i=m}^N s_t^i = 1$ for all t, the second inequality is equivalent to $\sum_{i=1}^{m-1} s_{t+1}^i < \sum_{i=1}^{m-1} s_t^i\ \forall m > M$. The result follows. ■

The second result provides sufficient conditions under which the household distribution of capital shares in two different economies are Lorenz comparable at a given point in time.

LEMMA A.2. *Consider two economies $h = 1, 2$ such that $\boldsymbol{s}_t^h = \boldsymbol{s}_t$. Suppose $k_t^{j,h} > k_t^{i,h}$ implies that $\gamma_{t+1}^{j,h} > \gamma_{t+1}^{i,h}$ for $h = 1, 2$. If $\Delta_{t+1}^1(i, j) < \Delta_{t+1}^2(i, j)\ \forall i, j$, where $\Delta_{t+1}^h(i, j) = (1 + \bar{\gamma}_{t+1}^h)^{-1} \cdot |\gamma_{t+1}^{i,h} - \gamma_{t+1}^{j,h}|$, $\boldsymbol{s}_{t+1}^1$ is Lorenz superior to $\boldsymbol{s}_{t+1}^2$.*

Proof. For each economy, arrange households in order of increasing period t capital stocks. Note that, in this new ordering, $\boldsymbol{s}_t^1$ will still equal $\boldsymbol{s}_t^2$. Now, because $k_t^{j,h} > k_t^{i,h}$ implies that $k_{t+1}^{j,h} > k_{t+1}^{j,h}$, this ordering also will arrange households in both economies in order of increasing capital stock in period $t+1$. Then, by assumption on $\Delta_{t+1}^h(i, j)$,

$$\left(\frac{s_{t+1}^{i,2}}{s_t^{i,2}} - \frac{s_{t+1}^{j,2}}{s_t^{j,2}}\right) < \left(\frac{s_{t+1}^{i,1}}{s_t^{i,1}} - \frac{s_{t+1}^{j,1}}{s_t^{j,1}}\right) \forall i < j.$$

Rearranging,

$$\left(\frac{s_{t+1}^{i,2}}{s_t^{i,2}} - \frac{s_{t+1}^{i,1}}{s_t^{i,1}}\right) < \left(\frac{s_{t+1}^{j,2}}{s_t^{j,2}} - \frac{s_{t+1}^{j,1}}{s_t^{j,1}}\right) \forall i < j.$$

To obtain a contradiction, assume that there is an $M < N$ such that $\sum_{i=1}^M s_{t+1}^{i,2} \ge \sum_{i=1}^M s_{t+1}^{i,1}$. Then, there must be some $i \le M$ for which $s_{t+1}^{i,2} - s_{t+1}^{i,1} \ge 0$. Because $s_t^{i,1} = s_t^{i,2}$ for all i, it follows that $s_{t+1}^{i,2} > s_{t+1}^{i,1}$ for all $i > M$. Therefore, $\sum_{i=1}^M s_{t+1}^{i,2} + \sum_{i=M+1}^N s_{t+1}^{i,2} > \sum_{i=1}^M s_{t+1}^{i,1} + \sum_{i=M+1}^N s_{t+1}^{i,1}$, which is impossible. Hence, $\sum_{i=1}^M s_{t+1}^{i,2} < \sum_{i=1}^M s_{t+1}^{i,1}$ for all $M < N$ and the result follows. ■

Note that Lemma A.2 makes a statement only about one-period-ahead Lorenz orderings. However, it does not restrict the current level of total capital stock or the growth rate of capital stock to be the same across the two economies.

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A.1. PROOF OF PROPOSITION 1

It follows from Alvarez and Stokey (1998) that $v(\cdot)$ exists and is increasing, strictly concave, and differentiable. The constraint set for the household is compact and convex. Hence, the policy functions are well defined. The solution to the household's problem must satisfy the Euler equations

$$\left(\tilde{c}_t^i\right)^{-\sigma} = \beta(1+r)\left(\tilde{c}_{t+1}^i\right)^{-\sigma}$$

for all $t \geq 0$. Guess the policy function to be linear in discretionary capital stock. The result follows from using the Euler equations and the constraints. ■

A.2. PROOF OF PROPOSITION 2

The result follows from the observation that if $k_0^i = \bar{k}_0$, then the growth rate of household i's capital stock will be given by equation (2) for all $t \geq 0$. ■

A.3. PROOF OF PROPOSITION 3

Proposition 1 implies that

$$(\bar{k}_t - \alpha/r) = \left\{[\beta(1+r)]^{\frac{1}{\sigma}}\right\}^t (\bar{k}_0 - \alpha/r).$$

By assumption, $k_0^i > \alpha/(a-\delta)\ \forall i$, which implies that $\bar{k}_0 > \alpha/r$. Then, $\beta(1+r) > 1$ implies that $\bar{k}_{t+1} > \bar{k}_t\ \forall t \geq 0$ and equation (2) implies that $(\bar{k}_{t+2} - \bar{k}_{t+1})/\bar{k}_{t+1} > (\bar{k}_{t+1} - \bar{k}_t)/\bar{k}_t\ \forall t \geq 0$. Also, because $\bar{k}_t \to \infty$ as $t \to \infty$, it follows that $(\bar{k}_{t+1} - \bar{k}_t)/\bar{k}_t \to \{[(1+r)\beta]^{\frac{1}{\sigma}} - 1\}$ as $t \to \infty$.

Suppose that for economies 1 and 2, $\alpha^1 > \alpha^2$ and $\bar{k}_t^1 < \bar{k}_t^2$ for some $t > 0$. Then, equation (2) implies that $(\bar{k}_{t+1}^1 - \bar{k}_t^1)/\bar{k}_t^1 \leq (\bar{k}_{t+1}^2 - \bar{k}_t^2)/\bar{k}_t^2$ and $\bar{k}_{t+1}^1 < \bar{k}_{t+1}^2$. Repeating this argument for $t+1, t+2$, and so on, we get $(\bar{k}_{t+1}^1 - \bar{k}_t^1)/\bar{k}_t^1 \leq (\bar{k}_{t+1}^2 - \bar{k}_t^2)/\bar{k}_t^2$ for all $\tau > t$. Because $\bar{k}_0^1 = \bar{k}_0^2$ and $\alpha^1 > \alpha^2$ implies that $(\bar{k}_1^1 - \bar{k}_0^1)/\bar{k}_0^1 < (\bar{k}_1^2 - \bar{k}_0^2)/\bar{k}_0^2$, it follows that $(\bar{k}_{t+1}^1 - \bar{k}_t^1)/\bar{k}_t^1 < (\bar{k}_{t+1}^2 - \bar{k}_t^2)/\bar{k}_t^2$ for all $t > 0$.

The proof of the final statement is similar. ■

A.4. PROOF OF PROPOSITION 4

With $(1+r)\beta > 1$, $\bar{k}_{t+1} > \bar{k}_t$ for all $t \geq 0$. Proposition 1 implies that for $k_t^j > k_t^i$ we must have $\gamma_{t+1}^j > \gamma_{t+1}^i$. It then follows from Lemma A.1 that $\boldsymbol{s}_t$ is Lorenz superior to $\boldsymbol{s}_{t+1}$ for all $t \geq 0$. Next, observe that $k_t^i \to \infty$ as $t \to \infty$ implies that $(\gamma_{t+1}^i/\bar{\gamma}_{t+1}) \to 1$ as $t \to \infty$. Hence, asymptotically, there is no change in the distribution of capital shares from one period to the next. ■

A.5. PROOF OF PROPOSITION 5

Proposition 1 shows that $k_t^{j,h} > k_t^{i,h} \Rightarrow k_{t+1}^{j,h} > k_{t+1}^{i,h}\ \forall h$. Now, by hypothesis, $\sum_{i=1}^m s_0^{i,2} < \sum_{i=1}^m s_0^{i,1}\ \forall m < N$. Because $\bar{k}_0^1 = \bar{k}_0^2$, it follows that $\sum_{i=1}^m k_0^{i,2} < \sum_{i=1}^m k_0^{i,1}\ \forall m < N$. Then, the decision rule for discretionary capital implies that $\sum_{i=1}^m k_1^{i,2} < \sum_{i=1}^m k_1^{i,1}\ \forall m < N$. By equation (2), $\bar{k}_1^1 = \bar{k}_1^2$. Hence, $\sum_{i=1}^m s_1^{i,2} < \sum_{i=1}^m s_1^{i,1} \forall m < N$, and, thus, $\boldsymbol{s}_1^1$ is Lorenz superior to $\boldsymbol{s}_1^2$. This logic applies for any $t > 1$ as well. ■

A.6. PROOF OF PROPOSITION 6

By equation (2), $\bar{k}_t^2 > \bar{k}_t^1 \; \forall \, t > 0$. Suppose for some t, $\boldsymbol{s}_t^2$ is Lorenz superior to $\boldsymbol{s}_t^1$. Consider another economy, denoted by the index 3, which has the same values for N, σ, β, and α as economies 1 and 2 but which has $\bar{k}_t^3 = \bar{k}_t^2$ and $\boldsymbol{s}_t^3 = \boldsymbol{s}_t^1$. Because $\boldsymbol{s}_t^2$ is Lorenz superior to $\boldsymbol{s}_t^1$, it is also Lorenz superior to $\boldsymbol{s}_t^3$. By Proposition 5, $\boldsymbol{s}_{t+1}^2$ is Lorenz superior to $\boldsymbol{s}_{t+1}^3$. Now, observe that Proposition 1 implies that

$$\Delta_{t+1}^h(i, j) = \left(\frac{\{[\beta(1+r)]^{1/\sigma} - 1\}\alpha}{Nr\bar{k}_{t+1}^h} \right) \left| \frac{1}{s_t^{i,h}} - \frac{1}{s_t^{j,h}} \right|.$$

Because $\boldsymbol{s}_t^1 = \boldsymbol{s}_t^3$ and $\bar{k}_{t+1}^3 > \bar{k}_{t+1}^1$, it follows that $\Delta_{t+1}^3(i, j) < \Delta_{t+1}^1(i, j)$. Therefore, by Lemma A.2 $\boldsymbol{s}_{t+1}^3$ is Lorenz superior to $\boldsymbol{s}_{t+1}^1$. Transitivity of Lorenz orderings implies that $\boldsymbol{s}_{t+1}^2$ is Lorenz superior to $\boldsymbol{s}_{t+1}^1$. Because $\boldsymbol{s}_0^1 = \boldsymbol{s}_0^2$ and $\bar{k}_0^2 > \bar{k}_0^1$, we conclude that $\boldsymbol{s}_1^2$ is Lorenz superior to $\boldsymbol{s}_1^1$. The result follows. ■

A.7. PROOF OF PROPOSITION 7

By Proposition 1, it follows that $\bar{k}_t^2 > \bar{k}_t^1$ for all $t > 0$. Next, suppose that $\boldsymbol{s}_t^2$ is Lorenz superior to $\boldsymbol{s}_t^1$ for some t. Then, following the steps of the preceding proof, we can show that $\boldsymbol{s}_{t+1}^2$ is Lorenz superior to $\boldsymbol{s}_{t+1}^1$. Now note that, in the initial period, $\bar{k}_0^1 = \bar{k}_0^2$. Because $\alpha^1 > \alpha^2$, it follows from Proposition 1 that $\bar{k}_1^2 > \bar{k}_1^1$. Then, $\Delta_1^2(i, j) < \Delta_1^1(i, j)$ for all i and j. Hence, by Lemma 2, $\boldsymbol{s}_1^2$ is Lorenz superior to $\boldsymbol{s}_1^1$.The result follows. ■