Robotica (2000) volume 18, pp. 209–211. Printed in the United Kingdom

Uncertainty configurations of parallel manipulators

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(Received in Final Form: August 27, 1999)

SUMMARY

This paper presents a new method of analyzing and calculating the uncertainty configurations of parallel manipulators. The main feature of the method that makes it attractive with respect to the methods available in the literature, is its ability to obtain simplified uncertainty configuration equations. It's very useful in real-time control. The application of the proposed method is illustrated in detail by two examples.

KEYWORDS: Parallel manipulator; Uncertainty configuration; Instantaneous motion.

1. INTRODUCTION

Parallel manipulators consist of multiple branches acting in parallel on a common payload platform. For a parallel manipulator to be capable of spatial motion each branch must be able to accommodate six degrees of freedom of task space motion. Parallel manipulators are superior to serial manipulators, which include higher stiffness, improved accuracy and dynamic characteristics[1-3]. These advantages stem from the fact that the base-distal joints of parallel manipulators do not have to be actuated, thereby reducing the manipulated mass; that the actuators of the branches act in parallel, sharing a common payload; and that errors due to inaccurate device manufacture or sensing act in parallel rather than in serial, thereby improving accuracy.

The special configurations of parallel manipulators can be classified as degeneracies related to the branches and uncertainty configurations of the parallel architecture[4]. In branch degeneracy a serial branch, and hence, the entire parallel manipulator is not capable of providing a required end effector motion. In an uncertainty configuration a parallel manipulator is not able to resist or apply a required end effector force. Near degenerate configurations a serial branch, and hence, the entire parallel manipulator has a very poor motion performance, i.e. the manipulator becomes incapable of producing end effector motion in the direction of a lost degree of freedom. Similarly, the force transmission performance of a parallel manipulator is very poor near uncertainty configurations, i.e. the manipulator cannot effectively resist or apply forces at the end effector in certain directions.

The degeneracy related to the branches can be identified by finding the roots of the determinant of the Jacobian matrices of the individual branches. For kinematically simple branches this reduces to a simple problem. Similarly, the uncertainty configurations of a parallel manipulator may be determined from the roots of the determinant of the matrix of the wrenches associated with the actuated joints of the parallel devices. However, for a parallel manipulator the matrix of wrenches is complex and it is not easy to find these roots.

Gosselin and Angeles[5] classified special configurations of closed-chain manipulators based on the singularities of Jacobian matrices obtained from differentiating the relationship between input and output coordinates and analyzed parallel manipulator singularities using the Jacobian of its constraint equations. They identified singular configurations of the platform as those that lead to a singular Jacobian matrix. The goal is to determine the location of all singular configurations in the workspace of the manipulator. Later, Ma and Angeles[6] examined architecture singularities of linear actuated fully parallel manipulators. Zlantanov, Fenton and Benhabib[7] presented a novel method for finding and classifying all the singularities of an arbitrary non-redundant mechanism.

The proposed technique is based on the velocity-equation formulation of kinematic singularity and the singularity classification introduced earlier by the authors. Criteria for singularity are derived and used to formulate a method for computing the singularity set and revealing its devision into singularity classes. The uncertainty configurations of parallel manipulators can be investigated by examining potential actuated-joint associated wrench system degeneracies using screw theory. For particular device classes line geometry considerations can be utilized. Hunt[3] utilized the principles of reciprocity and linear dependence of screws to classify the screw systems and studied special configurations of a variety of mechanisms. Merlet[8] and Hao and Mccarthy[9] utilized line geometry to identify the uncertainty configurations of parallel manipulators. Basu and Ghosal[10] present a geometric condition for platform-type, multi-loop, mechanisms and parallel manipulators, containing spherical joints on the platform, whose existence ensures singularitie in such mechanisms. The geometric condition is based on the concept of a common tangent. They show that this condition also implies that the determinants of certain matrices, formed by the differentiation of the loop-closure equations, are zero.

The aim of this paper is to present a new method for analyzing and calculating the uncertainty configurations of parallel manipulators. Based on the instantaneous motion of the moving platform, the uncertainty configuration conditions and simplified uncertainty configuration equations are

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https://doi.org/10.1017/S0263574799002088 Published online by Cambridge University Press

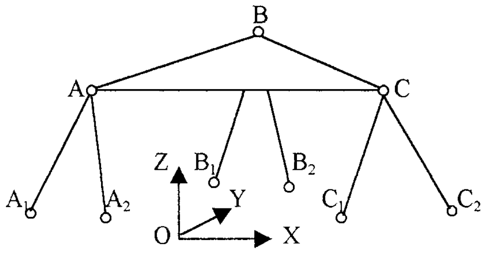

Fig. 1. 6-sps triangular platform parallel manipulator.

obtained for the 6-sps triangular platform and three-branch parallel manipulators.

2. UNCERTAINTY CONFIGURATIONS OF A 6-SPS TRIANGULAR PLATFORM PARALLEL MANIPULATOR

The triangular platform parallel manipulator is illustrated in Figure 1. The coordinate system O-XYZ is fixed to the base frame.

In uncertainty configurations the end effector of a parallel manipulator is instantaneously movable even when all of the actuated joints are locked. The instantaneous motion is restrained by structure constraints, i.e. velocity directions of points A, B and C are perpendicular to planes AA_1A_2, BB_1B_2 and CC_1C_2 respectively. Let $\mathbf{v}_A$, ν_A and Ω be the velocity vector of point A, its norm and the angular velocity vector of the moving platform, respectively.

$$\mathbf{v}_A = \nu_A \mathbf{d}_A \tag{1}$$

$$\Omega = [\omega_x \quad \omega_y \quad \omega_z]^T \tag{2}$$

where

$$\mathbf{d}_A = \frac{\mathbf{A_1A_2} \times \mathbf{A_1A}}{\|\mathbf{A_1A_2} \times \mathbf{A_1A}\|} \tag{3}$$

The velocities of point B and C are given as follows

$$\begin{aligned} \mathbf{v}_B &= \mathbf{v}_A + \Omega \times \mathbf{AB} \\ \mathbf{v}_C &= \mathbf{v}_A + \Omega \times \mathbf{AC} \end{aligned} \tag{4}$$

When the constraints are satisfied, we have the following equations

$$\begin{aligned} \mathbf{v}_B \cdot \mathbf{B}_1\mathbf{B} &= 0 \\ \mathbf{v}_B \cdot \mathbf{B}_1\mathbf{B}_2 &= 0 \\ \mathbf{v}_C \cdot \mathbf{C}_1\mathbf{C} &= 0 \\ \mathbf{v}_C \cdot \mathbf{C}_1\mathbf{C}_2 &= 0 \end{aligned} \tag{5}$$

Substituting (1), (2) and (4) into (5), we obtain

$$\begin{aligned} [\nu_A \mathbf{d_A} + \Omega \times \mathbf{AB}] \cdot \mathbf{B}_1\mathbf{B} &= 0 \\ [\nu_A \mathbf{d}_A + \Omega \times \mathbf{AB}] \cdot \mathbf{B}_1\mathbf{B}_2 &= 0 \\ [\nu_A \mathbf{d}_A + \Omega \times \mathbf{AC}] \cdot \mathbf{C}_1 C &= 0 \\ [\nu_A \mathbf{d}_A + \Omega \times \mathbf{AC}] \cdot \mathbf{C}_1\mathbf{C}_2 &= 0 \end{aligned} \tag{6}$$

Substituting (3) into (6), the following equation can be derived

$$\mathbf{JX} = \mathbf{0} \tag{7}$$

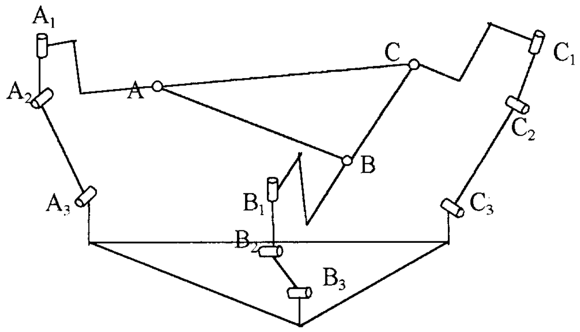

Fig. 2. Three-branch parallel manipulator.

where $\mathbf{0}$ is a zerovector and

$$\mathbf{X} = [\nu_A \quad \omega_x \quad \omega_y \quad \omega_z]^T \tag{8}$$

$$\mathbf{J} = \begin{bmatrix} (\mathbf{A}_1\mathbf{A}_2 \times \mathbf{A}_1\mathbf{A}) \cdot \mathbf{B}_1\mathbf{B} & (\mathbf{AB} \times \mathbf{B}_1\mathbf{B})^T \\ (\mathbf{A}_1\mathbf{A}_2 \times \mathbf{A}_1\mathbf{A}) \cdot \mathbf{B}_1\mathbf{B}_2 & (\mathbf{AB} \times \mathbf{B}_1\mathbf{B}_2)^T \\ (\mathbf{A}_1\mathbf{A}_2 \times \mathbf{A}_1\mathbf{A}) \cdot \mathbf{C}_1\mathbf{C} & (\mathbf{AC} \times \mathbf{C}_1\mathbf{C})^T \\ (\mathbf{A}_1\mathbf{A}_2 \times \mathbf{A}_1\mathbf{A}) \cdot \mathbf{C}_1\mathbf{C}_2 & (\mathbf{AC} \times \mathbf{C}_1\mathbf{C}_2)^T \end{bmatrix} \tag{9}$$

In equation (7), $\mathbf{X}$ is a nonzero vector, than we have

$$\det(\mathbf{J}) = 0 \tag{10}$$

where $\mathbf{J}$ is a 4×4 matrix, and hence the determination of the uncertainty configurations is greatly simplified.

3. UNCERTAINTY CONFIGURATIONS OF A THREE-BRANCH PARALLEL MANIPULATOR

The three-branch parallel manipulator is illustrated in Figure 2. The coordinate system O-XYZ is fixed to the base frame. The end effector is the moving platform ABC; the six input joints are A_1, A_2, A_3, B_2, B_3 and C_3.

In uncertainty configurations the end effector of the manipulator is instantaneously movable even when all of the input joints are locked. The instantaneous motion must satisfy structure constraints, i.e. velocity direction of point B is perpendicular to $\mathbf{B}_2\mathbf{B}$ and $\mathbf{B}_2\mathbf{B}_1$ and velocity direction of point C is perpendicular to $\mathbf{C}_2\mathbf{C}$ respectively. Let Ω be the angular velocity vector of the moving platform, the velocities of points B and C are derived as follows

$$\begin{aligned} \mathbf{v}_B &= \Omega \times \mathbf{AB} \\ \mathbf{v}_C &= \Omega \times \mathbf{AC} \end{aligned} \tag{11}$$

where

$$\Omega = [\omega_x \quad \omega_y \quad \omega_z]^T \tag{12}$$

When the constraints are satisfied, we have the following equations

$$\begin{aligned} \mathbf{v}_B \cdot \mathbf{B}_2\mathbf{B} &= 0 \\ \mathbf{v}_B \cdot \mathbf{B}_2\mathbf{B}_1 &= 0 \\ \mathbf{v}_C \cdot \mathbf{C}_2\mathbf{C} &= 0 \end{aligned} \tag{13}$$

Substituting (11) and (12) into (13), we obtain

$$\mathbf{JX} = 0 \tag{14}$$

where $\mathbf{0}$ is a zerovector and

$$\mathbf{X} = [\omega_x \quad \omega_y \quad \omega_z]^T \tag{15}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{AB} \times \mathbf{B}_2\mathbf{B} \\ \mathbf{AB} \times \mathbf{B}_2\mathbf{B}_1 \\ \mathbf{AC} \times \mathbf{C}_2\mathbf{C} \end{bmatrix} \tag{16}$$

In equation (14), $\mathbf{X}$ is a nonzero vector, then we have

$$\det(\mathbf{J}) = 0 \tag{17}$$

where $\mathbf{J}$ is a 3×3 matrix, and hence the determination of the uncertainty configurations is greatly simplified.

4. CONCLUSIONS

Based on the instantaneous motion of the moving platform, uncertainty configuration conditions and simplified uncertainty configuration equations are obtained. In the 6-sps triangular platform parallel manipulator and three-branch parallel manipulator, the matrices, which are used to determine the uncertainty configurations are 4×4 and 3×3 matrices, respectively, and hence the determinations of the uncertainty configurations, are greatly simplified.

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