PEF, **10** (1): 119–145, January, 2011. © Cambridge University Press 2010
doi:10.1017/S1474747210000077 First published online 14 October 2010

Optimal investment strategies and performance sharing rules for pension schemes with minimum guarantee

JACQUES PÉZIER
ICMA Centre, Henley Business School, University of Reading
(*e-mail*: j.pezier@icmacentre.ac.uk)

JOHANNA SCHELLER
ICMA Centre, Henley Business School, University of Reading
(*e-mail*: j.scheller@icmacentre.ac.uk)

Abstract

There is a potential conflict of interest between a pension fund sponsor and future pensioners when they share unequally in the pension fund performance. Thus, when a scheme offers a yearly guaranteed minimum return to pensioners, as is presently the case with German Pensionskassen, the sponsors cannot afford to invest in risky assets and consequently, pensioners end up with safe but very low expected returns. We examine optimal investment strategies for sponsors under alternative performance sharing rules and seek the rules that are most beneficial to pensioners. We find that the current yearly performance sharing rule imposed on Pensionskassen could be tilted in favor of sponsors without impairing the welfare of pensioners. We also find that the welfare of pensioners would be greatly enhanced if the guaranteed minimum return were applied to the cumulative return since inception of the scheme rather than to yearly returns. The ensuing credit risk taken by pensioners on sponsors could be kept to a minimum by proper regulation; this would induce sponsors to adopt safe constant proportionality portfolio insurance (CPPI) style investment strategies.

1 Introduction: choices in designs of pension schemes

We examine optimal investment performance sharing rules between future pensioners and sponsors/managers[1] of private pension schemes. Ideally, a performance sharing rule should align the interests of the sponsors and those of the pensioners, so that a sponsor seeking an investment strategy that maximizes his own utility also maximizes the utility of the pensioners. In reality, there are a variety of performance sharing

The contents of this paper are presented in good faith, and neither the authors, the ICMA Centre, nor the University, will be held responsible for any losses, financial or otherwise, resulting from actions taken on the basis of its contents. Any persons reading the paper are deemed to have accepted this.

[1] Although sponsors and fund managers are usually separate entities, we assume here that, within the performance sharing rules of a pension scheme, the fund manager acts for the benefit of the sponsor. We therefore refer to both parties as a single party. Performance sharing rules distribute the fund value between this party, referred to as either sponsor or manager, and the pensioners. We are not concerned here with the remuneration of the fund manager by the sponsor.

rules currently in use that leave some conflicts of interests between the two parties and result in equilibrium investment strategies that depend on information available to the two parties and their relative bargaining power.

In many European countries, the UK for instance, private pension schemes fall into two main categories, defined benefits (DB) and defined contributions (DC). A pensioner in a DC scheme bears the full performance risk of the fund into which he and his employer make regular contributions; the fund manager collects a fixed fee. By contrast, a pensioner in a DB company scheme bears no investment performance risk but takes a credit risk on the scheme's sponsoring firm that guarantees the defined benefits. Typically, the defined benefits are a fraction of final salary multiplied by the number of years in employment. Such DB schemes are referred to more specifically as 'final salary schemes'. The benefits from other DB schemes may be based on average salary or other more stable references. The DB scheme sponsor must make up any investment performance deficit with additional contributions but retains any performance surplus (usually by reducing or deferring contributions).[2] This performance risk is all the more visible in the UK and countries using IAS accounting standards now that new regulations[3] aimed at giving credit risk protection to DB pensioners[4] require that a DB fund surplus or deficit be reported by the sponsoring firm as a special item on its balance sheet. These regulations also require that the surplus or deficit be evaluated on a fair value basis as the mark-to-market value of the fund's assets less the present value of the fund's liabilities discounted at 'AA' Libor rates for matching maturities. Thus, the sponsoring firm's financial standing is affected by the volatility of both assets and liabilities in its pension scheme.[5]

An unintended consequence of these new regulations is that many firms found that their DB schemes were becoming too risky and therefore stopped offering them to new employees, closed existing schemes to new contributions, and induced existing members to transfer to alternative schemes.[6] The remaining DB schemes – and they will still represent the majority of company schemes in the UK and other countries for some years to come – are now managed more conservatively. Asset allocations have been tilted away from equities towards long-term bonds to match liabilities more closely and thus reduce surplus volatility. But long-term expected returns are thereby reduced. Thus, the new regulations designed to provide greater security

[2] See Blake (2006: 101–102 and 191–193).

[3] European regulations for private pension funds have multiple aims: harmonizing tax benefits to pensioners, giving greater flexibility in the choice of investment assets (introduction of equities, alternative investments, cross-border investments), and, at the same time, greater transparency (mark-to-market of assets and sometimes of liabilities) and greater protection of the pensioners against mismanagement and default of the fund sponsor on their commitments. In the UK, where the private pension sector is generally more developed than in continental Europe, new regulations and accounting standards (FRS17) became effective on 1 January 2005 after a transition period of several years.

[4] The situation of a DB scheme pensioner is precarious if the sponsoring firm defaults on its obligations. In case of bankruptcy of the sponsor/employer, pensioners may not only lose their jobs but also find that their pension benefits are reduced.

[5] See Pension Act 2004 (UK).

[6] According to a report issued by Aon on the 31 August 2009, a new wave of DB scheme closures can be observed as the cost for the private sector of providing final salary pensions has increased. The top 200 schemes in the UK suffer from a deficit of £78bn, up from £73bn in the month before. See Inman (2009).

(lesser credit risk) to DB pensioners resulted in fewer DB schemes being offered and an increase in the cost of the remaining schemes for the sponsoring firms. They may have also contributed to the decline of the equity markets in the few years leading to their implementation. So, pension regulations not only affect the balance of risks between pensioners and sponsors but also affect investment strategies and, ultimately, global market performance.

DB and DC schemes are but two instances in a wide spectrum of possible risk sharing rules, and neither may be ideal from the point of view of pensioners. Some intermediate performance sharing rules may be more attractive to pensioners because they would align better their interests with those of the sponsor. We analyze a particular class of performance sharing rules that guarantee a minimum return to the pensioners. Some governments allow this type of performance sharing rule and regulate them to ensure that the sponsors are able to meet their commitments. An illustration is provided by Pensionskassen[7] in Germany, which typically are life insurance companies. Pensionskassen guarantee at retirement date the contributions plus interest compounded at a fixed rate, currently set by law to at least 2.25% per year. Every year pensioners accumulate either this guaranteed minimum return on previous contributions or 90% of the fund's annual return, if higher.[8] In future, the guaranteed minimum may be linked to inflation, but for the moment it is simply raised by a flat 1% to 3.25% to compensate for inflation.[9]

The German Pensionskassen scheme is an instance of a generic type of an intermediate performance sharing rule between DB and DC rules, namely a DC scheme with a minimum guarantee for the pensioners and a share of the surplus for the sponsor. We shall not attempt to reflect the detailed workings of Pensionskassen, or limit ourselves to their current investment strategies, which currently use fixed income instruments almost exclusively. Rather, we shall examine how the choice of investment strategy by the fund manager depends on structure and choice of parameters of the performance sharing rule and, consequently, affects the benefits of pensioners. The onus is on the pension fund industry to propose, and on regulators to approve, performance sharing rules that benefit pensioners.

Performance sharing rules for pension schemes and related dynamic investment strategies have been studied in a variety of contexts. In one group of studies, the authors investigate regulatory schemes with defined minimum guarantees. They evaluate the advantages of introducing such guaranteed returns and examine the corresponding optimal investment strategies. Deelstra *et al.* (2003) search for optimal investment strategies for a DC scheme with a fixed minimum guarantee in a continuous time framework. They develop an analytic approach in the context of a complete financial market and illustrate their results with a numerical analysis. Boulier *et al.* (2001) consider a DC scheme with a minimum guaranteed stochastic interest rate. They develop a quantitative method to determine an optimal dynamic allocation among three different asset classes: cash, long-term bonds, and stocks.

[7] For more information on Pensionskassen, see Klatt (2003: 67–70).

[8] See Section 4 (3) Verordnung über die Mindestbeitragsrückerstattung in der Lebensversicherung, introduced 4 April 2008 (Germany).

[9] See Blome *et al.* (2007: 46).

Doskeland and Nordahl (2008) show that annual guarantees have a negative effect on the final wealth of the investor and that a lifetime guarantee is preferable, if a guarantee is required at all. We come to the same conclusion in this paper. Several other studies, such as Brennan (1993), analyze the effect of a minimum guarantee on investor's welfare. He analyzes, inter alia, bonus policies with reversionary bonuses of life insurance companies and shows them to be inefficient. Jensen and Sorensen (2001) demonstrate that a guaranteed minimum return may lead to a significant expected utility loss measured with a CRRA utility function. Consiglio *et al.* (2006) present a model to compare portfolio performance under different policy structures for with-profits funds. Hansen and Miltersen (2002) examine funds offering a guaranteed minimum rate of return and introduce a complex smooth surplus-sharing rule between the investor and the fund that is fair to both.

A second group of studies concentrates on the optimal design of pension fund schemes leading to optimal return profiles for investors. Some studies investigate how a minimum guarantee should be designed optimally. Deelstra *et al.* (2004) study the optimal design of a guarantee in a DC framework. By means of optimal control theory, they maximize the expected utility of the fund manager under the assumption of a power utility function but they do not provide numerical results. Doskeland and Nordahl (2008) also analyze how to optimally design traditional funds and pension funds with a guaranteed minimum rate of return. To find how to increase investors' welfare (measured with a certainty equivalent) they use numerical methods and find that, with a CRRA utility, they cannot justify the existence of a minimum guarantee. Pézier (2008) also finds that, with exponential utility functions, optimal portfolio returns are linear in asset returns; however, he finds that with power utilities, optimal portfolio returns may be convex, linear, or concave in the risky asset return depending on the sensitivity of the local risk aversion coefficient to wealth and the risk/return characteristics of the risky asset.

A third group of studies focuses on the optimal management of pension funds with dynamic investment strategies. A stochastic pension fund model is used by Cairns (1996) who considers a generalized constant proportion portfolio insurance to manage a pension fund in continuous time and gives numerical examples to show that a very good approximation to discrete time models is reached. For the funding level, he also derives the stationary distribution. Martinelli and Milhau (2009) examine dynamic allocation strategies for pension funds to find an integrated model for asset-liability management. They find that the cost of short-term funding constraints is unexpectedly low but the lack of dynamic risk management can be costly. Other studies compare the performance of static and dynamic investment strategies. Bertrand and Prigent (2005) analyze and compare two very common portfolio insurance strategies: an option based portfolio insurance (OBPI) and a constant proportion portfolio insurance (CPPI). They consider various criteria for their comparison and conclude that there is no dominant strategy either state-wisely or stochastically to the first order. Zagst and Kraus (2008) compare the same two portfolio insurance methods but consider their stochastic dominance up to third order criteria. They derive parameter conditions that lead to a second or third order dominance of the CPPI strategy.

In a recent study, Amenc *et al.* (2009) examine the impact of regulatory and institutional frameworks on pension fund management and show the challenges European regulatory developments pose to pension funds. Their two main findings are that if regulators would tolerate short-term risk, the pension system would be more stable and pension funds should further develop internal models to analyze investment strategies. This study reveals growing concerns about the influence of regulations on the management of pension funds, but, as seen in our literature overview, there are still few studies on what could be considered as optimal design of pension plans. Our paper addresses this issue.

The next two sections describe our problem setting. In Section 2, we define four notional assets available for investment, denoted Cash, Bond, Equity, and Market (a constant mix of Bond and Equity) and specify their price dynamics. In Section 3, we describe the yearly and cumulative performance sharing rules we want to compare. We also describe two types of investment strategies: constant value mix (CM) investment strategies and constant proportionality portfolio insurance (CPPI) style strategies. To complete the setting, we specify the expected utility criterion used to assess investment performance from the points of view of pensioners and sponsors. Section 4 presents the results of CM strategies under a yearly performance sharing rule; they reveal a sharp conflict of interest between pensioners and sponsors and poor results for both. Section 5 shows that a cumulative performance sharing rule alleviates this conflict and improves the results for both parties. Sections 6 and 7 revisit the sharing rules in Sections 4 and 5 but with CPPI style investment strategies. The main advantage of CPPI style strategies is to reduce the downside risk for both pensioners and fund managers. Under the cumulative performance sharing rule, there is no conflict of interest between these two parties and CPPI style strategies do not improve on the performance of the optimal CM strategy. But by reducing the risk of a loss for the fund manager, they reduce the credit exposure of the pensioners. We conclude in Section 8 on the advantages of introducing new pension schemes with profit sharing rules intermediate between the traditional DB and DC schemes, provided they are suitably designed and supervised, and on the relative advantages of CPPI strategies over CM strategies in the presence of a liability constraint.

2 Assets and asset price dynamics

We define four notional assets for investing pension contributions. We call them Cash, Bond, Equity, and Market. In the following, we give brief introductions into the theory of the modeling of those assets and present our approaches to it. The overarching assumption in this section is that Brownian motions can be used to model the uncertainty in the dynamics of financial assets. We use Brownian motions in the subsequent subsections for modeling interest rates and equity prices.

Interest rate modeling

Vasicek (1977) proposes a time-homogeneous interest rate model. He assumes that a spot rate, r, follows an Ornstein–Uhlenbeck (O–U) process under a real-world

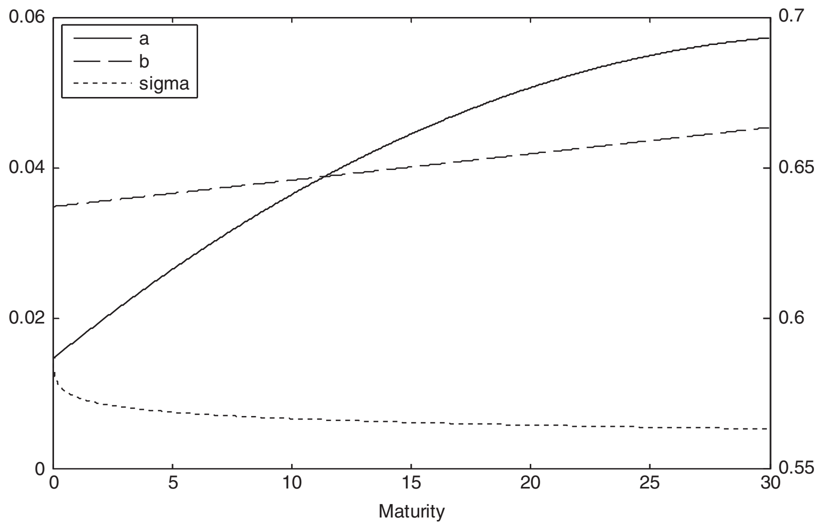

Figure 1. Interpolated Vasicek model parameters
(Right scale for parameter a, left scale for parameters b and σ_r)

probability measure

$$dr = a(b - r)dt + \sigma_r \, dW_r \qquad (1)$$

where r_0, a, b, and σ_r are positive constants and W_r is a Brownian process.

We calibrate the Vasicek model on the yields of the 30-year German government Bund, the 5-year BoBL,[10] and the 6-month Treasury Bill with daily data from January 2000 until June 2008.[11,12] The corresponding calibrated parameters a, b, and σ_r are associated with the modified durations (MD) of these instruments which are, approximately, 15 years, 4.5 years, and 6 months respectively.[13] Matching parameters for intermediate MDs are obtained by interpolation. We also find a correlation of 0.5 between the yields of the long-term bond and the 6-month Bill. These parameters are plotted in Figure 1. Although the O–U process allows negative yields, the probability of a negative yield is negligible with these parameters.[14]

An O–U process for an instantaneous rate could be used as a single factor model for the entire yield curve. However, such modeling would lack realism and flexibility. In our implementation, it would grossly underestimate the volatility of long-term bonds and imply perfect correlation between long-term and short-term yields.

We prefer a more empirical approach to the modeling of returns on a portfolio of bonds. We assume that the fund manager always takes positions in notional par bonds with yield r and modified duration MD so that over a short period dt the return

[10] Bundesobligation issued with five years to maturity.
[11] The data were downloaded from Bloomberg.
[12] January 2000 is when German bonds were converted from Deutsche Mark to Euros and interest rates became subject to ECB rather than Bundesbank policies. Former data would therefore be less relevant.
[13] The model calibration was conducted with maximum likelihood estimation.
[14] All calculations and simulations in this study are conducted with Matlab R2008a.

on a bond position to first order terms is

$$\frac{dP_t}{P_t} = -MD\,dr + r\,dt \qquad (2)$$

where P_t is the bond price at t.

We use this formula to define Cash returns with $MD=1$ and Bond returns with $MD=15$ when the fund's remaining life is greater or equal to 15 years and MD equal to the remaining fund's life during the last 15 years. In other words, the Cash and Bond assets are notional instruments with defined durations such that Cash corresponds to a one-year government Bill and Bond approximates first a 30-year par Bund and then, during the last 15 years of the fund, approximates shorter-term par Bunds or BoBLs. Adjusting the MD of the Bond asset with the time to maturity of the fund reduces the uncertainty in the cumulative performance of the Bond asset.

Equity market modeling

For the Equity asset, we assume that the price S_t follows a geometric Brownian motion

$$dS_t = S_t(\mu\,dt + \sigma_s\,dW_s) \qquad (3)$$

where S_0, μ, and σ_s are positive constants and W_s is a Brownian process. Over a time interval dt, the Equity return is

$$S_t = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma_s^2\right)t + \sigma_s W_s\right). \qquad (4)$$

We calibrate the Equity model parameters with daily data for the German DAX equity index from June 1980 until June 2008.[15,16] The estimated yearly parameters are $\mu=0.0904$ and $\sigma_s=0.2084$.

Finally, we define a single risky asset as a constant value mix of 60 % Bond and 40 % Equity. We call it the 'Market' asset. Indeed, the mix so defined has about the highest Sharpe ratio achievable with the Bond and Equity assets.

Equity was not significantly correlated with Cash and Bond during the period January 2000 to June 2008; we therefore take the Brownians W_s and W_r to be independent. A correlation between Bond and Equity performance could easily be introduced to reflect different circumstances.

Although we use historical data to calibrate our chosen price processes for our chosen assets, it does not necessarily mean that our choice of representative assets is ideal and that the corresponding processes and parameters lead to good forecasts of future returns. However, we found that these assets and processes are sufficiently representative to validate our general conclusions. Sensitivity analyses to the choice of parameters show that realistic variations would not change our conclusions about optimal performance sharing rules and the relative merits of CM and CPPI strategies. We evaluate the performance of a fund with alternative performance sharing rules

[15] The data were downloaded from Bloomberg.
[16] The DAX index is a total return index, including dividends.

and CM and CPPI strategies by simulating 10,000 paths of monthly Cash, Bond, Equity, and Market returns. This number of simulations proved to be sufficient to yield stable results.

3 Performance sharing rules, investment strategies, and preference criteria

We assume that 40 equal yearly contributions are made into a Fund (the 'Fund').[17] It does not matter for this study whether the contributions are made by a cohort of members of the scheme (the 'Pensioner'),[18] their employer, the state, or jointly by several contributors. These contributions are immediately and fully invested in the Fund. The Fund manager (the 'Manager') guarantees a minimum return on the contributions, or Floor, plus participation to the performance of the Fund if that participation exceeds the guaranteed minimum return. In reality, some management fees would probably be charged either directly on the contributions or on the current value of assets under management, or a combination of both.[19] For the sake of simplicity, we do not model any specific fee arrangement but recognize that the Manager will require a satisfactory return for sponsoring a scheme.

The performance sharing rule is characterized by two parameters: α, the guaranteed minimum return on contributions, and β, the percentage participation in the performance of the Fund. We understand that German Pensionskassen apply the sharing rule to yearly returns in the following way. Denote by V_t the value of the Fund at the beginning of year t before the yearly contribution c_t is received, F_t the guaranteed minimum value, or Floor, for the Pensioner, and R_t the share of the Fund value already attributed to the Pensioner. Then, at the beginning of year $t+1$, before a new contribution is made, the value of the Fund attributed to the Pensioner is

$$R_{t+1} = R_t + c_t + \max\{\alpha(F_t + c_t), \beta(V_{t+1} - (V_t + c_t))\} \tag{5}$$

the value attributed to the Manager, M_t, is

$$M_t = V_t - R_t \tag{6}$$

and the new value of the Floor is

$$F_{t+1} = (F_t + c_t)(1+\alpha) \tag{7}$$

with, at inception, $R_1 = V_1 = F_1 = 0$.

The value of the Floor at any time can be calculated from the contributions up to that point. For example, with uniform contributions c per year from $t=1$ to $t=T$, the Floor at $T+1$ is

$$F_{T+1} = c((1+\alpha)^T - 1)\frac{1+\alpha}{\alpha}. \tag{8}$$

[17] Our analysis could easily be extended to irregular contributions and early retirement from the Fund.
[18] For simplicity, we prefer to refer to the members of the scheme as Pensioner rather than 'future Pensioners', members, or investors.
[19] A realistic fee could be about 0.75% per year of assets under management.

On the other hand, the value of the Fund and the shares attributed to the Pensioner and the Manager for $t > 1$ are stochastic variables depending on investment performance.

According to this yearly performance sharing rule (YPSR), the Manager provides the Pensioner with a yearly in-the-money put option and a fraction β of a yearly call of same strike as the put on the performance of the Fund in return for a fraction $(1-\beta)$ of that call. This amounts to a series of cliquet options[20] with variable known strikes on the value of the Fund itself influenced by the choice of investment strategy. It could be unprofitable for the Manager unless he finds a suitably low risk and high yielding asset to invest in.

Alternatively, the same Floor could be guaranteed to the Pensioner with the excess cumulative (rather than yearly) performance of the Fund above the Floor being shared between the Pensioner and the Manager in the proportions $\beta : (1-\beta)$. That is, one could define the value of the Fund attributed to the Pensioner at the beginning of year $t+1$, before a new contribution is made as

$$R_{t+1} = F_{t+1} + \beta \max\{(V_{t+1} - F_{t+1}), 0\} \tag{9}$$

with F_{t+1} defined as before.

Thus, the Manager would provide a single 40-year American style put option with the monotonically increasing Floor defined by (7) rather than a more costly series of yearly put options.[21] This might induce the Manager to seek riskier investments with correspondingly higher expected returns and the pension scheme could, on balance, become more attractive to the Pensioner. We call this sharing rule the cumulative performance sharing rule (CPSR) as opposed to the previous yearly performance sharing rule (YPSR).

Among all imaginable investment strategies using Cash, Bond, Equity, and Market assets, we focus on CM and CPPI style strategies.[22]. Each strategy of either type is characterized by the parameter ω defining the allocation to the risky asset, the rest being allocated to Cash. With CM strategies, ω is the proportion of the total Fund value allocated to the risky asset. The Fund is rebalanced every month to maintain ω constant. Many traditional pension Fund management strategies are approximately CM strategies because pension Fund trustees often stipulate narrow ranges for allocations to major asset classes. CM strategies are contrarian strategies. Every month some of the risky asset is bought (sold) if it has underperformed (outperformed) the return on Cash. We use alternatively Bond, Equity, and Market for the risky asset.

But for the Manager the risk of providing a minimum guaranteed return to the pensioners could be excessive if he were to invest a fixed fraction of the Fund in a risky asset according to a CM strategy. If the risky asset performs badly, the Fund value could approach zero and the Manager would have to pay any minimum

[20] When the regular coupons of a bond are defined as options, these options are traditionally referred to as cliquet options.

[21] Pensioners wishing to collect early the value of their Fund could exercise this put, although one would expect that a secondary market would develop offering better prices, as it has been the case for many other long-term investment products.

[22] On the definition of CM and CPPI strategies, see Perold and Sharpe (1988).

guaranteed return out of his own resources. To reduce this risk, the Manager could find it preferable to manage dynamically his allocation to the risky asset so as to reduce it to zero if the Fund value falls to the minimum guaranteed return. In general, the performance of a dynamic strategy depends on the ability of the Manager to forecast the volatility and other dynamics of the risky asset he chooses. Modeling such capabilities to decide on the best choice of dynamic strategy would be complex and would reflect the timing and asset selection skills of the Manager. Instead, we consider a systematic CPPI style strategy, independent of the forecasting skills of the Manager. A CPPI strategy ensures that a minimum performance, or Floor – typically the minimum guaranteed return – is achieved by allocating to a risky asset a constant proportion (or multiplier) of the excess of the Fund value above the Floor, the excess is called Buffer. By analogy with the CM strategy, we denote the CPPI multiplier ω. We use exclusively Market for the risky asset and rebalance the Fund at monthly intervals. Should the Buffer become nil or negative, the Fund would be entirely invested in the relevant risk-free asset.[23] We implement the CM and the CPPI strategies both for a yearly and a cumulative performance sharing rule.

On a yearly basis (YPSR), the Buffer B_s for the CPPI strategy at time s, $t \leqslant s < t+1$, is

$$B_s = V_s - D_s \tag{10}$$

where, using previous notations, V_s is the value of the Fund at time s and D_s is the discounted value of the minimum due to the Pensioner at year $t+1$ before a new contribution is made, that is

$$D_s = ((F_t + c_t)(1+\alpha) + (R_t - F_t)) \frac{1}{(1+r_t)^{(t+1-s)}} \tag{11}$$

where r_t is the Cash rate at time s. The term in square brackets on the right-hand side of (11) is the new Floor value at year $t+1$ (before a new contribution c_{t+1} is made) plus the excess above the Floor already attributed to the Pensioner at year t.[24] At the end of the year, a new Buffer is calculated and the CPPI process is repeated. The Manager must make up any negative Buffer value to deliver the minimum benefits guaranteed to the Pensioner. If the leverage ω is not large, it is improbable that the Buffer would ever become negative.

On a cumulative basis (CPSR), the Buffer is defined as in (26) but the discount value D_s at time s, $t \leqslant s < t+1$, is defined as

$$D_s = (F_t + c_t)(1+\alpha)^{(T-t)} \frac{1}{(1+r)^{(T-s)}} \tag{12}$$

which is the discounted value of the minimum guaranteed to investors at maturity T from contributions already made at time t. The risk-free rate r is chosen as the

[23] See Black and Perold (1992) and Perold and Sharpe (1988).

[24] In reality, for risk-free discounting and investment during the period (t, $t+1$) we should use the Treasury Bill maturing at t, but for simplicity and with only a small loss of accuracy we use Cash, that is a one-year Treasury Bill.

relevant Bond yield (as defined before). Should the Buffer become nil or negative, the Fund would be fully invested at that yield until maturity.[25]

For the evaluation of the investment strategies, we rely on utility analysis and calculate the certainty equivalents (*CE*) of the CM and CPPI strategies for both the Manager and the Pensioner. Each *CE* should be interpreted as the minimum amount for sure that the beneficiary would be willing to receive at maturity instead of facing an uncertain terminal value. A *CE* is a primitive concept that encapsulates the risk attitude of the beneficiary. To evaluate *CE*s systematically we assume the risk attitudes of the Pensioner and of the Manager can be characterized by exponential utility functions[26]

$$u(V_T) = \lambda\left(1 - \exp\left(-\frac{V_T}{\lambda}\right)\right). \tag{13}$$

The single parameter λ, a local coefficient of risk tolerance, is usually in a range from 10 % to 25 % of the net worth of the beneficiary. For illustration, we assume $\lambda_P = 40$ for the Pensioner and $\lambda_M = 15$ for the Manager.[27]

Thus, the *CE*s are functions of the sharing rule type, its parameters α and β, the coefficients of risk tolerance of the beneficiaries, and the relevant investment strategy parameter ω. We assume that the Manager always adopts the investment strategy, $\omega^*(\alpha, \beta)$ that maximizes his *CE*. It should be of interest to the pension industry and to regulators to find out which choice of sharing rule parameters α and β maximizes the *CE* of the Pensioner in these circumstances. We shall say that a triplet (α, β, ω) defines a plan, a triplet $(\alpha, \beta, \omega^*)$ a Manager's optimal plan, and a triplet $(\alpha^*, \beta^*, \omega^*)$ a Pensioner's optimal plan.

4 Performance with constant mix strategies and yearly performance sharing rule

Figure 2 shows probability densities of the Fund value a year after the last of 40 units of yearly contributions is made and the Manager has implemented a CM strategy either all in Cash, all in Bond, or with $\omega = 30\,\%$ in Market (and 70 % in Cash). With the chosen asset price dynamics, these densities do not have simple analytical

[25] In reality, for risk-free discounting and investment between time s and maturity T, one should use the zero coupon of maturity $(T-s)$. In practice and with only a small loss of accuracy, we use the Bond of relevant modified duration as defined in Section 2.

[26] More general utility functions could be used, in particular utility functions that would justify the Pensioner's desire for a guaranteed minimum value. Exponential utilities can be regarded as a first approximation with constant risk tolerance to more general utility functions with risk tolerance varying as a function of wealth. So far the provision of minimum guarantees for pension funds, or of defined benefits, or the imposition of constraints on the riskiness of the assets has more to do with governments wanting to ensure that pensions are safe than with the recognition of pensioners' risk attitudes.

[27] Various empirical studies, for example Barsky *et al.* (1997), show that there is considerable heterogeneity in the risk attitude of individuals; they find coefficients of risk tolerance ranging from 1/15 to half of net worth. Bodie *et al.* (2009) in their widely used *Investments* textbook use a risk tolerance coefficient of 1/4 in their illustrations. Pensioners making 40 yearly contributions of one unit each might expect a final pension value of around 120 and if that is half of their total net worth then $\lambda_P = 40$ would represent 1/6 of their net worth. We choose $\lambda_M = 15$ for the Manager to correspond to 1/6 of his equity value as well. Indeed, in a steady state situation, with one Pensioner in each age group from 40 years before retirement to retirement, total AUM would be around 1,800; with fees of 1 % of AUM and expenses of 0.5 %, net income would be 9 per year and equity would have to be 90 for an ROE of 10 %. Our main conclusions would not be greatly affected even if these figures were changed by as much as 50 %.

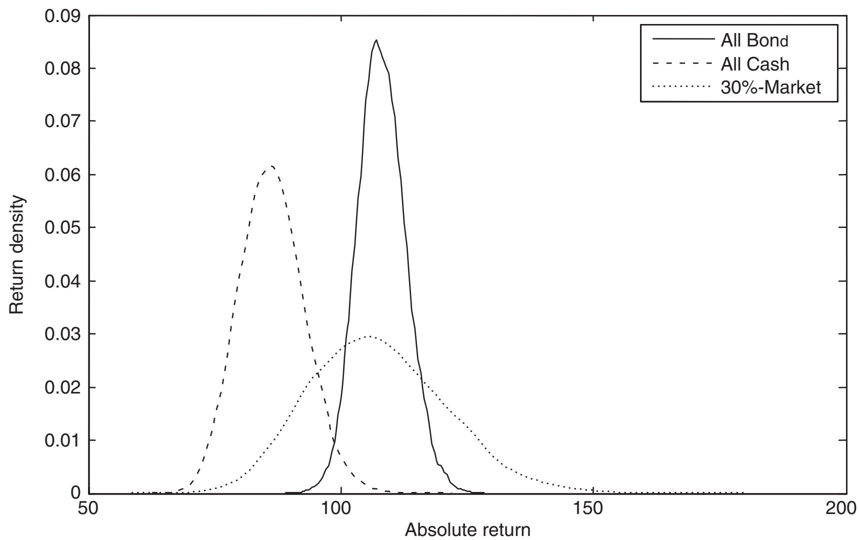

Figure 2. Final fund value distributions under various CM strategies and YPSR

forms; they are obtained by simulating 10,000 asset return scenarios. Their main statistics and those of an All Equity investment are reported in the second column of Table 1 under the heading 'Fund'.

The All Cash investment leads to an approximately normal distribution with low expected return (86.18) and low standard deviation (6.38). Both All Bond and 30%-Market investment strategies yield also approximately normal distributions but with higher expected values (108.22 and 107.38, respectively); the All Bond investment is also less risky than the All Cash investment, whereas the 30%-Market investment is more risky (standard deviation of 4.72, 6.38, and 13.44, respectively). The All Equity investment, on the other hand, yields a highly positively skewed final value distribution with much higher expected value (406.88) and standard deviation (622.37).

How these final Fund values are shared between the Manager and the Pensioner under the YPSR is critical. The key observation is that all of these investment strategies are unattractive to the Manager. There are only three combinations with positive yet insufficient expected returns:[28] the 30%-Market investment strategy with $\alpha = 2.25\%$, the All Cash investment strategy with $\alpha = 2.25\%$, and, marginally positive, the All Cash investment strategy with $\alpha = 3.00\%$. Moreover, these combinations are the least attractive to the Pensioner. It may surprise at first that the All Bond investment strategy, which leads to the least risky final Fund value distribution, is less attractive to Managers than the All Cash and the 30%-Market strategies. The reason is that the yearly returns of the Bond strategy are highly volatile and therefore less attractive for the Manager. Clearly, the interests of the Pensioner and the Manager

[28] As explained in the previous footnote, the Manager would look for an expected profit of around 9 or more.

Table 1. *Final performance with alternative CM strategies under YPSR*

(The final value of the Fund is shared between Pensioner and Manager according to a yearly performance sharing rule with guaranteed return α as shown and $\beta = 90\%$)

		Min return $\alpha = 2.25\%$		Min return $\alpha = 3.00\%$		Min return $\alpha = 3.75\%$	
Statistics	Fund	Pensioner	Manager	Pensioner	Manager	Pensioner	Manager
Floor		65.22		77.66		92.97	
All Cash investment strategy							
Mean	86.18	82.82	3.36	86.17	0.01	95.26	−9.08
Std. dev	6.38	5.12	1.46	3.88	2.83	1.84	4.89
Skewness	0.24	0.44	−0.76	0.80	−0.60	1.67	−0.20
Kurtosis	3.16	3.30	3.88	3.98	3.41	7.70	2.86
All Bond investment strategy							
Mean	108.22	113.81	−10.73	117.99	−14.91	124.56	−21.47
Std. dev	4.72	5.11	4.33	5.15	4.59	5.09	4.86
Skewness	0.17	0.25	−0.31	0.25	−0.26	0.27	−0.19
Kurtosis	3.02	3.10	3.12	3.07	3.06	3.07	3.03
All Equity investment strategy							
Mean	406.88	547.86	−140.98	552.88	−146.00	559.27	−152.39
Std. dev	622.37	652.39	140.73	651.76	140.75	650.93	140.79
Skewness	10.76	8.99	−3.75	9.01	−3.73	9.03	−3.70
Kurtosis	259.49	181.08	53.13	181.63	53.01	182.37	52.85
CM strategy with 70%-Cash/30%-Market							
Mean	107.38	104.86	1.61	108.11	−1.64	113.89	−7.42
Std. dev	13.44	10.69	3.44	9.87	4.41	8.52	5.88
Skewness	0.37	0.51	−0.35	0.60	−0.32	0.76	−0.26
Kurtosis	3.25	3.44	3.09	3.60	3.03	3.94	2.95

are in conflict and Managers cannot extract a sufficient return to sponsor these schemes.

For a more comprehensive comparison of the attractiveness of alternative sharing rules and investment strategies, we calculate their respective *CE*s for the Pensioner and the Manager. The *CE*s of the Pensioner and the Manager with the All Cash, All Bond, and 30%Market CM strategies are shown in Table 2. For each pension plan we calculate the average utility $EU = \frac{1}{n}\sum u$ over $n = 10{,}000$ scenarios and calculate the corresponding *CE*s using the exponential utility function

$$CE = -\lambda \ln\left(1 - \frac{EU}{\lambda}\right). \tag{14}$$

As expected, all *CE*s are lower than the means reported in Table 1 for the corresponding plans because of uncertainties in terminal values. The *CE*s of the Manager are negative except for the All Cash strategy and the 30%-Market strategy when $\alpha = 2.25\%$. They are most negative with the All Bond strategy because of the

Table 2. *Certainty equivalents of CM strategies All Cash, All Bond, and 30 %-Market under YPSR*

(Yearly performance sharing rule with Pensioner participation $\beta = 90\,\%$ in all cases)

	Min return $\alpha = 2.25\,\%$		Min return $\alpha = 3.00\,\%$		Min return $\alpha = 3.75\,\%$	
CM Strategy	Pensioner	Manager	Pensioner	Manager	Pensioner	Manager
All Cash	82.49	3.29	85.99	−0.27	95.22	−9.89
All Bond	113.49	−11.37	117.66	−15.63	124.24	−22.28
30 %-Market	103.49	1.21	106.95	−2.30	113.03	−8.61

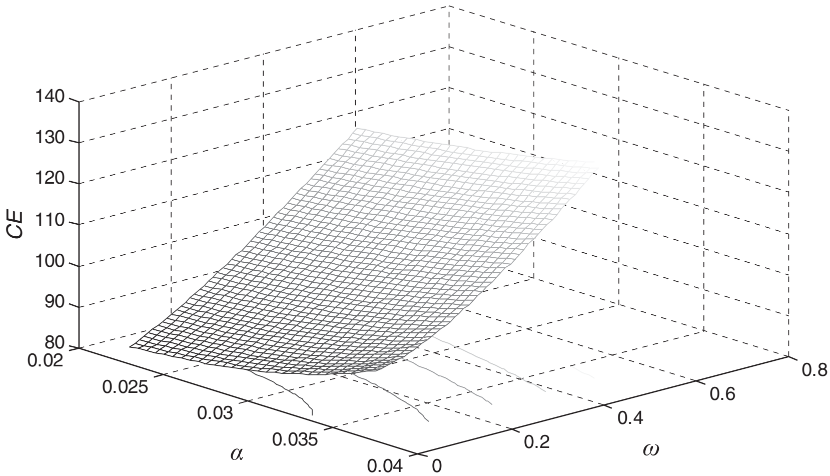

Figure 3. Certainty equivalent of the Pensioner with Cash/Market CM strategies under YPSR

relatively high volatility of Bond on a yearly basis. The *CE*s of the Pensioner are always above the Floor but unattractive with the All Cash strategy compared to the 30 %-Market strategy. They are highest for the All Bond strategy that is the least attractive from the Manager's perspective.

We therefore focus our attention on CM strategies with Cash and Market assets and explore the continuum of *CE* outcomes with Market allocations from 0 % to 40 % and guaranteed minimum rates of return from 2.25 % to 3.75 %. In all cases, the Pensioner participation is 90 %. The results are plotted in Figure 3 for the Pensioner and Figure 4 for the Manager.

The *CE* of the Pensioner is always well above the Floor. It increases moderately with the increase in the guaranteed minimum and it increases markedly with increasing Market allocation, especially if the guaranteed minimum is high. On the other hand, the *CE* of the Manager is positive only for very low Market allocations. It decreases moderately with an increasing guaranteed minimum. It increases slightly

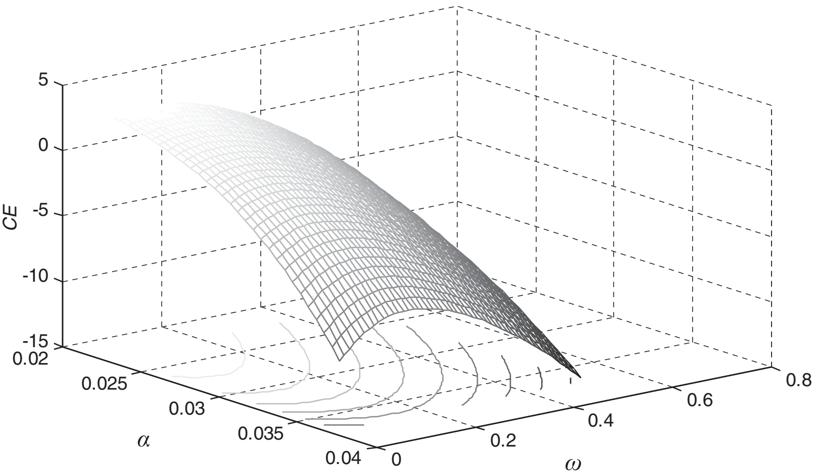

Figure 4. Certainty equivalent of the Manager with Cash/Market CM strategies under YPSR

with small market allocations, reaches a maximum with about 5–10 % allocated to Market, and then decreases more and more rapidly with higher Market allocations. This confirms the obvious conflict of interest between the investment strategy preferences of the Pensioner and the Manager. The former, being protected by the Floor, prefers the higher expected return brought by higher Market allocations, whereas the latter, providing the downside protection, prefers only a small Market allocation.

This conflict of interest could be reduced by modifying the sharing rule in favor of the Manager. This would induce him to adopt an investment strategy with a larger allocation to Market, thus benefiting the Pensioner as well. Figures 5 and 6 show the *CE*s of the Pensioner and the Manager as a function of the Pensioner participation, β, when the guaranteed minimum rate of return is $\alpha = 2.25\,\%$. The Pensioner always prefers both a greater participation, and a greater Market allocation as shown by the constant Market allocation curves in Figure 5. The Manager, on the other hand, prefers a lower participation going to the Pensioner, but his optimal level of Market allocation varies as a function of Pensioner participation. We see in Figure 6 that the Manager's optimal Market allocation is near zero when Pensioner participation is near 100 %, but increases to near 40 % when Pensioner participation decreases from 100 % to 55 %. In an oligopoly where the Manager would be able to optimize his choice of investment strategy for his own benefit and with little regards for the Pensioner, the Pensioner's *CE* would evolve as shown by the Manager's optimal allocation curve in Figure 5. It remains very flat with highs around 88 for $\beta = 100\,\%$ and $\beta = 55\,\%$ and a low around 85 for $\beta = 80\,\%$. An adequate performance for the Manager would be 10 or above and would be attainable only if Pensioner participation were no more than 75 %. On the other hand, if the Manager had to compete with others for the custom of the Pensioner, he would be able to reduce their upside participation down to 75 %, necessary for a viable business, but Pensioners could force the Manager to allocate 30 % to the Market asset because it would cost

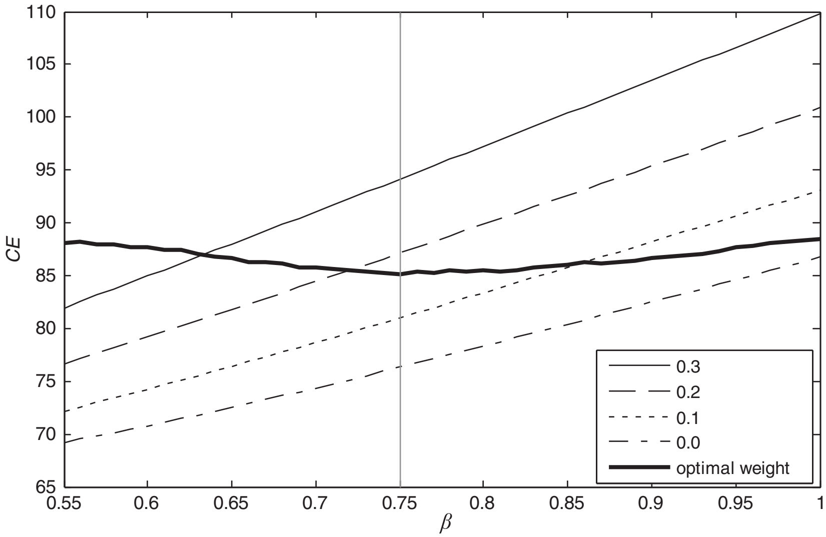

Figure 5. Certainty equivalent of the Pensioner as a function of the Pensioner's participation and the Market asset allocation under YPSR

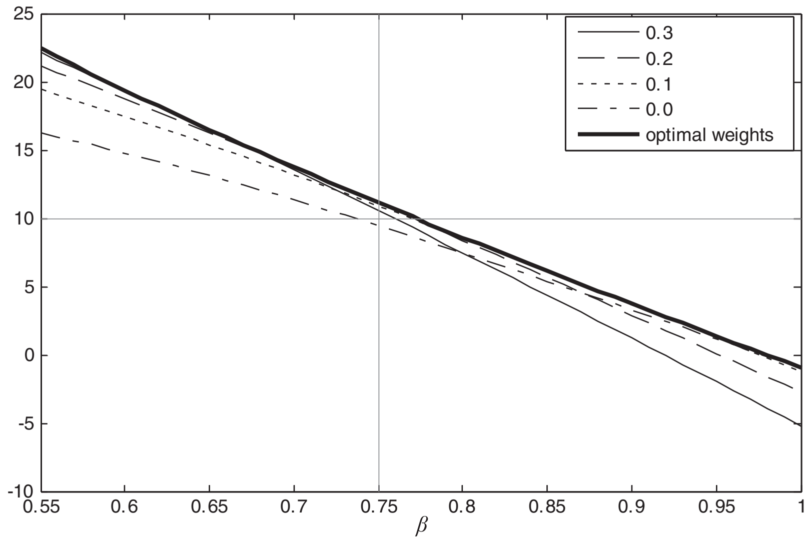

Figure 6. Certainty equivalent of the Manager as a function of the Pensioner's participation and the market asset participation under YPSR

Manager little to do so. Thus, the *CE*s would be at least 10 for the Manager and close to 95 for the Pensioner. That would be markedly better than under the current rule for Pensionskassen with a Pensioner participation of 90 %, which makes the business unviable for the Manager and still poor value (*CE* around 87) for the Pensioner. It remains that a Pensioner's *CE* of about 95 under the YPSR ($\alpha = 2.25\,\%$, $\beta = 75\,\%$,

Table 3. *Certainty equivalents of CM strategies All Cash, All Bond, and 30%-Market under CPSR*

(Cumulative performance sharing rule with Pensioner participation $\beta = 90\%$ in all cases)

	Min return $\alpha = 2.25\%$		Min return $\alpha = 3.00\%$		Min return $\alpha = 3.75\%$	
CM strategy	Pensioner	Manager	Pensioner	Manager	Pensioner	Manager
All Cash	83.67	2.08	84.92	0.84	86.45	−0.69
All Bond	103.70	4.29	104.94	3.05	106.47	1.52
30%-Market	101.40	4.16	102.65	2.91	104.18	1.38
All Market	136.64	10.23	137.89	8.99	139.42	7.46

$\omega = 30\%$) is still poor compared to what can be achieved with a CPSR rule, as we shall see next.

5 Performance with constant mix strategies and cumulative performance sharing rule

We found in Section 4 that the YPSR produces final value distributions for the Pensioner that are well above the guaranteed minimum value, or Floor. In fact, due to frequent participations in the yearly Fund performance, these distributions for All Cash, All Bond, and 30%-Market CM strategies are close to normality. If instead, one applies a CPSR with same Floor and same Pensioner participation, the corresponding final value distributions for the Pensioner are truncated distributions easily obtained from the final Fund value distributions in Figure 2. The Pensioner obtains 90% (or generally the participation β) of the final value above the Floor, if any, or the Floor when the final value of the Fund does not reach the Floor. The Manager obtains 10% (or generally $(1-\beta)$) of the excess of the final value of the Fund above the Floor, if any, or pays out to the Pensioner the shortfall when the final value of the Fund does not reach the Floor. These distributions are generally more attractive to the Manager and less so to the Pensioner than the distributions with matching investment strategies under the YPSR as we can see from Table 3, which should be compared with Table 2 for YPSR. We also add in Table 3 an All Market investment strategy, which is now attractive for the Manager.

We observe only small differences between YPSR and CPSR for an All Cash investment strategy, especially with low levels of α, because it is almost guaranteed to produce low volatility results greater than the guaranteed minimum return. On the other hand, the All Bond and the 30%-Market strategies become more attractive to the Manager and less so to the Pensioner. That is because the Bond asset has low final uncertainty (although it has relatively high yearly volatility). But the best CM strategy among the four displayed in Table 4 is clearly the All Market strategy. It is best both for the Pensioner and the Manager. It is also the only strategy that may appear sufficiently attractive for a Manager to sponsor such a scheme. The final Fund value distribution is plotted in Figure 7.

Table 4. *Final performance with CPPI strategy* ($\omega = 2$) *under YPSR*

(The final value of the Fund is shared between Pensioners and Manager according to a yearly performance sharing rule with guaranteed return α as shown and $\beta = 90\,\%$)

	Min return $\alpha = 2.25\,\%$			Min return $\alpha = 3.00\,\%$			Min return $\alpha = 3.75\,\%$		
Statistics	Fund	Pen.	Man.	Fund	Pen.	Man.	Fund	Pen.	Man.
Floor		65.22			77.66			92.97	
Mean	87.68	84.08	3.54	86.52	86.43	0.08	86.22	95.28	−9.06
Std. dev.	7.08	5.71	1.51	6.64	4.10	2.86	6.41	1.86	4.89
Skewness	0.27	0.44	−0.70	0.31	0.86	−0.58	0.25	1.68	−0.20
Kurtosis	3.06	3.18	3.77	3.22	4.05	3.39	3.18	7.75	2.87

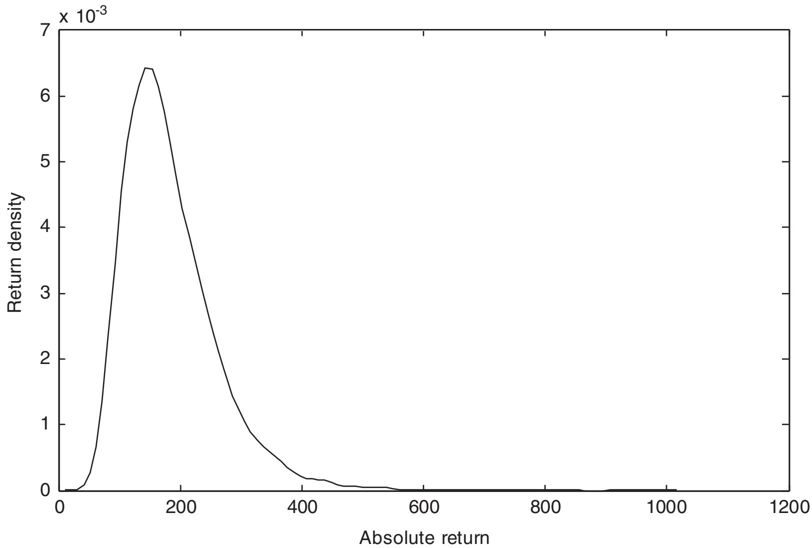

Figure 7. Final Fund value distribution with all market strategy under CPSR

The only drawback from the Pensioner's perspective is that under the All Market strategy there is a risk that the final Fund value does not exceed the Floor and therefore the Pensioner takes a credit risk on the Manager.

Figures 8 and 9 show the *CE*s of the Pensioner and the Manager for CM strategies with Cash and Market assets with Market allocations from 0 % to 100 % and guaranteed minimum rate of return on contributions from 2.25 % to 3.75 %. The Pensioner participation is still 90 %. These figures with the CPSR correspond to Figures 3 and 4 with the YPSR.

As before, the CE of the Pensioner increases moderately with the increase in the guaranteed minimum and it increases markedly with increasing Market allocation.

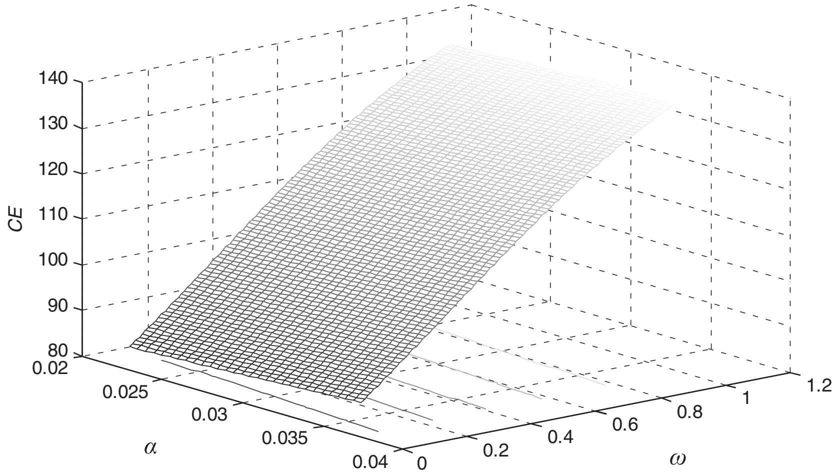

Figure 8. Certainty equivalent of the Pensioner with Cash/Market CM strategies under CPSR

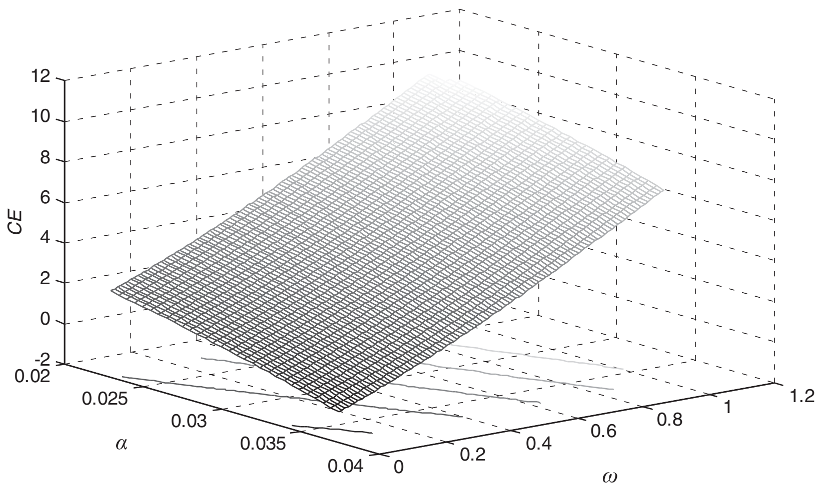

Figure 9. Certainty equivalent of the Manager with Cash/Market CM strategies under CPSR

But now the CE of the Manager also increases with increasing market allocations in the range $\omega = 0\%$ to 100%. It decreases moderately with an increasing guaranteed minimum, but remains mostly positive. Therefore, there is still some conflict of interest between the Pensioner and the Manager on the guaranteed minimum return, but both prefer higher Market allocations. Maximum *CE*s would be reached for allocations to Market beyond 100%, if the Manager were allowed to cash-leverage his investments. We conclude that under CPSR the All Market investment strategy, if cash-leverage is not allowed, offers the best achievable *CE*s for both the Pensioner and the Manager. The *CE*s of the Pensioner (above 136) are greatly superior to those

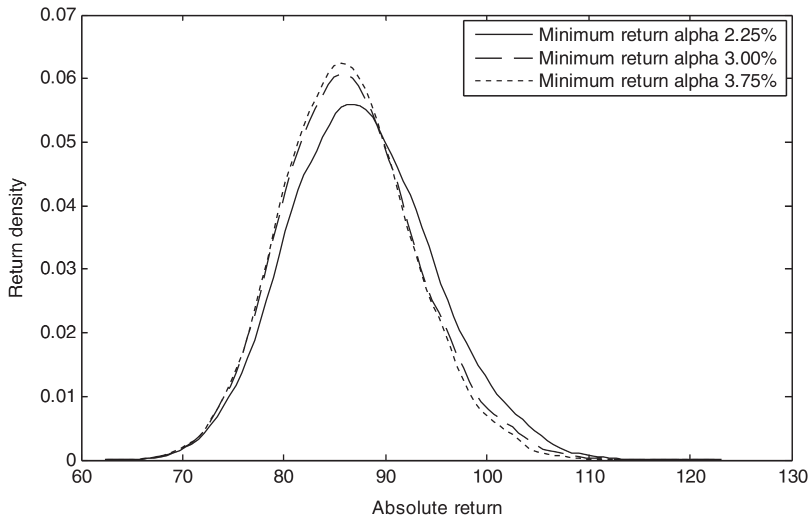

Figure 10. Final fund value distributions with CPPI strategy ($\omega = 2$) under YPSR

achievable with a YPSR. The CE of the Manager is above 10 when $\alpha = 2.25\,\%$ and could be kept above 10 for larger minimum guarantees if the upside participation were reduced slightly below 90 %.

6 Performance with CPPI strategies and yearly performance sharing rule

We now consider CPPI style investment strategies using the Cash and Market assets only. Specifically, the amount invested in the Market asset every month is a constant multiplier, ω, of the Buffer, the difference between the value of the Fund and the discounted minimum value of the benefits already promised to the Pensioner. The rest of the Fund is invested in Cash. This CPPI strategy is first used with the YPSR. The distributions of final Fund values are plotted in Figure 10 for a leverage $\omega = 2$. There is one distribution for each of the three guaranteed minimum returns on contribution, $\alpha = 2.25\,\%$, 3.00 %, and 3.75 %, because the investment strategy now depends on the level of the Floor. The expected final values are seen to decrease slightly with increasing Floors and so do the standard deviations. This is confirmed by the statistics in Table 4 in the columns headed 'Fund'. Clearly, the higher the Floor, the smaller the Buffer, and the lower the average investment in the Market asset. But the Buffer remains always very small so that all three distributions have approximately the same shape and statistics as the All Cash distribution examined in Section 4 (Figure 2 and Table 1); the expected values and standard deviations are only marginally greater than with the All Cash strategy.

The same conclusions can be drawn for the shares of the final Fund value attributed to the Pensioner and the Manager when the Manager's participation to the upside is $\beta = 90\,\%$. The corresponding statistics are also reported in Table 4 (columns headed 'Pens.' and 'Man.', respectively). Naturally, with increasing Floor levels the

Table 5. *Certainty equivalents of CPPI strategies under YPSR*

(Pensioners yearly surplus participation $\beta = 90\,\%$ in all cases)

	Min return $\alpha = 2.25\,\%$		Min return $\alpha = 3.00\,\%$		Min return $\alpha = 3.75\,\%$	
Multiplier	Pensioner	Manager	Pensioner	Manager	Pensioner	Manager
$\omega = 0$	82.49	3.29	85.99	−0.27	95.22	−9.89
$\omega = 5$	85.71	3.66	86.65	−0.14	95.27	−9.86
$\omega = 10$	88.84	3.73	87.40	−0.14	95.35	−9.84

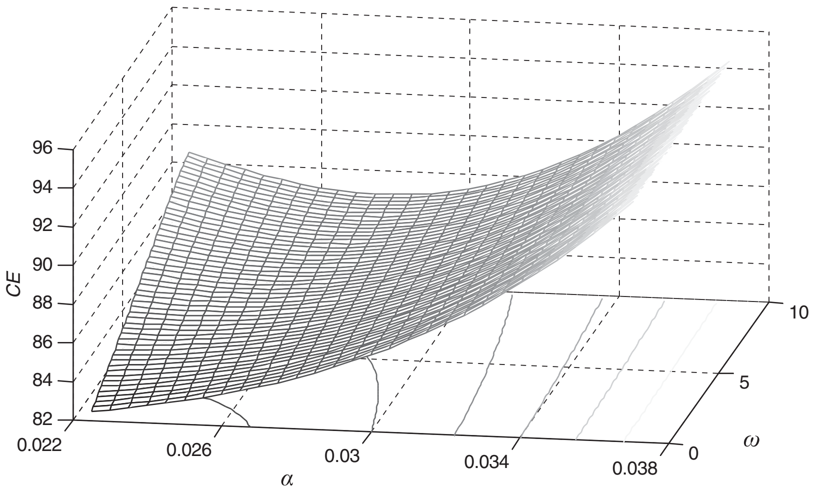

Figure 11. Certainty equivalent of the Pensioner with CPPI strategies under YPSR

expected values for the Pensioner increase and those for the Manager decrease. The standard deviations of these distributions are small so that the corresponding *CE*s, shown in Table 5, are barely below the relevant expected values. For ease of comparison, Table 5 repeats the results of the All Cash investment strategy of Section 4, which corresponds to a leverage $\omega = 0$, and adds the cases $\omega = 5$ and $\omega = 10$. As ω increases, both Pensioner's and Manager's *CE*s increase marginally; there is no conflict of interest. However, there is conflict of interest on the choice of minimum guarantee and, in all cases, the *CE*s of the Manager are too low to make the business viable under the YPSR.

A more complete picture of *CE*s for both Pensioner and Manager as a function of the minimum guaranteed rate α (from 2.25 % to 3.75 %) and the leverage ω (from 0 to 10) is given in Figures 11 and 12. The Pensioners surplus participation is 90 % in all cases. These figures should be compared to Figures 3 and 4 with CM strategies.

Because there is little if any conflict of interest between Manager and Pensioner about seeking a high investment leverage, there is little room for improving the

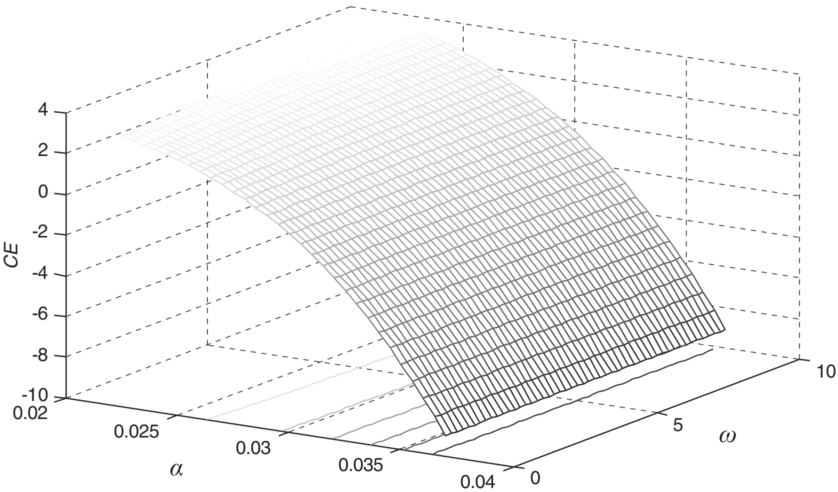

Figure 12. Certainty equivalent of the Manager with CPPI strategies under YPSR

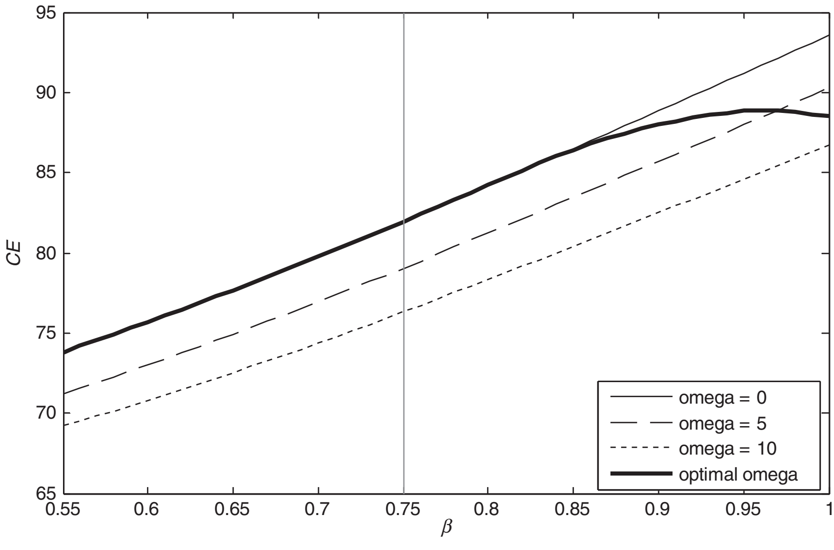

Figure 13. Certainty equivalent of the Pensioner with CPPI strategies as a function of the Pensioner's participation and the leverage under YPSR

Pensioner's *CE* by modifying the performance sharing rule in favor of the Manager so that the Manager would adopt a more profitable investment strategy. Figures 13 and 14 show the *CE*s of the Pensioner and the Manager under a CPPI strategy as a function of the Pensioner's participation, β, when the guaranteed minimum rate of return is $\alpha = 2.25\%$. In an oligopoly, the Manager would maximize his own *CE* by choosing the optimal ω. That optimal leverage is in the region of $\omega = 10$ when β is less

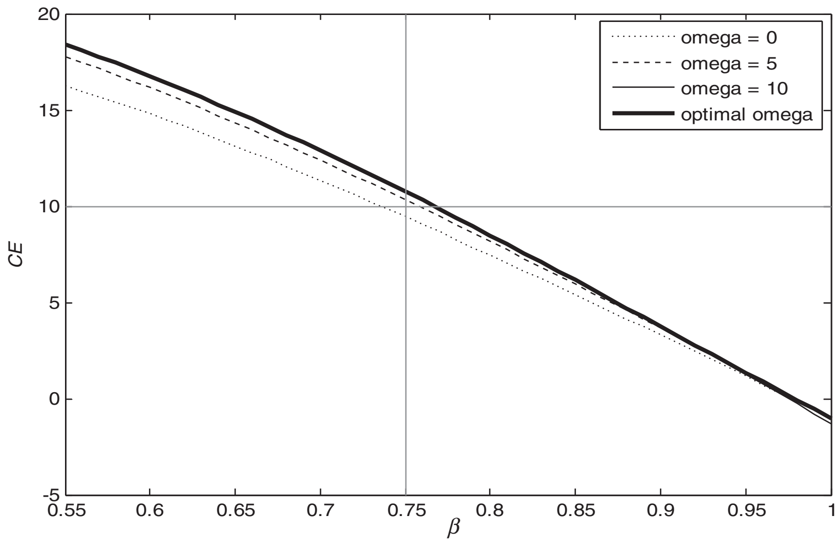

Figure 14. Certainty equivalent of the Manager with CPPI strategies as a function of the Pensioner's participation and the leverage under YPSR

than 85 % but then decreases to reach 5 when β reaches 97 % and drops below 5 when β reaches 100 %. Correspondingly, the Pensioner's *CE* reaches a maximum of about 89 when $\beta = 96\,\%$. But again, to make the plan viable for the Manager, say with a *CE* greater than 10, pension authorities should reduce the Pensioner's upside participation in the Fund performance to $\beta = 75\,\%$ or less, in which case the *CE* of the Pensioner under a CPPI strategy becomes inferior to the CE under a CM strategy with the Market asset. In a competitive market, the Pensioner might push the Manager to increase the multiplier ω to increase his CE. We conclude that if pension authorities want to design viable pension schemes with guaranteed minimum return on contributions and a YPSR, they should keep the guaranteed minimum return low (well below risk-free rates), and, assuming our parameters, offer an upside performance participation to the Pensioner of no more than 75 %, then the Manager is likely to adopt a CM investment strategy with around 30 % invested in the Market asset (the risky asset mix with maximum Sharpe ratio) and 70 % in Cash as we found in Section 4. CPPI strategies are not preferable under the YPSR.

7 Performance with CPPI strategies and cumulative performance sharing rule

We now investigate the use of CPPI style investment strategies with the CPSR using exclusively the Cash and Market assets as before. The *CE*s of the Pensioner and the Manager are shown in Table 6 for a selection of leverage factors from $\omega = 0$ to $\omega = 10$. In all cases, the Pensioner participation is $\beta = 90\,\%$. Table 6 should be compared to Table 3 for CM strategies with CPSR. When $\omega = 0$, we have an All Cash strategy showing the same results as in Table 3 for the corresponding All Cash investment. As we observed before, this plan is unattractive for the Manager, especially when the guaranteed minimum return is high.

Table 6. *Certainty equivalents of CPPI strategies under CPSR*

(CPSR with Pensioner participation $\beta=90\,\%$ in all cases)

	Min return $\alpha=2.25\,\%$		Min return $\alpha=3.00\,\%$		Min return $\alpha=3.75\,\%$	
CM strategy	Pensioner	Manager	Pensioner	Manager	Pensioner	Manager
$\omega=0$	83.67	2.08	84.92	0.84	86.45	−0.69
$\omega=2$	129.09	9.32	117.16	5.84	91.65	0.02
$\omega=4$	134.89	10.12	128.42	8.08	98.48	1.49
$\omega=6$	135.34	10.16	130.02	8.34	100.87	2.18
$\omega=8$	135.44	10.17	130.37	8.41	101.71	2.46
$\omega=10$	135.48	10.17	130.40	8.43	102.30	2.63

As ω increases, the average investment in the Market asset increases towards a 100 % self-imposed no-cash-leverage ceiling; both the Manager's and the Pensioner's *CE*s increase rapidly at first, especially when the guaranteed minimum return is low, and then reach a plateau. The reason is simple. At first, the size of the Buffer is limited by the value of the Fund; then it is limited by the no-cash leveraging constraint. When ω increases, the average amount invested in the Market asset increases. When $\omega=1$, the average investment in the Market asset is about 43 % of the Fund value, whereas when $\omega=2$, it reaches 87 %. Since we found in Section 5 that the All Market investment strategy is superior to all other CM strategies with lesser allocations to the Market asset, we can therefore expect the CPPI strategies with high ω to approach but never exceed the performance of the All Market strategy. With $\alpha=2.25\,\%$ and $\omega=10$, the *CE*s of the Pensioner and the Manager are 135.48 and 10.17 respectively, which comes close to the corresponding *CE*s of 136.64 and 10.23 with the All Market investment strategy (see bottom row of Table 3). With larger guaranteed minimum returns on contribution, the Buffers are smaller and the CPPI strategies run more distant seconds to the All Market strategy. The *CE*s of the Manager drop below 10 but could be restored at that level by reducing the Pensioner's upside participation below 90 %.

A fuller picture of *CE* variations as a function of the leverage ω from 0 to 20 and α from 2.25 % to 3.75 % is given in Figure 15 for the Pensioner and Figure 16 for the Manager. There is little gain to be found by increasing ω beyond 5. At that leverage level, both the Pensioner and the Manager lose if the guaranteed minimum return on contributions is increased because it reduces the Buffer. Only at values of ω well below 1 does the raising of the guaranteed minimum return increase slightly the Pensioner's *CE*.

So, under CPSR, a CPPI strategy with a leverage $\omega=5$ performs almost as well as the All Market strategy. The main advantage of the CPPI strategy is that the probability of the Buffer becoming negative is negligible; therefore, the Pensioner takes no material credit risk on the Manager compared to the small credit risk he takes under the All Market investment strategy.

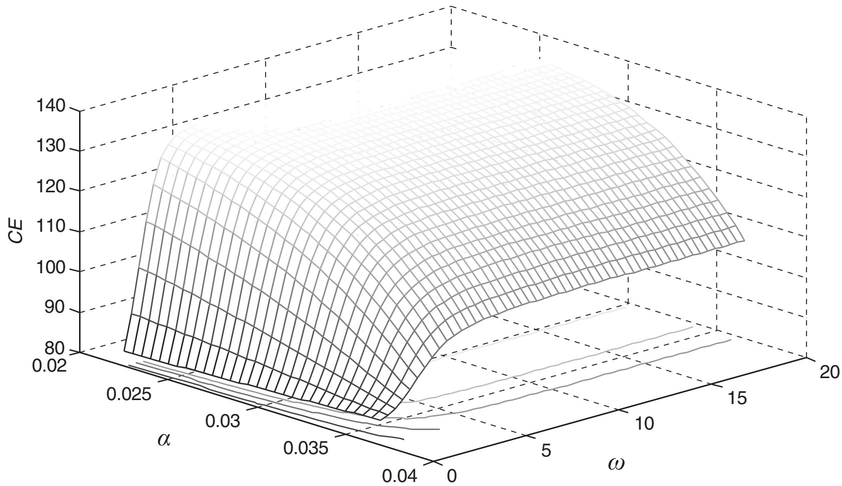

Figure 15. Certainty equivalent of the Pensioner with CPPI strategies under CPSR

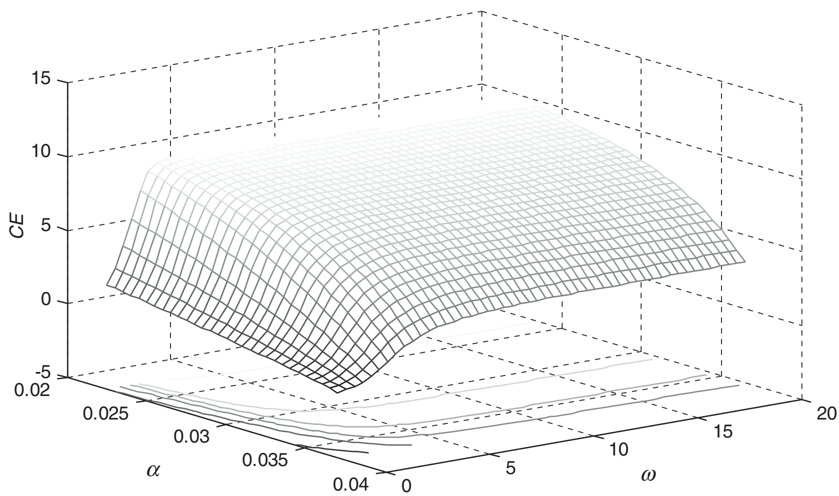

Figure 16. Certainty equivalent of the Manager with CPPI strategies under CPSR

8 Conclusions

There is room for new types of investment performance sharing rules between pensioners and scheme sponsors. In many countries, DB pension schemes that still represent the bulk of employers' schemes are now wound down because the risks have become too high for the sponsors. Individuals are still encouraged by governments, in the form of various tax incentives, to make provisions for their retirement, but with DC schemes they bear the full investment risk and may not be well equipped to make the appropriate investment choices. In these circumstances, it would seem appropriate for the pension industry to suggest, and for pension authorities to approve, intermediate schemes between defined benefits and defined contributions that would promise some degree of security to future pensioners together with attractive expected returns well above the risk-free rate. But poorly designed performance sharing rules between pensioners and sponsors can lead to conflicts of interest and poor performance for both; the design of adequate sharing rules necessitates an understanding of the associated optimal investment strategies.

To explore suitable performance sharing rules we consider a setting similar to that of the Pensionskassen in Germany. They offer a guaranteed minimum return on contributions and a participation to the upside performance of the fund above the guaranteed minimum return on a yearly basis. But we show that this type of scheme can be unattractive to both investors and sponsors. We find that a yearly performance sharing rule puts in direct conflict the interests of the pensioners and those of the fund managers. They would need to be revised to give managers a higher upside performance participation (at least 25% instead of the current 10%) as well as to keep the guaranteed minimum return well below the yearly risk-free rate. Even so, this type of yearly performance sharing rule would still favor too much investment in low risk assets and therefore would still offer relatively poor returns to the pensioners. The returns for the pensioners would be only marginally improved by a more adapted dynamic, CPPI style, investment strategy.

On the other hand, the returns for pensioners would be much more attractive if the performance sharing rule and the guaranteed minimum return on contributions were applied to the cumulative performance of the fund at maturity instead of yearly. A cumulative performance sharing rule gives greater incentive to the fund manager to invest in an asset mix offering a large Sharpe ratio and this improves the welfare of the pensioners as well. The main drawback of a cumulative rule is that it would expose pensioners to credit risks on fund managers. But this research shows that credit risks can be reduced to negligible proportions without significant loss of performance if managers implement dynamic, CPPI style, investment strategies. Regulators can ensure low credit risks by setting limits or penalties on the downside risks of managers.

The design of performance sharing rules between pensioners and fund managers is complex. Rules designed to enhance the welfare of pensioners may have the opposite effect by forcing fund managers to adopt less beneficial investment strategies for pensioners. Pension Fund authorities should consider fostering schemes that guarantee minimum returns for pensioners and limit their exposures to defaults of managers, but they should primarily look for schemes that limit conflicts of interest between pensioners and sponsors and be very careful in setting precise parameters, especially in the current turbulent financial markets. If significant conflicts of interest can be avoided, competition between fund managers and the desire to satisfy the specific needs of special groups of investors should be sufficient to determine the best parameters for guaranteed minimum return pension schemes within ranges approved by the authorities.

References

Amenc, N., Martellini, L., and Sender, S. (2009) Impact of regulations on the ALM of European pension funds. EDHEC Risk and Asset Management Research Centre.

Barsky, R. B., Juster, F. T., Kimball, M. S., and Shapiro, M. D. (1997) Preference parameters and behavioural heterogeneity: an experimental approach in the health and retirement study. *Quarterly Journal of Economics*, **112**: 537–579.

Bertrand, P. and Prigent, J. L. (2005) Portfolio insurance strategies: OBPI versus CPPI. *Finance*, **26**: 5–32.

Black, F. and Perold, A. F. (1992) Theory of constant proportion portfolio insurance. *Journal of Economic Dynamics and Control*, **16**(3–4): 403–426.

Blake, D. (2006) *Pension Finance*. Chichester: John Wiley & Sons Ltd.

Blome, S., Fachinger, K., Franzen, D., Scheuenstuhl, G., and Yermo, J. (2007) Pension fund regulation and risk management: results from an ALM optimisation exercise. OECD Working Papers on Insurance and Private Pensions 8, OECD.

Boulier, J. F., Huang, S. J., and Taillard, G. (2001) Optimal management under stochastic interest rates: the case of a protected pension fund. *Insurance: Mathematics and Economics*, **28**: 173–189.

Brennan, M. J. (1993) Aspects of insurance, intermediation and finance. *Geneva Papers on Risk and Insurance Theory*, **18**: 7–30.

Bodie, Z., Kane, A., and Marcus, A. J. (2009) *Investments*. Eighth Edition, New York: McGraw Hill.

Cairns, A. J. G. (1996) Continuous-time pension-fund modeling. *Proceedings of the 6th AFIR International Colloquium*, **1**: 609–624.

Consiglio, A., Saunders, D., and Zenios, S. A. (2006) Asset and liability management for insurance products with minimum guarantees: the UK case. *Journal of Banking and Finance*, **30**: 645–667.

Deelstra, G., Grasselli, M., and Koehl, P. F. (2003) Optimal investment strategies in the presence of a minimum guarantee. *Insurance: Mathematics and Economics*, **33**: 189–207.

Deelstra, G., Grasselli, M., and Koehl, P. F. (2004) Optimal design of the guarantee for defined contribution funds. *Journal of Economic Dynamics and Control*, **28**: 2239–2260.

Doskeland, T. and Nordahl, A. (2008) Optimal pension insurance design. *Journal of Banking and Finance*, **32**: 382–392.

Hansen, M. and Miltersen, K. R. (2002) Minimum rate of return guarantees: the Danish case. *Scandinavian Actuarial Journal*, **4**: 230–318.

Inman, P. (2009) Cost of final salary pension schemes tops £1tn. *The Guardian*, 1 September 2009.

Jensen, B. A. and Sorensen, C. (2001) Paying for minimum interest rate guarantees – who should compensate who? *European Financial Management*, **7**: 183–211.

Klatt, M. (2003) *Die Pensionskasse in der betrieblichen Altersvorsorge*. Karlsruhe: Verlag Versicherungswirtschaft GmbH.

Martellini, L. and Milhau, V. (2009) Measuring the benefits of dynamic asset allocation strategies in the presence of liability constraints. EDHEC Risk and Asset Management Research Centre.

Perold, A. F. and Sharpe, W. F. (1988) Dynamic strategies for asset allocation. *Financial Analysts Journal*, January–February: 16–27.

Pézier, J. (2008) Maximum certain equivalent excess returns and equivalent preference criteria – part ii: approximations and applications. DP2008-08, ICMA Centre, Henley Business School, University of Reading.

Vasicek, O. (1977) An equilibrium characterization of the term structure. *Journal of Financial Economics*, **5**: 177–188.

Zagst, R. and Kraus, J. (2008) Stochastic dominance of portfolio insurance strategies. HVB-Institute for Mathematical Finance, Technical University of Munich.