Linear algebra II by Frederick P. Greenleaf and Sophie Marques, pp. 312, £58.95 (paper), ISBN 978-1-47045-425-8, American Mathematical Society (2020)

This book (hereafter denoted *LA II*) is the second in a two-volume set of graduate-level linear algebra textbooks. The first volume [1] (*LA I*) started at the beginning with the definition and basic properties of a vector space (over an arbitrary field) and went on from there to discuss linear transformations, duality, determinants, diagonalization and inner product spaces (including some spectral theory of operators on such spaces).

The most notable omissions in *LA I* were canonical forms and general bilinear forms, and *LA II* wastes no time in getting to these. There are five chapters in this book; the first two deal with the Jordan form and its applications (to differential equations, and to operators on real spaces, using complexification as a tool); the third deals with bilinear, quadratic, and multilinear forms, including such topics as form-preserving groups (including the Lorentz group), tensors and multilinear forms, and Sylvester's Law of Inertia. The contents of *LA I* and these three chapters cover, quite well, many of the basic topics in a standard linear algebra graduate course. There are, however, some notable omissions, including other canonical forms (e.g. Smith or rational) and some basic topics in numerical linear algebra, such as eigenvalue location theorems.

The selection and arrangement of material might be seen by some instructors as a bit problematic. Many universities offer a one-semester course in graduate linear algebra, in which the Jordan form is invariably covered. Using these books would require the students to buy two texts for the course; it might have been better if the authors had omitted, or covered much more rapidly, the really elementary aspects of volume I, so as to allow all of the material described above to be covered in one volume. The second volume could then have been devoted to more advanced material and to applications.

After the first three chapters of *LA II*, the book changes tone. Each of the remaining two chapters is devoted to an area of mathematics that uses linear algebra, and develops these subjects fairly fully. Chapter 4 looks at differential geometry. The concept of a differentiable manifold is introduced and it is explained how linear algebra (specifically, multilinear forms) plays a role in this subject. Chapter 5 is an introduction to Lie theory via the study of matrix Lie groups, though the coverage is not limited to matrix Lie groups; general Lie groups are looked at as well. Because of this, chapter 5 does rely somewhat on the development of differential geometry in the first section of chapter 4, but the necessary background from that chapter is repeated, so as to make this chapter reasonably independent of the one preceding it.

These two final chapters struck me as somewhat less successful than the others. For one thing, they seemed rather out of place in a textbook on linear algebra; indeed, both chapters sometimes read almost like chapters in an analysis text. Sometimes this seemed to be a little awkward, as in the definition of a manifold in chapter 4: the authors define this to be a 'set of points' M on which open charts are defined satisfying the usual overlap conditions. But of course the very concept of 'open' is unclear unless we know what kind of structure already exists on M. Is M intended to be a subset of a Euclidean space? A topological space? The authors certainly do not make this clear in the definition.

As for chapter 5, I'm not sure it's a good idea to engage in this level of generality in a text on linear algebra, especially because matrix groups *per se* are already an excellent vehicle for studying Lie groups: they are nice, concrete objects, but general enough to convey a considerable amount of the flavour of Lie theory. Indeed, entire books have been written on the subject of Lie theory from a matrix group

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perspective: excellent examples are Brian Hall's *Lie Groups, Lie Algebras and Representations* and John Stillwell's *Naïve Lie Theory*. Both books do quite a bit with matrix groups without having to mention general manifolds; a similar discussion here would have been more elementary and linear-algebraic in nature. Of course, all this is a matter of personal taste: there may well be instructors out there who think a graduate course in linear algebra is an appropriate venue for discussions of differentiable manifolds or Lie groups, and for such people these two chapters should prove very useful.

The presence of chapters 4 and 5 do, however, make this book a useful reference source for people wanting a quick overview of these areas of mathematics without tackling more sophisticated texts.

Given the subject matter covered in this volume, it is not surprising that *LA II* is a somewhat more demanding text than is *LA I*. Nevertheless, it retains a number of useful pedagogical features of the first volume, and, in fact, adds a new one. The retained features include generally clear exposition, a decent number of examples, and a good number of exercises (including nicely chosen and useful True-False problems). As in the first volume, some exercises are embedded in the text itself, and others appear collected at the end of each chapter. Also as in volume I, no solutions are provided. The newly added feature is the presence of an overview for each chapter, generally several pages long, which helps set the stage for the more detailed discussion that is to follow. These overviews seemed to me to be informative and valuable.

In one respect, however, this second volume is problematic: Volume I had an Index, but this book does not. This, I think, is an inexcusable omission for any mathematics text (or, for that matter, *any* text). On the flip side, Volume I lacked a bibliography; this book has one, but it's very small (6 items), and is limited to books on the subject matter of the last two chapters, namely differential geometry and Lie theory. A list of some of the many books covering linear algebra at a reasonably sophisticated level would have been appreciated. The books listed are also fairly old (the most recent one was published in 2002).

So, to summarise and conclude: the eclectic selection of topics in *LA II* might pose problems with using this book as a stand-alone text for a traditional graduate course in linear algebra; it could more readily be used in conjunction with *LA I*, but that would require the students to purchase two texts. However, over and above the use of this book as a text, it would make an interesting reference source, especially for people who wish to see a relatively quick overview of differential geometry or Lie theory.

Reference

1. Frederick P. Greenleaf and Sophie Marques, *Linear Algebra I*, American Mathematical Society (2019).

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MARK HUNACEK
Iowa State University,
Ames, IA 50011, USA
e-mail: *mhunacek@iastate.edu*

Advanced calculus explored by Hamza Alsamraee, pp. 448, $26.99 (paper), ISBN 978-0-578-61682-7, Curious Math Publications (2019)

One of the most surprising things about this book is the age of its author. At the time of its publication Alsamraee was in his final year of secondary school. As incredible as I found this, in a writer who is so young come both advantages and drawbacks, and I shall come to both shortly. Despite the word ‘calculus’ in its title, *Advanced calculus explored*