Macroeconomic Dynamics, **16**, 2012, 657–659. Printed in the United States of America.
doi:10.1017/S1365100510000684

NOTE

A NOTE ON R&D SPILLOVERS IN AN ENDOGENOUS GROWTH MODEL WITH PHYSICAL CAPITAL, HUMAN CAPITAL, AND VARIETIES

MANUEL A. GÓMEZ
Universidad de A Coruña

TIAGO NEVES SEQUEIRA
Universidade da Beira Interior (DGE and CEFAGE-UBI)
and
INOVA, Universidade Nova de Lisboa

This note shows that Lemma 3 in the paper by Tiago N. Sequeira [R&D spillovers in an endogenous growth model with physical capital, human capital and varieties, *Macroeconomic Dynamics* (2011)] is insufficient to guarantee stability and provides an alternative sufficient condition for stability.

Keywords: Stability, Transitional Dynamics, R&D Spillovers

1. A COUNTEREXAMPLE

The model and notation, as well as the equation numbers, are taken from Sequeira (2011). His Lemma 3 provides a sufficient condition for saddle-path stability of the steady state: namely, $1/\alpha < 1 + (1 - \beta - \eta)/\eta$. However, the following counterexample shows that this condition is insufficient to guarantee stability.

Example 1

The parameterization $\beta = 0.12$, $\eta = 0.32$, $\alpha = 0.37$, $\xi = 0.035$, $\rho = 0.023$, $\theta = 2$, $\delta = 0.1$, and $\phi = 0.02$ fulfills the sufficient stability condition of Lemma 3 in Sequeira (2011). The steady state is $r^* = 0.0390$, $\chi^* = 0.2784$, $g_n^* = 0.0041$, and $\psi^* = 1.1468$. The eigenvalues of the coefficient matrix J of the linearized system (19) are $0.0008 \pm 0.1433i$, 6.3405 and 0.0310. Hence, the steady state is unstable.

Gómez acknowledges financial support from the Spanish Ministry of Science and Innovation through Grant ECO2008-04180. Sequeira acknowledges financial support from the Portuguese Ministry of Science, Technology and Higher Education, through FCT project PTDC/EGE-ECO/102238/2008. Address correspondence to: Tiago Neves Sequeira, Departamento de Gestão e Economia, Universidade da Beira Interior, 6200-001 Covilhã, Portugal; e-mail: sequeira@ubi.pt.

 1365-1005/11

Sequeira (2011) uses the Gershgorin disk theorem to derive his Lemma 3. He proves that disk (23) is contained in the left half of the complex plane and then argues that one eigenvalue lies in this disk. However, the sufficient condition should have included other conditions that guarantee that disk (23) is disjoint from all the other disks.[1] Though the proof of Lemma 3 can be completed by enforcing this requirement, the ensuing sufficient conditions for stability are rather complex.[2]

Therefore, in the next section, we provide an alternative Lemma 3 that yields a simple meaningful condition by using the Routh–Hurwitz theorem.

2. AN ALTERNATIVE LEMMA 3

LEMMA 3. *A sufficient condition to rule out the instability outcome is* $\alpha\beta(1-\phi) \geq (1-\alpha)\eta$.

Proof. Let J_{ij} denote the (i, j)-element in the coefficient matrix J of the linearized system (19). Lemma 2 shows that $J_{33} = \xi - (1-\phi)g_n^* > 0$ is an unstable root of J. The other three roots of J are those of the matrix $\bar{J}$ that results from eliminating the third row and column from J. Therefore, the number of stable roots of J is equal to that of $\bar{J}$. The characteristic equation of $\bar{J}$ is $p(\lambda) = -\lambda^3 + \Delta_2\lambda^2 - \Delta_1\lambda + \Delta_0 = 0$. Here, $\Delta_2 = \text{tr}(\bar{J})$ is the trace of $\bar{J}$,

$$\Delta_1 = J_{11}J_{22} - J_{14}J_{41} + (J_{11} + J_{22})\,J_{44}$$

is the sum of all 2×2 leading minors of $\bar{J}$, and $\Delta_0 = \det(\bar{J}) = \det(J)/J_{33} > 0$ is the determinant of $\bar{J}$, where the sign follows from Lemmas 1 and 2.

Using the Routh–Hurwitz theorem, the number of stable roots is equal to the number of variations of sign in the scheme

$$1 \qquad \text{tr}(\bar{J}) \qquad \Omega \equiv \Delta_1 - \det(\bar{J})/\text{tr}(\bar{J}) \qquad \det(\bar{J}).$$

We now show that a sufficient condition to rule out the case of no stable roots is that $\Delta_1 < 0$. If $\text{tr}(\bar{J}) < 0$, then there are two variations of sign in the preceding scheme—irrespective of the sign of Ω. If $\text{tr}(\bar{J}) > 0$, then $\Omega < 0$, so there are two variations in sign in the scheme. If $\text{tr}(\bar{J}) = 0$, we replace it with $\epsilon > 0$, and so, $\Omega = \Delta_1 - \det(\bar{J})/\epsilon$. Taking the limit as $\epsilon \to 0$, we have that $\Omega \to -\infty$ and, therefore, there are two variations in sign in the scheme. After simplification, we can obtain that

$$J_{11} + J_{22} = r^* - g_K^* + \frac{\eta(1-\alpha)}{\beta}r^* = \xi - (1-\phi)g_n^* + \frac{\eta(1-\alpha)}{\beta}r^* > 0,$$

where the first equality follows from (6), and the second results from (12). Because $J_{11}J_{22} < 0$, $J_{14}J_{41} > 0$, and $J_{44} = B_2(\xi + \phi g_n^*)/\phi$, using that $B_3 = -B_2 g_n^*$ by (13), a sufficient condition for $\Delta_1 < 0$ is $B_2 \leq 0$, which yields the desired result. ∎

It should be noted that the main implication of Lemma 3 in Sequeira (in press), the fact that stability is more likely the lower the markup, is maintained despite the incompleteness of the sufficient condition. The remaining contributions of the paper also remain valid.

NOTES

1. A similar argument also applies to the alternative sufficient condition given in note 3 in Sequeira (in press).
2. A complete proof is available upon request. Other sufficient conditions can be derived by means of the Gershgorin theorem; these are also available from the authors.

REFERENCE

Sequeira, Tiago N. (2011) R&D spillovers in an endogenous growth model with physical capital, human capital, and varieties. *Macroeconomic Dynamics* 15(2), 223–239.