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CONTRIBUTED PAPER

Sufficient mathematical conditions for identical estimation of the liability for remaining coverage under the general measurement model and premium allocation approach

Timothy Lee[1] and Antonie Jagga[2]

[1]Institute of Actuaries of Australia, Sydney, Australia and [2]Institute and Faculty of Actuaries, London, UK
Corresponding author: Timothy Lee; Email: timothy.a.lee@au.pwc.com

Abstract
Paragraph 53(a) of the new insurance accounting standard IFRS 17 suggests there is a relationship between the liability for remaining coverage ("LFRC") calculated under the general measurement model ("GMM") and premium allocation approach ("PAA"), although it is not immediately obvious how the two are related or could result in a similar estimate for the LFRC. This paper explores the underlying relationship between the GMM and PAA through the equivalence principle and presents a set of sufficient mathematical conditions that result in an identical LFRC when calculated under the GMM and PAA. An illustrative example is included to demonstrate how the sufficient conditions can be applied in practice and the optimisation opportunities offered to actuaries and accountants when conducting PAA eligibility testing.

Keywords: IFRS 17; premium allocation approach; PAA eligibility testing; equivalence principle; general measurement model; liability for remaining coverage

1. Introduction

Since the inception of *International Financial Reporting Standard 17 Insurance Contracts* or "IFRS 17" (Foundation, 2020) for reporting periods commencing 1 January 2023, many general insurers around the world have opted to apply the premium allocation approach ("PAA") simplification over the general measurement model ("GMM") when deriving an estimate for the liability for remaining coverage ("LFRC").

It is suggested in paragraph 53(a) of IFRS 17 that there is a relationship between the LFRC calculated under the GMM and PAA, although it is not immediately obvious after a perusal of the paragraphs in IFRS 17, the basis for conclusions (Foundation, 2017a) and supporting illustrative examples (Foundation, 2017b) what the relationship is. Given that the first criterion for the PAA to be applied is for the LFRC calculated under the PAA to not be materially different to the balance calculated under the GMM, it is of considerable value to IFRS 17 practitioners (whether that be actuaries or accountants) when conducting the PAA eligibility test, to first understand why the two measurement methods would produce the same or similar LFRC; and second but more importantly if there are certain modelling approaches that can reduce, if not completely eliminate, the discrepancies between the LFRC calculated under the GMM and PAA.

Whilst there have been numerous publications on the PAA eligibility test including those from the big four auditing firms (see, e.g. PwC (2019), Deloitte (2021), EY (2021) and KPMG (2020)), as well as an abundance of IFRS 17 literature by various actuarial organisations (see, e.g. Institute and Faculty of Actuaries (2018), Institute of Actuaries of Australia (2021) and Canadian Institute of

Actuaries (2020)), we have yet to see a formal piece of published research that satisfactorily answers our two questions. The purpose of this paper is to resolve these two questions, and consequently highlight the benefits to IFRS 17 practitioners in unlocking this knowledge.

The first question on why the two measurement methods could produce the same or similar LFRC can be answered informally by inspecting the paragraphs pertaining to the GMM and PAA contained in IFRS 17. This informal explanation is made rigorous when framed in a mathematical context when addressing the second question, which forms the body of this paper.

The critical observation to be made is in the comparison between the fundamental aspects of the insurance contract that is measured under the two approaches. The GMM is focused on estimating the "cost" of a group of insurance contracts by calculating the net fulfilment cash flows (which is roughly the claims cost) and the contractual service margin (which represents the unearned profit). On the other hand, the PAA is focused on estimating the LFRC using "premiums". The connection between the GMM and PAA becomes evident once it is observed that insurance premiums can be expressed as the sum of the cost of the contract. This is an example of a more general theory in insurance pricing where premiums are set to be equal to cost, known as the equivalence principle (see, e.g. Dickson *et al*., 2009). Therefore, despite the GMM and PAA appearing to calculate different quantities, there is a duality between the two due to the equivalence principle resulting in the same or similar LFRC.

Motivated by the theory on the equivalence principle, we endeavour to answer the second question by selecting mathematical and modelling assumptions that would create a perfect equality between the "premium" (i.e. PAA) and "cost" (i.e. GMM) sides. To this end, we have discovered a set of sufficient mathematical conditions (which we refer to as the "sufficient conditions model") that result in an identical LFRC when calculated under the GMM and PAA.

The remainder of this paper is structured as follows:

- Section 2 states and proves how our stated sufficient mathematical conditions produce a PAA LFRC identical to that using the GMM
- Section 3 applies our sufficient conditions model to an illustrative example to demonstrate how our model can be applied in practice
- Section 4 discusses the practical benefits of our model to IFRS 17 practitioners.

2. Sufficient Conditions Model

2.1. Mathematical Assumptions

In this section, we describe the sufficient mathematical assumptions to produce the same LFRC under both the GMM and PAA.

2.1.1. Insurance contracts

Suppose we have a group of (direct) insurance contracts without direct participation features, with initial recognition occurring at time 0, with integer reporting periods so that the balance date occurs at times 1, 2, 3, etc. Without loss of generality, we assume that the coverage period for the group of insurance contracts commences at time 0 and ends at time J, where J coincides with one of the integer balance dates. Further, we assume that all insurance contracts related to the group are in a single currency unit and are all recognised at initial recognition so that no other contracts are added to the group after initial recognition.

We assume that no contracts in our group are onerous at initial recognition, which paragraph 18 of IFRS 17 suggests is the default position unless facts and circumstances indicate otherwise for an entity applying the PAA.

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2.1.2. Discount rate

Paragraph 36 of IFRS 17 requires future cash flows to "reflect the time value of money and the financial risks related to those cash flows, to the extent that the financial risks are not included in the estimates of cash flows". For the purposes of our model, we assume a flat discount rate of i at initial recognition and the discount rate is unchanged on subsequent measurement. As discount rates typically vary with maturity in practice, Section 3.1 discusses techniques to approximate the discount impact via a flat discount rate.

2.1.3. Coverage units

According to paragraph B119(a) of IFRS 17 the number of coverage units in a group of insurance contracts represents "the quantity of insurance services provided by the contracts in the group". We let u_k denote the number of coverage units in our group of insurance contracts for services provided over reporting period k. For the purposes of recognising the amount of contractual service margin in the profit or loss at the end of reporting period k, we have adopted the following coverage unit allocation:

$$\frac{u_k}{\sum_{j=k}^{J} u_j (1+i)^{-(j-k)}}.$$

The pattern of allocation is similar to that provided in Example 2 of the IFRS 17 illustrative examples (i.e. IE12 to IE17) but has been adjusted for discounting as permitted under paragraph BC282 of the basis for conclusions, which provides the entity with a choice on whether to discount the coverage units.

2.1.4. Cash flows

For the purposes of our model, we assume that cash flows occur at integer time points, with T being the last time point a cash flow occurs. We define Ω to be the set of all different types of cash flows (e.g. premiums, payments, insurance acquisition cash flows, investment components and other cash flow examples as presented in paragraph B65 of IFRS 17) within the boundary of our group of insurance contracts and $C_{j,t}^{\omega}$ to represent the nominal (i.e. undiscounted) cash flow relating to cash flow type $\omega \in \Omega$ occurring at integer time point $t \leq T$, for claims incurred in reporting period $j \in [0, J]$. Here, $j = 0$ is understood to represent a cash flow unrelated to incurred claims cost which, in the context of our model, we have restricted to three types of cash flows: premiums paid for the policy, acquisition cash flows and investment component. Given these notations, the net nominal cash flow (defined as cash outflows less cash inflows) for our group of insurance contracts occurring at time t is represented as

$$\sum_{\omega \in \Omega} \sum_{j=0}^{J} (-1)^{\mathbf{1}\left(\omega \in \left\{Cash\ Inflows\right\}\right)} C_{j,t}^{\omega}.$$

We make a simplifying assumption that all cash flows relating to claims incurred at time $k = 1, \cdots, J$ can be expressed as a product of a measure of "exposure", which we have selected to be the number of coverage units over reporting period k, a "loss ratio" constant λ^{ω} reflecting the amount of cash outflow/inflow incurred over each reporting period and a payment pattern p_{t-k+1}^{ω} based on the time since the claim was incurred, where cash flows cannot occur until the claim has been incurred. Hence:

$$C_{k,t}^{\omega} = \lambda^{\omega} p_{t-k+1}^{\omega} u_k \mathbf{1}(t \geq k).$$

We observe that as T represents the last time point for a cash inflow or outflow for the group of contracts, it follows that $p_n^{\omega} = 0$ in the tail for $n = T - J + 2, \cdots, T$.

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For the purposes of paragraphs 38(c) and 55(a)(iii) of IFRS 17, which describe the derecognition of any asset for insurance acquisition cash flows and any other asset or liability recognised for cash flows related to the group of contracts at initial recognition, we assume that only insurance acquisition cash flows and premiums paid in advance are applicable, so the asset and liability are respectively represented as $\sum_{t<0} C_{0,t}^{Acq}$ and $\sum_{t<0} C_{0,t}^{Prem}$.

We also assume that once the cash flows have been determined at initial recognition, there will be no expectation changes so the estimated cash flows will remain the same at subsequent measurement.

2.1.5. Risk adjustment

We assume the risk adjustment for the group of insurance contracts is a proportion r_{t-j+1}^{ω} (dependent on the time from when a claim was incurred) of each discounted cash flow excluding those cash flows related to premiums paid for the policy, acquisition cash flows and investment component. Thus, the risk adjustment at balance date k is represented as

$$[Risk\ Adjustment]_k = \sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}(\omega\in\{Cash\ Inflows\})} r_{t-j+1}^{\omega} C_{j,t}^{\omega}(1+i)^{-(t-k)},$$

where $I = \{Prem, Acq, Invest\}$.

2.1.6. Premium allocation approach insurance revenue recognition

Paragraph B126 of IFRS 17 sets out the principles of selecting a pattern to recognise the insurance revenue under the PAA, requiring the "expected timing of incurred insurance service expenses" should the "expected pattern of release of risk during the coverage period" differ "significantly from the passage of time". In selecting a pattern for the expected timing of incurred insurance service expenses, we have assumed the expected timing of incurred insurance service for reporting period k is proportional ($\rho > 0$) to the quantity of insurance contract services provided by the group of contracts in reporting period k, which is reflected in the number of coverage units u_k. This assumption is appropriate provided there is a linear relationship between the quantity of services provided and release of risk. Hence, the insurance revenue recognised in reporting period k is represented as

$$\frac{\rho u_k}{\sum_{j=1}^{J}\rho u_j(1+i)^{-j}}\sum_{t\le T}\left(C_{0,t}^{Prem}-C_{0,t}^{Invest}\right)(1+i)^{-t\mathbf{1}(t>0)}.$$

Following the requirements of paragraph B126 of IFRS 17, we have excluded the investment component and have also adjusted the recognition pattern to reflect the time value of money and the effect of financial risk by assuming the group of insurance contracts have significant financing component as set out in paragraph 56 of IFRS 17. It is important to note that the presence of a financing component does not automatically lead to its recognition and in practice, any adjustment to the PAA LFRC is only required if the impact on the financing component is significant.

2.1.7. Amortisation of insurance acquisition cash flows

We have elected not to apply the practical expedient in paragraph 59(a) of IFRS 17 recognising insurance acquisition cash flows as expenses when the costs are incurred. Accordingly, paragraph 55(b)(iii) and B125 of IFRS 17 require insurance acquisition cash flows to be amortised in each period in a "systematic way on the basis of the passage of time". In meeting this requirement, we have selected a similar recognition pattern to Section 2.1.6. If $\alpha > 0$ is a constant proportion of the number of coverage units u_k representing a systematic allocation, then the insurance acquisition cash flow amortised over reporting period k is

$$\frac{\alpha u_k}{\sum_{j=1}^{J} \alpha u_j (1+i)^{-j}} \sum_{t \leq T} C_{0,t}^{Acq} (1+i)^{-t\mathbf{1}(t>0)}.$$

2.2. General Measurement Model

Let $[LFRC]_k^{GMM}$ represent the balance of the LFRC at the end of reporting period k as calculated under the GMM. It follows from paragraphs 32 and 40(a) of IFRS 17 that

$$[LFRC]_k^{GMM} = [FCF]_k + [CSM]_k, \tag{1}$$

where $[FCF]_k$ and $[CSM]_k$ are, respectively, the fulfilment cash flows balance and the contractual service margin balance at the end of reporting period k. In this section, we show that for $k = 0, \cdots, J$,

$$\begin{aligned}[FCF]_k = &\sum_{\omega \in I} \sum_{t=k+1}^{T} (-1)^{\mathbf{1}(\omega \in \{Cash\ Inflows\})} C_{0,t}^{\omega} (1+i)^{-(t-k)} \\ &+ \sum_{\omega \in \Omega \setminus I} \sum_{j=k+1}^{J} \sum_{t=1}^{T} (-1)^{\mathbf{1}(\omega \in \{Cash\ Inflows\})} (1+r_t^{\omega}) \lambda^{\omega} p_t^{\omega} u_j (1+i)^{-(t+j-k-1)},\end{aligned} \tag{2}$$

and

$$\begin{aligned}[CSM]_k = (1+i)^k \Bigg[&- \sum_{\omega \in I} \sum_{t=0}^{T} (-1)^{\mathbf{1}(\omega \in \{Cash\ Inflows\})} C_{0,t}^{\omega} (1+i)^{-t} \\ &- \sum_{\omega \in \Omega \setminus I} \sum_{j=1}^{J} \sum_{t=1}^{T} (-1)^{\mathbf{1}(\omega \in \{Cash\ Inflows\})} (1+r_t^{\omega}) \lambda^{\omega} p_t^{\omega} u_j (1+i)^{-(t+j-1)} + \sum_{t<0} \left(C_{0,t}^{Prem} - C_{0,t}^{Acq} \right) \Bigg] \\ &\times \left[1 - \frac{\sum_{j=1}^{k} u_j (1+i)^{-j}}{\sum_{j=1}^{J} u_j (1+i)^{-j}} \right],\end{aligned} \tag{3}$$

thereby, deriving a formula to calculate $[LFRC]_k^{GMM}$. Note that for $k > J$, trivially $[LFRC]_k^{GMM} = 0$ as there is no unexpired portion of the insurance coverage.

2.2.1. Fulfilment cash flows

To show that Equation (2) holds, we observe that, as described in paragraph 32(a) of IFRS 17, the fulfilment cash flows are comprised of three components:

1. Estimates of future cash flows
2. An adjustment to reflect the time value of money and the financial risks related to the future cash flows
3. A risk adjustment for non-financial risk.

The first two components are equivalent to the discounted future cash flows and applying the assumptions from Section2.1.4 to 2.1.2 respectively, the discounted future cash flows at balance date k is

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$$\sum_{\omega\in\Omega}\sum_{j\in\{0\}\cup\{k+1,\ldots,J\}}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{j,t}^{\omega}(1+i)^{-(t-k)},$$

whereas the risk adjustment at balance date k is as described in Section 2.1.5:

$$\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}r_{t-j+1}^{\omega}C_{j,t}^{\omega}(1+i)^{-(t-k)}.$$

Combining the two results and simplifying gives:

$$\begin{aligned}
[FCF]_k &= \sum_{\omega\in\Omega}\sum_{j\in\{0\}\cup\{k+1,\ldots,J\}}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}r_{t-j+1}^{\omega}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&=\sum_{\omega\in\Omega}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}r_{t-j+1}^{\omega}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&=\sum_{\omega\in I}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}r_{t-j+1}^{\omega}C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&=\sum_{\omega\in I}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_{t-j+1}^{\omega})C_{j,t}^{\omega}(1+i)^{-(t-k)}\\
&=\sum_{\omega\in I}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}\\
&\quad+\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_{t}^{\omega})\lambda^{\omega}p_{t}^{\omega}u_{j}(1+i)^{-(t+j-k-1)}
\end{aligned}$$

where we apply the assumptions made in Section 2.1.4 to simplify the last term

$$\begin{aligned}
&\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r^{\omega}_{t-j+1})C^{\omega}_{j,t}(1+i)^{-(t-k)}\\
&=\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r^{\omega}_{t-j+1})\lambda^{\omega}p^{\omega}_{t-j+1}u_j\mathbf{1}\big(t\geq j\big)(1+i)^{-(t-k)}\\
&=\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=j}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r^{\omega}_{t-j+1})\lambda^{\omega}p^{\omega}_{t-j+1}u_j(1+i)^{-(t-k)}\\
&=\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=1}^{T-j+1}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r^{\omega}_{t})\lambda^{\omega}p^{\omega}_{t}u_j(1+i)^{-(t+j-k-1)}\\
&=\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r^{\omega}_{t})\lambda^{\omega}p^{\omega}_{t}u_j(1+i)^{-(t+j-k-1)}.
\end{aligned}$$

2.2.2. Contractual service margin

To show that Equation (3) holds, we break the analysis down to consider the contractual service margin at initial recognition (i.e. $k=0$) and subsequent measurement (i.e. $k=1,\dots,I$).

Paragraph 38 of IFRS 17 explains the built up of the contractual service margin at initial recognition for groups of contracts that are not onerous is given by the negative of the sum of three components:

1. **Fulfilment cash flows at time 0:** Using the results from Section 2.2.1, we have

$$\begin{aligned}
[FCF]_0=&\sum_{\omega\in I}\sum_{t=1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C^{\omega}_{0,t}(1+i)^{-t}\\
&+\sum_{\omega\in\Omega\setminus I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r^{\omega}_{t})\lambda^{\omega}p^{\omega}_{t}u_j(1+i)^{-(t+j-1)}
\end{aligned}$$

2. **Cash flows at initial recognition:** Following from Section 2.1.4, this is

$$\sum_{\omega\in\Omega}\sum_{j=0}^{J}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C^{\omega}_{j,0}=\sum_{\omega\in I}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C^{\omega}_{0,0},$$

   where the last equality follows from noting that there cannot be any incurred claims cash flows (i.e. $\Omega\setminus I$) before the claim has occurred.
3. **Derecognition of certain assets and liabilities:** Following from Section 2.1.4, this is

$$-\sum_{t<0}C^{Prem}_{0,t}+\sum_{t<0}C^{Acq}_{0,t}.$$

Taking the negative of the sum of the three components shows that Equation (3) holds for $k=0$.

To show that Equation (3) holds for $k=1,\dots,I$, we apply paragraph 44 of IFRS 17 and deduce the result via an inductive argument. Paragraph 44 explains for insurance contracts without direct participation features, $[CSM]_{k+1}$ is calculated by summing the following components:

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1. **The carrying amount at the start of the reporting period:** As the carrying amount at the start of the reporting period is equal to the carrying amount at the end of the last reporting period, this is $[CSM]_k$.
2. **The effect of any new contracts added to the group:** For the purposes of our model, this amount is nil as we have assumed in Section 2.1.1 that all contracts in the group are recognised at initial recognition.
3. **Interest accreted on the carrying amount:** Following from Section 2.1.2, this amount is $i[CSM]_k$.
4. **Change in fulfillment cash flow relating to future service**: As we have assumed in Section 2.1.4 that there are no subsequent changes in fulfilment cash flow, this amount is nil.
5. **The effect of currency exchange differences**: As we have assumed in Section 2.1.1 that all contracts relate to the same single currency unit, this amount is nil.
6. **Amount recognised as insurance revenue**: Using the coverage units allocation described in Section 2.1.3, the reduction to the contractual service margin from recognising insurance revenue is

$$[CSM]_k(1+i)\frac{u_{k+1}}{\sum_{j=k+1}^{J} u_j(1+i)^{-(j-k-1)}}.$$

Combining the above components yields:

$$\begin{aligned}[CSM]_{k+1} &= [CSM]_k + i[CSM]_k - [CSM]_k(1+i)\frac{u_{k+1}}{\sum_{j=k+1}^{J} u_j(1+i)^{-(j-k-1)}}\\ &= [CSM]_k(1+i)\left[1-\frac{u_{k+1}}{\sum_{j=k+1}^{J} u_j(1+i)^{-(j-k-1)}}\right].\end{aligned}$$

Applying the inductive hypothesis for $[CSM]_k$, i.e. Equation (3), gives

$$\begin{aligned}[CSM]_{k+1} = (1+i)^k\Bigg[&-\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{\mathbb{1}(\omega\in\{Cash\ Inflows\})}C_{0,t}^{\omega}(1+i)^{-t}\\ &-\sum_{\omega\in\Omega\setminus I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{\mathbb{1}(\omega\in\{Cash\ Inflows\})}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-1)}+\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)\Bigg]\\ &\times\left[1-\frac{\sum_{j=1}^{k} u_j(1+i)^{-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}\right](1+i)\left[1-\frac{u_{k+1}}{\sum_{j=k+1}^{J} u_j(1+i)^{-(j-k-1)}}\right]\\ =(1+i)^{k+1}\Bigg[&-\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{\mathbb{1}(\omega\in\{Cash\ Inflows\})}C_{0,t}^{\omega}(1+i)^{-t}\\ &\sum_{\omega\in\Omega\setminus I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{\mathbb{1}(\omega\in\{Cash\ Inflows\})}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-1)}+\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)\Bigg]\\ &\times\left[1-\frac{\sum_{j=1}^{k+1} u_j(1+i)^{-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}\right],\end{aligned}$$

where we observe that

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$$\left[1-\frac{\sum_{j=1}^{k} u_j(1+i)^{-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}\right]\left[1-\frac{u_{k+1}}{\sum_{j=k+1}^{J} u_j(1+i)^{-(j-k-1)}}\right] = \frac{\sum_{j=k+1}^{J} u_j(1+i)^{-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}\cdot\frac{\sum_{j=k+2}^{J} u_j(1+i)^{-(j-k-1)}}{\sum_{j=k+1}^{J} u_j(1+i)^{-(j-k-1)}}$$
$$= \frac{\sum_{j=k+2}^{J} u_j(1+i)^{-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}$$
$$= 1-\frac{\sum_{j=1}^{k+1} u_j(1+i)^{-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}.$$

By mathematical induction, we have shown that Equation (3) holds for $k = 1, \ldots, I$.

2.3. Premium Allocation Approach

Let $[LFRC]_k^{PAA}$ represent the balance of the LFRC at the end of reporting period k as calculated under the PAA. In this section, we show for $k = 0, \ldots, J$,

$$[LFRC]_k^{PAA} = -\sum_{\omega\in I}\sum_{t=0}^{k}(-1)^{\mathbf{1}\left(\omega\in\{Cash\ Inflows\}\right)}C_{0,t}^{\omega}(1+i)^{k-t} + \sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)(1+i)^k$$
$$+\frac{\sum_{j=1}^{k} u_j(1+i)^{k-j}}{\sum_{j=1}^{J} u_j(1+i)^{-j}}\sum_{\omega\in I}\sum_{t\leq T}(-1)^{\mathbf{1}\left(\omega\in\{Cash\ Inflows\}\right)}C_{0,t}^{\omega}(1+i)^{-t\mathbf{1}(t>0)}. \tag{4}$$

Similarly, trivially $[LFRC]_k^{PAA} = 0$ for $k > J$ as there is no unexpired portion of the insurance coverage.

To show that Equation (4) holds, we again employ mathematical induction and separately consider the case for initial recognition (i.e. $k = 0$) and subsequent measurement (i.e. $k = 1, \ldots, I$).

Paragraph 55(a) of IFRS 17 explains $[LFRC]_0^{PAA}$ is comprised of three components:

1. **Premiums received at initial recognition**: Using the notation in Section 2.1.4, this amount is $C_{0,0}^{Prem}$.
2. **Less any acquisition cash flows at initial recognition**: This amount is $-C_{0,0}^{Acq}$.
3. **Plus/minus any amounts from the derecognition of certain assets and liabilities:** Following from Section 2.1.4, this is

$$\sum_{t<0} C_{0,t}^{Prem} - \sum_{t<0} C_{0,t}^{Acq}.$$

Combining the components, we have

$$[LFRC]_0^{PAA} = C_{0,0}^{Prem} - C_{0,0}^{Acq} + \sum_{t<0} C_{0,t}^{Prem} - \sum_{t<0} C_{0,t}^{Acq}$$
$$= -\sum_{\omega\in I}(-1)^{\mathbf{1}\left(\omega\in\{Cash\ Inflows\}\right)}C_{0,0}^{\omega} + \sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right),$$

where we note that $C_{0,0}^{Invest} = 0$ at initial recognition (i.e. no investment returns at time 0). Hence, Equation (4) holds for $k = 0$.

To show that Equation (4) holds for $k = 1, \ldots, I$, we apply paragraph 55(b) of IFRS 17 and deduce the result via an inductive argument. Paragraph 55(b) explains the carrying amount at the end of the reporting period $[LFRC]_{k+1}^{PAA}$ is the sum of the following components:

1. **The carrying amount at the start of the reporting period:** As the carrying amount at the start of the reporting period is equal to the carrying amount at the end of the last reporting period, this is $[LFRC]_k^{PAA}$

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2. **Plus premiums received in the period:** This amount is $C_{0,k+1}^{Prem}$
3. **Minus insurance acquisition cash flows:** This amount is $-C_{0,k+1}^{Acq}$
4. **Plus any amounts relating to the amortisation of the insurance acquisition cash flows**: Following from Section 2.1.7, the amount amortised over reporting period $k+1$ is

$$\frac{\alpha u_{k+1}}{\sum_{j=1}^{J} \alpha u_j (1+i)^{-j}} \sum_{t \le T} C_{0,t}^{Acq} (1+i)^{-t\mathbf{1}(t>0)}$$

5. **Plus any adjustment to a financing component:** We have taken this to be the interest accretion and the amount is $i[LFRC]_k^{PAA}$
6. **Minus the amount recognised as insurance revenue:** Following from Section 2.1.6, the amount recognised over the reporting period $k+1$ is

$$-\frac{\rho u_{k+1}}{\sum_{j=1}^{J} \rho u_j (1+i)^{-j}} \sum_{t \le T} \left(C_{0,t}^{Prem} - C_{0,t}^{Invest} \right) (1+i)^{-t\mathbf{1}(t>0)}$$

7. **Minus any investment component:** This amount is $-C_{0,k+1}^{Invest}$.

Combining the above components gives:

$$\begin{aligned}
[LFRC]_{k+1}^{PAA} &= [LFRC]_k^{PAA} + C_{0,k+1}^{Prem} - C_{0,k+1}^{Acq} \\
&\quad + \frac{\alpha u_{k+1}}{\sum_{j=1}^{J} \alpha u_j (1+i)^{-j}} \sum_{t \le T} C_{0,t}^{Acq} (1+i)^{-t\mathbf{1}(t>0)} + i[LFRC]_k^{PAA} \\
&\quad - \frac{\rho u_{k+1}}{\sum_{j=1}^{J} \rho u_j (1+i)^{-j}} \sum_{t \le T} \left(C_{0,t}^{Prem} - C_{0,t}^{Invest} \right) (1+i)^{-t\mathbf{1}(t>0)} - C_{0,k+1}^{Invest} \\
&= (1+i)[LFRC]_k^{PAA} - \sum_{\omega \in I} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,k+1}^{\omega} \\
&\quad + \frac{u_{k+1}}{\sum_{j=1}^{J} u_j (1+i)^{-j}} \sum_{\omega \in I} \sum_{t \le T} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,t}^{\omega} (1+i)^{-t\mathbf{1}(t>0)}.
\end{aligned}$$

Applying the inductive hypothesis for $[LFRC]_k^{PAA}$, i.e. Equation (4), gives

$$\begin{aligned}
[LFRC]_{k+1}^{PAA} &= (1+i)\Bigg[-\sum_{\omega \in I} \sum_{t=0}^{k} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,t}^{\omega} (1+i)^{k-t} + \sum_{t<0} \left(C_{0,t}^{Prem} - C_{0,t}^{Acq} \right) (1+i)^k \\
&\quad + \frac{\sum_{j=1}^{k} u_j (1+i)^{k-j}}{\sum_{j=1}^{J} u_j (1+i)^{-j}} \sum_{\omega \in I} \sum_{t \le T} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,t}^{\omega} (1+i)^{-t\mathbf{1}(t<0)} \Bigg] \\
&\quad - \sum_{\omega \in I} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,k+1}^{\omega} \\
&\quad + \frac{u_{k+1}}{\sum_{j=1}^{J} u_j (1+i)^{-j}} \sum_{\omega \in I} \sum_{t \le T} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,t}^{\omega} (1+i)^{-t\mathbf{1}(t>0)} \\
&= -\sum_{\omega \in I} \sum_{t=0}^{k+1} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,t}^{\omega} (1+i)^{(k+1)-t} + \sum_{t<0} \left(C_{0,t}^{Prem} - C_{0,t}^{Acq} \right) (1+i)^{k+1} \\
&\quad + \frac{\sum_{j=1}^{k+1} u_j (1+i)^{(k+1)-j}}{\sum_{j=1}^{J} u_j (1+i)^{-j}} \sum_{\omega \in I} \sum_{t \le T} (-1)^{\mathbf{1}\left(\omega \in \{Cash\ Inflows\}\right)} C_{0,t}^{\omega} (1+i)^{-t\mathbf{1}(t>0)}.
\end{aligned}$$

Thus, by mathematical induction, we have shown that Equation (4) holds for $k = 1, \ldots, I$.

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2.4. Proof of Identical Liability for Remaining Coverage

In this section, we show that Equations (1) and (4) are equivalent, thereby demonstrating the mathematical conditions described in Section 2.1 provide sufficient conditions such that the LFRC produced under the GMM and PAA are identical.

We start with Equation (1) and show that it is equal to Equation (4):

$$
\begin{aligned}
[LFRC]_k^{GMM} &= \sum_{\omega\in I}\sum_{t=k+1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)} \\
&+\sum_{\omega\in\Omega\backslash I}\sum_{j=k+1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)} \\
&+(1+i)^k\Bigg[-\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-t} \\
&-\sum_{\omega\in\Omega\backslash I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-1)}+\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)\Bigg] \\
&\times\left[1-\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\right] \\
&=\sum_{\omega\in I}\sum_{t=k+1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)} \\
&+\sum_{\omega\in\Omega\backslash I}\sum_{j=k+1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)} \\
&-\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)} \\
&-\sum_{\omega\in\Omega\backslash I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)} \\
&+\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)(1+i)^k \\
&+\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}} \\
&+\sum_{\omega\in\Omega\backslash I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)}\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}} \\
&-\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)(1+i)^k\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}.
\end{aligned}
$$

We note the following simplifications. First:

$$
\begin{aligned}
&\sum_{\omega\in I}\sum_{t=k+1}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}-\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)} \\
&=-\sum_{\omega\in I}\sum_{t=0}^{k}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{k-t}.
\end{aligned}
$$

Second, noting $C_{0,t}^{Invest} = 0$ for $t < 0$ we have:

$$\begin{aligned}
&\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-(t-k)}\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\\
&-\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)(1+i)^{k}\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\\
&=\sum_{\omega\in I}\sum_{t=0}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-t\mathbf{1}(t>0)}\frac{\sum_{j=1}^{k}u_j(1+i)^{k-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\\
&+\sum_{\omega\in I}\sum_{t<0}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-t\mathbf{1}(t>0)}\frac{\sum_{j=1}^{k}u_j(1+i)^{k-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\\
&=\frac{\sum_{j=1}^{k}u_j(1+i)^{k-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\sum_{\omega\in I}\sum_{t\le T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-t\mathbf{1}(t>0)}.
\end{aligned}$$

Finally:

$$\begin{aligned}
&\sum_{\omega\in\Omega\setminus I}\sum_{j=k+1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)}\\
&-\sum_{\omega\in\Omega\setminus I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)}\\
&+\sum_{\omega\in\Omega\setminus I}\sum_{j=1}^{J}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}u_j(1+i)^{-(t+j-k-1)}\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\\
&=-\sum_{\omega\in\Omega\setminus I}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}(1+i)^{-(t-k-1)}\sum_{j=1}^{k}u_j(1+i)^{-j}\\
&+\sum_{\omega\in\Omega\setminus I}\sum_{t=1}^{T}(-1)^{1\left(\omega\in\left\{Cash\ Inflows\right\}\right)}\\
&(1+r_t^{\omega})\lambda^{\omega}p_t^{\omega}(1+i)^{-(t-k-1)}\sum_{j=1}^{J}u_j(1+i)^{-j}\frac{\sum_{j=1}^{k}u_j(1+i)^{-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}=0.
\end{aligned}$$

Therefore:

$$\begin{aligned}
[LFRC]_k^{GMM}&=-\sum_{\omega\in I}\sum_{t=0}^{k}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{k-t}+\sum_{t<0}\left(C_{0,t}^{Prem}-C_{0,t}^{Acq}\right)(1+i)^{k}\\
&+\frac{\sum_{j=1}^{k}u_j(1+i)^{k-j}}{\sum_{j=1}^{J}u_j(1+i)^{-j}}\sum_{\omega\in I}\sum_{t\le T}(-1)^{\mathbf{1}\left(\omega\in\left\{Cash\ Inflows\right\}\right)}C_{0,t}^{\omega}(1+i)^{-t\mathbf{1}(t>0)}\\
&=[LFRC]_k^{PAA}.
\end{aligned}$$

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3. Illustrative Example

3.1. Model Assumptions

The purpose of our illustrative example is to demonstrate that the sufficient conditions model developed in Section 2 extends beyond theoretical interest and is of practical use. We have constructed our example with the intention of it being simplistic enough to allow the reader to work through the key concepts in the calculations, but also sophisticated enough to illustrate the versatility, extensibility and amenability of the model to many real-life circumstances.

Table 1 lists the policy and claims information for our illustrative example, which is a group consisting of two multi-year insurance contracts. Contract A is a shorter contract with a more simplistic cash flow structure than contract B. Whilst the policy information is provided at a contract level, the claims assumptions have been provided at a higher level (in this case the group level), reflecting the fact that many of these quantities are calibrated at a more aggregated level using aggregated claims methods. We have endeavoured to present the assumptions in a format that would typically be available to a general insurance reserving actuary after some data extractions (information from policy/claims system as well as finance and general ledger extracts) or outputs from an actuarial valuation exercise.

In putting the assumptions together, we make the following comments to illustrate how our sufficient conditions model and these assumptions can be extended or tailored to accommodate a more complicated scenario.

- We have assumed two contracts in the group with different cash flow patterns for premium and acquisition cash flows, to demonstrate that the sufficient conditions model is agnostic to the number of contracts as well as to the number and timing of premium receipts and payment of acquisition cash flows.
- Despite publications challenging the suitability of using earned premiums for the coverage units (see TRG, 2018)), we have observed that this continues to be a popular choice for many

Table 1. Illustrative Example Liability for Remaining Coverage Assumptions

Assumptions	Details
Contract A	Coverage period: 3 years
	Premium received: $9000 received at inception
	Acquisition cash flow: $300 paid at initial recognition
Contract B	Coverage period: 5 years
	Premium received: 5 receipts of $1000 paid at inception and at the start of each year
	Acquisition cash flow: 5 payments of $50 paid at inception and at the start of each year
Others	Acquisition cash flow: $1000 asset to be derecognised at initial recognition
	Loss ratio: 60% of earned premiums
	Gross claims payment pattern: uniform rate of 50% per year (i.e. claim fully settled after 2 years of being incurred)
	Claims handling expenses: 5% of gross claim payments
	Non-reinsurance recoveries: 10% of gross claim payments
	Risk adjustment: 10% of discounted net claim payments (net of claims handling expenses, non-reinsurance recoveries)
	Discount rate: 5% per year (also assume there exists significant financing component)
	Coverage units: earned premium of the group

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insurers. Partly influenced by its pervasiveness, but mostly because earned premiums and loss ratios are readily understood and available metrics for an insurer, we consider it more suitable for demonstration purposes. However, other coverage units (such as sum insured) can be used, and the implicit mechanical relationships that hold for earned premiums and loss ratios can be adapted for other coverage units and the corresponding "loss ratio" proxy λ^{ω}.

- For simplicity of calculations, we have assumed a uniform payment pattern. This assumption can be easily relaxed without impacting the sufficient conditions model. In practice, we expect a more sophisticated payment pattern to be applied, possibly derived from traditional actuarial run-off triangle techniques.
- We have assumed a flat discount rate applies across all maturities. In practice, the discount rate varies with maturities. However, we observe that one common approach that enables the use of a flat discount rate is to use the single implied discount rate computed from the projected cash flows and yield curves. Where the variability across the yield curve is small, this provides a reasonable approximation in interest rate accretion calculations. We have also assumed the insurance contracts have a significant financing component to demonstrate the recognition and application of the significant financing component assumption in the sufficient conditions model.
- One immediate limitation of our sufficient conditions model is the assumption that no other contracts are added to the group after initial recognition. We have required this condition because a weighted-average approach to the locked-in rate would result in interest rate accretion calculations that cause discrepancies in the LFRC between the GMM and PAA. In practice, insurance contracts are typically written throughout the year. One modelling approach to deal with this limitation is to conduct "mid-point" calculations and assume that initial recognition occurs at the middle of the year (rather than at the beginning of the year) and to calculate all insurance contracts to be added to the group at that point in time. In this illustrative example, we have applied mid-point calculations to demonstrate that our sufficient conditions model can be adapted with relative ease to accommodate this case. From a modelling perspective, we reflect that mid-point calculations generally provide a good approximation in cases where contracts are relatively homogenous and are uniformly written throughout the year.
- In Section 2, we assumed that the LFRC assumptions do not change throughout the life of the group of contracts and the assumptions selected in Table 1 reflect this condition. This condition is to avoid complications in the contractual service margin arising from changes in fulfilment cash flows relating to future service on subsequent measurement (paragraph 44(c) of IFRS 17) which would trigger discrepancies in the LFRC between the GMM and PAA. However, in practice, changes to yield curves as well as cash flow assumptions are common as more information on the contracts and economic conditions becomes available. Such changes would be the subject of the second criterion of the PAA eligibility test, which requires no significant variability in the fulfillment cash flows that would affect the measurement of the LFRC during the period before a claim is incurred (paragraph 54 of IFRS 17). Notwithstanding this, it is our view that because the sufficient conditions model provides identical LFRC balances in circumstances where there are no changes in assumptions throughout the life of the group of contracts, it would also be the superior model to apply where there are changes in assumptions, as the principle is to reduce the discrepancies between the GMM and PAA (within the realms of interpreting IFRS 17). To this end, we have achieved much success (which has also been supported by anecdotal evidence from our colleagues) in applying our framework as a basis for testing for paragraph 54 reasonably expected scenarios. In general, we have found that the overall discrepancies in the LFRC between GMM and PAA that arise have been much smaller when the base case produces an identical LFRC than methods where discrepancies also arise in the base case. In the latter

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case, the discrepancies would not only include those arising from testing the reasonably expected scenarios, but also the inherent discrepancies from the base case. In applying our sufficient conditions model to assess for reasonably expected scenarios, we have kept the general structure of the model unchanged, and only altered the stressed assumptions (for example, perturbing the "loss ratio" λ^{ω}, payment pattern p^{ω}_{t-k+1} or discount rate i on subsequent measurement). We have found that, in general, the more changes made to the sufficient conditions model, the larger the discrepancies in the LFRC between the GMM and PAA. Ultimately, our sufficient conditions model provides sufficient conditions to achieve identical LFRC in the base case and is of value in testing the reasonably expected scenarios, but does not provide sufficient conditions in the testing of the reasonably expected scenarios. Invariably the reasonably expected scenarios tested are specific to the claim characteristics of the insurance products under consideration and materiality thresholds determined, making it difficult for the development of a set of sufficient conditions.
- In this example, we have assumed yearly balance dates. However, the sufficient conditions model can be adapted to produce monthly balance dates, which would be highly beneficial to insurers which require more frequent reporting of financial results for management purposes.

3.2. Results

Applying the modelling assumptions set out in Section 2 and Table 1, we have provided a more detailed view of the assumptions by time point. Table 2 shows a typical extract of the intermediate calculations involved in deriving the final balances of the LFRC.

In interpreting Table 2, we note that initial recognition is understood to occur at the middle of the first year (time point 0.5) and premiums and acquisition cash flows are understood to occur at the start of each subsequent year, whereas claim payments are understood to occur at the end of each year.

Earned premiums have been calculated at a contract level as at each subsequent measurement balance date (i.e. time points 1, 2, 3, . . .) using the passage of time. The earned premiums for the group is the sum of the earned premiums for each contract, which is consistent with paragraph B119(a) of IFRS 17 where the number of coverage units in a group is determined by considering the number of coverage units provided under each contract in the group.

We have divided up both the gross and net payments according to when the claim is incurred. It is useful to consider the claim payments using this subdivision because the claim payments in the LFRC only reflect those where a claim has not been incurred. Claim payments for claims that have been incurred are reflected in the liability for incurred claims.

Both the contractual service margin earning pattern for the GMM and the amortisation and earning pattern under the PAA follow the formulae presented in Section 2, with suitable adjustments to reflect a time 0.5 initial recognition.

Table 3 shows the LFRC calculated under the GMM at each time point, with detailed breakdowns of the fulfillment cash flows and contractual service margin.

The calculations are based on the formulae presented in Section 2, with suitable adjustments to reflect a time 0.5 initial recognition. Similarly, Table 4 shows the corresponding calculations using the PAA.

Based on our model we observe that the LFRC is identical for the GMM and PAA. Therefore, our illustrative example has provided numerical confirmation in support of our sufficient conditions model. Of more importance, we hope that the illustrative example has provided insight on how the sufficient conditions model can be applied in practice.

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Table 2. Detailed View of Modelling Assumptions by Time Period

Year	Before Initial Recognition	Initial Recognition	Subsequent Measurement										Total
		0.5	1	2	3	4	5	6	7	8	9	10	
Premiums		10,000		1,000	1,000	1,000	1,000						14,000
Contract A		9,000											9,000
Contract B		1,000		1,000	1,000	1,000	1,000						5,000
Acquisition cash flow	1,000	350		50	50	50	50						1,550
Asset	1,000												1,000
Contract A		300											300
Contract B		50		50	50	50	50						250
Earned premiums			2,000	4,000	4,000	2,500	1,000	500					14,000
Contract A			1,500	3,000	3,000	1,500							9,000
Contract B			500	1,000	1,000	1,000	1,000	500					5,000
Gross claims payments													
Contract A - claims incurred year 1			225	450	225								900
Contract A - claims incurred year 2				450	900	450							1,800
Contract A - claims incurred year 3					450	900	450						1,800
Contract A - claims incurred year 4						225	450	225					900
Contract B - claims incurred year 1			75	150	75								300
Contract B - claims incurred year 2				150	300	150							600
Contract B - claims incurred year 3					150	300	150						600
Contract B - claims incurred year 4						150	300	150					600
Contract B - claims incurred year 5							150	300	150				600
Contract B - claims incurred year 6								75	150	75			300

(*Continued*)

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Table 2. (*Continued*)

Year	Before Initial Recognition	Initial Recognition	Subsequent Measurement										Total
		0.5	1	2	3	4	5	6	7	8	9	10	
Net claim payments													
Claims incurred in year 1			285	570	285								1,140
Claims incurred in year 2				570	1,140	570							2,280
Claims incurred in year 3					570	1,140	570						2,280
Claims incurred in year 4						356	713	356					1,425
Claims incurred in year 5							143	285	143				570
Claims incurred in year 6								71	143	71			285
GMM - CSM earning pattern			15.6%	35.2%	51.8%	64.0%	67.7%	100.0%					
PAA - amortisation/earning pattern			16.0%	32.0%	32.0%	20.0%	8.0%	4.0%					

Table 3. Results of GMM LFRC

	Initial Recognition	Subsequent Measurement					
Year	0.5	1	2	3	4	5	6
Fulfilment Cash Flows							
Discounted premiums	−3,634	−3,723	−2,859	−1,952	−1,000		
Discounted acquisition cash flows	182	186	143	98	50		
Discounted net claim payments	6,791	5,873	3,994	2,021	764	259	
Risk adjustment	679	587	399	202	76	26	
Closing Balance	4,019	2,923	1,677	368	−110	285	0
Contractual Service Margin							
Initial Recognition							
Fulfilment cash flows at initial recog.	−4,019						
Cash flow at initial recognition	9,650						
Derecognition of asset/liability	−1,000						
Subsequent Measurement							
New contracts							
Interest accreted		114	200	136	69	26	9
Changes in fulfilment cash flows							
Currency exchange differences							
Insurance revenue		−741	−1,482	−1,482	−926	−370	−185
Closing Balance	4,631	4,005	2,723	1,378	521	176	0
Liability for Remaining Coverage	**8,650**	**6,928**	**4,400**	**1,746**	**411**	**461**	**0**

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Table 4. Results of PAA LFRC

	Initial Recognition	Subsequent Measurement					
	0.5	1	2	3	4	5	6
Initial Recognition							
Premiums received	10,000						
Acquisition cash flow	−350						
Derecognition of asset/liability	−1,000						
Subsequent Measurement							
Premiums received			1,000	1,000	1,000	1,000	
Acquisition cash flow			−50	−50	−50	−50	
Amortisation of acquisition cash flow		245	490	490	306	123	61
Finance component		214	394	267	135	68	23
Insurance revenue		−2,181	−4,362	−4,362	−2,726	−1,090	−545
Investment component							
Closing balance	**8,650**	**6,928**	**4,400**	**1,746**	**411**	**461**	**0**

4. Practicalities of Our Sufficient Conditions Model

Despite the mathematical appeal of our sufficient conditions model, ultimately the LFRC is influenced by the accounting policies adopted by an insurer. Notwithstanding, we observe that the insights gained from our sufficient conditions model offer IFRS 17 practitioners with the following set of opportunities:

- Our sufficient conditions provide a mathematical lens on which to refine accounting policies where the objective is to minimise the gap between the GMM and PAA when computing the LFRC
- Given our sufficient conditions model eliminates all discrepancies between the GMM and PAA, it provides a benchmark on which to test the mathematical optimality of different accounting policies and provide management with a framework to quantify and articulate the impact of each assumption on the LFRC
- Our sufficient conditions model has great potential in reducing the amount of tedious recalibration when performing PAA eligibility testing, as a large part of the testing could be reduced to observing whether the sufficient conditions have been satisfied (a much simpler task) compared to a less scalable approach of a full recalibration every time an assumption is tweaked
- Whilst we have developed the sufficient conditions model for direct insurance contracts, the ideas presented in this paper can, with suitable modifications and subject to complexity of contracts, be extended to reinsurance contracts held, thereby arming an insurer opting for the PAA with a unified modelling approach to address the vast majority of insurance contracts covered under IFRS 17

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Data Availability. The data (from illustrative example) and code (formulae) are as presented in the body of the paper.

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