

## Pruss's ontological arguments

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**Abstract:** First, I suggest that it is possible to make some further improvements upon the Gödelian ontological arguments that Pruss develops. Then, I argue that it is possible to parody Pruss's Gödelian ontological arguments in a way that shows that they make no contribution towards 'lowering the probability of atheism and raising the probability of theism'. I conclude with some remarks about ways in which the arguments of this paper can be extended to apply to the whole family of Gödelian ontological arguments.

The outline of my paper is as follows. In the first part, I construct an ontological argument which, I claim, is an improvement upon the arguments that Pruss produces in the preceding paper (347–353). In the second part, I argue that this ontological argument – and, by extension, each of the arguments that Pruss develops in his paper – is plainly not a successful argument. In particular, I point out that there are, after all, effective parodies of Pruss's arguments. While I don't have space to develop the further argument in this paper, I claim that the preceding observations can be developed into a general critique of Gödelian ontological arguments.

### Further improvements

We might construct an ontological argument beginning with the following two premises:

- Premise 1 If  $A$  is a positive property, then  $\sim A$  is not a positive property.
- Premise 2 If  $\Delta$  is a set of properties all of which are positive, and  $\Delta$  entails  $A$ , then  $A$  is a positive property.

Given these two premises, it is easy to prove the following lemma:

Lemma 1 If  $\Delta$  is a set of properties all of which are positive, then it is possible that there is an object that possesses all of the properties in  $\Delta$ .

Here is the proof. Suppose that  $\Delta$  is a set of properties all of which are positive, that  $A$  is one of the positive properties in  $\Delta$ , and that  $\Delta/A$  is the set of properties that is obtained by removing  $A$  from  $\Delta$ . Suppose that it is not possible that there is an object that possesses all of the properties in  $\Delta$ . If it is not possible that there is an object that possesses all of the properties in  $\Delta$ , then, necessarily, any object that possesses all of the properties in  $\Delta/A$  must possess the property  $\sim A$ . But that's just to say that  $\Delta/A$  entails  $\sim A$ . Contradiction! (Why? Well,  $\Delta/A$  is a set of properties all of which are positive,  $A$  is positive, and yet  $\Delta/A$  entails  $\sim A$ . By Premise 2, since  $\sim A$  is entailed by a set of properties all of which are positive,  $\sim A$  is itself positive; but, by Premise 1, if  $A$  is positive, then  $\sim A$  is not positive.)

If we now give ourselves one more premise:

Premise 3 The following properties are all positive: necessary existence, essential omnipotence, essential omniscience, and essential perfect goodness

then, using Lemma 1, it is a very short step to the conclusion that there is a necessarily existent, essentially omnipotent, essentially omniscient, and essentially perfectly good being. For, if it is possible that there is a necessarily existent, essentially omnipotent, essentially omniscient, and essentially perfectly good being, then, by the characteristic S5 axiom, it is the case that there is a necessarily existent, essentially omnipotent, essentially omniscient, and essentially perfectly good being.

The arguments that Pruss develops are not quite this argument, but they are very close to it. Clearly, we could weaken our second premise without damaging the argument that we sketched above:

Premise 2' If  $\Delta$  is a *finite* set of properties all of which are positive, and  $\Delta$  entails  $A$ , then  $A$  is a positive property.

However, with this weaker premise, we cannot get to the conclusion that there is a being that has *all* of the positive properties, a conclusion that we can reach from Premise 2, provided that there *is* a set consisting of all of the positive properties. This fact about our argument mirrors the difference between Pruss's proofs using his Axiom F3 ('The property of having all strongly positive properties is a positive property', 350), and his Axiom F4 ('If  $A$  and  $B$  are strongly positive and compossible, then their conjunction is positive', 351). It is worth noting that Pruss objects against his Axiom F3 that 'it is more than just a formal claim about the

logic of positive properties' (350), whereas this complaint could hardly be lodged against our Premise 2.

Pruss also objects against his Axiom F3 that it 'is quite a strong claim [that all positive properties are possibly co-exemplified] and one might worry whether an intuition that so quickly entails it can be that plausible' (350). However, in order to determine whether Pruss is well-placed to make this complaint against his Axiom F3, we would do well to look at the justification that Pruss offers on behalf of the other half of his counterpart to our Premise 2:

Axiom F2 If A is positive and A entails B, then B is positive.

What reason might there be to accept Axiom F2 that is not also reason to accept our Premise 2? And is it true that Pruss provides us with some reason to accept Axiom F2 that is not also reason to accept our Premise 2?

Pruss canvasses four interpretations of the notion of a positive property, and argues that, on each of these interpretations, his first two axioms are at least *prima facie* plausible (and, in some cases, necessary). We shall consider each of these interpretations in turn.

*Excellence/greatness/value*

... one might take a positive property to be one that in no respect detracts from any respect of the excellence (or greatness or value, depending on how we prefer to phrase it) of the entity that has the property but whose negation does detract from some respect of the excellence (or greatness or value) of the possessor. ... . The correctness of F1 on the excellence, goodness, and greatness reading is clear. ... . Moreover, if a property doesn't detract from the excellence (or goodness or greatness) of an entity, then anything it entails had better not detract from it either. On the other hand, if a property detracts from the excellence (or goodness or greatness) of an entity, so does any property that entails that property. Hence if  $\sim A$  detracts from excellence (etc.), and A entails B, then  $\sim B$  detracts from excellence (etc.), since  $\sim B$  entails  $\sim A$  by contraposition. This yields Axiom F2 on the excellence, goodness and greatness interpretations. (347–348)

Although it is not entirely clear, I think that the proposed interpretation of the notion of a positive property is as follows:

$A(x)$  is a positive property iff necessarily, for all  $y$ , if  $y$  has  $A(x)$ , then having  $A(x)$  in no way detracts from any respect of the excellence/greatness/value of  $y$ , and if  $y$  has  $\sim A(x)$ , then having  $\sim A(x)$  does in some way detract from some respect of the excellence/greatness/value of  $y$ .

Pruss's argument for his Axiom F2, on this interpretation, has two parts. The first part is just this: 'if a property doesn't detract from the excellence (or goodness or greatness) of an entity, then anything it entails had better not detract from it either' (348). To say the least, it is quite unclear why we should suppose that this claim is any more secure than the claim that: 'if a set of properties doesn't detract from the excellence (or goodness or greatness) of an entity, then any

property that the set entails had better not detract from it either'. Neither claim is much of an argument; but, at this point, we're only interested in their relative merits.

The second part of Pruss's argument for his Axiom F2 on the excellence/goodness/value interpretation is this: '[I]f a property detracts from the excellence (or goodness or greatness) of an entity, so does any property that entails that property. Hence if  $\sim A$  detracts from excellence (etc.), and  $A$  entails  $B$ , then  $\sim B$  detracts from excellence (etc.), since  $\sim B$  entails  $\sim A$  by contraposition' (348) This time, it is not clear why we should suppose that Pruss's argument is any more secure than the argument that 'If a property detracts from the excellence (or goodness or greatness) of an entity, so does any property that entails that property. But, if *every* property in  $\Delta$  is such that its negation does in some way detract from some respect of excellence/greatness/value, and  $\Delta$  entails  $A$ , then  $\sim A$  also detracts from excellence/greatness/value. Why? Because  $\sim A$  entails that it is not the case that, for all  $F$  in  $\Delta$ ,  $F$ ; and *that* property – its not being that case that, for all  $F$  in  $\Delta$ ,  $F$  – plainly detracts from excellence/greatness/value *given* that every property in  $\Delta$  is such that its negation does in some way detract from some respect of excellence/greatness/value.'

In light of the above considerations, it seems to me to be reasonable to conclude that the considerations that Pruss advances on behalf of his Axiom 2 are no more compelling than the parallel considerations that can be advanced on behalf of my Premise 2. Of course, this leaves it open that there might be some other considerations that favour his Axiom 2 above my Premise 2. However, it seems to me that, until any such further considerations are advanced, it is reasonable to conclude that there is no reason to prefer his Axiom 2 above my Premise 2, on the excellence/greatness/value interpretation of positive properties.

### *Limitation*

... one might take a positive property to be one that does not entail any limitation but whose negation does ... . The correctness of F1 on the ... no-limitation reading is clear. ... Exactly the same reasoning [as for the excellence/greatness/value interpretation] shows that if a property does not entail any limitation but its negation does, the same holds for any property that it entails. (347–348)

In this case, I think – though, again, I'm not entirely sure – that the proposed interpretation of the notion of a positive property is as follows:

$A(x)$  is a positive property iff necessarily, for all  $y$ , if  $y$  has  $A(x)$ , then having  $A(x)$  in no way entails that  $y$  is limited in any respect, and if  $y$  has  $\sim A(x)$ , then having  $\sim A(x)$  does entail that  $y$  is limited in some respect.

Suppose, first, that  $A(x)$  is a property that does not entail limitation. If  $A(x)$  entails  $B(x)$ , then it is clear that  $B(x)$  cannot entail limitation (else, by the transitivity of entailment,  $A(x)$  would entail limitation). Moreover, it makes no

difference to this argument if we consider, instead, a set of properties, none of which entails limitation. So, in this case, the arguments are both good and plainly one a par.

Suppose, on the other hand, that  $\sim A(x)$  does entail limitation. Since  $A(x)$  entails  $B(x)$ , it is clear that  $\sim B(x)$  must entail limitation (because, by contraposition,  $\sim B(x)$  entails  $\sim A(x)$ ). In the parallel case, if the negation of every member of  $\Delta$  entails limitation, then it must be that  $\sim A$  entails limitation, because  $\sim A$  entails that it is not the case that for all  $F$  in  $\Delta$ ,  $F$  (and, for each member of  $\Delta$ , its negation entails limitation). Again, in this case, it seems that both arguments are good, and that we have no more reason to accept the one than we do to accept the other.

### *Leibniz*

One might start with a somewhat Leibnizian structure of the space of properties, on which there are some basic properties that are mutually compatible (e.g. because they are logically independent of each other) and then count a property as positive provided that it is entailed by at least one of the basic properties. ... on the Leibnizian interpretation,  $F_1$  follows from the compatibility of the basic properties. ... And closure under entailment is trivial on the Leibnizian interpretation, so  $F_2$  follows once again. (347–348)

Clearly, we could propose an alternative Leibnizian interpretation according to which a property is positive provided that it is entailed by at least one set of the basic properties (or, if we prefer, by the set of all of the basic properties). It is no less obvious that our Premise 1 and Premise 2 will be satisfied in this interpretation than it is that Pruss's  $F_1$  and  $F_2$  are satisfied in his Leibnizian interpretation.

### *Maydole*

'[F]ollowing Robert Maydole's discussion of "perfections", one might take a positive property to be one that it is better to have than not ... Maydole gives arguments for  $F_1$  and  $F_2$  on his interpretation (Maydole (2003), 302)' (347–348).

Elsewhere, (Oppy (2004, 2007)), I have argued at length for the falsity of  $F_2$  on Maydole's interpretation of positive properties. In short, the core of that argument is as follows. Maydole should be understood to be claiming that  $A(x)$  is a positive property iff necessarily, for all  $y$ , if  $y$  has  $A(x)$ , then it is better that  $y$  has  $A(x)$  than it is that  $y$  fails to have  $A(x)$ . But, if we consider a disjunctive property, one of whose disjuncts is a property that is better to have than not, and the other of whose disjuncts is a property that it is worse to have than not, then it seems wrong to say that, necessarily, for all  $y$ , if  $y$  has the disjunctive property, then it is better than not that  $y$  has the disjunctive property. For, surely, whether it is better than not that  $y$  has the disjunctive property depends upon which of the disjuncts of the disjunctive property are possessed by  $y$ . If  $y$  possesses the disjunctive property only because  $y$  possesses the disjunct that it is worse to have than not, then surely we should not say that, in the case of  $y$ , the disjunctive property is

better to have than not. And yet the disjunctive property is *entailed* by the disjunct that, *ex hypothesi*, it is better to have than not. So F<sub>2</sub> fails under Maydole's interpretation of positive property.

Of course, for our present purpose, it doesn't matter whether this objection to Maydole is cogent. Even if Maydole's interpretation survives this objection, it is plainly true that we do no worse if we replace Pruss's Axiom 2 with our Premise 2, making use of Maydole's interpretation of the notion of a positive property.

### *Conclusion*

The upshot of this examination of Pruss's discussion of possible interpretations of the notion of a positive property is that, for all that he says, there is no reason to prefer his Axiom 2 to our Premise 2. It is true, of course, that his Axiom 2 is strictly weaker than our Premise 2. But, on the one hand, he then needs to introduce an additional axiom in order to get out some of the conclusions that we get out using just our Premise 2. And, on the other hand, at least for all that we have been able to discover so far, there is no independent motivation for Axiom 2 that is not also motivation for our Premise 2: apart from the fact that Axiom 2 is weaker than Premise 2, there is nothing else that favours Axiom 2 above our Premise 2. So, I think, there is some reason to prefer the ontological argument that we have constructed to the ontological arguments that Pruss offers in his paper.

### **An effective parody**

What should someone who does not believe that there is a necessarily existent, essentially omnipotent, essentially omniscient, and essentially perfectly good being say in response to the above ontological argument? Discussing his own variants of the argument, Pruss claims that:

Whether the arguments are sound will, I think, depend on a deeper analysis of the nature of positivity. But the premises all appear at least somewhat plausible, are largely independent of the premises of non-ontological arguments such as the cosmological argument or the argument from religious experience, and hence the Gödelian arguments should further lower the probability of atheism and increase that of theism. (352)

Moreover, Pruss also claims that, 'the full Anderson axiom is essential to the parodies [of Gödel's ontological argument] in Oppy (1996, 2000), while the present argument does not appear to be subject to those parodies (though I have no argument that other parodies cannot be constructed)' (350).

I think that a good place to start to think about the merits of *our* Gödelian ontological argument is with the following parody.

By stipulative definition, a *natural* property is a property whose instantiation in no way entails the existence of any supernatural entities, or the holding of supernatural states of affairs, or the like, but the instantiation of whose negation

does in some way entail the existence of supernatural entities, or the holding of supernatural states of affairs, or the like. Given this definition of what it is to be a natural property, the following two premises are at least plausible:

- Premise 1' If A is a natural property, then  $\sim A$  is not a natural property.  
 Premise 2' If  $\Delta$  is a set of natural properties, and  $\Delta$  entails A, then A is a natural property.

In fact, Premise 1' is immediate from the definition of natural property; and Premise 2' seems guaranteed by the transitivity of entailment.

In order to derive a suitably 'naturalistic' conclusion, we need only to add the following premise:

- Premise 3' The following property is natural: having no world-mate that is a necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being.

At least by 'naturalistic' lights, it seems highly plausible to claim that the instantiation of this property in no way implies the existence of any supernatural entities, or the holding of supernatural states of affairs, or the like, but that the instantiation of the negation of this property does in some way imply the existence of supernatural entities, or the holding of supernatural states of affairs, or the like, since the instantiation of the negation of this property entails the quintessentially supernaturalistic claim that there is a necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being.

Premise 1' and Premise 2' together entail that, for any set of natural properties, it is possible that there is some entity that instantiates all of the properties in that set. Hence, given Premise 3', we have that it is possible that there is something that inhabits a possible world in which there is no necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being. But, by S5, if it is possible that there is no necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being, then there *is* no necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being. QED.

QED? Not really. Anyone who thinks that there is a necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being also thinks that it is *impossible* for there to be an entity that has no world-mate that is a necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being. But a necessarily uninstantiated property entails all properties. Consequently, someone who thinks that there is a necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being must object to the conjunction of the premises of the argument: if Premise 3' is true, then it is not the case that the natural properties are non-trivially closed under entailment; and if the natural properties are non-trivially

closed under entailment, then, contrary to initial appearances, it cannot be that the property of having no world-mate that is a necessarily existent, essentially omnipotent, essentially omniscient, essentially perfectly good being is natural.

Which way you go depends upon what you are prepared to say about the ‘possibility’ that there are necessarily uninstantiated natural properties. If you hold that it is impossible for there to be necessarily uninstantiated natural properties, then you reject Premise 3’; if, on the other hand, you hold that it is possible for there to be necessarily uninstantiated natural properties, then either you reject Premise 2’, since necessarily uninstantiated natural properties entail all properties, including properties that are not natural, or else you reject Premise 1’ because the negation of a necessarily uninstantiated natural property is natural. If our ‘naturalistic’ ontological argument has a superficial plausibility, this is only because, when we consider things from the standpoint of the naturalist, we overlook the considerations that would be raised by necessarily uninstantiated natural properties (even though, by the lights of the naturalist, there are – and can be – no such properties).

Now, of course, what goes for the parody argument also goes for the Gödelian ontological argument that was being parodied. If someone supposes that there is no necessarily existent, essentially omniscient, essentially omniscient, essentially perfectly good being, then that person will either reject Premise 3 or at least one of Premises 1 and 2, depending upon a decision that is to be made about the possibility that there are necessarily uninstantiated positive properties. If you hold that it is impossible for there to be necessarily uninstantiated positive properties, then you reject Premise 3; if, on the other hand, you hold that it is possible for there to be necessarily uninstantiated positive properties, then you reject at least one of Premises 1 and 2.

In deciding what to say about Premise 3 and Premises 1 and 2, it is crucial for the opponent of the argument to first make a decision about what to say about the case of necessarily uninstantiated positive properties. If our Gödelian ontological argument has a superficial plausibility, this is only because, when we consider things from the standpoint of the theist, we overlook the considerations that would be raised by necessarily uninstantiated positive properties (even though, by the lights of the theist, there are – and can be – no such properties).

If my argument to this point is good, then we can safely conclude that Pruss was right to observe that he had no argument ‘that other parodies [to his argument] could not be constructed’ (350). Of course, Pruss is also right to note that his Gödelian ontological argument is not subject to the parody that I constructed for Anderson’s formulation of Gödel’s ontological argument. But it should not be surprising that different ontological arguments are subject to different kinds of parodies: this is an already familiar lesson from the history of the discussion of ontological arguments (cf. Oppy (1995), chapter 11).



Furthermore, if my argument to this point is good, then we can conclude that it is not really to the main point to suppose that a better developed view of the soundness of Pruss's argument must await 'a deeper analysis of the nature of positivity' (352). It seems reasonable to suppose that Pruss's argument is sound just in case there is a necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being; and that the naturalist's parody of Pruss's argument is sound just in case there is no necessarily existent, essentially omnipotent, essentially omnipotent, essentially perfectly good being. But, if that's right, then neither argument gets us anywhere. And, in particular, neither argument has the consequence that it raises or lowers the probability of either theism or atheism.

### Concluding remark

There is unfinished business that cannot be taken up in the present paper. First, I claim that my critique of Maydole (2003) actually extends to all of the interpretations that give substantive content to the notion of a positive property (e.g. to Pruss's excellence/greatness/value and limitation interpretations). Second, I claim that my critique of Pruss actually extends to all Gödelian ontological arguments: in the light of considerations about necessarily uninstantiated positive properties, opponents of these arguments can always either reasonably reject some of the axioms or premises which say directly that certain properties are positive properties (Pruss's 'non-formal' axioms), *or* else reasonably reject some of the axioms or premises which impose general constraints on positive properties (Pruss's 'formal' axioms). However, development of these points will need to wait for another occasion.

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