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# DEATH AND CAPITAL

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In a life-cycle model with dynastic households, parents value the transfer of tangible assets to their offspring in the event of premature death. This raises the subjective reward from investing in them relative to intangible human capital and tilts investment choice away from the latter. These effects of mortality on human capital risk and relative investment can translate into divergent growth paths, delayed transition from physical to human capital accumulation, and a dampened response to mortality shock in developing countries.

**Keywords:** Mortality, Human Capital Risk, Altruism, Growth

## 1. INTRODUCTION

We study the effect of adult mortality in a life-cycle economy with dynastic households. Specifically, we emphasize a novel channel through which it affects the pattern of investment and economic development.

There are multiple ways in which mortality is linked to household consumption and savings decisions. A commonly studied one is its negative effect on the enjoyment of future utilities because of which households prioritize present consumption and invest less. This can discourage economic development as in Ram and Schultz (1979), Gersovitz (1983), Chakraborty (2004), Lorentzen et al. (2008), and Jayachandran and Lleras-Muney (2009), among others.

That assets can be passed down generations mitigates the problem for altruistic households. Yet, not all assets can be readily bequeathed. Physical assets such as capital, land, and livestock are tangible and transferable in a way that human capital is not. This introduces a difference in how parents subjectively value investment in physical assets relative to human capital. The difference is particularly salient under lifetime uncertainty if parents value unintended bequests.

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As a result, the risk of premature death tilts portfolio choice toward tangible investment.[1] It follows that the predominant form of asset accumulation in developing countries with high mortality will be physical capital; patterns of investment and production shift toward human capital when life-cycle uncertainty falls with economic development.

Two stylized facts are relevant for this result. First, mortality declined sharply in the late 19th and early 20th centuries in the West due, in large measure, to exogenous improvements in public health and medicine [Wrigley and Schofield (1981), Szreter (1988), Dobson (1997), and Cutler and Miller (2005)]. Second, as documented by Abramovitz and David (2000), Goldin and Katz (2001), and Galor and Moav (2004), there was a concomitant transition from physical capital to human capital as the primary engine of growth. These two transitions become related in our model: during the initial stages of development, high mortality is accompanied by investment in transferable assets (physical capital, land) while in later stages, lower mortality from (possibly exogenous) health improvements is accompanied by investment in human capital.[2]

There are two distinct mechanisms through which mortality influences the accumulation of human capital in the model. First, high mortality lowers the expected return on human capital vis-a-vis physical capital. Second, high mortality makes human capital investment riskier. The latter may induce risk-averse agents to underinvest in human capital even when the expected return to human capital rises. To the extent that technological progress is complementary to human capital, high mortality may therefore result in delayed adoption of modern technologies when they are available.[3]

Evidence of the differential effect of mortality is discernible even in contemporary experiences. For example, Fortson (2011) argues that while the growth effect of the HIV epidemic in sub-Saharan Africa has been ambiguous, it had a definite negative effect on schooling and human capital formation. This would suggest that the loss of output from lower human capital formation was attenuated by other effects; a shift toward physical assets is one possibility. Put differently, the tendency of altruistic families to overaccumulate physical assets under life-cycle uncertainty suggests that the cost of epidemic shocks can be relatively lower in developing countries that face already-high mortality risks.

Indeed adjusting the portfolio of asset stocks for consumption smoothing purposes in the face of idiosyncratic income (not necessarily mortality) shocks is not uncommon in developing countries where insurance mechanisms are weak. Examining data from rural India, Jacoby and Skoufias (1997) find that seasonal fluctuations in income were accompanied by seasonal fluctuations is children's school attendance where child labor was used as a mechanism to smooth consumption instead of borrowing. Based on a study of consumption and investment behavior of Indian farmers, Rosenzweig and Wolpin (1993) conclude that when hit by adverse weather conditions, farmers are more likely to sell their livestock than jewelry or land. Similar self-insurance mechanism for consumption smoothing are reported by Janzen and Carter (2013) in the context of Kenya.

It is well known in the literature that adult mortality affects the return to investment and growth.[4] Many of these studies look at either the relationship between mortality and the effective rate of time preference or a single productive asset. Even when both physical and human capital are considered, the difference in their inheritability does not play a role. Razin (1976) is an early contribution that links mortality to the choice between human capital and other investments. His analysis is restricted to exogenous factor prices and impure altruism. In a dynamic general equilibrium framework, asset returns respond to factor accumulation and incentives change over time. By identifying clearly general equilibrium effects that amplify the portfolio choice margin and how resource sharing within households responds to mortality, our work emphasizes the role of the mortality in the transition from physical capital- to human capital-based development.

Also related is Minamimura and Yasui's (2019) recent work in which high mortality delays the transition from physical to human capital by lowering the latter's expected return. Human capital risk has no role in their story. Yet risk is a central element of human capital acquisition. First, the inalienability of human capital limits the scope for diversification [Levhari and Weiss (1974)]. Second, that same inalienability also restricts the investment choice of firms that offer various insurance instruments (e.g., annuities and life insurance) for household risk diversification. Therefore, as long as mortality risk is high, physical capital will continue to be a major channel for investment, either directly by households or indirectly by insurance providers who earn their return on the capital market.

The structure of the paper is as follows. The following section presents the overall framework. Section 3 analyzes household decisions and the portfolio allocation problem. In section 4, we study corner equilibria in which households invest in only one asset. The general equilibrium analysis of Section 5 looks at the growth effects of the mortality transition and other empirical implications. Section 6 concludes.

## 2. STRUCTURE OF THE ECONOMY

In a discrete-time overlapping-generations economy, a unit measure of agents are born every period. Each agent potentially lives for two periods, "youth" and "middle-age." She lives in youth for sure but survives into middle-age with a constant (exogenous) probability $p \in [0, 1]$. She gives birth to a single offspring in youth (before the mortality shock is realized) and does not do wage work, implicitly spending her time raising the child, managing assets, and acquiring human capital, if at all.

A young agent receives an endowment in the first period from her parent that is used for own consumption and asset accumulation. There are two income-generating assets she can invest in: tangible physical capital and intangible human capital. Physical capital is transferable across agents while human capital is not. If the agent survives into middle age, she earns capital income and labor income from the investments made in youth. She consumes a part $\theta$ of this

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income and transfers the remainder to her offspring as *intended* bequest. If she dies prematurely, instead, the offspring inherits the tangible asset of the parent, income from which constitutes her first period endowment. We call this *unintended* or *accidental* bequest. The altruistic agent derives utility from both forms of bequests.

## 2.1. Preferences

Agents have identical preferences. The expected lifetime utility $V_t$ of a young adult at $t$ with income endowment $y_t$ received either as intended or unintended bequest, both of which bring utility, is

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}. \quad (1)$$

Here $\beta \in (0, 1)$ is the subjective discount rate, $\gamma > 0$ represents the intensity of parental altruism, and consumption utility from death has been normalized to zero. Standard Inada conditions apply to $u$. Even though altruism is pure in that parents care about their offspring's lifetime welfare, they do not necessarily discount their offspring's lifetime utility at the same rate as they discount their own future consumption. It may be plausibly assumed that $\gamma \leq \beta$.

## 2.2. Income and Endowment

All individuals are born with the same level of innate skills normalized to zero. We also assume that the physical capital stock depreciates completely within one generation.

Let $w_t$ denote the wage rate per unit of human capital (skill) and $\rho_t$ denote the return on physical capital in period $t$. Suppose a young agent has invested $x_t$ units in physical capital and $e_t$ units in human capital in the first period of her life. Assuming full depreciation, future asset levels are $k_{t+1} = x_t$ for physical capital and, if she survives, $h_{t+1} = e_t$ for human capital.

Hence, should the agent survive to middle age, with probability $p$, she will earn the income $w_{t+1}e_t + \rho_{t+1}x_t$ and share $1 - \theta_t \in [0, 1]$ proportion of it with her offspring. On the other hand if she dies prematurely, with probability $1 - p$, the offspring inherits the entire physical capital stock $x_t$ and receives the income $\rho_{t+1}x_t$. Thus, the first period endowment of the offspring is stochastic, given by

$$y_{t+1} = \begin{cases} (1-\theta_t)\,(\rho_{t+1}x_t + w_{t+1}e_t) & \text{w.p. } p, \\ \rho_{t+1}x_t & \text{w.p. } 1-p. \end{cases}$$

Note the asymmetry in the endowment process: the offspring does not get to access any portion of the parent's labor earnings in case of premature death while she receives the entire capital income.

We anticipate that in equilibrium, $\theta_t < 1$. Since consumption in youth comes out of parental income, the agent will always choose to share with her offspring because of the Inada condition on $u(c_{1t+1})$. One way to allow for a no-sharing equilibrium is to introduce an independent source of income, like labor earnings, in youth. As in standard dynastic models, no-sharing would occur for low values of $\gamma$. For $\theta_t = 1$, the offspring does not materially benefit from parental survival but still receives the entire capital income from parental death. Expectation of this unintended bequest brings utility to the parent (as long as $\gamma > 0$) even when she does not leave intended bequest. It is this margin that differentially affects the perceived return on physical capital[5] and makes the decision problem different from the standard dynastic household framework where an inoperative (intended) bequest motive breaks the intergenerational link.

### 2.3. Aggregate Production

The unique final good is produced from aggregate capital $K$ and labor $H$ using a technology $F(K, H)$ that is CRS and subject to diminishing marginal products. Perfectly competitive goods and factor markets imply the standard factor pricing relations

$$\rho_t = F_K(K_t, H_t) \text{ and } w_t = F_H(K_t, H_t) \tag{2}$$

for all $t$, for the rental rate of capital and wage per unit of human capital, respectively. The implicit interest rate $r_t = \rho_t - 1$ applies to saving and investment in period $t-1$ that yields capital at the beginning of $t$.

## 3. HOUSEHOLD OPTIMIZATION

We start by studying how mortality affects the portfolio allocation problem in partial equilibrium where rates of return to physical and human capital are exogenous to household decisions. In fact these returns are taken to be time-invariant, $\rho_t = \rho$, $w_t = w$ for all $t$, a conjecture verified later as general equilibrium outcomes for two aggregate technologies.

As noted above, a generation-$t$ agent has the stochastic endowment $y_t$. It depends on parental survival whose realization we denote by $z_{t-1} \in \{a, d\}$ corresponding to "alive" and "deceased" respectively:

$$y_t \equiv y(z_{t-1}) = \begin{cases} (1-\theta_{t-1})\,(\rho x_{t-1} + w e_{t-1})\,, & \text{if } z_{t-1} = a, \\ \rho x_{t-1}, & \text{if } z_{t-1} = d. \end{cases} \tag{3}$$

Given $y_t$, her decision problem then is

$$V_t(y_t) = \max\,\{u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}\,(y_{t+1})\}$$

subject to

$$c_{1t} + x_t + e_t = y_t,$$
$$c_{2t+1} = \theta_t(\rho x_t + w e_t),$$

and (3) led one period forward. Expectations are taken with respect to $y_{t+1}$ which depends on $z_t \in \{a, d\}$ that is *i.i.d.* across agents belonging to generation $t$.

There are two sources of uncertainty in the model. The first period endowment received by an agent depends on the realization of parental mortality. But it is known to the agent by the time she takes consumption and investment decisions. Given the endowment, utilities from second period consumption and bequest left to the progeny are also uncertain: they depend on the realization of own mortality shock. The agent's decisions will, therefore, depend on her attitude toward risk. In what follows, we assume that agents are risk-averse with preferences taking the CRRA/CES form

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \ \ \sigma > 0. \tag{4}$$

The coefficient of risk aversion is $\sigma$ while the intertemporal elasticity of substitution is $1/\sigma$. As $\sigma \to 1$, $u(c) \to \ln c$. A strictly positive $\sigma$ indicates risk aversion and ensures strict concavity of the objective function and positive saving/investment at all values of $p$.[6]

Physical capital, because of its ready transferability, is a relatively safe asset: it generates utility in both states of nature. Human capital, on the other hand, is riskier in that it generates utility only when the agent survives to middle age. A forward-looking young agent decides on her optimal asset portfolio after taking into consideration the future risk and return of the two assets. Two alternative institutional setups are considered. In the first, $\theta$ is exogenously given, for example, by social customs and convention. In the second case, parents optimally determine how much to share with their offspring taking into account associated trade-offs. While there is additional insight to be gained in how lifetime uncertainty affects $\theta$, the fundamental portfolio allocation problem is not sensitive to whether or not $\theta$ is exogenous.

### 3.1. Portfolio Choice Under Exogenous $\theta$

For exogenous $\theta$ and (4), optimization with respect to $x_t$ and $e_t$ yields the following first-order conditions (FOCs) in an interior optima:

$$c_{1t}^{-\sigma} = p\beta\theta\rho\, c_{2t+1}^{-\sigma} + \gamma E_t\left[\frac{\partial V_{t+1}}{\partial x_t}\right],$$
$$c_{1t}^{-\sigma} = p\beta\theta w\, c_{2t+1}^{-\sigma} + \gamma E_t\left[\frac{\partial V_{t+1}}{\partial e_t}\right]$$

with the corresponding Envelope conditions

$$E_t\left[\frac{\partial V_{t+1}}{\partial x_t}\right] = p(1-\theta)\rho\,(c_{1t+1}|_{z_t=a})^{-\sigma} + (1-p)\rho\,(c_{1t+1}|_{z_t=d})^{-\sigma},$$
$$E_t\left[\frac{\partial V_{t+1}}{\partial e_t}\right] = p(1-\theta)w\,(c_{1t+1}|_{z_t=a})^{-\sigma}.$$

As is common to this class of dynamic programming problems with homothetic preferences and full depreciation of capital, we exploit the *guess and verify*

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method to solve for investment decisions. Let us conjecture that the investments are proportional to the endowment received, such that

$$\left.\begin{aligned} x_t &= \mu y_t(z_{t-1}) \\ e_t &= \nu y_t(z_{t-1}) \end{aligned}\right\} \text{ for all } t;\ z_{t-1} \in \{a, d\},$$

where $\mu$ and $\nu$ are the (yet unknown) investment propensities in physical and human capital respectively. Denoting by $y_t^a$ and $y_t^d$ the endowments for $z_{t-1} = a$ and $z_{t-1} = d$, we then have the consumption functions $c_{1t}|_{z_{t-1}=a} = (1-\mu-\nu)y_t^a$, $c_{1t}|_{z_{t-1}=d} = (1-\mu-\nu)y_t^d$, $c_{2t+1}|_{z_{t-1}=a} = \theta\,(\rho\mu + w\nu)\,y_t^a$, and $c_{2t+1}|_{z_{t-1}=d} = \theta\,(\rho\mu + w\nu)\,y_t^d$. Using these in the FOCs above gives a pair of equations that solve for the investment rate $\mu$ and $\nu$:

$$\begin{aligned} (1-\mu-\nu)^{-\sigma} = {} & p\beta\rho\theta^{1-\sigma}(\rho\mu+w\nu)^{-\sigma} \\ & + p\gamma\rho(1-\theta)^{1-\sigma}(\rho\mu+w\nu)^{-\sigma}(1-\mu-\nu)^{-\sigma} \\ & + (1-p)\gamma\rho(\rho\mu)^{-\sigma}(1-\mu-\nu)^{-\sigma}, \end{aligned} \tag{5}$$

$$\begin{aligned} (1-\mu-\nu)^{-\sigma} = {} & p\beta w\theta^{1-\sigma}\,(\rho\mu+w\nu)^{-\sigma} \\ & + p\gamma w\,(1-\theta)^{1-\sigma}\,(\rho\mu+w\nu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma}. \end{aligned} \tag{6}$$

The solutions are constant, as conjectured:

$$\mu = \left[\gamma(1-p)\right]^{1/\sigma}\frac{1}{\rho}\left(\frac{w\rho}{w-\rho}\right)^{1/\sigma}, \tag{7}$$

$$\nu = \frac{(p\gamma w)^{1/\sigma} + (p\beta w)^{1/\sigma}}{w + (p\beta w)^{1/\sigma}} - \left(\frac{\rho + (p\beta w)^{1/\sigma}}{w + (p\beta w)^{1/\sigma}}\right)\mu. \tag{8}$$

Note that the solution to $\mu$ makes sense only if $w > \rho$. It is easy to see why it is necessary. If $w < \rho$, physical capital dominates human capital—it is less risky and yields a return which is at least as high—and no investment in human capital would occur; neither (7) nor (8) would apply. The actual restriction required for $\nu > 0$ is, however, tighter, and analyzed in Section 4.

From (7) and (8), in an interior equilibrium and given factor prices, $\mu$ is a decreasing function of $p$ while $\nu$ is an increasing function of $p$. In other words, since investment in both comes out of the same endowment, a higher survival probability shifts investment toward human capital. Interestingly, the investment propensities are independent of $\theta$. Societies where close-knit family ties and social customs dictate income sharing within the household differ from others only in terms of investment *level*, not propensities. The homotheticity of $u(c)$ ensures that the intertemporal decisions—saving and investment—depend on relative consumption across periods, not their levels. This separates it from within-period sharing of income, that is, the consumption level achieved by the parent in middle age. Moreover, since a fraction of *total* income is being shared when the realized state is $z_t = a$, the parent's subjective costs and benefits are symmetric for physical and human capital investment within the period. Hence $\theta$ does not affect the trade-off between physical and human capital.

### 3.2. Portfolio Choice Under Endogenous $\theta$

Suppose now that parents also optimize over how much of middle-age income to share with their offspring. Similar to before, in an interior optimum, we have the FOCs

$$c_{1t}^{-\sigma} = p\beta\theta_t\rho\, c_{2t+1}^{-\sigma} + \gamma E_t\left[\frac{\partial V_{t+1}}{\partial x_t}\right],$$

$$c_{1t}^{-\sigma} = p\beta\theta_t w\, c_{2t+1}^{-\sigma} + \gamma E_t\left[\frac{\partial V_{t+1}}{\partial e_t}\right],$$

$$p\beta\rho c_{2t+1}^{-\sigma} x_t + \gamma E_t\left[\frac{\partial V_{t+1}}{\partial e_t}\right] = 0,$$

for $x_t$, $e_t$, and $\theta_t$, and the corresponding Envelope conditions

$$E_t\left[\frac{\partial V_{t+1}}{\partial x_t}\right] = p(1-\theta_t)\rho\,(c_{1t+1}|_{z_t=a})^{-\sigma} + (1-p)\rho\,(c_{1t+1}|_{z_t=d})^{-\sigma},$$

$$E_t\left[\frac{\partial V_{t+1}}{\partial e_t}\right] = p(1-\theta_t)w\,(c_{1t+1}|_{z_t=a})^{-\sigma},$$

$$E_t\left[\frac{\partial V_{t+1}}{\partial \theta_t}\right] = -p\,(c_{1t+1}|_{z_t=a})^{-\sigma}\,(\rho x_t)\,.$$

Apply again the *guess and verify* method to solve for optimal $x_t$, $e_t$, and $\theta_t$ by conjecturing that decision rules are proportional to endowment, $x_t = \mu y_t(z_{t-1})$ and $e_t = \nu y_t(z_{t-1})$, where $\mu$ and $\nu$ are to be determined. Substituting these into the FOCs, $(\mu, \nu, \theta_t)$ solve

$$\begin{aligned}(1-\mu-\nu)^{-\sigma} = {} & p\beta\rho\theta_t^{1-\sigma}\,(\rho\mu + w\nu)^{-\sigma} \\ & + p\gamma\rho\,(1-\theta_t)^{1-\sigma}\,(\rho\mu + w\nu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma} \\ & + (1-p)\gamma\rho\,(\rho\mu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma},\end{aligned} \tag{9}$$

$$\begin{aligned}(1-\mu-\nu)^{-\sigma} = {} & p\beta w\theta_t^{1-\sigma}\,(\rho\mu + w\nu)^{-\sigma} \\ & + p\gamma w\,(1-\theta_t)^{1-\sigma}\,(\rho\mu + w\nu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma},\end{aligned} \tag{10}$$

$$\beta\theta_t^{-\sigma}\,(\rho\mu + w\nu)^{-\sigma} = \gamma\,(1-\theta_t)^{-\sigma}\,(\rho\mu + w\nu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma}. \tag{11}$$

The first two are identical to the exogenous $\theta$ case. Hence the solutions for $\mu$ and $\nu$ will be the same, (7) and (8) that are independent of $\theta_t$. Then from (11), the optimal value of $\theta_t$ is

$$\theta_t = \frac{(1-\mu-\nu)(\beta/\gamma)^{1/\sigma}}{1 + (1-\mu-\nu)(\beta/\gamma)^{1/\sigma}} = \theta, \tag{12}$$

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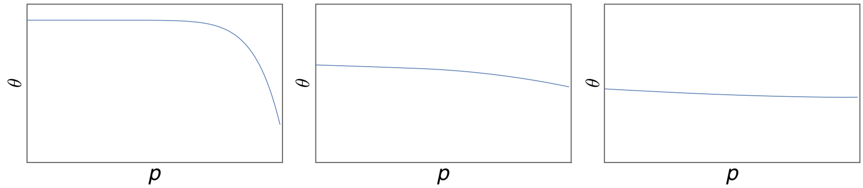


**FIGURE 1.** Optimal $\theta$ for $\sigma \in \{0.1, 0.5, 1.3\}$.

which is time-invariant. The constancy of $\theta$ follows from the constancy of $\mu$ and $\nu$, which of course is possible because of homothetic preferences, full depreciation of capital, and time-invariant factor returns.

*3.2.1. Log specification.* To understand how these choices depend on $p$, let's first consider log preferences ($\sigma = 1$) for which analytically simple expressions can be obtained:

$$\mu = \gamma(1-p)\left(\frac{w}{w-\rho}\right), \tag{13}$$

$$\nu = \frac{\gamma + p\beta}{1+p\beta} - \mu, \tag{14}$$

$$\theta = \frac{(1-\mu-\nu)(\beta/\gamma)}{1+(1-\mu-\nu)(\beta/\gamma)}. \tag{15}$$

Here $\mu + \nu$ is clearly increasing and $\theta$ falling in $p$.

*3.2.2. General specification.* For the more general CES function with $\sigma \neq 1$, we rely on numerical results. Figure 1 presents three cases based on $\sigma$ that show $\theta$ to be monotonically decreasing in $p$.[7] Where $\sigma$ matters is in determining the responsiveness of $\theta$ to $p$: for higher values, $\theta$ falls more slowly with $p$. To understand why, rewrite the FOCs in terms of consumption

$$(c_{1t})^{-\sigma} = p\beta\theta\rho\,(c_{2t+1})^{-\sigma} + \gamma p(1-\theta)\rho\,(c_{1t+1}|_{z_t=a})^{-\sigma} + \gamma(1-p)\rho\,(c_{1t+1}|_{z_t=d})^{-\sigma}, \tag{16}$$

$$(c_{1t})^{-\sigma} = p\beta\theta w\,(c_{2t+1})^{-\sigma} + \gamma p(1-\theta)w\,(c_{1t+1}|_{z_t=a})^{-\sigma}, \tag{17}$$

$$\beta(c_{2t+1})^{-\sigma} = \gamma\,(c_{1t+1}|_{z_t=a})^{-\sigma}. \tag{18}$$

Using the offspring's budget constraint and investment rules in (18), we get the optimality condition

$$\frac{\theta}{(1-\theta)(1-\mu-\nu)} = \frac{c_{2t+1}}{c_{1t+1}|_{z_t=a}} = \left(\frac{\beta}{\gamma}\right)^{1/\sigma}$$

that shows how the parent allocates consumption between herself and her offspring. The higher is $\beta$ relative to $\gamma$, the more does the parent want to consume relative to her offspring and higher will $\theta$ be. Second, as long as $\beta > \gamma$, lower $\sigma$

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increases $\theta$, ignoring for now its effect on investment propensities. Here $\sigma$ plays the role of the inverse of the elasticity of substitution between parent's and offspring's consumption: a lower $\sigma$ means higher substitutability and the parent responds by shifting consumption toward herself since $\beta > \gamma$.

That leaves the $1 - \mu - \nu$ term which comes from the offspring's investment behavior. Since investment propensities are invariant across generations, we can use the parent's optimality conditions to understand how $\mu$ and $\nu$ change in response to $p$ and $\sigma$. Use (18) in (16) and (17) to obtain the no-arbitrage condition

$$p\beta\rho\,(c_{2t+1})^{-\sigma} + \gamma(1-p)\rho\,(c_{1t+1}|_{z_t=d})^{-\sigma} = (c_{1t})^{-\sigma} = p\beta w\,(c_{2t+1})^{-\sigma}. \qquad \textbf{(19)}$$

The first equality captures consumption smoothing via physical capital, and the second via human capital. Given $\sigma$, as $p$ rises, if investments do not change, the average future utility at the margin from human capital [right-hand side of (19)] goes up proportionately more than the average future utility from physical capital [left-hand side of (19)]. Therefore, agents will switch from physical to human capital, causing $\nu$ to rise and $\mu$ to fall. What happens to $\mu + \nu$ is unclear from the algebra alone. Intuitively, however, the higher relative price of consumption in youth due to higher $p$ must prompt the offspring to shift toward consumption in middle age. This can only happen if she saves a higher fraction of her endowment; that is, $\mu + \nu$ rises. Anticipating this, the parent will partially compensate by increasing the offspring's share, $1 - \theta$; that is, $\theta$ must fall. The magnitude of the response of consumption will depend on $\sigma$ which, here, is tied to risk aversion. Under higher risk aversion (higher $\sigma$), even when the expected return from human capital dominates that from physical capital, the offspring will shift less toward human capital for a given increase in $p$ and the parent needs to compensate less; $\theta$ falls by less.

To summarize, $p$ lowers $\theta$, shifting more resources toward future generations as a way to compensate for lower consumption in youth. The investment propensities $\mu$ and $\nu$, however, do not depend on $\theta$ whether or not the latter is determined optimally. Hence the basic trade-off whereby higher longevity favors human capital investment over physical capital is robust to the institutional arrangement guiding inter-generational resource sharing.

## 4. PORTFOLIO CHOICE WITH CORNER SOLUTIONS

A necessary condition for the interior optima in Section 3 is $w > \rho$. Otherwise physical capital dominates human capital for sure and all saving is channelized to physical capital alone. It is conceivable, however, that physical capital dominates even when this condition is satisfied. This is likely if the survival probability ($p$) is low enough for risk-averse agents to shy away from the riskier asset, human capital.

We derive the Kuhn–Tucker conditions associated with the household's optimization problem. Inequality constraints for $\theta$ are not necessary. Similar to why

$\theta = 1$ can be ruled out (Section 2), the Inada condition on the parent's own middle-age utility precludes $\theta = 0$ as long as $\sigma > 0$. This also means, since endogenously chosen $\theta$ is always in the interior, its expression is identical to (12) above except for the different values of $\mu$ and $\nu$ depending on the cases below.

The choices of $x_t$ and $e_t$ are associated with the Kuhn–Tucker conditions[8]:

$$\begin{aligned} -(c_{1t})^{-\sigma} + p\beta\theta\rho\,(c_{2t+1})^{-\sigma} + \gamma p(1-\theta)\rho\,(c_{1t+1}|_{z_t=a})^{-\sigma} \\ +\gamma(1-p)\rho\,(c_{1t+1}|_{z_t=d})^{-\sigma} \le 0, \\ -(c_{1t})^{-\sigma} + p\beta\theta w\,(c_{2t+1})^{-\sigma} + \gamma p(1-\theta)w\,(c_{1t+1}|_{z_t=a})^{-\sigma} \le 0. \end{aligned}$$

Whenever either of the above holds with *strict* inequality, the optimal value of the corresponding choice is zero. Conversely, whenever $x_t$ or $e_t$ is positive, the corresponding Kuhn–Tucker condition holds with strict equality.

As before, conjecture that the investment functions are proportional to the endowment, $x_t = \mu y_t(z_{t-1})$ and $e_t = \nu y_t(z_{t-1})$, $z_{t-1} \in \{a, d\}$, which leads to

$$\begin{aligned}(1-\mu-\nu)^{-\sigma} \ge p\beta\rho\theta^{1-\sigma}(\rho\mu+w\nu)^{-\sigma} + p\gamma\rho\,(1-\theta)^{1-\sigma}\,(\rho\mu+w\nu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma} \\ + (1-p)\gamma\rho\,(\rho\mu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma},\end{aligned} \quad \textbf{(20)}$$

$$(1-\mu-\nu)^{-\sigma} \ge p\beta w\theta^{1-\sigma}(\rho\mu+w\nu)^{-\sigma} + p\gamma w\,(1-\theta)^{1-\sigma}\,(\rho\mu+w\nu)^{-\sigma}\,(1-\mu-\nu)^{-\sigma}. \quad \textbf{(21)}$$

Obviously $\mu$ and $\nu$ cannot simultaneously be zero: since $\sigma > 0$, the right-hand side of both (20) and (21) would violate the inequalities. For the same reason, $\mu$ cannot be zero as the third term on the right-hand side of (20) would then go to infinity.[9] Therefore only two possibilities arise: (i) $\mu, \nu > 0$ and (ii) $\mu > 0, \nu = 0$.

### 4.1. Case (i): $\mu, \nu > 0$

Both (20) and (21) hold with equality and we get back the interior values of $\mu$ and $\nu$, derived earlier in (7) and (8). These choices are consistent with case (i) if and only if $\nu > 0$, that is,

$$\left[\gamma(1-p)\right]^{1/\sigma}\frac{1}{\rho}\left(\frac{w\rho}{w-\rho}\right)^{1/\sigma} < \frac{(p\gamma w)^{1/\sigma} + (p\beta w)^{1/\sigma}}{\rho + (p\beta w)^{1/\sigma}}, \quad \textbf{(22)}$$

which can be interpreted as a restriction of the form $p > \hat{p}$ given $\sigma$, $\beta$, $\gamma$, $\rho$, and $w$.

### 4.2. Case (ii): $\mu > 0, \nu = 0$

Only (20) holds with equality. Setting $\nu = 0$, we get

$$(\rho\mu)^{\sigma} - p\beta\rho\theta^{1-\sigma}(1-\mu)^{\sigma} = p\gamma\rho\,(1-\theta)^{1-\sigma} + (1-p)\gamma\rho, \quad \textbf{(23)}$$

which implicitly defines the corner solution for $\mu$. An analytical solution is possible only under log utility (see in the following).

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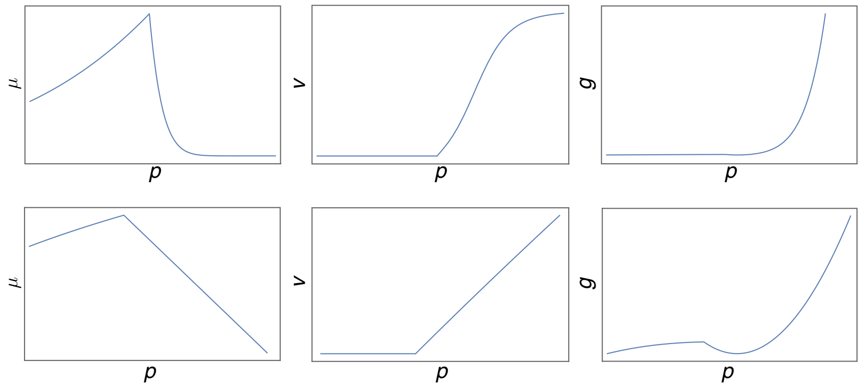


**FIGURE 2.** Effects of $p$ under linear production. $\sigma = 0.1$ (top row) and $\sigma = 1$ (bottom row).

### 4.3. An Example: Log Utility

When $\sigma = 1$, cases (i) and (ii) are characterized as follows:
**Case (i):** $\mu, \nu > 0$
Here we get the interior solutions for $\mu$ and $\nu$ in (13) and (14) above while (22) becomes

$$\rho < \frac{p\gamma + \{1 - \gamma(1-p)\}p\beta}{\gamma + p\beta} w < w. \tag{24}$$

We noted earlier that investment in human capital requires $w > \rho$. Condition (24) shows that the requirement for human capital investment is tighter. In fact, it can be rewritten as $\left[p\gamma + \{1 - \gamma(1-p)\}p\beta\right]/(\gamma + p\beta) > \rho/w$, where the left-hand side is an increasing function of $p$ for plausible values of the parameters. Therefore, given all other parameter values and factor prices, this condition requires $p$ to be higher than some threshold value $\hat{p}$.
**Case (ii):** $\mu > 0$, $\nu = 0$
In this case, we have

$$\mu = \frac{\gamma + p\beta}{1 + p\beta}. \tag{25}$$

Unlike in the interior solution where $\mu$ was a decreasing function of $p$ as the agent shifted in favor of human capital investment, here $\mu$ is increasing in $p$. The reason is obvious: higher $p$ incentivizes future consumption to which agents respond by investing more. When they invest in physical capital alone, it goes up. When they invest in human capital too, they scale back physical capital investment and scale up human capital investment more than one for one because the non-transferability of human capital becomes less of a concern at the margin. This length-of-life effect [Chakraborty (2004)] is not specific to logarithmic utility, and applies to the general case of (23) too as shown in Figure 2.

These solutions are obtained under exogenously given $w$ and $\rho$. In general equilibrium, investment will determine factor returns, which in turn may rule out some cases. Before turning our attention to that, it is important to note that we have so far assumed away the availability of life insurance. The appeal of life insurance policies is to allow altruistic parent to circumvent the problem of non-transferability of human capital [Fischer (1973)]. So the relevant question is to what extent investment in the risky asset (human capital) along with life insurance (which allows agents to diversify bequest risks) can substitute for investment in the safe asset (physical capital). The Appendix shows that it does not make a qualitative difference to the basic result that mortality has a differential effect on tangible versus intangible investment.

## 5. GENERAL EQUILIBRIUM AND DYNAMICS

In general equilibrium, the core intuition from above generalizes but there are additional factors to consider. For example, an increase in $p$ will have a demographic effect as more people survive, and this will shift out the aggregate supply of human capital. Moreover, complementarity between physical and human capital matters: higher human capital investment from higher $p$ will raise the return to physical capital, encouraging its accumulation too.

### 5.1. Aggregation

By the law of large numbers, a $p$ fraction of each cohort survives into middle age. Dynasties in our model will have heterogeneous parental survival histories. Since parental survival makes the initial endowment stochastic, this will generate within-cohort wealth inequality. What allows us to ignore this heterogeneity is the linearity of decision rules due to homothetic preferences and full depreciation of capital. Specifically, we can track the macroeconomic behavior by focusing on an “average agent” each period, that is an agent with the average endowment $\bar{y}_t \equiv Ey_t$, where $y_t$ follows the process (3).

With a slight abuse of notation, we continue to denote this average agent’s holding of the two assets by $k$ and $h$. Since $L_t = L_0 = 1\ \forall t$, aggregate capital stocks are simply

$$K_t = k_t,\ H_t = ph_t. \tag{26}$$

We saw earlier that decision rules take the stationary forms $x_t = \mu y_t$, $e_t = \nu y_t$, and $\theta_t = \theta$ under constant factor prices. But agents may not always invest in human capital. When they do not, for $p \leq \hat{p}$, $\mu$ is implicitly given by (23), $\nu = 0$, and $\theta$ is given by (12). This means $k_{t+1} = \mu(1 - p\theta)\rho k_t$ and $h_{t+1} = 0$ because basic labor productivity has been normalized to zero. On the other hand, for $p > \hat{p}$, decisions are given by (7), (8), and (12) and the agent’s future capital stocks are

$$k_{t+1} = x_t = \mu\bar{y}_t = \mu\left[(1 - p\theta)\rho k_t + p(1 - \theta)wh_t\right], \tag{27}$$

$$h_{t+1} = e_t = \nu\bar{y}_t = \nu\left[(1 - p\theta)\rho k_t + p(1 - \theta)wh_t\right]. \tag{28}$$

Using these in (26), we can see that the aggregate physical-to-human capital ratio when there is investment in the latter is time-invariant

$$\frac{K_{t+1}}{H_{t+1}} = \frac{\mu}{p\nu} \equiv \kappa \;\forall t. \tag{29}$$

We use two specifications of the aggregate technology $F(K,H)$, both capable of generating endogenous growth, and study the effect of $p$ on relative investment and the growth rate of output.[10] Both technologies lead to constant equilibrium rental and efficiency wage rates along the dynamic path, as was assumed above.

## 5.2. Linear Production

Consider first a linear technology that is additive in capital and labor:

$$Y_t = aK_t + bH_t,$$

with $a$, $b > 0$, $b > a$, and factor returns to capital and labor independent of each other. Following (2), these returns are $\rho_t = a$, $w_t = b \;\forall t$. Despite $b = w > \rho = a$, the agent may not invest in human capital if condition (22) is not met ($p \leq \hat{p}$). In that case, $H_t = 0$ and aggregate production is entirely physical capital based, $Y_t = aK_t$. From (27) and equilibrium factor prices,

$$K_{t+1} = \mu \bar{y}_t = a\mu(1 - p\theta)K_t$$

is sufficient to describe the behavior of the aggregate economy. Hence, the economy's growth factor is

$$g = \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = a\mu(1 - p\theta), \tag{30}$$

that is assumed to exceed unity. For $p > \hat{p}$, on the other hand, $Y_t = H_t[a\kappa + b]$. Then from (28), we have

$$H_{t+1} = p\nu\left[(1 - p\theta)\rho\kappa + (1 - \theta)w\right]H_t$$

and

$$g \equiv \frac{Y_{t+1}}{Y_t} = \frac{H_{t+1}}{H_t} = p\nu\left[(1 - p\theta)a\kappa + (1 - \theta)b\right]. \tag{31}$$

Though a higher $p$ lowers $\theta$, across a range of numerical simulations including those reported below, $p\theta$ is increasing in $p$. Therefore, below $\hat{p}$, while the length-of-life effect working through $\mu$ encourages physical capital accumulation, $p\theta$ discourages it. Effectively below $\hat{p}$, the loss of accidental bequest from higher parental survival lowers the accumulation of physical capital.

Above $\hat{p}$, additional effects are at work. First is the positive human capital effect working through $\nu p$: the greater willingness to invest in human capital as people live longer and the supply effect from having more skilled workers survive. Counteracting it is the slower accumulation of physical capital, operating through $\kappa$, as households shift toward human capital. We know, however, that

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since $\mu + \nu$ is increasing in $p$, overall investment rises. Then there is the negative effect, the $1 - p\theta$ term, of lower accidental bequests that is counteracted by higher intended bequests, the $1 - \theta$ term, from the parent's labor income. How these competing effects play off each other depends on the parameter space.

We turn to numerical simulations using empirically reasonable parameter values that ensure positive growth. The human capital effects of $p$ turn out to dominate. Figure 2 shows two representative examples based on $\sigma = 0.1$ (near linearity) and $\sigma = 1$ (log).[11] The kink in both occurs at $\hat{p}$. Below $\hat{p}$, higher survival encourages physical capital accumulation and growth. Above $\hat{p}$, there is rising substitution in favor of human capital, and *growth* accelerates. Two aspects of these figures are worth pointing out. The transition of survival from below $\hat{p}$ to above is accompanied by a growth slowdown as households prioritize human capital over physical in their portfolio allocation: $\mu$ falls sharply as $\nu$ rises gradually. Growth picks up when the latter is a strong enough force, at higher values of $p$. This result is in line with Minamimura and Yasui's (2019) empirical finding that a higher initial level of human capital increases the likelihood of higher income per capita from a decrease in mortality. Here the effect is on the growth rate.

Second, note the convexity of the $g(p)$ function. It shows how substantial the contribution of a mortality transition can be to economic growth in the long run. Shorter bursts of mortality reduction, on the other hand, may or may not have appreciable economic payoffs. Until $p$ reaches the threshold $\hat{p}$, there is little boost to growth. A corollary to this is that a reduction in $p$ at higher levels of survival leads to proportionately higher growth loss than an equivalent reduction at lower levels of $p$. In effect, above $\hat{p}$, a relatively low-$p$ society self-insures against mortality shocks by allocating more toward transferable assets.[12] As a result, a mortality shock that lowers $p$, such as an HIV or ebola outbreak affecting the adult population, will cost less in terms of growth in high-mortality (low $p$) environments than in low-mortality ones (high $p$). This prediction is based on aggregate output; the growth rate of output per capita (or its level) will fall less because of a smaller population base.

## 5.3. Cobb–Douglas Production

Now consider the Cobb–Douglas technology

$$Y_t = F(K_t, H_t) = AK_t^{\alpha} H_t^{1-\alpha},$$

where $A > 0$ and $\alpha \in (0, 1)$. Human capital investment now is necessarily positive; that is, the restriction $pw < \rho < \left[\gamma + 1 - \gamma(1-p)\right] pw/(\gamma + p\beta)$ from Table A1 (Appendix) always binds. For if it did not, the scarcity of human capital would drive wages up sufficiently until investing in it became worthwhile.[13] In perfectly competitive markets, factor prices are again constant, $w_t = (1-\alpha)A\kappa^{\alpha}$ and $\rho_t = \alpha A\kappa^{\alpha-1}$ $\forall t$ because of (29).

Because factor prices now respond to mortality and investment behavior, they can *amplify* the effect of mortality on investment and growth. Figure 3 illustrates

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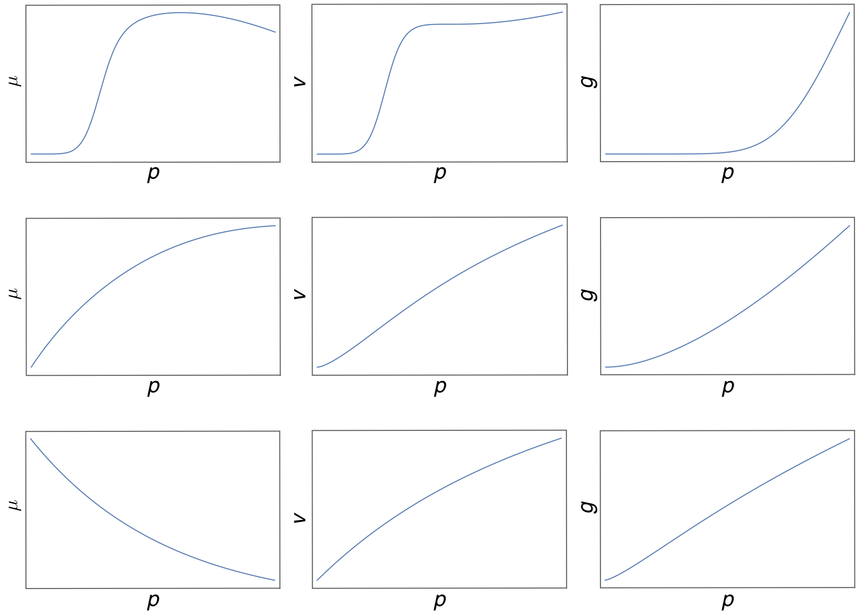


**FIGURE 3.** Effects of $p$ under Cobb–Douglas production. $\sigma = 0.1$ (top row), $\sigma = 0.5$ (middle row), and $\sigma = 1$ (bottom row).

the general equilibrium effects of higher $p$. Investment propensities, except in the case of log, depend on how factor prices respond to higher survival. As $\kappa$ is decreasing in $p$, $\rho$ rises and $w$ falls with $p$. Hence, from (7), the effect of equilibrium prices is to drive up $\mu$ when $p$ rises even as the direct effect is to lower it.

For $\sigma = 0.1$ and $\sigma = 0.5$ in Figure 3, the general equilibrium price effect dominates such that $\mu$ is increasing in $p$ over much of the domain. Observe how, for $\sigma = 0.1$, $\mu$ and $\nu$ remain low (but positive) at low values of $p$. This is due to the high degree of intertemporal substitution at $\sigma = 0.1$: households strongly prioritize present consumption at low $p$ values. In contrast, for log preference, on the other hand, $\mu$ steadily falls with $p$ as factor prices do not affect it. Across all these figure $g$ is increasing in $p$. The convexity of $g(p)$ is again apparent for the first two cases, weaker for the log case due to the absence of amplifying general equilibrium price effects.[14]

## 5.4. Discussion

Taken together, the results in this paper emphasize a new mechanism through which health and mortality affect the macroeconomy.

Much has been written about the changes unleashed by widespread mortality reductions—spanning child and infant survival to adult survival—in late 19th- and early 20th-century Western Europe [Cutler et al. (2006)]. To that literature, we add the possibility that those mortality reductions catalyzed the skill-biased technological innovations that followed. Abramovitz (1993) observes, for example,

> In the nineteenth century, technological progress was heavily biased in a physical capital-using direction. ... In the twentieth century, however, the physical capital-using bias weakened; it may have disappeared altogether. The bias shifted in an intangible (human and knowledge) capital-using direction and produced the substantial contribution of education and other intangible capital accumulation to this century's productivity growth...

A common explanation for why the technological bias changed is the supply side effect working through directed technological change. In Galor and Moav's (2004) theory, the supply of skilled labor increased during the second phase of industrialization due to a technological difference in the accumulation of physical and human capital. Specifically, the return to human capital production is bounded above at zero investment because of which, in a physical capital scarce economy, its returns are dominated. In other words, households do not invest in human capital until industrialization has proceeded far enough to make returns on the two capitals comparable. For us, a high enough mortality rate is sufficient to discourage human capital investment without appealing to production function asymmetries. That means a mortality transition can lower the threshold level of capital accumulation at which human capital investment becomes rewarding. Indeed, the physical-to-human capital transition can begin even if returns to human capital initially remain low or unaffected by technological change.

Human capital risk, stemming from mortality risk, drives this transformation. In traditional societies, the family often plays a central role in diversifying the risk associated with physical assets. In effect, intergenerationally linked families act like annuity markets that make physical capital a safe asset as shown in Chakraborty and Das (2019). The family does not have to be large for this to work; all that is needed is they derive some utility from knowing survivors will inherit their (tangible) assets. But the family's role is limited regarding human capital risk because of its non-transferability. This is in sharp contrast to Minamimura and Yasui's (2019) assumption that all human capital risk is fully diversified within large families and the main effect of mortality is lost labor.[15]

Several implications for developing countries follow from our work. If newer technologies in the 20th century have been skill-intensive in keeping with the skill base of the advanced economies, their adoption in the developing world would depend on mortality. For instance, an increase in the return to human capital, from the flow of skill-intensive technologies, in a low-$p$ country would have a more muted response on skill acquisition compared to a high-$p$ country. High mortality, in other words, works as a barrier to technology adoption. One testable implication is that countries with similar exposure to foreign technologies, say, through shared colonial history or global trade, would differ in their adoption of skill-biased technologies depending on their adult mortalities. Globally, there is substantial variation in adult mortality, as documented by Rajaratnam et al. (2010), for such a test to be feasible.

The theory also demonstrates how a high mortality society partially insures itself by disproportionately investing in tangible assets. A mortality shock would curtail labor supply but leave tangible assets relatively unaffected as they can be productively used by survivors. It stands to reason then that if output per worker depends on factor intensity, the economic impact of a mortality shock will not be large as is commonly feared. The same shock hitting a low mortality society, on the other hand, can deplete skilled labor and cause significant economic damage. Preliminary quantitative evidence in Chakraborty and Pérez-Sebastián (2018) point to the relevance of this mechanism in regions that were affected by the HIV/AIDS crisis.

Beyond economic growth, our work bears upon the demographic transition. Lifetime uncertainty affects fertility choice and the willingness to invest in child human capital. Parents who expect their offspring to have short life spans and face the same non-transferability problem of intangible assets as themselves have little incentive to invest in child quality. Under the usual quantity–quality trade-off, fertility rates will be high which, conditional on child survival, will further incentivize tangible investment. The demographic transition, in this interpretation, becomes tightly linked to the adult health transition through a mechanism different from those emphasized in the literature such as Kalemli-Ozcan et al. (2000), Soares (2005), and Aksan and Chakraborty (2014).

## 6. CONCLUSION

Our study of the effect of mortality on economic development makes two contributions. First, we show that intergenerational wealth transfer can mitigate the investment loss that can occur from future consumption uncertainty. Second, lifetime uncertainty introduces human capital risk because of which mortality intensifies investment in tangible assets that can be passed on to survivors. These results have implications for long-run growth, convergence, and technology adoption.

An interesting avenue to extend this work is the long-run effects of mortality when parental altruism develops to respond to the environment. For example, if altruism requires parental time investment, developing a sufficiently high altruism comes at the opportunity cost of time devoted to building human capital. Thus altruism is likely to be high when parents are engaged in occupations that are less skill intensive, like primary production. At the same time, high mortality itself makes investment in physical capital more profitable than human capital following the logic of this paper. In the initial stages of development, these two mechanisms work in tandem so that high mortality leads to a concentration of production in the primary sector and a high degree of altruism that sustains that production pattern for a long time until some disruption, for instance, an exogenous improvement to mortality or the arrival of technologies, breaks the cycle.

## NOTES

1. The premise that human capital investment has an inherently non-diversifiable idiosyncratic risk component is not new in the literature; see, for example, Levhari and Weiss (1974), Eaton and Rosen (1980), Krebs (2003), and Gottardi et al. (2015). Much of this literature identifies the non-diversifiable risk with unemployment risk, whereas here it comes from life-cycle uncertainty and the non-transferability of human capital. The latter has deeper consequences for household decisions beyond the production side arbitrage based on expected returns.

2. For example, 2010 life expectancy (at birth) in Swaziland was 53.6 years while that in Iceland was 81.8 years (http://www.who.int/gho/mortality_burden_disease/life_tables/situation_trends/en/). More relevant to our work is working-age mortality. In 2010, the mortality rate of 15-year-old men dying before reaching the age 60 was 76.5% in Swaziland, highest in the world, compared to 6.5% in Iceland, lowest in the world [Rajaratnam et al. (2010)].

3. The inverse relationship between risk aversion and technology adoption is established in the literature. This literature differentiates between production risks and consumption risks. Indeed attempts at consumption smoothing by risk averse households can hinder technology adoption as documented by Dercon and Christiaensen (2011).

4. See Blackburn and Cipriani (1998), Kalemli-Ozcan et al. (2000), Bhattacharya and Qiao (2007), Zhang and Zhang (2009), Boucekkine and Laffargue (2010), Chakraborty et al. (2016), Yasui (2016), and Gehringer and Prettner (2019) for various theoretical mechanisms.

5. The difference is amplified if physical capital does not fully depreciate and the offspring puts it to further use. Note that we assumed full depreciation of $h$ because of its inalienability: it is lost when the parent dies.

Household models in the Beckerian tradition often assume that parental human capital positively affects the productivity of children's human capital investment often through an externality. Incorporating that kind of intergenerational effect introduces a different wedge: lower $p$ implies a greater loss of learning opportunities for children that does not apply to inherited tangible assets. Depending on model specification, that may adversely affect parental investment in own or child's human capital.

6. Under linear utility, risk-neutral agents may choose to consume their entire endowment in youth if $p$ is low enough which will, within one period, drive all generations to zero consumption.

7. The next section shows that investments may be at a corner. But $\theta$ is always given by (12) for appropriate values of $\mu$ and $\nu$ that are taken into account in Figure 1.

8. Since the objective function is concave and the constraint functions are either linear or concave, these Kuhn–Tucker conditions are necessary and sufficient.

9. To elaborate, second period consumption is $c_{2t+1} = \theta(\rho\mu + w\nu)y_t$. If $\mu$ and $\nu$ are simultaneously zero, $c_{2t+1}$ goes to zero which can never be optimal for CRRA preferences that satisfy the Inada conditions. Likewise if $\mu$ is zero, accidental bequest $\rho\mu y_t$ is zero which, again, cannot be optimal.

10. As capital depreciates fully and the capital–labor ratio is time-invariant, there is no adjustment period to the balanced growth path. If $p$ is steadily increasing over time, however, the growth of aggregate output has to be balanced against the growing population to arrive at the growth of per capita output.

11. Results are qualitatively similar for exogenous $\theta$.

12. Below $\hat{p}$, on the other hand, no productive asset is lost from mortality shocks as offspring inherit parental physical capital.

13. If raw labor were normalized to a positive number, it is possible, under suitable parametric restrictions for $\nu = 0$ to be sustained in general equilibrium. This produces results similar to the additive technology case above.

14. Results for $\sigma > 1$ are qualitatively similar to log for both production functions.

15. Even in a family-based framework, the family has to be large enough for the law of large numbers to apply. Besides, the institution of the family itself undergoes a transformation as fertility falls and extended family connections get tenuous. These forces are sure to disrupt traditional risk-diversification avenues.

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# APPENDIX A: LIFE INSURANCE

Continue to assume factor returns ($\rho$ and $w$) are exogenously given constants and consider only the case of exogenous $\theta$. This is not restrictive as we have seen endogenizing $\theta$ does not alter investment plans.

The Kuhn–Tucker analysis tells us that when physical and human capitals are the only assets available, the agent invests in physical capital for sure and may or may not invest in human capital. This is because physical capital alone allows the transfer of resources in case of premature parental death. Yet in modern societies with reasonably developed financial markets, there exist instruments like life insurance whose role is exactly that, to allow agents to transfer resources to their survivors. This begs the question: to what extent does our conclusion depend on the absence of life insurance? We proceed to show it does not.

Suppose an agent has the option of investing in life insurance with the objective of transferring a part of her total earnings (from physical as well as human capital) to her offspring even in the event of premature death. Life insurance firms operate on a no-profit no-loss basis and invest the funds in the capital market. The returns from this are transferred to those whose parents have died prematurely. Young agents whose parents are alive to make an end-of-the-period intended bequest get nothing. Since human capital is inalienable, the only investment vehicle available to life insurance companies is physical capital which means for every unit invested in life insurance, the policy payout is $\rho/(1-p)$.

Let $s_t$ denote a young agent's purchase of life insurance. The decision problem at time $t$ is to maximize expected lifetime utility (1) now subject to

$$c_{1t} + x_t + e_t + s_t = y_t,$$

$$c_{2t+1} = \theta(\rho x_t + we_t),$$

and

$$y_{t+1} = \begin{cases} (1-\theta)\,(\rho x_t + we_t), & \text{if } z_t = a, \\ \rho\left(x_t + \dfrac{s_t}{1-p}\right), & \text{if } z_t = d, \end{cases}$$

where accidental bequests now include life insurance payouts. Conjecturing that the decision rules take the form $x_t = \mu y_t(z_{t-1})$, $e_t = \nu y_t(z_{t-1})$, and $s_t = \lambda y_t(z_{t-1})$, where $z_{t-1} \in \{a, d\}$, we get the Kuhn–Tucker conditions

$$(1-\mu-\nu-\lambda)^{-\sigma} \geq p\beta\rho\theta^{1-\sigma}\,(\rho\mu + w\nu)^{-\sigma} + p\gamma\rho\,(1-\theta)^{1-\sigma}\,(\rho\mu + w\nu)^{-\sigma}\,(1-\mu-\nu-\lambda)^{-\sigma} + (1-p)\gamma\rho\left(\rho\mu + \frac{\rho\lambda}{1-p}\right)^{-\sigma}(1-\mu-\nu-\lambda)^{-\sigma}, \tag{A32}$$

$$(1-\mu-\nu-\lambda)^{-\sigma} \geq p\beta w\theta^{1-\sigma}(\rho\mu + w\nu)^{-\sigma} + p\gamma w\,(1-\theta)^{1-\sigma}(\rho\mu + w\nu)^{-\sigma}\,(1-\mu-\nu-\lambda)^{-\sigma}, \tag{A33}$$

$$\gamma\rho\left(\rho\mu + \frac{\rho\lambda}{1-p}\right)^{-\sigma} \leq 1, \tag{A34}$$

for $x_t$, $e_t$, and $s_t$ respectively.

Because our aim is to demonstrate robustness of results with respect to life insurance, we work with log utility that yields explicit expressions for $\mu$, $\nu$, and $\lambda$; optimality conditions for the general CRRA case are analytically intractable. As before, $\mu$ and $\nu$ cannot simultaneously be zero as that will violate Kuhn–Tucker conditions (A32) and (A33). Likewise, $\mu$ and $\lambda$ cannot simultaneously be zero as that will violate (A32) and (A34). That leaves four possibilities: (i) $\mu, \lambda > 0$, $\nu = 0$, (ii) $\nu, \lambda > 0$, $\mu = 0$, (iii) $\mu, \nu > 0$, $\lambda = 0$, and (iv) $\mu > 0$, $\nu, \lambda = 0$.

## CASE (I): $\mu, \lambda > 0$, $\nu = 0$

Conditions (A32) and (A34) hold with equality. Substituting (A34) in (A32) and $\sigma = 1$, we get

$$\frac{1}{(1-\mu-\lambda)} = \frac{p\beta}{\mu} + \frac{p\gamma}{\mu}\frac{1}{(1-\mu-\lambda)} + \frac{(1-p)}{(1-\mu-\lambda)}$$

and simplifying, $\lambda = -\left[\beta(1-\gamma)(1-p)\right]/(1+p\beta) < 0$, which is not possible. Therefore, Case (i) cannot be an equilibrium outcome.

## CASE (II): $\nu, \lambda > 0$, $\mu = 0$

In this case, (A33) and (A34) hold with equality. Setting $\nu = 0$, we get $\lambda = \gamma(1-p)$ and $\nu = \left[p\gamma + p\beta\left\{1-\gamma(1-p)\right\}\right]/(1+p\beta)$. The Kuhn–Tucker inequality (A32) will be

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**TABLE A1.** Equilibrium outcomes under life insurance

| Parameters | Decisions |
|---|---|
| $\rho \geq \left[\frac{\gamma+\{1-\gamma(1-p)\}\beta}{\gamma+p\beta}\right]pw$ | $\mu > 0;\ \nu, \lambda = 0$ |
| $pw < \rho < \left[\frac{\gamma+[1-\gamma(1-p)]\beta}{\gamma+p\beta}\right]pw$ | $\mu > 0,\ \nu > 0;\ \lambda = 0$ |
| $\rho \leq pw$ | $\mu = 0;\ \nu > 0, \lambda > 0$ |

satisfied if and only if

$$pw \geq \rho.$$

An increase in $p$ lowers $\lambda$ and raises $\nu$.

## CASE (III): $\mu, \nu > 0, \lambda = 0$

In this case (A32) and (A33) hold with equality. Setting $\lambda = 0$ and solving, we get back the interior solution derived earlier in equations (13) and (14) for which inequalities (A33) and (A34) are satisfied if

$$pw < \rho < \frac{p\gamma + [1-\gamma(1-p)]p\beta}{\gamma + p\beta}w.$$

## CASE (IV): $\mu > 0, \nu, \lambda = 0$

Only (A32) holds with equality. Setting $\nu, \lambda = 0$ and solving, $\mu = (\gamma + p\beta)/(1 + p\beta)$ for which inequality (A33) is satisfied if

$$\rho \geq \frac{p\gamma + \{1-\gamma(1-p)\}p\beta}{\gamma + p\beta}w.$$

Inequality (A34) is always satisfied.

These results are collected in Table A1. It is clear that even when life insurance policies are available, agents do not opt for such policies unless $p \geq \rho/w$. Given $w$ and $\rho$, societies with high mortality (i.e., low $p$) are less likely to satisfy this condition. It is only when mortality improves substantially that we find agents switching from physical capital investment to a combination of human capital and life insurance. Even so, note that those life insurance purchases ultimately finance physical capital accumulation: in effect, insurance providers invest in capital on behalf of households. And as without insurance, higher $p$ shifts investment from physical capital (life insurance) to human.