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Robust position-force control of robot manipulator in contact with linear dynamic environment

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SUMMARY

The paper presents a control scheme for simultaneous control of position and force of robot manipulator in contact with an elastodynamic environment. The control makes the assumption that interaction force between the robot and environment is adequately modeled by a second-order linear model with constant coefficients, and its implementation requires the knowledge of only boundary values of the environment parameters. It is shown that, provided that robot dynamics is exactly modeled, the scheme ensures asymptotic convergence of errors along nominal trajectories characterized by constant prescribed interaction forces and constant prescribed velocities along the contact surface.

KEYWORDS: Robot control; Position-force control; Lyapunov stability.

I. INTRODUCTION

Many robot tasks pose a requirement for regulating a specified contact force between the robot and environment. A common solution to this problem was hybrid position/force control,[1,2] where the space of task coordinates $\mathbf{p}$ is partitioned into two subspaces, i.e. $\mathbf{p} = [\mathbf{x}; \mathbf{y}]$, $\dim \mathbf{x} = n_x$, $\dim \mathbf{y} = n_y$, and $\dim \mathbf{p} = n_x + n_y = n$ (see Fig. 1). Then, the objective of control is to provide tracking of prescribed nominal trajectory $\mathbf{x}^0$, while simultaneously maintaining interaction forces $\mathbf{f}_y$, along directions $\mathbf{y}$, at prescribed values $\mathbf{f}_y^0$.

A majority of works in hybrid position/force control rely on assumption that the interaction force is a linear function of deflection of the contact surface. This property has been commonly utilized to develop independent or semi-independent control for $\mathbf{x}$ and $\mathbf{y}$ coordinates. However, there are applications characterized by significant elastodynamics of the environment interacting with a robot[3,4] for which the simple spring model of the environment is inadequate. A direct consequence of dynamic terms is coupling between $\mathbf{x}$ and $\mathbf{y}$ subspaces. For such systems, hybrid position/force control methods are not adequate anymore.

The problem of stabilization of robot manipulators in contact with dynamic environment has been receiving increased attention. Several model-based control schemes have been proposed for simultaneous position-force control.[5] However, their applicability has been severely limited by the factors such as quality of the contact model and uncertainties in its parameters.

In this paper, a new control scheme is proposed to overcome the problem of uncertainty in the parameters of a dynamic environment. The scheme consists of a decoupling compensator of robot dynamics and a set of simple position/force PID regulators in the task space. Contrary to the schemes that rely on the assumption that environment parameters are sufficiently accurately modeled, the implementation of the proposed control requires only a knowledge of boundary values of model parameters.

Despite of its simple structure, it is shown that, with an adequate selection of PID gains, the proposed scheme ensures asymptotic stability of a nominal trajectory characterized by the constant velocity $\dot{\mathbf{x}}^0$ and constant interaction force $\mathbf{f}_y^0$. Stability conditions are rigorously developed and it is shown that the domain of attraction, for which asymptotic convergence of errors is guaranteed, can be enlarged as desired by increasing appropriately the PID gains.

II. MODEL OF ROBOT IN CONTACT WITH DYNAMIC ENVIRONMENT

In the general case, dynamics of a rigid-body robot can be represented in fixed reference task space by the model:

$$\mathbf{H}_R(\mathbf{p})\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\dot{\mathbf{p}} + \mathbf{g}(\mathbf{p}) + \mathbf{f} = \boldsymbol{\tau}_p \tag{1}$$

where $\mathbf{p}$ is the vector of task coordinates, $\mathbf{f}$ is the corresponding vector of generalized interaction forces, and $\boldsymbol{\tau}_p$ is the vector of generalized driving forces. If we denote a manipulator Jacobian as $\mathbf{J}(\mathbf{p})$, then driving forces $\boldsymbol{\tau}_q$ at manipulator joints are uniquely determined from the equivalent driving forces $\boldsymbol{\tau}_p$ in task coordinates as:

$$\boldsymbol{\tau}_q = \mathbf{J}^T(\mathbf{q})\boldsymbol{\tau}_p \tag{2}$$

It is known that matrices $\mathbf{H}_R(\mathbf{p})$ and $\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})$ possess an important structure that is used by many authors in control design:

(i) Matrix of inertia $\mathbf{H}_R(\mathbf{p})$ is symmetric, positive definite, and bounded. Thus,

$$\|\mathbf{H}_R(\mathbf{p})\| \le \lambda_{\max}[\mathbf{H}_R(\mathbf{p})] \tag{3}$$

where $\lambda_{\max}[\mathbf{H}_R(\mathbf{p})]$ is the maximum eigenvalue of $\mathbf{H}_R(\mathbf{p})$. Due to boundedness of the matrix of inertia, $\lambda_{\max}[\mathbf{H}_R(\mathbf{p})]$ represents some finite constant.

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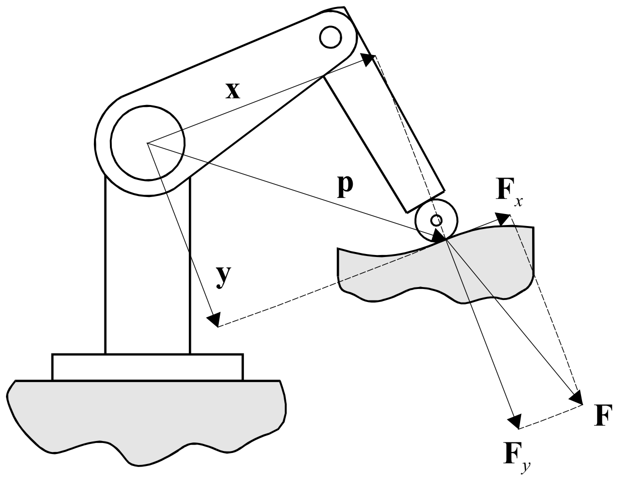

Fig. 1. Simultaneous position-force control.

(ii) Matrix $\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})$ is bounded in $\mathbf{p}$ and linearly dependent on $\dot{\mathbf{p}}$, that is:

$$\|\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\| \leq K_C \|\dot{\mathbf{p}}\| \tag{4}$$

for some finite K_C.

(iii) With a proper choice of $\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})$,

$$\frac{d}{dt}\{\mathbf{H}_R(\mathbf{p})\} = \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}) + \mathbf{C}^T(\mathbf{p}, \dot{\mathbf{p}}) \tag{5}$$

With sufficient degree of generality, the force of interaction $\mathbf{f}$ can be analyzed by employing the second-order nonlinear model:[4]

$$\mathbf{f} = \mathbf{M}(\mathbf{p}, \dot{\mathbf{p}})\ddot{\mathbf{p}} + \mathbf{L}(\mathbf{p}, \dot{\mathbf{p}}) \tag{6}$$

where $\mathbf{M}(\mathbf{p}, \dot{\mathbf{p}})$ and $\mathbf{L}(\mathbf{p}, \dot{\mathbf{p}})$ are nonlinear matrix functions of $\mathbf{p}$, $\dot{\mathbf{p}}$. However, in many practical situations, (6) can be simplified to:

$$\mathbf{f}_x = \mathbf{f}_x(\dot{\mathbf{x}}, \mathbf{f}_y) \tag{7}$$

$$\mathbf{f}_y = \mathbf{M}\ddot{\mathbf{y}} + \mathbf{B}\dot{\mathbf{y}} + \mathbf{K}(\mathbf{y} - \mathbf{y}_e) \quad \text{for} \quad \mathbf{f}_y \geq \mathbf{0} \tag{8}$$

where $\mathbf{f} = [\mathbf{f}_x; \mathbf{f}_y]$ and $\mathbf{f}_x(\dot{\mathbf{x}}, \mathbf{f}_y)$ is a nonlinear vector function of $\dot{\mathbf{x}}$, $\mathbf{f}_y$.

In this work, (7, 8) is considered as the model of the contact force. Furthermore, all parameters $\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}$, $\mathbf{y}_e$ are assumed constant, whereas the matrices $\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}$ are assumed in diagonal form, that is,

$$\mathbf{M} = \text{diag}(m_j), \quad \mathbf{B} = \text{diag}(b_j), \quad \mathbf{K} = \text{diag}(k_j)$$

for $j = 1, \ldots, n_y$.

III. CONTROL ALGORITHM

The algorithm is intended for applications where the objective is to keep both the interaction force $\mathbf{f}_y$ and velocity $\dot{\mathbf{x}}$ at constant prescribed values, i.e.

$$\dot{\mathbf{x}}^0(t) = \text{const} \tag{9}$$

$$\mathbf{f}_y^0(t) = \text{const}, \quad \dot{\mathbf{y}}^0(t) = \mathbf{0} \tag{10}$$

With the proposed scheme, driving forces in task space are generated as:

$$\boldsymbol{\tau}_p = \hat{\mathbf{C}}(\mathbf{p}, \dot{\mathbf{p}})\dot{\mathbf{p}}^0 + \hat{\mathbf{g}}(\mathbf{p}) + \Delta\boldsymbol{\tau}_p \tag{11}$$

where $\hat{\mathbf{C}}(\mathbf{p}, \dot{\mathbf{p}})$ and $\hat{\mathbf{g}}(\mathbf{p})$ are estimates of corresponding matrices in model (1), $\dot{\mathbf{p}}^0 = [\dot{\mathbf{x}}^0; \mathbf{0}]$ is the nominal velocity, and:

$$\Delta\boldsymbol{\tau}_p = [\Delta\boldsymbol{\tau}_x; \Delta\boldsymbol{\tau}_y] \tag{12}$$

are PID regulators in position-controlled and force-controlled directions:

$$\Delta\boldsymbol{\tau}_x = \mathbf{f}_x - \mathbf{K}_{Dx}\Delta\dot{\mathbf{x}} - \mathbf{K}_{Px}\Delta\mathbf{x} - \mathbf{K}_{Ix}\int \Delta\mathbf{x}\, dt \tag{13}$$

$$\Delta\boldsymbol{\tau}_y = \mathbf{f}_y^0 - \mathbf{K}_{Dy}\Delta\dot{\mathbf{y}} - \mathbf{K}_{Py}\Delta\hat{\mathbf{y}} - \mathbf{K}_{Iy}\int \Delta\mathbf{f}_y\, dt \tag{14}$$

with errors defined as:

$$\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^0; \quad \Delta\dot{\mathbf{x}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}^0$$
$$\Delta\hat{\mathbf{y}} = \mathbf{y} - \hat{\mathbf{y}}^0; \quad \Delta\dot{\mathbf{y}} = \dot{\mathbf{y}} - \dot{\mathbf{y}}^0 = \dot{\mathbf{y}}$$
$$\Delta\mathbf{f}_y = \mathbf{f}_y - \mathbf{f}_y^0$$

Gain matrices in (13, 14) are assumed constant and diagonal, i.e.

$$\mathbf{K}_{Dx} = \text{diag}(K_{Dxi}), \quad \mathbf{K}_{Px} = \text{diag}(K_{Pxi}),$$
$$\mathbf{K}_{Ix} = \text{diag}(K_{Ixi})$$

for $i = 1, \ldots, n_x$, and:

$$\mathbf{K}_{Dy} = \text{diag}(K_{Dyj}), \quad \mathbf{K}_{Py} = \text{diag}(K_{Pyj}),$$
$$\mathbf{K}_{Iy} = \text{diag}(K_{Iyj})$$

for $j = 1, \ldots, n_y$.

Note that the component $\mathbf{f}_x$ of interaction force is simply compensated in (11), whereas (12) incorporates a reference force $\mathbf{f}_y^0$ and integral of force error $\Delta\mathbf{f}_y$. Note also that (12) employs the estimate $\Delta\hat{\mathbf{y}}$ of the position error. This is inevitably since the reference position $\mathbf{y}^0$:

$$\mathbf{y}^0 = \mathbf{K}^{-1}\mathbf{f}_y^0 + \mathbf{y}_e \tag{15}$$

depends on static characteristics of the model (8) and is generally unknown. Thus, approximate reference position $\hat{\mathbf{y}}^0$ is computed from estimated static parameters as:

$$\hat{\mathbf{y}}^0 = \hat{\mathbf{K}}^{-1}\mathbf{f}_y^0 + \hat{\mathbf{y}}_e \tag{16}$$

IV. STABILITY ANALYSIS

Assume that manipulator dynamics is exactly compensated by (11), i.e.

$$\hat{\mathbf{C}}(\mathbf{p}, \dot{\mathbf{p}}) = \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}), \quad \hat{\mathbf{g}}(\mathbf{p}) = \mathbf{g}(\mathbf{p})$$

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Then the control (11–14), combined with manipulator dynamics (1), yields the closed-loop system:

$$\mathbf{H}_R(\mathbf{p})\Delta\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\Delta\dot{\mathbf{p}} + \begin{bmatrix} \mathbf{K}_{Dx}\Delta\dot{\mathbf{x}} + \mathbf{K}_{Px}\Delta\mathbf{x} + \mathbf{K}_{Ix}\int\Delta\mathbf{x}\,dt \\ \mathbf{K}_{Dy}\Delta\dot{\mathbf{y}} + \mathbf{K}_{Py}\Delta\hat{\mathbf{y}} + \Delta\mathbf{f}_y + \mathbf{K}_{Iy}\int\Delta\mathbf{f}_y\,dt \end{bmatrix} = \mathbf{0} \tag{17}$$

By introducing:

$$\tilde{\mathbf{K}}_{Dy} = \mathbf{K}_{Dy} + \mathbf{K}_{Iy}\mathbf{M} + \mathbf{B} \tag{18}$$

$$\tilde{\mathbf{K}}_{Py} = \mathbf{K}_{Py} + \mathbf{K}_{Iy}\mathbf{B} + \mathbf{K} \tag{19}$$

$$\boldsymbol{\sigma}_x = \Delta\mathbf{x} + 2\mathbf{K}_{Px}^{-1}\mathbf{K}_{Ix}\int\Delta\mathbf{x}\,dt \tag{20}$$

$$\boldsymbol{\sigma}_y = \Delta\mathbf{y} + 2\tilde{\mathbf{K}}_{Py}^{-1} \cdot \left[\mathbf{K}_{Iy}\left(\int\Delta\mathbf{f}_y\,dt - \mathbf{M}\Delta\dot{\mathbf{y}} - \mathbf{B}\Delta\mathbf{y}\right) - \mathbf{K}_{Py}\Delta\mathbf{y}^0\right] \tag{21}$$

where $\mathbf{M}$, $\mathbf{B}$, $\mathbf{K}$ are matrices of the environment model (8), $\Delta\mathbf{y}^0$ is the error in reference position:

$$\Delta\mathbf{y}^0 = \hat{\mathbf{y}}^0 - \mathbf{y}^0 \tag{22}$$

and by accounting that, due to linearity of interaction force (8),

$$\Delta\mathbf{f}_y = \mathbf{M}\Delta\ddot{\mathbf{y}} + \mathbf{B}\Delta\dot{\mathbf{y}} + \mathbf{K}\Delta\mathbf{y} \quad \text{for} \quad \mathbf{f}_y \geq \mathbf{0} \tag{23}$$

the closed-loop system (17) is transformed to:

$$[\mathbf{H}_R(\mathbf{p}) + \operatorname{diag}(\mathbf{O}, \mathbf{M})]\Delta\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\Delta\dot{\mathbf{p}} + \begin{bmatrix} \mathbf{K}_{Dx}\Delta\dot{\mathbf{x}} + \frac{1}{2}\mathbf{K}_{Px}\Delta\mathbf{x} + \frac{1}{2}\mathbf{K}_{Px}\boldsymbol{\sigma}_x \\ \mathbf{K}_{Dy}\Delta\dot{\mathbf{y}} + \frac{1}{2}\tilde{\mathbf{K}}_{Py}\Delta\mathbf{y} + \frac{1}{2}\tilde{\mathbf{K}}_{Py}\boldsymbol{\sigma}_y \end{bmatrix} = \mathbf{0} \tag{24}$$

With the additional substitutions:

$$\boldsymbol{\sigma} = [\boldsymbol{\sigma}_x; \boldsymbol{\sigma}_y] \tag{25}$$

$$\mathbf{H}(\mathbf{p}) = \mathbf{H}_R(\mathbf{p}) + \operatorname{diag}(\mathbf{O}, \mathbf{M}) \tag{26}$$

$$\mathbf{K}_D = \operatorname{diag}(\mathbf{K}_{Dx}, \tilde{\mathbf{K}}_{Dy}) \tag{27}$$

$$\mathbf{K}_P = \operatorname{diag}(\mathbf{K}_{Px}, \tilde{\mathbf{K}}_{Py}) \tag{28}$$

(24) is further simplified to:

$$\mathbf{H}(\mathbf{p})\Delta\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\Delta\dot{\mathbf{p}} + \mathbf{K}_D\Delta\dot{\mathbf{p}} + \frac{1}{2}\mathbf{K}_P\Delta\mathbf{p} + \frac{1}{2}\mathbf{K}_P\boldsymbol{\sigma} = \mathbf{0} \tag{29}$$

It is readily seen that $\Delta\mathbf{p} = \mathbf{0}$, $\Delta\dot{\mathbf{p}} = \mathbf{0}$ implies $\boldsymbol{\sigma} = \mathbf{0}$. Therefore, a trivial solution of (29) is determined by the condition $\Delta\mathbf{p} = \mathbf{0}$, $\Delta\dot{\mathbf{p}} = \mathbf{0}$.

Now, we may assert the following statement on the stability of the control:

There exist a continuous range of control parameters $\mathbf{K}_{Dx}, \mathbf{K}_{Px}, \mathbf{K}_{Ix}, \mathbf{K}_{Dy}, \mathbf{K}_{Py}, \mathbf{K}_{Iy}$ *for which the trivial solution* $\Delta\mathbf{p} = \mathbf{0}$, $\Delta\dot{\mathbf{p}} = \mathbf{0}$, $\boldsymbol{\sigma} = \mathbf{0}$ *of the closed-loop system (29) is asymptotically stable in the sense of Lyapunov when* $t \to +\infty$. *Furthermore, the domain of attraction of the trivial solution can be made arbitrarily large by a proper choice of control parameters.*

To prove the statement and investigate conditions for asymptotic stability, consider the following scalar function:

$$V(\mathbf{p}, \Delta\mathbf{p}, \Delta\dot{\mathbf{p}}, \boldsymbol{\sigma}) = \frac{1}{2}\Delta\dot{\mathbf{p}}^T\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}} + \frac{1}{4}\Delta\mathbf{p}^T\mathbf{K}_P\Delta\mathbf{p} + \frac{1}{4}\boldsymbol{\sigma}^T\mathbf{K}_P\boldsymbol{\sigma} + 2\Delta\mathbf{p}^T\mathbf{D}\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}} \tag{30}$$

where $\mathbf{D}$ is a constant diagonal matrix:

$$\mathbf{D} = \mathbf{K}_P^{-1}\mathbf{K}_I \tag{31}$$

and:

$$\mathbf{K}_I = \operatorname{diag}(\mathbf{K}_{Px}, \mathbf{K}_{Iy}\mathbf{K}) \tag{32}$$

From the assumption that $\mathbf{M}$ is positive and diagonal, it follows that matrix $\mathbf{H}(\mathbf{p})$ in (29) is positive definite, bounded, and:

$$\lambda_{\max}[\mathbf{H}(\mathbf{p})] \leq \lambda_{\max}[\mathbf{H}_R(\mathbf{p})] + \lambda_{\max}(\mathbf{M}) = \lambda_{\max}[\mathbf{H}_R(\mathbf{p})] + \max_{j=1,\ldots n_y} m_j \tag{33}$$

where $\lambda_{\max}[\mathbf{H}(\mathbf{p})] > 0$ denotes the maximum eigenvalue of the equivalent matrix of inertia $\mathbf{H}(\mathbf{p})$.

It is readily verified that $V(\cdot)$ is continuous in $\mathbf{p}$, $\Delta\mathbf{p}$, $\Delta\dot{\mathbf{p}}$, $\boldsymbol{\sigma}$ and $V(\mathbf{p}, \mathbf{0}, \mathbf{0}, \mathbf{0}) \equiv 0$. Furthermore, since:

$$\Delta\dot{\mathbf{p}}^T\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}} = [\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}]^T\mathbf{H}^{-1}(\mathbf{p})[\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}] \geq \lambda_{\max}^{-1}[\mathbf{H}(\mathbf{p})] \cdot \|\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}\|^2$$

and:

$$\Delta\mathbf{p}^T\mathbf{D}\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}} \geq -D_{\max} \cdot \|\Delta\mathbf{p}\| \cdot \|\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}\|$$

where:

$$D_{\max} = \max\left\{\max_i \frac{K_{Ixi}}{K_{Pxi}}, \max_j \frac{K_{Iyj}k_j}{K_{Pyj} + K_{Iyj}b_j + k_j}\right\} \tag{34}$$

for $i = 1, \ldots, n_x$, $j = 1, \ldots, n_y$, the following lower bound of V is established:

$$V \geq \frac{1}{2}\mathbf{z}_1^T\begin{bmatrix} \frac{1}{2}K_{P\min} & -2D_{\max} \\ -2D_{\max} & \lambda_{\max}^{-1}[\mathbf{H}(\mathbf{p})] \end{bmatrix}\mathbf{z}_1 + \frac{1}{4}K_{P\min}\|\boldsymbol{\sigma}\|^2 \tag{35}$$

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where:

$$\mathbf{z}_1 = [\|\Delta\mathbf{p}\|; \quad \|\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}\|]$$

and:

$$K_{P\min} = \min\left\{\min_i K_{Pxi}, \quad \min_j K_{Pxj} + K_{Iyj}b_j + k_j\right\} \tag{36}$$

Thus, V is positive definite in $\|\Delta\mathbf{p}\|$, $\|\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}\|$, $\|\boldsymbol{\sigma}\|$ provided that:

$$K_{P\min} > 8D_{\max}^2 \cdot \lambda_{\max}[\mathbf{H}(\mathbf{p})] \tag{37}$$

Since:

$$K_{P\min} > \min\left\{\min_i K_{Pxi}, \quad \min_j K_{Pyj} + k_j\right\} \tag{38}$$

$$D_{\max} < \max\left\{\max_i K_{Ixi}/K_{Pxi}, \quad \max_j K_{Iyj}\right\} \tag{39}$$

the condition (37) can always be satisfied by selecting sufficiently large proportional gains and/or small integral gains. Besides, $D_{\max}$ does not increase when proportional gains increase. Therefore, for sufficiently high values of the gains,

$$V \geq \frac{1}{4}\left(K_{P\min} - K_{P\min}^0\right) \cdot \|\Delta\mathbf{p}\|^2 \tag{40}$$

where:

$$K_{P\min}^0 = 8D_{\max}^2\lambda_{\max}[\mathbf{H}(\mathbf{p})] \tag{41}$$

The total time derivative of V is given by:

$$\begin{aligned}\dot{V} &= \Delta\dot{\mathbf{p}}^T\mathbf{H}(\mathbf{p})\Delta\ddot{\mathbf{p}} + \frac{1}{2}\Delta\dot{\mathbf{p}}^T[\mathbf{C}(\mathbf{p},\dot{\mathbf{p}}) + \mathbf{C}^T(\mathbf{p},\dot{\mathbf{p}})]\Delta\dot{\mathbf{p}} \\ &\quad + \frac{1}{2}\Delta\dot{\mathbf{p}}^T\mathbf{K}_P\Delta\mathbf{p} + \frac{1}{2}\cdot\dot{\boldsymbol{\sigma}}^T\mathbf{K}_P\boldsymbol{\sigma} + 2\Delta\mathbf{p}^T\mathbf{D}\mathbf{H}(\mathbf{p})\Delta\ddot{\mathbf{p}} \\ &\quad + 2\Delta\mathbf{p}^T\mathbf{D}[\mathbf{C}(\mathbf{p},\dot{\mathbf{p}}) + \mathbf{C}^T(\mathbf{p},\dot{\mathbf{p}})]\Delta\dot{\mathbf{p}} + 2\Delta\dot{\mathbf{p}}^T\mathbf{D}\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}\end{aligned}$$

Since $\mathbf{M}$ is constant, from (5, 26) it follows:

$$\dot{\mathbf{H}}(\mathbf{p}) = \mathbf{C}(\mathbf{p},\dot{\mathbf{p}}) + \mathbf{C}^T(\mathbf{p},\dot{\mathbf{p}})$$

Besides, from the construction of $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_x; \boldsymbol{\sigma}_y]$, it is easily verified that:

$$\dot{\boldsymbol{\sigma}} = \Delta\dot{\mathbf{p}} + 2\mathbf{D}\Delta\mathbf{p}$$

Therefore,

$$\begin{aligned}\dot{V} &= \Delta\dot{\mathbf{p}}^T\left[\mathbf{H}(\mathbf{p})\Delta\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p},\dot{\mathbf{p}})\Delta\dot{\mathbf{p}} + \frac{1}{2}\mathbf{K}_P\Delta\mathbf{p} + \frac{1}{2}\mathbf{K}_P\boldsymbol{\sigma}\right] \\ &\quad + 2\Delta\mathbf{p}^T\mathbf{D}\left[\mathbf{H}(\mathbf{p})\Delta\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p},\dot{\mathbf{p}})\Delta\dot{\mathbf{p}} + \frac{1}{2}\mathbf{K}_P\boldsymbol{\sigma}\right] \\ &\quad + 2\Delta\dot{\mathbf{p}}^T\mathbf{C}(\mathbf{p},\dot{\mathbf{p}})\mathbf{D}\Delta\mathbf{p} + 2\Delta\dot{\mathbf{p}}^T\mathbf{D}\mathbf{H}(\mathbf{p})\Delta\dot{\mathbf{p}}\end{aligned}$$

The time derivative of V along (29) is:

$$\begin{aligned}\dot{V} &= -\Delta\dot{\mathbf{p}}^T[\mathbf{K}_D - 2\mathbf{D}\mathbf{H}(\mathbf{p})]\Delta\dot{\mathbf{p}} - \Delta\mathbf{p}^T\mathbf{K}_I\Delta\mathbf{p} \\ &\quad - 2\Delta\dot{\mathbf{p}}^T[\mathbf{K}_D - \mathbf{C}(\mathbf{p},\dot{\mathbf{p}})]\mathbf{D}\Delta\mathbf{p}\end{aligned}$$

Since, from (4),

$$\|\mathbf{C}(\mathbf{p},\dot{\mathbf{p}})\| \leq K_C\|\Delta\dot{\mathbf{p}}\| + K_C\|\dot{\mathbf{p}}^0\|$$

the following upper bound of $\dot{V}$ is obtained:

$$\begin{aligned}\dot{V} &\leq -\{K_{D\min} - 2D_{\max}\lambda_{\max}[\mathbf{H}(\mathbf{p})]\}\cdot\|\Delta\dot{\mathbf{p}}\|^2 \\ &\quad - K_{I\min}\|\Delta\mathbf{p}\|^2 + 2D_{\max}\left\{K_{D\max} + K_C\|\dot{\mathbf{p}}^0\|\right\}\cdot\|\Delta\mathbf{p}\| \\ &\quad \cdot\|\Delta\dot{\mathbf{p}}\| + 2D_{\max}K_C\|\Delta\mathbf{p}\|\cdot\|\Delta\dot{\mathbf{p}}\|^2\end{aligned} \tag{42}$$

where:

$$K_{D\min} = \min\left\{\min_i K_{Dxi}, \quad \min_j K_{Dyj} + K_{Iyj}m_j + b_j\right\} \tag{43}$$

$$K_{D\max} = \max\left\{\max_i K_{Dxi}, \quad \max_j K_{Dyj} + K_{Iyj}m_j + b_j\right\} \tag{44}$$

$$K_{I\min} = \min\left\{\min_i K_{Ixi}, \quad \min_j K_{Iyj}k_j\right\} \tag{45}$$

Equivalently,

$$\dot{V} \leq -\mathbf{z}_2^T\mathbf{W}\mathbf{z}_2 + 2D_{\max}K_C\|\Delta\mathbf{p}\|\cdot\|\Delta\dot{\mathbf{p}}\|^2 \tag{46}$$

where:

$$\mathbf{z}_2 = [\|\Delta\mathbf{p}\|; \|\Delta\dot{\mathbf{p}}\|]$$

$$\mathbf{W} = \begin{bmatrix} K_{D\min} - 2D_{\max}\lambda_{\max}(\mathbf{H}) & -D_{\max}(K_{D\max} + K_C\|\dot{\mathbf{p}}^0\|) \\ -D_{\max}(K_{D\max} + K_C\|\dot{\mathbf{p}}^0\|) & K_{I\min}\end{bmatrix}$$

It is seen that the matrix $\mathbf{W}$ is positive definite for:

$$\begin{aligned}&K_{I\min}\{K_{D\min} - 2D_{\max}\lambda_{\max}[\mathbf{H}(\mathbf{p})]\} \\ &\quad > D_{\max}^2\{K_{D\max} + K_C\|\dot{\mathbf{p}}^0\|\}^2\end{aligned} \tag{47}$$

If (47) is satisfied, then the minimum eigenvalue of $\mathbf{W}$ is $\lambda_{\min}(\mathbf{W}) > 0$. By taking into account the inequality (40), the total time derivative of V along (29) is:

$$\begin{aligned}\dot{V} &\leq -\lambda_{\min}(\mathbf{W})\|\mathbf{z}_2\|^2 + 2D_{\max}K_C\|\Delta\mathbf{p}\|\cdot\|\Delta\dot{\mathbf{p}}\|^2 \\ &\leq -\lambda_{\min}(\mathbf{W})\|\mathbf{z}_2\|^2 + \frac{2D_{\max}K_C\sqrt{2V}}{\sqrt{K_{P\min} - K_{P\min}^0}}\|\Delta\dot{\mathbf{p}}\|^2 \\ &\leq -\lambda_{\min}(\mathbf{W})\|\Delta\mathbf{p}\|^2 \\ &\quad - \left\{\lambda_{\min}(\mathbf{W}) - \frac{2D_{\max}K_C\sqrt{2V}}{\sqrt{K_{P\min} - K_{P\min}^0}}\right\}\|\Delta\dot{\mathbf{p}}\|^2\end{aligned}$$

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Therefore, $\dot{V} < 0$ as long as the above expression in curly braces is negative. Thus, we may conclude that the scalar function V, defined by (30), is a Lyapunov function of the second kind provided that control gains meet the conditions (37) and (47). Equivalently, the closed-loop system is asymptotically stable along the trajectory (9,10). The domain of attraction is:

$$V < \lambda_{\min}^2(\mathbf{W}) \frac{K_{P\min} - K_{P\min}^0}{8 D_{\max}^2 K_C^2} = \frac{\lambda_{\min}^2(\mathbf{W}) \cdot \lambda_{\max}(\mathbf{H})}{K_C^2} \left(\frac{K_{P\min}}{K_{P\min}^0} - 1 \right)$$

and it can be increased at will by increasing $\lambda_{\min}(\mathbf{W})$ and the ratio $K_{P\min}/K_{P\min}^0$.

It remains to demonstrate that inequalities (37) and (47), indeed, always have a range of solutions. To this end, assume that control gains in (13–14) are taken in the form:

$$K_{Pxi} = K_{Pyj} = \alpha^2 K_{P0}$$
$$K_{Dxi} = K_{Dyj} = \alpha^2 K_{D0}$$
$$K_{Ixi} = \alpha K_{P0}^2 / K_{D0}$$
$$K_{Iyj} = \frac{1}{\alpha} K_{P0} / K_{D0}$$

for $i = 1, \ldots, n_x$, $j = 1, \ldots, n_y$ and some constant $\alpha > 0$. Then, from the definitions (34, 36, 43–45), we have:

$$K_{P\min} > \alpha^2 K_{P0}$$
$$K_{D\min} > \alpha^2 K_{D0}$$
$$K_{D\max} = \alpha^2 K_{D0} + \frac{1}{\alpha} \frac{K_{P0}}{K_{D0}} m_{\max} + b_{\max}$$
$$D_{\max} < \frac{1}{\alpha} \frac{K_{P0}}{K_{D0}}$$
$$K_{I\min} > \frac{1}{\alpha} \frac{K_{P0}}{K_{D0}} \cdot \min(\alpha^2 K_{P0}, k_{\min})$$

where $m_{\max}$, $b_{\max}$, $k_{\min}$ are the bounds of the corresponding coefficients in the model of the contact force (8), i.e.

$$m_{\max} = \max_j m_j, \quad b_{\max} = \max_j b_j, \quad k_{\min} = \min_j k_j$$

Now, the stability conditions (37, 47) become:

$$\alpha^4 \frac{K_{D0}^2}{K_{P0}} > 8 \lambda_{\max}(\mathbf{H}) \tag{48}$$

$$\left\{ \alpha^3 \frac{K_{D0}^2}{K_{P0}} - 2\lambda_{\max}(\mathbf{H}) \right\} \cdot \min(\alpha^2 K_{P0}, k_{\min}) > \left\{ \alpha^2 K_{D0} + \frac{1}{\alpha} \frac{K_{P0}}{K_{D0}} m_{\max} + b_{\max} + K_C \|\dot{\mathbf{p}}^0\| \right\}^2 \tag{49}$$

and it is seen that (48,49) can always be satisfied by selecting a sufficiently large α.

V. CONCLUDING REMARKS

This paper was concentrated on the proof of stability of the proposed control scheme. Nonetheless, there was no attempt to analyze the influence of the control gains on the quality of the control. Yet, it was shown that stable responses can be achieved by a range of control gains, and very broad guidelines were established for selecting the actual values of the gains. Further work in this direction is desired. Particularly, one may expect that better performances could be achieved if nonlinear gains were employed, i.e. the gains that were dependent on the current position of the closed-loop system in the error space.

The proof of stability implicitly rely on the assumption that a manipulator is always in contact with the environment. Actually, from the proof it follows that, with sufficiently small initial position and velocity errors, the errors remain bounded, and thus the error in interaction force is bounded as well. Consequently, the assumption of the permanent contact is valid for a sufficiently large nominal force $\mathbf{f}_y^0$, although its magnitude was not quantified in this work.

A weak point of the scheme is its dependence on the estimates of matrices in the robot model (1), particularly the dependence on the matrix $\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})$. However, from (11), the influence of $\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})$ on the control signal increase with $\dot{\mathbf{p}}^0$ and it could be neglected with sufficiently small nominal velocities.

The applicability of the scheme is limited to tasks where the goal is the maintenance of constant or semi-constant forces and velocities in task space. This constraint is not too strong, since many practical tasks fall in this category. As a payoff, the scheme is advantageous for such tasks because it offers a firm proof of stability.

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