Theory and Applications

Valuation of biodiversity within a north–south trade model

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ABSTRACT. Under a General Equilibrium model of International Trade, industrialized countries export capital intensive goods, while developing countries export natural resource intensive goods. Biodiversity is viewed as the number of species conserved while producing these goods. Higher conservation increases demand, but lowers goods supply. Consumers value biodiversity as the weighted sum of all the different species. If producers of both goods conserve more species, the South's terms of trade will rise in relation to the North's. Furthermore, we believe that a switch in consumer preferences, to a more homogeneous valuation of the species, is likely. This change would drop the South's terms of trade. Therefore, under these circumstances, this region is facing a risk. In conserving additional species, the South would be better off, both because its terms of trade increases and because the risk associated with a switch in preferences decreases.

1. Introduction

Contrasts between patterns of production in different countries have been pointed out in the literature. Less developed countries are intensive in natural resources and developed countries are intensive in capital, both physical and human. This is often quoted in relation to the successful economic growth followed by the knowledge-based economies (e.g., Asian Dragons) in contrast with the resource-based economies (e.g. Latin-American or African countries). Under a free trade agreement,¹ developed countries export capital intensive goods and conversely, developing countries export goods intensive in natural resources.²

Biodiversity is viewed as the number of conserved species in the ecosystem which support the production of each one of these two goods.

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- 1 For transition from autarchy to free trade effects, see Grossman and Krueger (1991), Perroni and Wigle (1994) as well as Copeland and Scott (1994).
- 2 To what extent the South has a real comparative advantage in the production of resource intensive goods or it exports them due to ill-defined property rights and, consequently to insufficiently internalized externalities, see Chichilnisky (1991b).

A basic assumption here is that the capital intensive goods conserve more species than the resource intensive ones. An agent's utility stems not only from the consumed quantity, but also from the quality of the environment where the goods have been produced. Environmental quality is measured by biological diversity. Thus, the demand function is affected by biodiversity. On the other hand, there is a trade-off between species conservation and production. The supply of goods is negatively related to biodiversity.

Following Blamey and Common (1992), we divide the value of the species into four categories: *use value*, the individual's present or planned use of the species; *axistence value*, tied in with the ethical or intrinsic value of the species; *option value*, stemming from the certainty that the species will be available for use in the future (likely to be positive when uncertainty on the supply side is large and the resource is unique, i.e., most of the endangered species) and *quasi-option value*, the expected value of information gained from delaying an irreversible decision. Agents aggregate utilities coming from all species within this ecosystem. Even if the value of any particular species can change, we assume that the lump sum remains constant. This is why we use a distribution function for weights.

Weitzman, in (1992) and (1993) defines a value of diversity function in which the species are ordered according to the genetic distance between them. The choice of this distance is arbitrary and does not consider benefits like use or existence values. We assume that individuals rank species, giving them different weights according to the utility they accrue to them. This utility includes not only the value associated with genetic diversity but also the four categories of value described above.

The current number of named and recorded species is not known. Frequently cited estimates range from 1.4 million in 1978 (Southwood, 1978) to 1.8 million in 1988 (Stork, 1988). Estimations of the total number are even more uncertain. The total ranges from 3 million or 5 million to 30 million or more than 50 million in 1988 (May, 1988) or from 10 million to 100 million in 1996 (Simpson and Sedjo, 1996). Given this uncertainty, agents incorporate an infinite range of species in their utility functions. Known species are considered, regardless of whether they are currently producing utility or not, since they could be useful in the future. Unknown species that could be discovered in the future are also valued.

The number of recorded species is continuously growing (e.g., regarding the arachnid and crustacean species, May, (1988) shows that half of the total up to 1970 were recorded since 1960). The estimated total of living species is also rising rapidly. Recently discovered uses for new and old species, generates new utility (e.g., new drugs). Knowledge of how species interact in an ecosystem enhances the certainty of the negative effect that the extinction of any species would have on this ecosystem (e.g., today we know that many micro-organisms act as decomposers in the soil and leaf litter which is crucial to the

³ In addition to direct consumption, several services are provided by the species: absorbing wastes, mitigating floods and droughts, stabilizing the climate and maintaining the carbon cycle, ecotourism..., see Gretchen Daily (1996) for a more complete list of ecosystem services.

functioning of ecosystems). A characteristic of ecological systems is their complexity. They are usually represented by non-linear systems which display multiple stable and unstable equilibria. When the system is in the neighborhood of an unstable equilibrium, a small perturbation may have catastrophic consequences. Resilience is defined as the magnitude of disturbance that can be absorbed before the system changes its structure. A loss of functional diversity causes the resilience of the ecosystem to decrease. Thus, biodiversity conservation is an insurance against catastrophic changes. The crucial matter is the kind of species that are affected and the role they have in maintaining ecosystem resilience. Under particular conditions, previously independent species can develop affinities to others which gives them a key role in the future structure of the ecosystem (see Holling et al., 1995 and Perrings, 1995). If agents considered the chances of the environmental condition changing, currently low-valued species could develop a more significant role.

At the same time, an emerging moral or ethical value for any single species is appearing. Moreover, we assume that the low-valued species are actually being depleted faster and their populations are, therefore, becoming scarcer with respect to the high-valued ones (see Swanson, 1993). The level of information which can be expected from the exploration of the low-valued species is much higher than it is from the higher-valued ones. This is the result of their relative obscurity (see Swanson, 1995). We assume that the low-valued species are the less used ones.

We assume that these forces will probably be active, at least in the near future. This will increase 'option', 'quasi-option' and 'existence' value for the low-valued species in contrast with the high-valued ones. Therefore, the weights assigned to the set of species will be more homogeneously distributed. Following Rowthorn and Brown, 1995, we can also argue that expenditure on education, as to why we should care more about species preservation, can influence the parameters of the utility function.

Although the value of the global ecosystem remains unchanged, a more homogeneous distribution of weights means that more species must be conserved to get the same utility. That is, the value of biodiversity has grown.

The South exports goods intensive in natural resources, and conserves fewer species than the North. The marginal utility of conserving an additional species is then greater in the South. Therefore, the South has the comparative advantage in conserving species. That is, under general conditions, if the South decides to conserve more species while producing the goods it exports, its terms of trade will rise (e.g., from ecological farming and all kinds of ecological products, such as cosmetics, pesticides, etc). This is true even if the North also decides to increase the number of conserved species by the same amount.

The other aspect is the effect of a more homogeneous distribution of weights among species. Here, we are concerned with a change in preferences which implies not a higher valuation of the global ecosystem, but of biodiversity. The South will be worse off after this possible change, so it bears an associated risk from the current situation (in recent decades, the developing countries have seen how their terms of trade have been reduced due to the drop in the international price of agricultural products). Under some conditions, by increasing the number of conserved species, the South reduces this kind of risk. At the same time, its relative price rises.

The outline of the paper is as follows: in Section 2, we introduce a general equilibrium model of North–South trade in which conserved biodiversity enters as part of the utility function. Assuming that the number of conserved species affects the demand for goods, Section 3 shows the effect of a variation in this number upon the terms of trade. At the same time, it displays how a flatter distribution of weights would reduce the terms of trade in the South. In Sections 4 and 5, we carry out the same study but also take into account the effect on the supply side. Section 6 simulates the behavior of the South's relative price when producers of both goods increase the number of conserved species by the same amount. Finally, we present conclusions in Section 7.

2. Biodiversity as a source of utility

We review an Arrow–Debreu general equilibrium model of North–South trade, following Chichilnisky (1991a). Biodiversity enters the utility function as the welfare obtained from keeping the populations of different species within an ecosystem unaltered.

Developed and developing countries produce and consume two goods, *A* and *B*. These goods are produced by two inputs, natural resources and capital (by which we mean both physical and human capital). *A* is intensive in capital while *B* is intensive in natural resources. Therefore, one region is capital intensive and exports product *A*, while the other is resource intensive and exports product *B*. Each region is incompletely specialized in one product.

An agent's utility comes not only from the quantity consumed, but also from the conserved biodiversity. The latter is measured by the number of species whose population remains unaltered in the ecosystem which supports the production of the consumer goods. Furthermore, the marginal utility of conserved species decreases as its number grows.

An agent decides the consumption of either product *A* or *B* considering the impact of his decision on biodiversity. This leads to a non-separable utility function between marketed goods and biodiversity, connecting with green marketing literature (see Polonsky and Mintu-Wimsatt, 1995 and Mintu-Wimsatt and Lozada, 1996). This idea is displayed by the following Cobb–Douglas utility function.

$$U(A,B) = A^{l(nA)}B^{l(nB)}$$

$$l(n) = \int_0^n (e^{-x/\mu})/\mu dx = 1 - e^{-n/\mu}.$$
 (1)



Figure 1.

Here, n_A (n_B) denotes the number of species whose population is not affected by the production of product *A* (*B*). It measures the conserved biodiversity under *A* (*B*). An individual ranks the infinite set of species from high to low value. These values decrease exponentially. Hence, the total value for the ecosystem is the weighted mean of this infinite set of species. To aggregate species, we specify an exponential distribution function, *l*(*n*). It is a concave, growing function of *n* and it is governed by the expected value, μ , which indicates the homogeneity of the distribution of weights. Therefore, the exponent in the utility function *l*(n_A) (*l*(n_B)) measures the value of the $n_A(n_B)$ conserved species under the production function of product *A* (*B*).

Additionally, since *B* is intensive in natural resources and *A* in knowledge, from now on we will assume that *A*'s production is more respectful than *B*'s, in terms of conserved biodiversity. That is, $n_A > n_B$.

Under this utility specification, two assertions are true:

• If parameter μ increases, the number of conserved species, $n_A(n_B)$, must rise to maintain the exponents and, thus, the utility function as constant. That is, concern about biodiversity rises. Notice that a flatter curve does not imply a higher value of the ecosystem but this value is more homogeneously distributed among species.⁴

⁴ We use an exponential distribution function to weigh the set of species. Therefore,

by definition $\int_0^\infty (e^{-x/\mu})/\mu dx = 1$, regardless of the value of μ .

• Additionally, given that l(n) is concave, together with the assumption that $n_A > n_{R'}$ then $l'(n_A) < l'(n_B)$.

Two additional assumptions are considered. First, the number of conserved species is characteristic of the product but not the region where it is produced; $n_A(n_B)$ does not change from one region to the other. However, since the North and the South export *A* and *B* respectively, n_A is basically connected with the North and n_B with the South. Second, biodiversity brings utility to consumers, regardless of whether it is conserved in the region they live in or not. Therefore, a consumer in the North will equally increase his utility when biodiversity conservation increases in his country or in another part of the globe, and vice versa.

Consumers maximize their utility, matching the quotient of marginal utilities and the quotient of prices

$$U'_{B}(A,B)/U'_{A}(A,B) = \theta A^{D}/B^{D} = P_{B}/P_{A'}$$
 (2)

where $\theta = l(n_B)/l(n_A)$; it measures the relative value of the n_B conserved species against n_A . Since $n_A > n_B$, then $0 < \theta < 1$. This equation holds in both regions, given that n_A, n_B, P_A and P_B do not change from the North to the South.

Conversely, on the supply side, parameters defining the production function and supply of inputs differ from the North to the South. Let us consider the South's economy in what follows.

Both goods are produced by a fixed coefficients production function

$$A^{S} = \min (E^{A}/a_{2}K^{A}/c_{2}),$$

$$B^{S} = \min (E^{B}/a_{1}K^{B}/c_{1}), a_{1}, a_{2}, c_{1}, c_{2} > 0.$$
(3)

Input *E* represents natural resources and *K* human and physical capital; the exponent indicates the product in which the input is employed. We assume a dual technology; *A* is capital intensive while *B* is resource intensive. Thus, $D = a_1c_2 - a_2c_1$ is always positive. The more dual the technologies are, the greater the value of *D*. Under competitive markets we show the relationship between input and output prices

$$P_{E} = (c_{2}P_{B} - c_{1}P_{A})/D, (4)$$

$$r = (a_1 P_A - a_2 P_B) / D.$$
(5)

The supply of each factor is defined as a positive slope function of the price plus an autonomous supply, \overline{E} and \overline{K} respectively

$$E^{S} = \alpha \cdot P_{F} + \overline{E}, \tag{6}$$

$$K^{S} = \beta \cdot r + \overline{K}, \qquad \alpha, \beta > 0.$$
(7)

The demand for the factor stems from the desired production of the two goods. In equilibrium all markets clear

$$E^{D} = E^{A} + E^{B} = a_{2}A^{S} + a_{1}B^{S}$$

$$K^{D} = K^{A} + K^{B} = c_{2}A^{S} + c_{1}B^{S}$$

$$B^{S} = (c_{2}E^{D} - a_{2}K^{D})/D.$$
(8)

$$A^{S} = (a_{1}K^{D} - c_{1}E^{D})/D.$$
(9)

$$E^D = E^S. (10)$$

$$K^D = K^S. \tag{11}$$

$$B^S = B^D + X_p^S. \tag{12}$$

$$A^D = A^S + X^D_A. aga{13}$$

$$P_B X_B^S = P_A X_A^D. \tag{14}$$

The gap between supply and demand is either exported if positive, or imported if negative. Thus, X_B^S is the amount of *B* produced but not consumed in the South, and X_A^D is the amount of *A* consumed but not produced in the South. The same equations apply for the North, with different parameter values.⁵ Moreover, equilibrium in international markets, ensures equal prices in both regions. Besides, the quantity exported by one region matches the quantity imported by the other

$$P_{\mathcal{B}}(S) = P_{\mathcal{B}}(N). \tag{15}$$

$$P_A(S) = P_A(N). \tag{16}$$

$$X_B^S(S) = X_B^D(N). (17)$$

Since (14) is satisfied in both regions and, at the same time, (15) to (17), then it is easy to show that

$$X_A^D(S) = X_A^S(N).$$
⁽¹⁸⁾

Using Walras' law, $P_BB + P_AA = rK + P_EE = Y$, together with equation (2), and assuming a price normalization, $P_A = 1$, we deduce the demand for each product in equilibrium

$$A^{D} = (rK + P_{E}E)/(1 + \theta), \qquad (19)$$

$$B^{D} = \theta(rK + P_{E}E) / [(1 + \theta)P_{B}].$$
⁽²⁰⁾

These are functions of θ , which is negatively related to demand of *A*, but positively related to demand of *B*. Again, the same reasoning applies for the North.

Equations (4), (5), (6), (7), equilibrium equalities and price normalization lead to demand and supply of goods *A* and *B*. These are functions of the relative price, $P_{B'}$ and parameters in the model. Focusing on product *A*, in equilibrium, the excess of supply in the North matches the excess of demand in the South, as shown in equation (18) and, therefore, the world's excess of demand for *A*, denoted by $F(P_{B'})$, equals zero

⁵ The North is characterized by lower α and higher β. Furthermore, the autonomous supply of capital \overline{K} is not lower, and the autonomous supply of natural resources \overline{E} , not larger than in the South, $\overline{K}(N) \ge \overline{K}(S)$ and $\overline{E}(N) \le \overline{E}(S)$. Therefore, the supply of capital is higher and the supply of natural resources lower in the North than the South. That explains why, from the standpoint of input supply, the South exports *B* and the North *A*.

$$A^{D}(S) - A^{S}(S) = A^{S}(N) - A^{D}(N) \Rightarrow F(P_{B}) \equiv P_{B}^{2}\{(\Gamma(N) + \Gamma(S))/(1 + \theta)\} + P_{B}\{[\Phi(N) + \Phi(S) + (\theta - 1) (\Psi(N) + \Psi(S))]/(1 + \theta)\} - \theta[\Lambda(N) + \Lambda(S) + \Omega(N) + \Omega(S)]/(1 + \theta) = 0,$$
(21)

where $\Lambda = (\alpha c_1^2 + \beta a_1^2)/D^2$, $\Omega = (a_1 \overline{K} - c_1 \overline{E})/D$, $\Psi = (\alpha c_1 c_2 + \beta a_1 a_2)/D^2$, $\Gamma = (\alpha c_2^2 + \beta a_2^2)/D^2$ and $\Phi = (c_2 \overline{E} - a_2 \overline{K})/D$. Notice that in both regions, coefficients Λ , Ψ and Γ above are positive by definition. Ω and Φ represent the supply of A and B respectively, when the prices of input capital and natural resources are zero. We suppose the autonomous supplies of input \overline{K} and \overline{E} take values that ensure Ω and Φ to be simultaneously positive.⁶

A unique positive root of equation (21) always exists, which will be *B*'s relative price in equilibrium, P_B^* . This depends not only on parameters in the model but also on the ratio θ . From P_B^* , we obtain the equilibrium value of all endogenous variables in this model. Hence we have completely settled the general equilibrium model up to the number of conserved species in the North and the South, as well as the parameter μ which defines the function of weights.

3. Effect of $n_{A'}$, n_{B} and μ over the terms of trade, P_{B}^{*}

Effect of n_A , n_B

In this section we prove that, if producers of *B* increase the number of conserved species, $n_{B'}$ then the utility and also the quotient of marginal utilities grows. Consequently, by (2) the relative price, $P_{B'}^*$ rises. The same reasoning applies for producers of *A*. Therefore, by conserving more species, producers can raise the price of the goods they produce. Moreover, we show that this effect is stronger when applied by producers of *B*. The proof relies on the different marginal utilities of biodiversity, which is greater for product *B*. This good is mainly produced in the South, while *A* is produced in the North. Therefore we can say that the South would have a 'comparative advantage' in increasing the number of conserved species and thus rising the relative price.

We calculate the derivative of P_B with respect to the number of conserved species under goods *A* and *B*.⁷

$$\frac{\partial P_B}{\partial n_A} = \frac{\partial P_B}{\partial \theta \cdot \partial \theta} \frac{\partial \theta_A}{\partial n_A} < 0, \frac{\partial P_B}{\partial n_B} = \frac{\partial P_B}{\partial \theta \cdot \partial \theta} \frac{\partial \theta_B}{\partial n_B} > 0, \frac{\partial P_B}{\partial n_A} + \frac{\partial P_B}{\partial \theta_B} = \frac{\partial P_B}{\partial \theta \cdot \partial \theta} \frac{\partial \theta_A}{\partial n_A} + \frac{\partial P_B}{\partial \theta \cdot \partial \theta} \frac{\partial \theta_B}{\partial n_B} > 0.$$
(22)

Proof in Appendix A.

If the industry of the exported product in one region reduces the number of damaged species, then its terms of trade will rise. When the exporting

- ⁶ How close \overline{K} and \overline{E} must be depends on the coefficients of Leontief; i.e., how intensive each good is in capital or in natural resources. The more intensive productions are $(a_1 >> c_1 \text{ and } c_2 >\geq a_2)$, the larger the distance between the autonomous supply of inputs \overline{K} and \overline{E} .
- ⁷ Notice that $P_{B'}$ is the inverse of *A*'s relative price. Thus, the first inequality implies that a larger n_A reduces the relative price of *B*, or accordingly, increases the relative price of *A*.



sectors in both regions reduce the number of damaged species, the rise in the South's terms of trade is greater than the North's.

Equation (22) remains positive as long as inequality $n_A > n_B$ holds. If both figures grow by the same quantity then, although the gap between them remains constant, the relative gap tends toward zero. The 'comparative advantage' for the South goes to zero as the number of conserved species tends to infinity.⁸ Therefore, as n_A and n_B increase by the same quantity, the variation in P_B^* is always positive, although, it converges to zero (Figure 2). The speed of this convergence depends on parameters of the model.⁹

Effect of μ

We suppose that a shift in preferences is likely. Specifically, we assume that in the future, individuals will probably turn to a flatter scheme of weights and hence, to a more homogeneous valuation of the set of species. This change is displayed by a higher μ . A more homogeneous distribution implies that the weights of low-valued species grow in relation to high-valued ones (Figure 1). The exponent $l(n_B)$, which confers very high weights to a small rank of species, will drop in comparison with $l(n_A)$, which takes into account

⁸ Assuming n_A and n_B increase by the same amount, we can express the polynomial (21) as a function of n_A and λ , where $n_B = n_A - \lambda$. The positive root of this equation is the equilibrium price, $P_B^*[n_A, \lambda, \mu, \zeta]$, where $\zeta = (a_1, a_2, c_1, c_2, \overline{E}, \overline{K}, \alpha, \beta)$. Its limit when n_A goes to infinity tends to a positive constant

$$\begin{split} &\lim_{n_{A}\to\infty} \ P_{B}^{*} = - \left[\Phi(N) + \Phi(S)\right] / \left[2(\Gamma(N) + \Gamma(S))\right] + \\ &\sqrt{\left(\left[\Phi(N) + \Phi(S)\right] / \left[2(\Gamma(N) + \Gamma(S))\right]\right)^{2} + \left[\Omega(N) + \Omega(S) + \Lambda(N) + \Lambda(S)\right] / \left[\Gamma(N) + \Gamma(S)\right]} \right] > 0, \end{split}$$

and therefore the derivative of this equilibrium price with respect to n_A would tend to zero as $n_A \rightarrow \infty$.

⁹ In particular our simulations will show that the higher μ is, the lower the speed of convergence.

a wider range of species. Thus, a higher μ reduces the ratio θ and consequently the relative price of *B*. As long as *B*'s production affects more species than *A*'s, the South is bearing the risk of this drop in its relative price

$$\partial P_{\rm B}/\partial \mu = \partial P_{\rm B}/\partial \theta \cdot \partial \theta / \partial \mu < 0. \tag{23}$$

Proof in Appendix A.

As the number of conserved species by producers of *B* increases, the South is better off, firstly, because its relative price grows, even if producers of *A* increase n_A by the same amount and, secondly, because the risk associated with a larger μ , is reduced.

However, this conclusion does not consider the effect of this greater ecological production function on the supply side. We are implicitly assuming that firms can turn the way they produce into a more ecological one at no cost. A technological jump in both sectors allows firms to produce exactly as they did before but conserving a larger number of species.

4. Biodiversity also affect supply of goods

If producers decide to conserve a larger number of species, this would require additional quantity of inputs. This increment would be used either to achieve a more environmentally respectful technology or to reduce wastes and pollution. Thus, supply of goods shows the trade-off between species and production. This is shown by non-constant Leontief coefficients, which grow exponentially with the number of conserved species for each product. In the North

$$a_1^N(n_B) = a_1(N) \exp[n_B]; a_2^N(n_A) = a_2(N) \exp[n_A]$$

$$c_1^N(n_B) = c_1(N) \exp[n_B]; c_2^N(n_A) = c_2(N) \exp[n_A]$$
(24)

and likewise in the South.¹⁰ Note that $c_2^N(n_A)/a_2^N(n_A) = c_2(N)/a_2(N)$ as well as $a_1^N(n_A)/c_1^N(n_A) = a_1(N)/c_1(N)$, i.e., the number of conserved species does not affect the degree of intensity with which inputs are used.¹¹ Equations (8), (9) and (24) show that, everything else remaining constant, the supply of one product is negatively related to the number of undisturbed species. This formulation reflects the trade-off of biodiversity against production.

Using the model described in (2)–(20), but considering the new Leontief coefficients in (24), we obtain, again, a two-order polynomial for *A*'s excess of demand

$$\begin{split} F(P_B) &= A^D(N) + A^D(S) - [A^S(N) + A^S(S)] = \\ & 1/(2 - \mathbb{A} - \mathbb{B}) \left\{ P_B^2[(\Gamma(N) + \Gamma(S)) (1 - \mathbb{A})] / \exp[2n_B] + \right. \\ P_B[(\Phi(N) + \Phi(S)) (1 - \mathbb{A}) \exp[n_A] + (\Psi(N) + \Psi(S)) (\mathbb{A} - \mathbb{B})] / \exp[n_A + n_B] + \end{split}$$

¹⁰ Although the real specification needs another scale parameter, $\eta : a_2(n_A) = a_2 \exp[n_A/\eta]$, $a_1(n_B) = a_1 \exp[n_B/\eta]$, we normalize taking $\eta = 1$. The demand side remains unaffected by this normalization.

 $l(n) = 1 - \exp[-(n/\eta)/(\mu/\eta)] = 1 - \exp[-n/\mu],$

- then, the ratio $\theta = l(n_B)/l(n_A)$ does not depend on η .
- ¹¹ We are implicitly assuming a symmetric increase of both inputs in order to get a larger number of conserved species.

$$((\mathbb{B} - 1) [(\Lambda(N) + \Lambda(S)) + (\Omega(N) + \Omega(S)) \exp [n_A]]) / \exp[2n_A]] = a[n_A, n_B, \mu, \zeta] P_B^2 + b[n_A, n_B, \mu, \zeta] P_B + c[n_A, n_B, \mu, \zeta],$$

where

$$0 < \mathbb{A} = \exp\left[-n_A/\mu\right] < \mathbb{B} = \exp\left[-n_B/\mu\right] < 1,$$
$$0 < \mathbb{A} < \mathbb{B} \to 0 \text{ as } n_A > n_B \to \infty.$$

a [·] is positive and *c* [·] is negative; then the polynomial has a unique positive root. This root is the equilibrium terms of trade, $P_{B'}^*$ from which the complete model can be solved.

5. Effect of n_A , n_B and μ over the terms of trade, P_B^*

Effect of μ *over the terms of trade,* P_{B}^{*}

What would now be the response of the terms of trade to a switch in preferences, such that the value provided by the species set becomes more equally distributed? It is easy to see that in this case it is also true that

$$\partial P_{\rm p}/\partial \mu > 0.$$

As in Section 3.2, a more homogeneous valuation of species, higher μ , leads to a drop in P_B^* and consistently to a worse trade position for the South. Given our belief that this change in preferences is likely, the South is obliged to reduce the gap between the number of conserved species under its exported product, *B*, and the North's, *A*. This must occur in order to reduce the current risk of a decrease in P_B^* .

Effect of n_A *and* n_B *over the terms of trade* P_B^*

At this point we assume that biodiversity affects not only demand but also the supply of goods. The question arising is again the same: if producers of *B* conserve more species, would P_B^* rise, even if producers of *A* increase the number of conserved species as well?

As in the previous section, the relative price, $P_{B'}^*$ increases with n_B and decreases with n_A (see proof in Appendix A). The remaining question is whether the effect of n_B is stronger than the effect of n_A over $P_{B'}^*$ or the reverse

$$\partial F(P_B) / \partial n_A + \partial F(P_B) / \partial n_B = a'[n_A, n_{B'}, \mu, \zeta] P_B^2 + b'[n_A, n_{B'}, \mu, \zeta] P_B + c'[n_A, n_{B'}, \mu, \zeta].$$
(25)

The sign of this polynomial is unknown. Assumption $n_A > n_B$ still holds and we look for an additional condition under which (25) is negative. That is, the terms of trade is more strongly affected by n_B than by n_A .

Unfortunately this is not a simple question. For the particular case $n_A = n_B$, we find a condition which guarantees equation (25) to be negative. Under this condition, but in the general case, $n_A > n_B$, we prove that the

equilibrium price, P_B^* , is below an upper bound. This bound is given by the limiting value of P_B^* when both quantities, n_A and n_B , grow continuously by the same amount towards infinity. Therefore, if the producers of both goods increase equally conserved biodiversity, the terms of trade for the South will be higher at the limit. The problem is to prove that this is a monotonous process; we show this point by numerical simulations.

Proposition 1 In the particular case in which $n_A = n_{B'}$ a necessary and sufficient condition for equation (25) to be negative is given by

$$([\Omega(N) + \Omega(S)] / [\Phi(N) + \Phi(S)])^2 > [\Lambda(N) + \Lambda(S)] / [\Gamma(N) + \Gamma(S)].$$
(26)

Proof in Appendix B.

If we skip, for the sake of tractability, the difference between North and South's parameters, and simplify (26), we get

$$[(a_1\overline{K}-c_1\overline{E})/(c_2\overline{E}-a_2\overline{K})]^2 > (\alpha c_1^2 + \beta a_1^2)/(\alpha c_2^2 + \beta a_2^2),$$

which avoids the following extreme situations (either jointly in the North and the South, or with one of the regions offsetting the other):

- 1. $a_1 c_1$ small enough in comparison with $c_2 a_2$, which, according to (3) implies that *B* is proportionally less intensive in natural resources than *A* is in capital.
- 2. On the R.H.S. α multiplies c_1^2 in the numerator and c_2^2 in the denominator and, likewise, β multiplies a_1^2 in the numerator and a_2^2 in the denominator. Therefore, for a sufficiently small α and large β , the R.H.S. would be large enough to deny this condition. Notice that α is the slope of the supply of natural resources, while β is the slope of capital supply.

Denying both items means that product *B* has to be 'sufficiently' intensive in natural resources in comparison with how intensive *A* is in capital.

When inequality (26) holds, if producers of *A* and *B* increase the number of conserved species equally, the terms of trade will grow in the South, P_B^* . This condition is fully determined by parameters in the model, and does not depend on the current number of conserved species for either product.

For that to be so, we have supposed the number of conserved species under the production of *A* and *B* to be the same, $n_A = n_B$. Nevertheless, our model assumes $n_A > n_B$. In order to study this general case, we follow two steps:

Step 1

Thereafter, we assume that producers adopt the existing technology. This technology allows producers of both goods to increase the number of conserved species in one unit each time. As a result, $n_A - n_B = \lambda$ remains constant and equation (25) depends only on variable n_A and parameter λ . In this first step, we prove that the variation of P_B^* tends toward zero as n_A tends toward infinity.

The equilibrium price, $P_{B'}^*$ is the positive root of $F(P_B) = 0$ while P'_B is the positive root of $\partial F(P_B) / \partial n_A = 0.^{12}$

Proposition 2 The limit of $P_{B'}^*$ when n_A and n_B grow equally, is given by expression $\Omega \exp [-\lambda]/\Phi$, which depends only on the gap λ between the number of conserved species in the North and the South, and the parameters in the model.

Proof in Appendix B.

Remark 1 Likewise, it is easy to prove that the limit of $P'_{B'}$ i.e., the positive root of $\partial F(P_B) / \partial n_A = 0$, equals the same value, $\Omega \exp [-\lambda] / \Phi$.

Therefore, at the limit, the positive root of equation $F(P_B) = 0$ (which is the equilibrium price P_B^*) and the positive root of $\partial F(P_B)/\partial n_A = 0$ converge to the same constant. This being so, $\partial F(P_B^*)/\partial n_A$ converges toward zero. This means that the variation of P_B^* tends towards zero as n_A (and hence n_B) tends toward infinity.

Step 2

We cannot prove that the equilibrium price, $P_{B'}^*$ rises continuously with additional conserved species. However, we demonstrate that P_B^* converges towards the limiting value, $\Omega \exp [-\lambda]/\Phi$, from below.

Proposition 3 Under sufficient condition (26), the equilibrium price P_B^* always remains below its limiting value, $\Omega \exp [-\lambda]/\Phi$. This is true no matter what the value of n_A is.

Proof in Appendix B.

From the propositions above, condition (26) ensures the relative price P_B^* to be an increasing function of n_A in the particular case, $n_A = n_B$. In the general case, $n_A > n_B$, this condition implies that P_B^* is lower than an upper bound, $\Omega \exp[-\lambda]/\Phi$, and converges to this value from below. A numerical simulation in Table 4 shows that this is a monotonous process, that is, P_B^* grows continuously as n_A and n_B grow by the same amount.

6. Modelling and simulation

Following Chichilnisky (1991a), we set the parameters in Table 1.

The supply of natural resources is greater in the South (larger α and \overline{E}), while the supply of capital is larger in the North (larger α and \overline{E}). At the same time, *A*'s production is intensive in capital and *B*'s intensive in natural resources. Duality, measured by *D*, is higher in the South. Condition (26) is satisfied, which means that product *B* is intensive enough

¹² Note that only when the independent term in (25) is positive, equation $\partial F(P_B)/\partial n_A = 0$ has a positive root; otherwise this curve would never cross the P_B axis. But in this last case the derivative would always be negative. The problem arises in the first case, where this derivative can be positive or negative, depending on the value of P_B^* .

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	α	β	<i>a</i> ₁	a ₂	<i>C</i> ₁	<i>C</i> ₂	\overline{K}	\overline{E}
North	6	9.7	2	0.015	1	1.7	6	3
South	75	0.025	4.5	0.02	0.01	3	3	6

in natural resources, in comparison with how intensive product A is in capital.

From these data we get the relative price, $P_{B'}^*$ its derivative when n_A and n_B grow equally, and the derivative with respect to μ . We compare situations with low and high gaps between n_A and n_B ($\lambda = 0.1$ and 0.8) and we distinguish for different values of μ (0.1, 0.5, 1 and 4).

An increment in μ reduces the relative price in the South, as shown in (23). However, the larger n_A and n_B are, the less sharp the drop in P_B^* . Thus, the South's borne risk decreases.

On the other hand, the smaller the gap between n_A and $n_{B'}$ the larger the relative price in the South. This is true only in relative terms. If the difference between n_A and n_B does not change in absolute terms, the greater n_A and n_B are, the lower the relative gap between them and therefore, the lower the relative price, P_B^* .

Finally, as expected from equation (25) and sufficient condition (26), as n_A and n_B increase, so too does the relative price P_B^* . However, its rate of growth becomes lower and lower, which involves a process of convergence.

If n_A and n_B grow continuously, the South will be better off for two reasons: firstly, because the relative gap between these two figures is being reduced, the relative price, P_B^* , is increasing. Secondly, because the effect of a rise in μ is less which is less harmful for the South.

A simulation is carried out for the parameters above. We assume n_A and n_B grow one unit at a time; hence we can fix $n_B = n_A - \lambda$ and go on with the simulation considering only the variable n_A . Regardless of how big the gap λ is, the equilibrium price converges to $\Omega/(\Phi \exp [\lambda])$. Different values of λ are considered to study this convergence process.

The outcome in Appendix C shows the relative price, P_B^* , as n_A grows. The process stops when the marginal increment of P_B^* becomes lower than a tolerance level. Two scenarios are considered: a large gap between n_A and n_B ($\lambda = .8$) and a small one ($\lambda = .1$). For both cases, several values of μ are observed; from very uneven distributions of weights ($\mu = .1$) to more

μ	$\frac{\partial P_{B}}{\partial n_{A}}$ + $\frac{\partial P_{B}}{\partial n_{B}}$	$\partial P_{B} / \partial \mu$	P_B^*	μ	$\partial P_{B} / \partial n_{A}$ + $\partial P_{B} / \partial n_{A}$	∂Р _В /∂µ	P_B^*
0.1	1.39004	-1.37663	0.346153	0.1	1.85936	-1.15353	0.742477
0.5 1	0.872266	-0.033893	0.137204	1	1.83347	-0.007333 -0.017424	0.628089
4	0.719844	-0.002000 0.9 $n = 0.10$	0.098184	4	$1.80145_{n} =$	-0.001112 0.2 $n = 0.10$	0.59772 $\lambda = 0.1$
$n_A = 0.2, n_B = 0.1(K = 0.0)$				"A	$0.2, m_B = 0.1$	n 0.1)	

Table 2

μ	$\partial P_{B} / \partial n_{A}$	$\partial P_{B} / \partial \mu$	P_B^*	μ	$\partial P_{B} / \partial n_{A}$	$\partial P_{B}/\partial \mu$	P_B^*
	$+ \partial P_{B} / \partial n_{B}$				$+ \partial P_{B} / \partial n_{B}$		
0.1	0.0490426	-0.022897	0.496351	0.1	0.092361	-0.000050	1.00019
0.5	0.197418	-0.18835	0.418818	0.5	0.107746	-0.034602	0.991785
1	0.20492	-0.071715	0.359448	1	0.11473	-0.019457	0.97822
4	0.18032	-0.003482	0.299706	4	0.114083	-0.001921	0.959501
$n_A = 1.5, n_B = 0.7(\lambda = 0.8)$				$n_A = 1.5, n_B = 1.4(\lambda = 0.1)$			

Table 3

homogeneous ones (μ = 2). All cases display convergence to the limit value from below. This process is monotonous, so the relative price P_B^* grows continuously.

Patterns differ on scale and speed of convergence. The smaller the gap, λ , the larger the initial and final values for P_B^* . However, at the same time, the relative gap between them is smaller. On the other hand, a homogeneous distribution of weights, a large μ , implies a slow convergence process. Therefore, the higher movements in the relative price result in high differences in the number of conserved species, large λ , and very unequal distribution of weights, small μ . At that time the South has greater 'comparative advantage'.

7. Conclusions

A general equilibrium model is used to characterize the North–South trade relationship. Biodiversity also enters this model. An agent's utility depends not only on consumed quantities but also on the number of conserved species under the production function of each product. Marginal utility of biodiversity decreases with the level of currently conserved species. At the same time, biodiversity is negatively related to the supply of goods.

We have deduced the necessary conditions under which the South is better off when producers of both goods increase the number of conserved species by the same amount. The South's relative price, $P_{B'}^*$ increases with the number of conserved species under production of *B*. Likewise, it decreases with the number conserved under *A*. Given that n_B is lower than n_A , the marginal effect of the former is stronger. Therefore, if both figures are jointly increased, the terms of trade grows in the South. Moreover, this process converges as the number of conserved species in both regions becomes larger. This being so, the comparative advantage for the South, although positive, tends towards zero as n_A and n_B go towards infinity.

The second issue concerns a likely change in preferences. We assume that individuals' preferences will move to a more egalitarian valuation of species. This shift will drop the South's terms of trade and this region is therefore at risk from this kind of change in preferences. Note that the lump sum of weights for the infinite set of species remains constant and equal to one. This means that the valuation of the global environment remains unchanged. However, given that more species are needed to get the same utility as before the change in preferences, the importance of biodiversity has been increased. From both points of view, the South will be better off increasing the number of conserved species. Its terms of trade will rise to a higher extent than the North's. As we have already seen, the South has a 'comparative advantage' conserving biodiversity. At the same time, the risk of a drop in the South's terms of trade, as a result of a flatter distribution of weights, decreases.

The main feature of this model is that it takes the utility provided by biodiversity into account. This presupposes that the markets 'perfectly' internalize a desire for biodiversity conservation through the consumer's utility function, which is non-separable between marketed goods and biodiversity. We are aware of the failures of the market to do so (see Panayotou 1996). Some of the benefits of diverse species are flows which are hardly captured by individuals or even by governments. Although this is not a realistic assumption, the conclusions are still true but to the extent to which the markets can, indeed, internalize this desire.

Consumers care about biodiversity, both in their region and abroad. A more realistic specification and a future line of research would distinguish consumers in the North, who actually care about the environment in the South, and consumers in the South, who are less concerned about the same problems in the North. Another extension could refer to the relationship between Leontief's coefficients and the number of conserved species. The latter could be assumed to increase at the expense of a non-symmetrical increment in both inputs, but by a greater amount in capital than in natural resources.

References

- Blamey, R.K. and M. Common (1992), 'Sustainability and the limits to pseudo market valuation', in M. Lockwood and T. DeLacy, eds., Valuing Natural Areas: Application and Problems of the Contingent Valuation Method, Johnstone Centre: Charles Sturt University, Albury, 117–146.
- Chichilnisky, G. (1991a), 'A general equilibrium theory of North–South trade', in W. Heller, D. Starrett, and R. Starr, eds., *Equilibrium Analysis*, vol. II, Cambridge University Press, pp. 3–56.
- Chichilnisky, G. (1991b), 'North–South trade and the global environment', *American Economic Review* 84 (4, September): 851–873.
- Copeland, Brian R. and M. Scott Taylor (1994), 'North–South trade and the environment', *The Quarterly Journal of Economics*, August: 755–787.
- Gretchen Daily (1996), 'Ecological issues statement on ecosystem services', Stanford University Department of Biological Sciences.
- Grossman, Gene M., and Alan B. Krueger (1991), 'Environment impacts of a North American Free Trade Agreement', NBER Working Paper No. 3914, November.
- Holling, C. S. *et al.* (1995), 'Biodiversity in the functioning of ecosystems: an ecological synthesis', in Charles Perrings, Karl-Göran Mäler, Carl Folke, C. S. Holling, and Bengt-Owe Jansson, eds., *Biodiversity Loss*, Cambridge: Cambridge University Press, 44–84.
- Jakobsson, Kristin M. and Andrew K. Dragun (1996), Contingent Valuation and Endangered Species, New Horizons in Environmental Economics, Edward Elgar.
- Krugman, Paul (1979), 'A model of innovation, technology transfer, and the world distribution of income', *Journal of Political Economy* **87**(21): 253–266.

May, R. M. (1988), 'How many species are there on earth?', *Science* 247: 1441–1449. Mintu-Wimsatt, A. and Hector R. Lozada, eds., (1996), 'Green marketing in a unified Europe', *Journal of Euromarketing* **5**(3), New York and London: Haworth Press, Internal Business Press.

- Panayotou, Theodore (1996), 'Green markets: the economics of sustainable development', International Center for Economic Growth and the Harvard Institute for International Development.
- Perrings, C. (1995), 'Biodiversity conservation as insurance', in Timothy M. Swanson, ed., *The Economics and Ecology of Biodiversity Decline: The Forces Driving Global Change*, Cambridge: Cambridge University Press, pp. 69–77.
- Perroni, Carlo and Randall M. Wigle (1994), 'International trade and environmental quality: how important are the linkages?', *Canadian Journal of Economics* **27** (3, August): 551–567.
- Polonsky, Michael J. and A. Mintu-Wimsatt (1995), *Environmental Marketing: Strategies, Practice, Theory, and Research, Binghampton, NY: The Hayworth Press.*
- Rowthorn, B. and G. Brown (1995), 'Biodiversity economic growth and the discount rate', in Timothy M. Swanson, ed., *The Economics and Ecology of Biodiversity Decline: The Forces Driving Global Change*, Cambridge: Cambridge University Press, pp. 25–39.
- Simpson, R. D. and R. A. Sedjo (1996), 'Valuing biodiversity for use in pharmaceutical research', *Journal of Political Economy* **104**(1): 163–185.
- Southwood, T. R. E. (1978), 'The components of diversity', in L. A. Mound and N. Waloff, eds., *Diversity of Insect Faunas*, Oxford: Blackwell.
- Stork, N. E. (1988), 'Insect diversity: facts, fiction and speculation', *Biol. J. Linn. Soc.* **35**, 321–337.
- Swanson, T. (1993), 'Endangered species', Economic Policy, April: 183–205.
- Swanson, T. (1995), 'The international regulation of biodiversity decline: optimal policy and evolutionary product', in Charles Perrings, Karl-Göran Mäler, Carl Folke, C. S. Holling, and Bengt-Owe Jansson, eds., *Biodiversity Loss*, Cambridge: Cambridge University Press, pp. 225–259.
- Weitzman, M. L. (1992), 'On diversity', *Quarterly Journal of Political Economics* 107: 363–406.
- Weitzman, M. L. (1993), 'What to preserve? An application of diversity theory to crane conservation', *Quarterly Journal of political Economics* **108**: 157–184.

Appendix A: Derivatives of P_B with respect to n_A , n_B and μ

Derivatives when biodiversity only affects utility

To calculate these derivatives,¹³ we first calculate the partial derivative of P_B with respect to the ratio between exponents in the utility function, θ .

$$\partial P_B / \partial \theta = - \left[\partial F(P_B) / \partial \theta \right] / \left[\partial F(P_B) / \partial P_B \right] =$$
(A.1)

$$-1/(1+\theta)[-P_B^2\Gamma+P_B(2\Psi-\Phi)-(\Lambda+\Omega)]/[2P_B\Gamma+(\Phi+(\theta-1)\Psi)]>0.$$

In equation (19), demand of *A* is a negative function of θ . Additionally supply of *A*, in equation (9), does not depend on this ratio. Hence, the excess of demand, given by polynomial (21) is necessarily negatively related to θ , and so, the numerator in the expression above is positive. On the other hand, in equilibrium, *A*'s excess of demand, *F*(*P*_B), presents a positive slope when pictured against the price *P*_B (Figure 3), and the denominator in (A.1) is consequently positive. Showing this point is straightforward, since the relative price in equilibrium is the positive root of equation (21), which is always greater than – $(\Phi + (\theta - 1)\Psi)/2\Gamma$. *P*_B is therefore positively related to θ .

Effect of n_A and n_B

First we obtain the derivative of θ to changes in n_A and n_B . Since $n_A > n_{B'}$ the positive effect given an increase in n_B is stronger than the negative one caused by an additional unit of n_A .

$$\begin{aligned} \partial \theta / \partial n_A &= -e^{-n_A/\mu} (1 - e^{-n_B/\mu}) / [\mu (1 - e^{-n_A/\mu})^2] < 0. \\ \partial \theta / \partial n_B &= e^{-n_B/\mu} / [\mu (1 - e^{-n_A/\mu})] > 0. \\ \partial \theta / \partial n_A &+ \partial \theta / \partial n_B &= (e^{-n_B/\mu} - e^{-n_A/\mu}) / [\mu (1 - e^{-n_A/\mu})^2] > 0. \end{aligned}$$

which proves inequality (22).

Effect of µ

Finally we prove that derivative of θ with respect of μ is negative, provided $n_A > n_B$.

$$\begin{aligned} \partial \theta / \partial \mu &= [n_A e^{-n_A/\mu} - n_B e^{-n_B/\mu} + (n_B - n_A) e^{-(n_A + n_B)/\mu}] / [\mu^2 (1 - e^{-n_A/\mu})^2] \\ &\quad \partial \theta / \partial \mu < 0 \Leftrightarrow \\ n_A e^{-n_A/\mu} / (1 - e^{-n_A/\mu}) < n_B e^{-n_B/\mu} / (1 - e^{-n_B/\mu}). \end{aligned}$$

Since $n_A > n_B$, all we must show is that $\phi(x) = x \exp[-x/\mu]/(1 - \exp[-x/\mu])$ is a decreasing function for any value of μ .

$$\phi'(x) = (\exp[x/\mu] - x/\mu \exp[x/\mu] - 1)/(\exp[x/\mu] - 1)^2.$$

Let us call $y = x/\mu$, then $\phi'(x)$ is negative if, and only if, the numerator is negative

¹³ Without loss of generality, and to simplify notation, we gather together parameters for the North and South.

$$\exp[y](1-y) - 1 < 0. \tag{A.2}$$

y is always greater than zero, such that two cases arise:

- $y \ge 1$: inequality (A.2) holds.
- $y \in (0, 1)$: by Taylor expansion of exp [-y] about y = 0

 $\exp[-y] \simeq 1 - y + y^2/2 \Rightarrow \exp[-y] > (1-y).$

• which proves that inequality (A.2) also holds.

A larger μ implies a lower relative price for the South, P_{B}^{*} .

Derivatives when biodiversity also affects supply of goods

$$\partial P_{\rm B}/\partial x = -\left[\partial F(P_{\rm B})/\partial x\right]/\left[\partial F(P_{\rm B})/\partial P_{\rm B}\right].$$

 A^D is a growing function and A^S a decreasing function for positive values of P_B . Therefore, the denominator in the expression above, $\partial F(P_B)/\partial P_B$, is always positive at the equilibrium price

Effect of μ

Therefore, to find out how P_{B} varies when μ grows, we calculate

$$\partial F(P_{\rm B})/\partial \mu = \partial A^D/\partial \theta \cdot \partial \theta / \partial \mu - \partial A^S / \partial \mu > 0. \tag{A.3}$$

 $\partial A^D/\partial \theta$ is negative by (19), additionally, we have proved that $\partial \theta/\partial \mu$ is also negative. Finally, considering (9) and (24), A^S does not depend on μ . In consequence, $\partial F(P_B)/\partial \mu$ is positive. The relative price in the South will drop after an increment in μ .

Effect of n_A and n_B

Gathering together parameters for the North and the South in order to simplify notation, we present expressions for A^D and A^S

$$A^{D} = (1 - \mathbb{A}) \Theta / (2 - \mathbb{A} - \mathbb{B})$$
$$A^{S} = \Omega \exp [-n_{A}] + \Lambda \exp [-2n_{A}] - P_{B}\Psi \exp [-n_{A}] \exp [-n_{B}]$$

where $\Theta = P_B^2 \Gamma \exp \left[-2n_B\right] + P_B\left[(\exp \left[-n_B\right]\Phi - 2\Psi \exp \left[-n_A - n_B\right])\right] + (\Lambda \exp\left[-2n_A\right] + \Omega \exp\left[-n_A\right]).$

Demand for product *A* has to be positive and a growing function of the relative price, $P_{B'}$ at the equilibrium price. Therefore, $\Theta > 0$ and $2AP/2P_{-} =$

$$(1 - \mathbb{A})/(2 - \mathbb{A} - \mathbb{B})[2P_B\Gamma \exp[-2n_B] + (\exp[-n_B]\Phi - 2\Psi \exp[-n_A - n_B])] > 0,$$

which implies

$$(\exp\left[n_{A}\right]\Phi - 2\Psi) > 0. \tag{A.4}$$

1. $\partial F(P_B) / \partial n_B$

Effect of n_B over A's demand and supply

$$\partial A^S / \partial n_B = \Psi \exp[-n_A] \exp[-n_B] > 0.$$

We can solve the derivative of Θ with respect to n_B .

$$\partial \Theta / \partial n_{\scriptscriptstyle B} = -P_{\scriptscriptstyle B} \{ 2P_{\scriptscriptstyle B} \Gamma \exp \left[-2n_{\scriptscriptstyle B} \right] + \left[(\exp \left[-n_{\scriptscriptstyle B} \right] \Phi - 2\Psi \exp \left[-n_{\scriptscriptstyle A} - n_{\scriptscriptstyle B} \right]) \right] \},$$

which is negative, since the term in brackets has the sign of $A^{D's}$ slope with respect to $P_{B'}$ which has to be necessarily positive at the equilibrium price. Hence

$$\partial A^D / \partial n_B = -(1-\mathbb{A})\mathbb{B}\Theta / [(2-\mathbb{A}-\mathbb{B})^2\mu] + (1-\mathbb{A})/(2-\mathbb{A}-\mathbb{B})\partial \Theta / \partial n_B < 0.$$

This proves that $\partial F(P_B)/\partial n_B < 0$. A larger n_B leads to a larger relative price of product *B*, P_B^* .

2. $\partial F(P_B) / \partial n_A$

Since Θ is positive

$$\begin{split} \partial F(P_B) / \partial n_A &= \\ 1 / [\mu(2 - \mathbb{A} - \mathbb{B})^2] \{ P_B^2[\Gamma(\mathbb{A} - \mathbb{A}\mathbb{B})/\exp\left[2n_B\right]] + \\ P_B[\Phi \exp[n_A](\mathbb{A} - \mathbb{A}\mathbb{B})/(\exp\left[n_A\right]\exp\left[n_B\right]) + \\ \Psi[2(\mathbb{A}\mathbb{B} - \mathbb{A}) + \mu(\mathbb{B} - \mathbb{A})(2 - \mathbb{A} - \mathbb{B})]/(\exp\left[n_A\right]\exp\left[n_B\right])] + \\ \Lambda(1 - \mathbb{B})[2(2 - \mathbb{A} - \mathbb{B})\mu + \mathbb{A}]/\exp\left[2n_A\right] + \\ \Omega(1 - \mathbb{B})[(2 - \mathbb{A} - \mathbb{B})\mu + \mathbb{A}]/\exp\left[n_A\right]] = \\ 1 / [\mu(2 - \mathbb{A} - \mathbb{B})^2] \{\Theta + \Psi\mu(\mathbb{B} - \mathbb{A})(2 - \mathbb{A} - \mathbb{B})/(\exp\left[n_A\right]\exp\left[n_B\right]) + \\ \Lambda(1 - \mathbb{B})2(2 - \mathbb{A} - \mathbb{B})\mu/\exp\left[2n_A\right] + \Omega(1 - \mathbb{B})(2 - \mathbb{A} - \mathbb{B})\mu/\exp\left[n_A\right]\} = \\ 1 / [\mu(2 - \mathbb{A} - \mathbb{B})^2]\Theta > 0 \end{split}$$

An increment in n_A , reduces the relative price P_B and, hence, increases the relative price for product *A*.

Appendix B: Proofs of propositions

Proof of proposition 1

When $n_A = n_{B'}$ the excess of demand for product A and its partial derivative with respect to n_A respectively are second-order polynomials

$$F(P_B) = [\Gamma P_B^2 + \Phi \exp[n_A]P_B - (\Lambda + \Omega \exp[n_A])]/(2 \exp[2n_A]).$$

$$\partial F(P_B)/\partial n_A = \left[-2\Gamma P_B^2 - \Phi \exp\left[n_A\right]P_B + (2\Lambda + \Omega \exp\left[n_A\right])\right]/(2 \exp\left[2n_A\right]).$$

Both polynomials have a unique positive root, P_B^* and P' respectively. If we could always set the parameters in which $P_B^* > P'$ then we would have found a condition which ensures that $\partial F(P_B)/\partial n_A$ is negative at the equilibrium price P_B^* .

We define coefficients, $a = \Phi \exp[n_{A}]/(2\Gamma)$, $b = \Phi \exp[n_{A}]/(4\Gamma) = a/2$, $\alpha = [2\Lambda + \Omega \exp[n_A]]/(2\Gamma)$, and $\epsilon = \Omega \exp[n_A]/(2\Gamma)$.

We want to prove the following inequality

$$P^* = -a + \sqrt{a^2} + \alpha + \epsilon > P' = -b + \sqrt{b^2} + \alpha,$$

$$\sqrt{4b^2 + \alpha + \epsilon} - \sqrt{b^2 + \alpha} > a - b = b.$$

After some manipulation this is equivalent to

 $\epsilon^2 > 4b^2(\alpha - \epsilon)$.

$$\begin{split} (\Omega \exp{[n_A]}/(2\Gamma))^2 &> 4(\Phi \exp{[n_A]}/(4\Gamma))^2[(2\Lambda + \Omega \exp{[n_A]})/(2\Gamma) \\ &- \Omega \exp{[n_A]}/(2\Gamma)], \end{split}$$

and we get sufficient condition (26)

 $([\Omega(N) + \Omega(S)]/[\Phi(N) + \Phi(S)])^2 > [\Lambda(N) + \Lambda(S)]/[\Gamma(N) + \Gamma(S)].$



Figure 3



Figure 4

Proof of proposition 2

Assuming a continuous equal increment, in n_A and n_B , we can set $n_B = n_A - \lambda$. Let us then write $F(P_B)$ as a function of n_A and λ

$$\begin{split} &1/[(2 - \mathbb{A} - \mathbb{A} \exp [\lambda/\mu]) \exp [2n_A - 2\lambda]] \{P_B^2 \Gamma(1 - \mathbb{A}) + P_B[\Phi(1 - \mathbb{A}) \exp [n_A] + \Psi \mathbb{A}(1 - \exp [\lambda/\mu])] \exp [-\lambda] + \\ & (\mathbb{A} \exp [\lambda/\mu] - 1)(\Lambda + \Omega \exp [n_A]) \exp [-2\lambda] \} = \\ & P_B^2 g_2 + P_B g_1 + g_0, \text{ where } g_i = g_i (n_A, \lambda, \mu, \zeta). \end{split}$$

The equilibrium price is given by $P_B^* = -g_1/(2g_2) + ([g_1/(2g_2)]^2 - g_0/g_2)^{1/2}$. In order to obtain its limiting value when $n_A \to \infty$ we multiply and divide this price by expression $g_1/(2g_2) + ([g_1/(2g_2)]^2 - g_0/g_2)^{1/2}$

$$P_B^* = [-g_0/g_2]/[g_1/(2g_2) + ([g_1/(2g_2)]^2 - g_0/g_2)^{1/2}]$$

If we multiply the numerator and the denominator by exp $[-n_A]$ and calculate the limit of this quotient as the quotient of limits, we get the limit of P_B^* when $n_A \rightarrow \infty$, only as a function of λ

$$\lim_{\substack{n_A \to \infty \\ n_A \to \infty}} P_B^* = \\ \lim_{\substack{n_A \to \infty \\ n_A \to \infty}} \{ [-g_0/g_2 \exp [-n_A] \} / \lim_{\substack{n_A \to \infty \\ n_A \to \infty}} \{ [g_1/(2g_2) + ([g_1/(2g_2)]^2 - g_0/g_2)^{1/2}] \exp [-n_A]) = \\ \Omega \exp [-\lambda] / \Phi \end{cases}$$

Proof of proposition 3 We try to prove

$$P_B^* = -g_1/(2g_2) + ([g_1/(2g_2)]^2 - g_0/g_2)^{1/2} < \Omega \exp[-\lambda]/\Phi;$$

- $g_0/g_2 - \Omega \exp[-\lambda]g_1/(\Phi g_2) < (\Omega \exp[-\lambda]/\Phi)^2.$

After some simplification, this inequality gives

$$\begin{split} \Lambda(1 - \mathbb{A} \exp \left[\lambda/\mu\right]) / [\Gamma(1 - \mathbb{A})] + \Omega \mathbb{A} \left(1 - \exp \left[\lambda/\mu\right]\right) (\Phi \exp \left[n_A\right] - \Psi) / [\Gamma(1 - \mathbb{A})\Phi] < [\Omega/\Phi]^2. \end{split}$$

At this point it is easy to see that when sufficient condition (26) holds, so too does the inequality above for the general case, because by inequality (A.4), the second term in the L.H.S., is negative.

Appendix C: Simulation

Table 4, shows the value of P_B^* against n_A when n_A and n_B are increasing by the same amount. In the first column, the gap between n_A and n_B is large, while in the second column it is small. \overline{n}_A is the number of conserved species after which the increase in the relative price, P_B^* , as a consequence of an additional unit of n_A and n_B , is lower than the tolerance.

	Table 4	
$n_A = 0.9, n_B = 0.1,$	$n_A = 0.2, n_B = 0.1$	
P_B^{\star}	P_B^{\star}	
Fig. 5.1.a $\mu = 0.1(\overline{n}_A = 152)$ Fig. 5.2.a $\mu = 0.5(\overline{n}_A = 152)$	Fig. 5.1.b $\mu = 0.1(\overline{n}_A = 166)$ Fig. 5.2.b $\mu = 0.5(\overline{n}_A = 166)$	
Fig. 5.3.a $\mu = 1(\overline{n}_A = 158)$ Fig. 5.4.a $\mu = 2(\overline{n}_A = 290)$	Fig. 5.3.b $\mu = 1(\overline{n}_{A} = 166)$ Fig. 5.4.b $\mu = 2(\overline{n}_{A} = 266)$	



Figure 5.1a



Figure 5.2b



Figure 5.4a



Figure 5.4b

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