Macroeconomic Dynamics, **6**, 2002, 496–522. Printed in the United States of America.
DOI: 10.1017.S1365100501010021

THE INFLATION TAX, VARIABLE TIME PREFERENCE, AND THE BUSINESS CYCLE

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This paper investigates the impact of anticipated inflation on features of the business cycle in the presence of recursive but intertemporally dependent tastes. Intertemporal dependence is induced by the presence of a variable or endogenous individual rate of time preference. Quantitative experiments indicate that variability in the rate of time preference can enhance the contribution of monetary shocks to the fluctuations of real variables. Another implication of the variable-time-preference model is that, unlike the fixed time preference model, the business cycle features in high inflation and low inflation economies can be very different. The contribution of monetary shocks to fluctuations increases partly because endogenous time preference accentuates inflation-tax effects, which are already present in the standard framework because of the presence of cash-in-advance constraints. The change in the relative role of monetary shocks is also related to how variable time preference alters the effects of technology shocks, which can be quantitatively or qualitatively different in comparison to the standard model, depending on the parameters of the model.

Keywords: Inflation, Business Cycles, Monetary Shocks, Variable Time Preference

1. INTRODUCTION

There are several discussions of the monetary nonneutralities that arise in neoclassical growth models with money introduced via cash-in-advance constraints on the purchases of consumption goods, and sometimes on the purchases of labor and capital. The prediction of these models, that money growth and output are negatively correlated in the long run, has considerable empirical support. See, for example, Kormendi and Meguire (1985), Summers and Heston (1991), Levine and Renelt (1992), Fischer (1993), and Barro (1995). The mechanism that leads to a negative correlation between money growth and output in cash-in-advance models is typically described as follows: Anticipated monetary shocks cause substitution

This paper is based on material from a chapter of my doctoral dissertation at Southern Methodist University, Dallas, Texas, USA. I am indebted to Greg Huffman, Nathan Balke, Jim Dolmas, Joe Haslag, Hideo Konishi, Lisa Magnani, Thomas Osang, E. Juerg Weber, and Mark Wynne for helpful comments on an earlier version of this paper. I also thank two anonymous referees and an associate editor of this journal for their helpful and constructive comments. I take responsibility for any errors in the paper. Address correspondence to: Radhika Lahiri, School of Economics and Finance, Faculty of Business, Queensland University of Technology, GPO Box 2434, 2 George Street, Brisbane, Queensland 4001, Australia; e-mail: r.lahiri@qut.edu.au.

out of goods subject to the cash constraint toward noncash goods such as leisure. Consequently, there is a decline in output. Such *anticipated inflation* or *inflation tax* effects have been discussed extensively by Stockman (1981), Greenwood and Huffman (1987), Cooley and Hansen (1989, 1991, 1992) among others.

The role of anticipated inflation or inflation tax effects in generating monetary nonneutralities in neoclassical growth models with cash-in-advance constraints has been assessed in many different ways. In addition to studies that confine their attention to the effects of inflation/money growth on the deterministic stationary state of the economy, there are also analyses of nonneutralities on the *transition path* to the steady state, such as that of Abel (1985). More recently, economists have assessed the quantitative significance of inflation-tax effects by examining whether these effects lead to a larger contribution of monetary shocks to the *fluctuations* of real variables. In one of the first papers in the tradition of equilibrium business cycle models with money, Cooley and Hansen (1989) address this issue by comparing the cyclical properties of a benchmark real-business-cycle model with that of the model modified to incorporate money via a cash-in-advance constraint on consumption purchases. They conclude that the contribution of monetary shocks to fluctuations of real variables is insignificant relative to technology shocks.

This paper is another exploration of monetary nonneutralities arising due to the inflation-tax effects of money growth. The model of this paper extends that of Cooley and Hansen (1989) by introducing variability in the representative agent's rate of time preference. Specifically, the discount factor applied to future utility is decreasing in contemporaneous utility, reflecting an increase in the agent's *impatience* as he or she feels better off. Preferences, which are still recursive but not time additive, are formulated according to Epstein's (1983) concept of *stationary cardinal utility*. Epstein postulates conditions under which such preferences are consistent with expected utility, and conditions that ensure the dynamic stability of models that incorporate such preferences.

An extension of the standard cash-in-advance framework to allow for variable or *endogenous* time preference has several motivations. One motivation, which would be applicable to similar extensions of *any* dynamic general equilibrium framework, is obvious. The flexible-time-preference framework is a more general one, and nests the fixed-discount-factor framework as a special case. It is therefore desirable to check whether conclusions of standard frameworks are robust to this generalization. In the context of monetary dynamic general equilibrium models of the type studied in this paper, we are interested in whether the percentage contribution of monetary shocks to the volatility of output is enhanced or diminished by incorporating variable time preference. Other issues of interest are the implications of endogenous time preference for the long run effects of money growth, and for the transition dynamics of real variables following a money growth shock. A priori, it is reasonable to expect that monetary nonneutralities will be *enhanced* in the variable-time-preference framework. Loosely speaking, the underlying intuition is as follows: In an environment in which monetary injections and real activity are negatively correlated due to the presence of inflation-tax effects, the utility decline associated with the money shock is followed by an increase in the discount factor. This would make individuals more patient, and consequently more tolerant of sacrifices in current consumption benefits, producing a larger negative correlation between money growth and real activity.

To address these issues, we calibrate the model economy to match second moment properties of postwar U.S. data. Volatility of real output in the model with the variance of the technology shock approximately set to zero is computed. This volatility, expressed as a percentage of the output volatility in the model with both money and technology shocks, is a measure of the contribution of monetary shocks to the fluctuations in output. It turns out that the assumption of endogenous time preference can enhance this contribution considerably, although it remains much smaller than that of technology shocks. Furthermore, increasing the mean growth rate of monetary shocks (where monetary shocks are assumed to follow an autoregressive process), affects the cyclical features of the variable-time-preference economy. This stands in sharp contrast to the fixed time preference framework, in which cyclical features are unaffected by an increase in the average growth rate of money.

To understand the mechanisms that alter the relative role of monetary shocks in the variable-discount-factor economy, we conduct some additional experiments. Exploring these mechanisms requires that we look at the responses of variables to *both* monetary and technology shocks, and how these responses change as we change the degree of variability in the rate of time preference. Some interesting qualitative and quantitative differences emerge in the more general framework. Two factors appear to be responsible for altering the relative contribution of monetary shocks in fluctuations of variables. First, in the case of monetary shocks, inflation-tax effects on the transition path to the steady state are accentuated in the presence of endogenous time preference. Second, in the case of technology shocks, endogeneity in the rate of time preference seems to have the effect of enhancing the *income effects* of technology shocks. For a range of parameters, these income effects tend to cancel out the *substitution effects* of technology shocks, thus diminishing their contribution to the fluctuations in variables. In fact, for a sufficiently high degree of variability in the rate of time preference, the dynamic responses of variables to productivity shocks can be somewhat paradoxical, since the income effects of the shock tend to dominate the substitution effects. The assumption of subjective discount factors then imposes some constraints on the values of some of the preference parameters at which the model is calibrated. However, the conclusions regarding the role of monetary shocks still go through for the set of parameters that are acceptable from this point of view. Specifically, even if we restrict the parameters to allow for a very small degree of variability in the rate of time preference, the contribution of monetary shocks is enhanced without diminishing the model's ability to mimic features of the data.

Section 2 presents some further motivation for incorporating endogenous time preference and reviews some of the related literature. Section 3 presents the economic environment. Section 4 analyzes the steady state of the variable-time-preference model and compares it with that of the fixed time preference model. The analysis in this section provides a backdrop to the analysis of quantitative experiments presented in Section 5. Section 6 concludes.

2. BACKGROUND AND MOTIVATION

Although incorporating endogenous time preference to any framework is an exercise of interest as a sensitivity check, it is not intended merely for that purpose. Insight into several unresolved issues is likely to emerge. To elaborate on this point, endogenous-time-preference models have the appealing feature that they allow for intertemporal dependence in tastes by relaxing the restrictive assumption of time additively separable preferences. Specifically, in this framework, increases in current consumption lead to declines in the subjective weights assigned to future consumption benefits. As a result, this assumption typically has had interesting implications in equilibrium models: it changes the implications of fiscal policy in Dolmas and Wynne (1998) and enhances the ability of the small open economy model of Mendoza (1991) to match features of international business cycles.

There are also other extensions, somewhat similar in spirit to the endogenous-time-preference assumption, that involve invoking some form of intertemporal dependence in tastes to explain a diverse set of phenomena. In Becker and Murphy (1988), for example, period utility is postulated to depend on some measure of past consumption, in addition to current consumption, so that addictive behavior may be consistent with utility maximization by rational consumers. On the other hand, the business cycle model of Kydland and Prescott (1982) assumes current utility to be affected by past *leisure* choices, in order to generate more realistic labor-market fluctuations. Constantinides (1990) uses habit formation in consumption to explain high excess returns to equities observed in U.S. data.

Some researchers also make an empirical case for abandoning the assumption of intertemporal independence in preferences. Tests of Euler equations arising from standard time additive fixed-discount-factor representations generally have led to strong statistical rejections. In addition, certain implications of time additive preferences are inconsistent with cross-sectional behavior of consumption and income growth. Some of this literature is briefly surveyed by Obstfeld (1990). There is also direct empirical evidence of *variability* in rates of time preference, such as that of Lawrance (1991), though its implications for *endogeneity* in rates of time preference are not obvious.

To be more specific, Lawrance uses PSID data to compute rates of time preference of different labor income groups. She finds that rates of time preference are higher among groups with low levels of labor income. This evidence cannot, however, be interpreted as conflicting with the assumption that the discount factor is decreasing in utility (or that the rate of time preference is increasing in utility). In Lawrance's work, rates of time preference are identified using Euler equations of *fixed*-discount-factor environments. Furthermore, as the dynamic responses (to monetary shocks) analyzed later in this paper illustrate, a positive correlation with the discount factor and real wages is possible even though the underlying assumption is that the discount factor is decreasing in utility.

There is also some disagreement among economists regarding the intuitive plausibility of this assumption. Friedman (1969), for example, does not believe that the rate of time preference can be systematically related to the level of consumption along a constant consumption path.[1] As a critique of the *Friedman rule*, which is stated as an equality between a specific rate of deflation and a constant rate of time preference, Stein (1970) points out that it is not possible to infer a constant optimal rate of monetary expansion by looking at historical data. This is because the rate of discount is an unknown that depends on socioeconomic factors, and changes over time. Stein suggests that time preference should decrease over time as the representative agent's utility increases. This is also the view of Fisher (1930) who discusses various factors on which the rate of time preference may or may not depend, and postulates that time preference or "impatience" is associated with lower levels of real income and, consequently, utility.

The assumption that the rate of time preference decreases as utility increases would be implied by the assumption that the discount factor, $\beta(.)$, is increasing in utility. As mentioned earlier, this is contrary to what is assumed in this paper and in other general equilibrium models that assume an endogenous time preference structure. Although this assumption may seem difficult to defend a priori, it is a necessary condition for the dynamic stability of these models. Furthermore, it is useful in eliminating some innately implausible implications of the fixed time preference assumption.

The assumption that $\beta'(u) < 0$, often referred to as the *increasing marginal impatience* assumption, is thus imposed in works that theoretically analyze optimal growth economies with recursive but not time additive preferences. [See, e.g., Uzawa (1968), Epstein (1983, 1987), and Lucas and Stokey (1984)]. Lucas and Stokey (1984) provide an example illustrating the stark theoretical predictions arising in fixed-discount-factor environments. In an economy in which consumers have different but *fixed* discount factors, all of the economy's long run wealth ends up with the most patient consumer. So that a nondegenerate steady state emerges for the economy, the discount factor must be decreasing in utility.[2] It is also quite natural, by similar analogy, to argue that the increasing marginal impatience assumption be imposed in small open economy models, so that the economy's long run debt position is well defined. Not surprisingly, we find that this assumption is imposed in the heterogeneous-agent equilibrium models, such as that of Gomme and Greenwood (1995), and open economy equilibrium models, such as that of Mendoza (1991).

Finally, Epstein (1987) provides two additional arguments in favor of the assumption $\beta'(.) < 0$. Loosely speaking, one of the arguments runs as follows: Since an increase in *aggregate future utility* may imply an increase in future consumption, it is reasonable to assume that present consumption will be given more weight in the event *aggregate future utility* increases. In other words, the rate of time preference increases with an increase in *future utility*. Increasing marginal impatience is in turn *implied* by the above hypothesis. Another argument is that the assumption of increasing marginal impatience "is *equivalent* to the implied preference ordering over random consumption paths exhibiting an aversion to correlation in the consumption levels in any two periods." To the extent that this aversion is intuitively plausible, it provides additional support for the increasing marginal impatience assumption.

3. THE ECONOMIC ENVIRONMENT

The economy described below is a version of the monetary business cycle model of Cooley and Hansen (1989), with endogenous time preference introduced via a discount factor that depends on utility. There is a continuum of identical infinitely lived households, with preferences formulated in accordance with Epstein's (1983) notion of *stationary cardinal utility*. The representative household in this economy therefore desires to maximize expected lifetime utility given by

$$E\left\{\sum_{t=0}^{\infty}\left[\prod_{\tau=0}^{t-1}\beta(u(c_\tau, 1-h_\tau))\right]u(c_t, 1-h_t)\right\}, \tag{1}$$

where $\beta(u(c_t, 1-h_t))$ must be of the form $e^{-\phi(u(c_t,1-h_t))}$ and $u(c_t, 1-h_t)$ represents the household's period-t momentary utility, defined over consumption c_t and leisure $1-h_t$. The function u must be negative, strictly increasing with $\ln(-u)$ convex in the composite consumption-leisure good. It is also required that ϕ be positive, increasing, strictly concave and that $u'e^{\phi(u)}$ be nonincreasing.[3] The endogenous discount factor attached to $u(c_t, 1-h_t)$ (i.e., the term $\prod_{\tau=0}^{t-1} e^{-\phi(u(c_\tau,1-h\tau))}$), incorporates an impatience effect: An increase in current-period utility causes the household to discount future periods more heavily.

The functional forms for the period utility and discount functions are described as follows:

$$u(c_t, 1-h_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma} - Bh_t, \tag{2}$$

$$\beta(u(c_t, 1-h_t)) = e^{-(\eta+\tau u)}, \qquad 0 < \tau < 1, \tag{3}$$

where the specification for the momentary utility function is consistent with the "indivisible labor" assumption and nests the Cooley and Hansen (1989) log utility specification as the case in which $\sigma = 1$.

Households enter period t with nominal money balances m_{t-1} carried over from the previous period. The government augments these money balances with a lump-sum transfer equal to the increase in money supply, where the aggregate money supply M_t is determined according the following rule[4]:

$$M_t = g_t M_{t-1}. \tag{4}$$

Thus the total amount of money balances held by a household at the beginning of period t is the amount

$$m_{t-1} + (g_t - 1)M_{t-1}. \quad (5)$$

There is a cash-in-advance constraint on the purchase of the nonstorable consumption good, which ensures that money will be valued in equilibrium. Expenditure on the consumption good, therefore cannot exceed the total money balances available to the household; that is,

$$p_t c_t \leq m_{t-1} + (g_t - 1)M_{t-1}. \quad (6)$$

The growth rate of money, g_t, evolves according to

$$\log(g_{t+1}) = \alpha \log(g_t) + \xi_{t+1}, \quad (7)$$

where ξ_{t+1} is i.i.d normal with mean $(1-\alpha)\log(\bar{g})$ and variance σ_ξ^2, and $\log(\bar{g})$ represents the unconditional mean of $\log(g_t)$.

The representative firm in the economy hires labor and capital from the households to produce a composite consumption-investment good. There is a standard neoclassical aggregate production function of the Cobb–Douglas form, which combines capital (K_t) and labor input (H_t) to yield output (Y_t):

$$Y_t = e^{z_t} K_t^\theta H_t^{1-\theta}, \quad (8)$$

where θ is the factor income share of capital and e^{z_t} represents a shock to technology in period t. The random variable z_t follows the process

$$z_{t+1} = \gamma z_t + \varepsilon_{t+1}, \quad (9)$$

where ε_{t+1} is an i.i.d. random variable with mean zero and variance σ_ε^2.

The competitive firm maximizes profit, which is given by $Y_t - w_t H_t - r_t K_t$. The variables w_t and r_t represent the wage and rental rates paid for the use of labor and capital services of the households. The first-order conditions for the firm's profit maximization problem imply that w_t and r_t are given by

$$w_t = (1-\theta)e^{z_t} K_t^\theta H_t^{-\theta}; \quad (10)$$

$$r_t = \theta e^{z_t} K_t^{\theta-1} H_t^{1-\theta}. \quad (11)$$

In every period t, household expenditures consist of consumption (c_t), investment (i_t), and the amount of money balances (m_t/p_t) that are to be carried over to the next period. These expenditures must not exceed total household income, which is the sum of income earned from labor and capital services, money balances carried over from the previous period, and the lump-sum monetary transfer from the government. Households therefore maximize expected lifetime utility subject to (6) and a sequence of budget constraints of the following form:

$$c_t + x_t + \frac{m_t}{p_t} \leq w(z_t, K_t, H_t)h_t + r(z_t, K_t, H_t)k_t + \frac{m_{t-1} + (g_t - 1)M_{t-1}}{p_t}, \tag{12}$$

where household investment expenditure in period t is given by

$$x_t = k_{t+1} - (1 - \delta)k_t. \tag{13}$$

In equation (13), k_t is the household's capital stock in period t, and δ is the rate at which the capital stock depreciates.

For a value of g greater than 1, both M_t and p_t will grow without bound. To make the household's problem stationary, some of the variables need to be transformed. To that end, we define $\hat{m}_t = m_t/M_t$ and $\hat{p}_t = p_t/M_t$. Furthermore, stationary cardinal utility permits a recursive formulation of the household's problem described above. The problem, then, can be restated as

$$\begin{aligned} V(z_t, g_t, \hat{m}_{t-1}, k_t, K_t) = \max_{c_t, h_t, \hat{m}_t, k_{t+1}} \{u(c_t, 1 - h_t) + \beta(u(c_t, 1 - h_t)) \\ \times E_t[V(z_{t+1}, g_{t+1}, \hat{m}_t, k_{t+1}, K_{t+1})/z_t, g_t, \hat{m}_{t-1}k_t, K_t]\}, \end{aligned} \tag{14}$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\hat{m}_t}{\hat{p}_t} = w(z_t, K_t, H_t)h_t + r(z_t, K_t, H_t)k_t + \frac{g_t - 1 + \hat{m}_{t-1}}{g_t \hat{p}_t}, \tag{15}$$

and

$$c_t = \frac{g_t - 1 + \hat{m}_{t-1}}{g_t \hat{p}_t}. \tag{16}$$

In addition, the representative household's decisions must be consistent with the laws of motion for the aggregate state variables, given by

$$K_{t+1} = (1 - \delta)K_t + X_t, \tag{17}$$

$$z_{t+1} = \gamma z_t + \varepsilon_{t+1}, \tag{18}$$

$$\log(g_{t+1}) = \alpha \log(g_t) + \xi_{t+1}, \tag{19}$$

as well as the economywide aggregate decision rules perceived by the households:

$$\begin{aligned} H_t &= H(z_t, g_t, K_t), \\ X_t &= X(z_t, g_t, K_t), \text{ and} \\ \hat{P}_t &= \hat{P}_t(z_t, g_t, K_t). \end{aligned} \tag{20}$$

In equilibrium, aggregate per capita quantities turn out to be equal to the choices of the representative household. In particular, it must be the case that $h_t = H_t, k_t = K_t$,

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$x_t = X_t$, and $\hat{m}_{t-1} = \hat{m}_t = 1$. Since the cash-in-advance constraint is assumed to be binding in equilibrium, we also have $c_t = 1/\hat{P}_t$.

4. ANALYSIS OF THE STEADY STATE

A comparison of the deterministic steady states of the benchmark fixed-discount-factor model, and the model with endogenous time preference clarifies the intuition underlying the differences between the two models. Consider first the model in which the discount factor, and hence the rate of time preference, is fixed. The household's problem in this case is the same as (14), with $\beta(u(c_t, 1-h_t))$ replaced by a fixed discount factor β. After eliminating c_t and h_t using the constraints (15) and (16), the Euler equation with respect to $\hat{m}_t$ is

$$\beta E_t \frac{u_1(c_{t+1}, 1-h_{t+1})}{g_{t+1}\hat{p}_{t+1}} = \frac{u_2(c_t, 1-h_t)}{\hat{p}_t w(z_t, K_t, H_t)}. \tag{21}$$

Substituting for $\hat{p}_t = p_t/M_t$, equation (21) can be written as

$$\beta E_t \frac{u_1(c_{t+1}, 1-h_{t+1})}{\pi_{t+1}} = \frac{u_2(c_t, 1-h_t)}{w(z_t, K_t, H_t)}, \tag{21'}$$

where $\pi_{t+1} = p_{t+1}/p_t$. The equation above illustrates the inefficiency imposed due to the presence of money in this economy. The presence of inflation clearly distorts the agent's consumption–leisure decision—higher inflation implies a lower *equilibrium* marginal utility of leisure in period t per unit real wages relative to the expected marginal utility from period $t+1$ consumption. In other words, it is now optimal for the agent to have a lower level of period $t+1$ consumption, and a higher level of leisure than in an economy in which there is no money, or the cash-in-advance constraint is not binding.

In the steady state of this economy, the above condition is equivalent to the following:

$$\frac{\beta u_1(c, 1-h)}{u_2(c, 1-h)} = \frac{g}{w(z, K, H)}. \tag{22}$$

Equation (22) looks very similar to a labor market-clearing condition, although the underlying condition from which it is derived is an *intertemporal*, not and *intratemporal*, efficiency condition. It is, however, clear from (22) that a higher growth rate of money implies a higher marginal utility of consumption relative to marginal utility of leisure per unit wage rate in the steady state, which in turn indicates a lower steady state level of consumption and hours. This is the standard outcome of a cash-in-advance model in which agents, in the face of inflation, substitute consumption of the cash good, which is subject to an inflation tax, for leisure, which is not. This model thus implies that high inflation rates are associated with low employment, a result that supports Friedman's (1977) conjecture of an upward-sloping Phillips curve.

Similarly, the Euler equation for capital is given by

$$\beta E_t[r(z_{t+1}, K_{t+1}, H_{t+1}) + 1 - \delta] = \frac{u_2(c_t, 1 - h_t)w(z_{t+1}, K_{t+1}, H_{t+1})}{u_2(c_{t+1}, 1 - h_{t+1})w(z_t, K_t, H_t)}.$$

The equation above equates the discounted expected return from capital to the marginal rate of substitution between current and future leisure times the ratio of future to current wages. From (21′), it is clear that in a monetary economy the expected rate of return to capital will be positively related to π_{t+2}/π_{t+1}, on the transition path toward the economy's steady state. In the nonstochastic steady state, however,

$$r(z, K, H) - \delta = \frac{1}{\beta} - 1. \tag{23}$$

Equation (23) equates the steady state real interest, adjusted for depreciation, to the fixed rate of time preference. Under the assumption of constant returns to scale, r and w can be written in the form

$$f'(\kappa) = r, \tag{24}$$

$$f(\kappa) - \kappa f'(\kappa) = w, \tag{25}$$

where κ is the capital/labor ratio and $f(\kappa)$ is the production function expressed in per capita terms. Since the capital/labor ratio is determined from (23), it is clear from (23), (24), and (25) that κ, r, and w are unaffected by the growth rate of money.[5] However, for a given capital/labor ratio, lower steady state hours imply a lower capital stock. In a version of this model with inelastic labor, of course, money growth would have no effect on the long run capital stock.

Now, consider the model with an endogenous discount factor. The corresponding Euler equations for this case are given by

$$\beta(u(c_t, 1 - h_t))E_t \frac{u_1(c_{t+1}, 1 - h_{t+1})}{\hat{p}_{t+1} g_{t+1}}[1 + \beta'(u(c_{t+1}, 1 - h_{t+1}))\, E_{t+1} V(t+2)]$$
$$= \frac{u_2(c_t, 1 - h_t)}{\hat{p}_t w(t)}[1 + \beta'(u(c_t, 1 - h_t))E_t V(t + 1)], \tag{26}$$

$$\beta(u(c_t, 1 - h_t))E_t \left\{ \frac{u_2(c_{t+1}, 1 - h_{t+1})}{w(t + 1)}[r(t + 1) + 1 - \delta] \right.$$
$$\left. \times [1 + \beta'(u(c_{t+1}, 1 - h_{t+1}))\, E_{t+1} V(t+2)] \right\}$$
$$= \frac{u_2(c_t, 1 - h_t)}{w(t)}[1 + \beta'(u(c_t, 1 - h_t))E_t V(t + 1)], \tag{27}$$

where $V(j)$, $w(j)$, and $r(j)$ are shorthand notation for the period-j value function, wage, and rental rate, respectively. Note that these conditions nest the fixed discount

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factor model as a special case in which $\beta'(u) = 0$. In the nonstochastic steady state, we have

$$\beta(u(c, 1-h))\frac{u_1(c, 1-h)}{u_2(c, 1-h)} = \frac{g}{w(z, K, H)}, \quad \textbf{(28)}$$

and,

$$r(z, K, H) - \delta = \frac{1}{\beta(u(c, 1-h))} - 1. \quad \textbf{(29)}$$

For a given level of $\beta(.)$, an increase in the money growth rate implies that consumption and hours fall, just as in the fixed-discount-factor case. In the endogenous-time-preference model, however, the fall in consumption causes a decline in utility, and in the presence of increasing impatience, a rise in the endogenous discount factor. On the other hand, the decline in hours exerts an opposite influence: the discount factor will fall as a result of the increase in utility generated by a higher level of leisure. What, then, is the net effect on $\beta(.)$? The answer is clearly sensitive to the specification of functional forms for u and $\beta(.)$, but for a reasonably broad class of specifications, one could expect an increase in the discount factor. It is then no longer possible to rule out the case in which consumption and hours, and hence output, are higher compared to the fixed time preference model. Similarly, lower levels of output, consumption, and hours are also possible.

The increase in money growth will also have a different impact on the capital market, as described by equation (29). An increase in $\beta(.)$ indicates a lower real interest rate, and therefore a higher capital/labor ratio than in the fixed time preference case, whereas the opposite result obtains in the event of a decrease in $\beta(.)$. Consequently, the impact on the capital stock is also ambiguous: the model can now generate a capital stock that is higher or lower than in the fixed-discount-factor case, while it is also possible that a positive money growth and capital correlation, or what is often termed as the *Tobin effect*, obtains.

It would therefore seem that the assumption of a constant discount factor is "necessary" to produce a long run decline in consumption, hours worked, capital stock, and output as a result of an increase in the monetary growth rate. The more general model with variable time preference, on the other hand, can nest long run outcomes that have been observed in a wide variety of frameworks in the money-and-growth literature—viz. Tobin (1965), Sidrauski (1967), and Stockman (1981).

5. ANALYSIS OF QUANTITATIVE EXPERIMENTS

In this section we analyze the results of several numerical experiments designed to assess the relative role of monetary shocks in the fixed and flexible discount factor economies. The artificial economies described above are used to compute second moment summaries and dynamic responses to monetary growth shocks, using a numerical approximation of the respective models. The technique used to obtain this approximation is the *method of parameterized expectations*, due to Den Haan and Marcet (1990), and is described in the Appendix.

To conduct quantitative experiments, values are assigned to the parameters of preferences and technology, following the convention of choosing parameters based on observed features of the data. To that end, and to maintain comparability with the study of Cooley and Hansen (1989), which is the benchmark economy here, most of the parameters chosen are the same as those used in that paper. In particular, we set $\theta = 0.36$, $B = 2.86$, $\alpha = 0.48$, and $\gamma = 0.95$, and $\sigma = 1$.[6] The parameter η is set to ensure that the steady state value of the discount function coincides with the fixed discount factor β, where $\beta = 0.99$. We also set the standard deviations of the technology and money growth shocks equal to 0.00721, and 0.009 respectively, as in Cooley and Hansen (1989).

This leaves us with the question of how to calibrate the additional parameter τ introduced in the variable-time-preference model. Stability conditions dictate that we choose a value in the range $0 \leq \tau \leq 1$, where $\tau = 0$ corresponds to the special case of fixed time preference. This is somewhat problematic since the choice of functional form for $\beta(.)$ has, so far, been guided strictly by the local stability conditions described in Section 3. Pending empirical research on this subject, it is difficult to say whether the specification considered above, or, for that matter, any value we may choose to assign to the parameter τ, is "reasonable." The approach we follow here is to look at a range of values of τ in which the business cycle features of the endogenous time-preference model provide an acceptable match to the data, and examine the contribution of monetary shocks in this range. It turns out that this range is roughly given by values of τ between 0 and 0.026, which implies a relatively small degree of variability in the rate of time preference. In what follows, we primarily focus on the cyclical features and dynamic responses of variables in this range.

Table 1 presents the second moment summaries of the fixed- and variable-time-preference models. The first panel corresponds to the value of $\tau = 0$, which is the special case of fixed time preference. The second and third panels represent the variable-time-preference economies for $\tau = 0.01$ and $\tau = 0.02$, respectively. Table 2 presents the corresponding second moments of postwar U.S. data, taken from Cooley and Hansen (1989). In all of the cases presented, cyclical features of the artificial economies have qualitative properties similar to the data: Investment and hours fluctuate more than output, while consumption, capital, and productivity are less volatile compared to output. Consumption, investment, hours, and productivity are procyclical, and the price level is countercyclical. The capital stock is weakly correlated with output. Moving from fixed to flexible time preference increases the volatility of consumption and makes it a little more procyclical, which is a movement away from the data. However, the *relative* volatility of consumption improves as we increase τ from 0 to 0.01. Investment volatility declines with increases in τ in this range, but it becomes less procyclical, which is an improvement in comparison with the fixed-time-preference case. The standard deviation of the capital stock, its relative volatility, and its correlation with output are also closer to the data in the flexible-time-preference model. In the case of work effort, there is an initial improvement in this range, in the sense of reducing its fluctuations, and

TABLE 1. Second moment summaries of fixed and variable time preference models

Cyclical properties	Time preference								
	Fixed ($\tau = 0$)			Endogenous ($\tau = 0.01$)			Endogenous ($\tau = 0.02$)		
Series (x)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)
Output (y)	2.0546	1.0000	1.0000	1.9454	1.0000	1.0000	1.7788	1.0000	1.0000
Consumption	0.8300	0.4040	0.7451	0.8848	0.4548	0.7726	0.9524	0.5354	0.8108
Investment	6.6053	3.2149	0.9693	5.9982	3.0833	0.9623	5.0901	2.8615	0.9481
Capital stock	0.5287	0.2573	0.0780	0.4725	0.2429	0.0893	0.3941	0.2216	0.1035
Hours	1.5242	0.7419	0.9824	1.3598	0.6990	0.9754	1.1087	0.6232	0.9637
Productivity	0.6259	0.3046	0.8903	0.6878	0.3536	0.9000	0.7697	0.4327	0.9231
Price level	0.8300	0.4039	−0.7451	0.8848	0.4548	−0.7726	0.9516	0.5354	−0.8108
Discount factor	—	—	—	0.0083	0.0042	0.5250	0.0141	0.0079	0.1798

TABLE 2. Quarterly U.S. time series

Series	Standard deviation	Standard deviation relative to output	Correlation with output
Output	1.74	1.00	1.00
Consumption	0.81	0.46	0.65
Investment	8.45	4.85	0.91
Capital stock	0.38	0.22	0.28
Hours	1.41	0.81	0.86
Productivity	0.89	0.51	0.59
Price level (CPI)	1.59	0.91	−0.48
(GNP deflator)	0.98	0.56	−0.53

Source: Cooley and Hansen (1989).

TABLE 3. Percentage contribution of money shocks to fluctuations in the range $\tau \in [0, 0.026]$

	τ						
	0	0.01	0.02	0.026	0.03	0.04	0.05
Output	2.7	2.6	3.3	3.6	4.8	6.6	9.4
Consumption	62.1	58.6	54.0	51.3	49.6	45.5	41.8
Investment	10.1	21.5	24.9	28.1	31.0	42.1	63.6
Capital stock	12.2	13.9	16.1	18.2	20.1	28.0	47.7
Hours	5.1	5.4	5.6	9.9	12.3	24.3	83.0
Productivity	5.7	5.6	5.8	5.9	6.0	5.9	5.8
$\beta(.)$	0.0	54.2	63.6	66.9	64.8	51.3	36.7

making it a little less procyclical to output. However, the decline in fluctuations is quite rapid as τ increases; as we see in the case of $\tau = 0.02$, the volatility of hours is lower than that observed in the data. The standard deviation and relative volatility of productivity improve, but it becomes more procyclical as τ increases. On the whole, however, although one cannot claim an unambiguous improvement of the model's features in the presence of endogenous time preference, the match with the data is not a bad one.

Table 3 presents the contribution of monetary shocks to fluctuations in the range $\tau \in [0, 0.026]$, in addition to some other values of τ. The percentage contribution of monetary shocks to the fluctuations in a particular variable is computed as follows: We simulate the models with the standard deviation of the technology shock set approximately equal to zero and express the volatilities obtained as a percentage of the volatilities in the model in which both shocks have the standard volatilities. We find that, in the range of $\tau \in [0, 0.05]$, the contribution of monetary shocks to fluctuations in output, investment, the capital stock, and work effort increases very significantly with increases in τ. However, in the range that is interesting from an empirical point of view, this contribution is still relatively small compared to that of technology shocks, although it is much larger than in the fixed time preference case. It is, of course, possible to get a much larger contribution of monetary shocks for some ranges of values of τ, but the variable-time-preference model does not provide a good match for the data in those ranges. Although we do not report business cycle properties in great detail for higher values of τ, a summary description, presented in Figures 1 and 2, is sufficient to give an idea of what happens to the cyclical properties of economic aggregates as variability of time preference increases. In Figures 1 and 2, we are looking at a much larger range of values of τ, that is, the range of values in $\tau \in [0, 0.4]$. Clearly, the implication of these results is that τ should be calibrated to allow for a relatively *small* degree of variability in the rate of time preference. Nevertheless, even for small values of τ the contribution of monetary shocks is predicted to be higher than that implied by constant time preference models of the business cycle.

We now examine the factors responsible for increasing the relative contribution of monetary shocks in the presence of endogenous time preference. Although this has something to do with the way endogenous time preference affects dynamic responses to monetary shocks, in what follows we shall see that it has more to do with the way variables respond to *technology shocks* in this economy. Figures 3 and 4, respectively, present dynamic responses of variables to temporary monetary and technology shocks for values of τ equal to 0, 0.01, and 0.02. First, consider the responses to monetary shocks for the fixed time preference case of $\tau = 0$. A 1% increase in money growth leads to a decline in consumption work effort and output, and an increase in the capital stock. The typical interpretation is that agents substitute out of activities subject to the inflation tax, that is, consumption and work effort, toward those that are not, such as leisure and, to a lesser extent, capital accumulation. Lower work effort results in a decline in output. The real wage rate, or the marginal product of labor increases, as expected, due to the increase in the capital/labor ratio.

In the case of endogenous time preference, providing an interpretation of this type becomes a little awkward, because the consumption and leisure decision affects the discount factor and vice versa. However, the following interpretation may be considered plausible for the cases presented in Figure 3. The decline associated with consumption leads to a net decline in utility even though leisure increases, causing an increase in the discount factor on impact of the monetary shock. The temporary nature of the shock ensures that this increase in patience toward future utility is short-lived, and the discount factor goes back to the steady state very quickly. This means that the representative agent cares a lot about utility in the period immediately after the shock, but not so much about utility in subsequent periods. Consequently, investment increases very sharply on impact, and there is a higher decline in work effort in the second period compared to the fixed time preference model, reflecting a higher preference for leisure in period 2. The decline in second-period work effort implies lower output even though the capital stock is

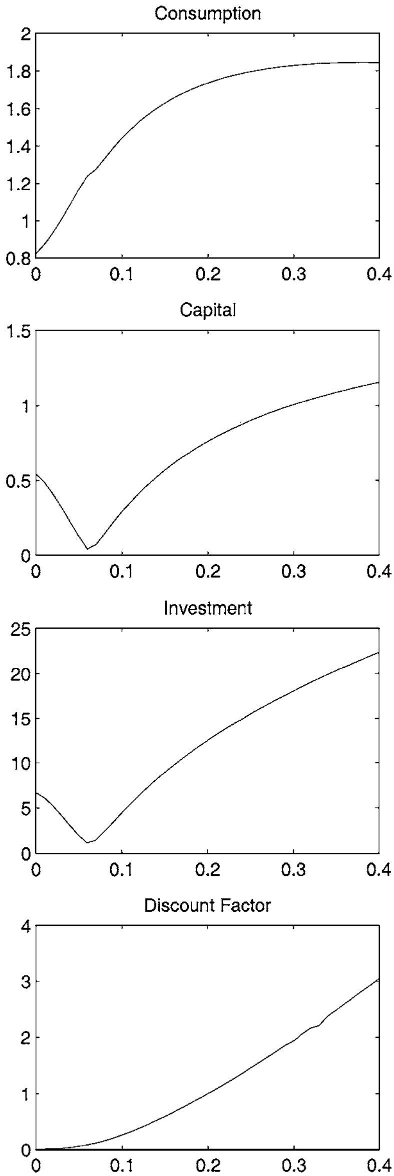

Figure 1. Percentage standard deviations of variables for values of τ in [0, 0.04].

higher relative to steady state. Productivity, as reflected by the real wage rate, is consequently also higher in the second period following the shock.

These effects, to a lesser extent are also at work in subsequent periods. For the endogenous time-preference cases presented in Figure 3, one may therefore conclude that the inflation-tax effects are stronger on the transition path than on

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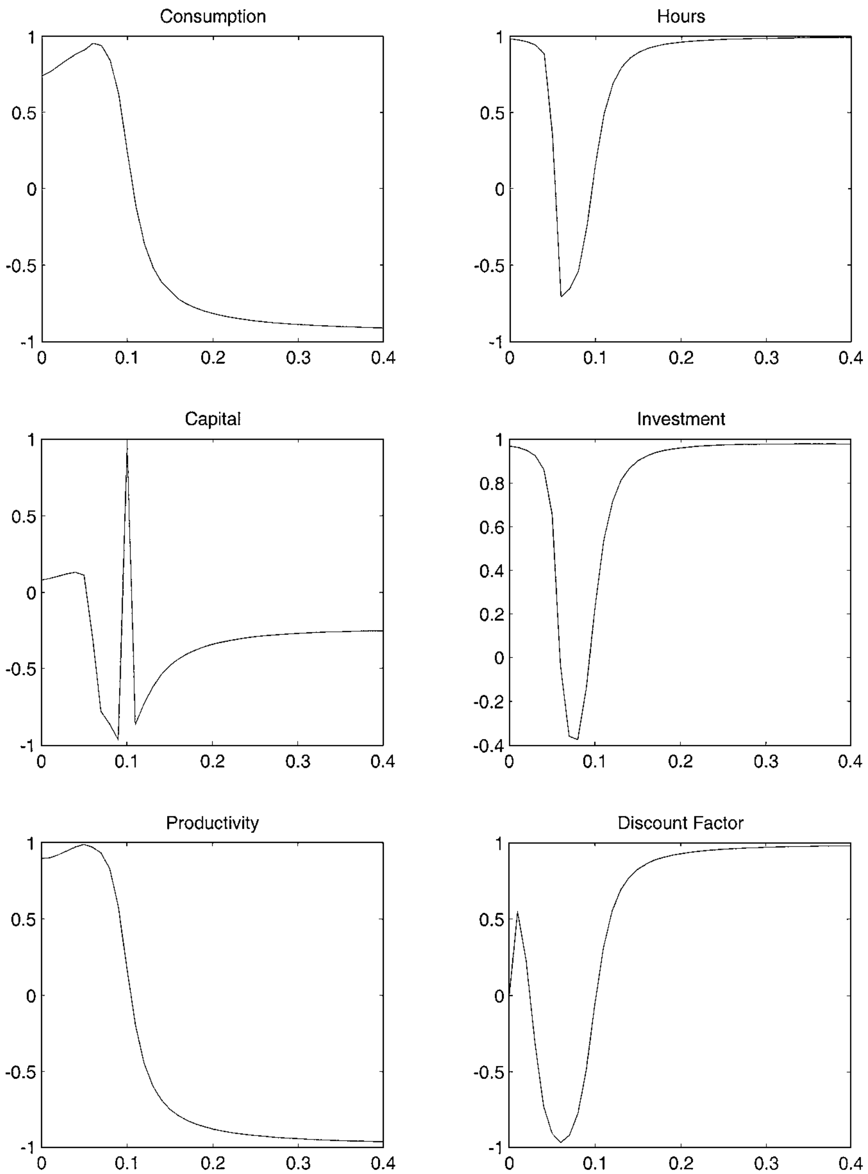

FIGURE 2. Correlations with output for values of τ in [0, 0.4].

impact. This feature also could be responsible for enhancing the contribution of monetary shocks to fluctuations in variables. Another interesting characteristic of the more general model is that the adjustment toward the steady state, of variables such as output, work effort, and real wages, can be nonmonotonic. This

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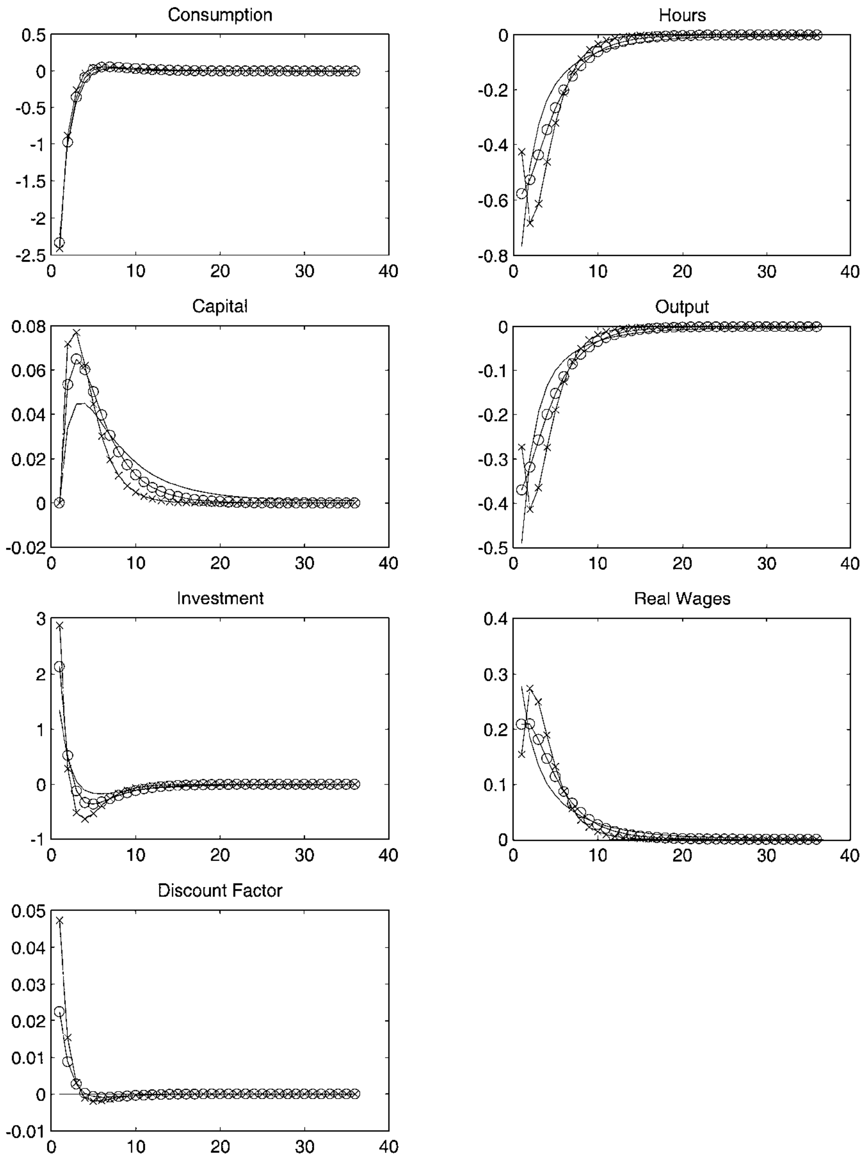

Figure 3. Dynamic responses to a 1% temporary shock to the monetary growth rate ($\alpha = 0.48$): (- - -) $\tau = 0$; (-x-) $\tau = 0.02$; (-o-) $\tau = 0.01$.

is related to the discount factor's adjustment toward the steady state: The agent becomes more patient relative to the steady state on impact, but is relatively impatient immediately before returning to the steady state level of the time preference. The variable-time-preference framework is therefore richer in the sense

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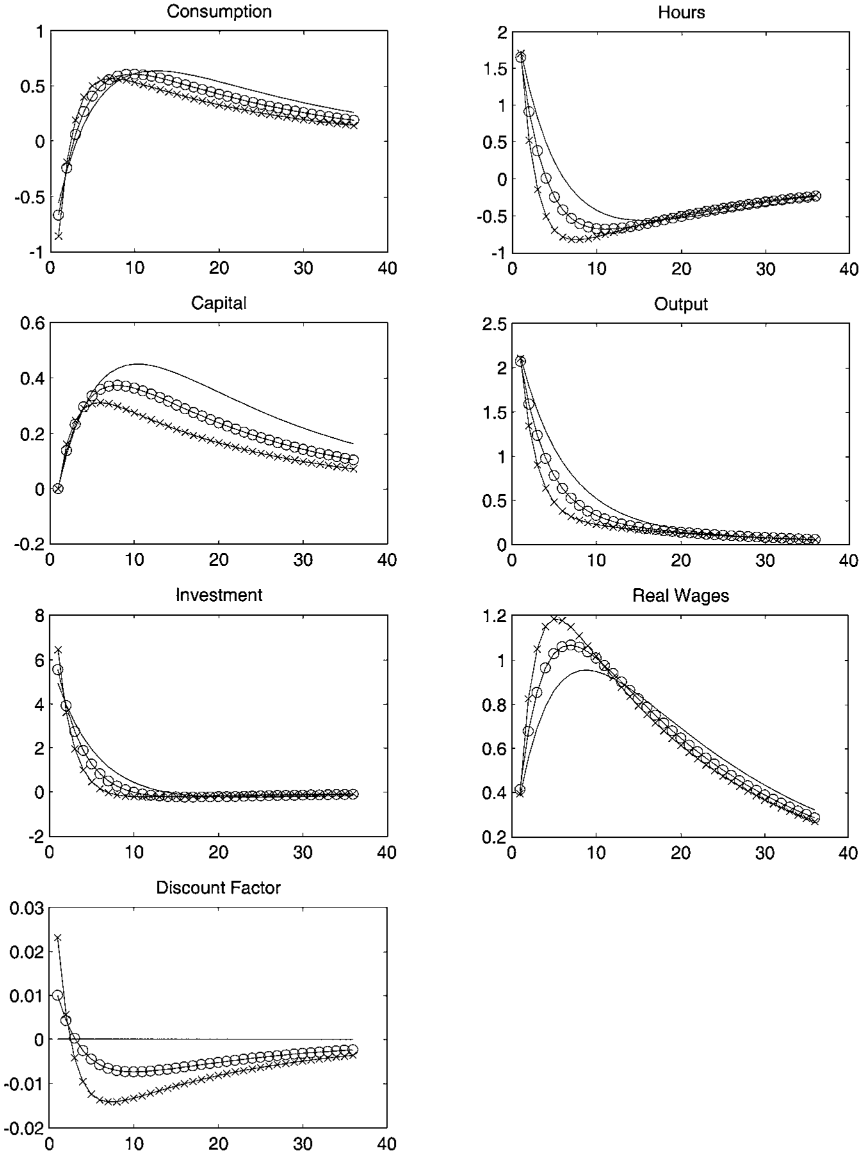

Figure 4. Dynamic responses to a 1% temporary shock to technology ($\gamma = 0.95$): (- - -) $\tau = 0$; (-x-) $\tau = 0.02$; (-o-) $\tau = 0.01$.

that it has the potential to generate a *cyclical* pattern of response to exogenous shocks. One can also view this as another sense in which the contribution of monetary shocks in fluctuations is greater relative to the fixed time preference model.

Next, we consider the responses of variables to technology shocks, which are presented in Figure 4. In all cases presented, there is a decline in consumption on impact of the technology shock. Since the shock is temporary, this can be attributed to consumption smoothing. Also, work effort and investment increase in order to take advantage of the temporary increase in productivity. Consequently, output increases on impact, and the capital stock is higher in the next period. In the variable-time-preference model, quantitative differences emerge because of the way the discount factor responds to changes in consumption and leisure. Initially, the decline in leisure and consumption causes the discount factor to increase. This increase in patience is reflected in a larger decline in consumption and a larger increase in investment in the first period of the shock. Later, as consumption and leisure increase, the discount factor declines rapidly, and the agents' *impatience* is reflected in the larger declines observed in work effort, investment, capital, and output. Put differently, the impatience observed in later periods appears to enhance the income effects of the technology shocks—people prefer to work less, leading to larger declines in output, consumption, and investment. This effectively reduces the persistence of the shock, leading to a somewhat lower contribution to fluctuations in variables.

Let us now briefly consider what happens in the variable-time-preference economy beyond the empirically plausible range of $\tau \in [0, 0.026]$. An analysis of higher levels of variability in the discount factor is not the focus of this paper; a summary description is presented here to provide some further intuition about the mechanisms that make monetary shocks play a more important role in this framework. The income-impatience effect described earlier is in fact the key to the dramatic variation in the model's quantitative properties as τ increases. Initially, in the empirically plausible range, substitution effects continue to dominate income effects, but to a lesser extent than in the fixed time preference model. Later, for an intermediate range of values of τ approximately between 0.026 and 0.06, the income-impatience effects tend to cancel out substitution effects. Consequently, the response to the technology shock of variables such as work effort is of a small magnitude. In this range, therefore, the percentage contribution of monetary shocks is very high. As τ increases further, however, the income-impatience effects begin to dominate the substitution effects. As a result, technology shocks become important again, although in a different sense relative to the fixed time preference economy. For relatively large values of τ, the response to technology shocks can in fact be quite paradoxical. For example, the income-impatience effect becomes so large that the response of work effort, and consequently output, is a highly negative one. This variation of responses to technology shocks is the reason why second moment features presented in Figures 1 and 2 change so much with increases in τ.

Another way of examining the role of monetary shocks is to compare the cyclical features of economies with different *average* growth rates of money. Tables 4 and 5 present the cyclical properties of the cases $\tau = 0$, $\tau = 0.026$, and $\tau = 0.05$, for an average growth rate of money $g = 1.05$ and $g = 1.15$, respectively. In the case

Table 4. Cyclical properties of fixed and variable time preference economies where $g = 1.05$

Property	Time preference								
	Fixed ($\tau = 0$)			Endogenous ($\tau = 0.026$)			Endogenous ($\tau = 0.05$)		
Series (x)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)
Output (y)	2.0546	1.0000	1.0000	1.6585	1.0000	1.0000	1.1041	1.0000	1.0000
Consumption	0.8300	0.4040	0.7451	0.9968	0.6010	0.8340	1.1784	1.0673	0.9125
Investment	6.6053	3.2149	0.9693	4.4073	2.6574	0.9338	1.8589	1.6837	0.6361
Capital stock	0.5287	0.2573	0.0780	0.3372	0.2033	0.1109	0.1151	0.1043	0.1049
Hours	1.5242	0.7419	0.9824	0.9275	0.5592	0.9524	0.1474	0.1335	0.2007
Productivity	0.6259	0.3046	0.8903	0.8251	0.4975	0.9395	1.0842	0.9820	0.9911
Price level	0.8300	0.4039	−0.7451	0.9988	0.6010	−0.8340	1.1784	1.0673	−0.9125
Discount factor	—	—	—	0.0177	0.0107	−0.1241	0.0580	0.0525	−0.9061

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TABLE 5. Cyclical properties of the fixed and variable time preference economies where $g = 1.15$

Property	Time preference								
	Fixed ($\tau = 0$)			Endogenous ($\tau = 0.026$)			Endogenous ($\tau = 0.05$)		
Series (x)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)	$s\,dx$	$\frac{s\,dx}{s\,dy}$	Corr(x, y)
Output (y)	2.0546	1.0000	1.0000	1.6588	1.0000	1.0000	1.1213	1.0000	1.0000
Consumption	0.8300	0.4040	0.7451	0.9970	0.6009	0.8345	1.1717	1.0449	0.9120
Investment	6.6053	3.2149	0.9693	4.4548	2.6857	0.9331	1.9457	1.7352	0.6555
Capital stock	0.5287	0.2573	0.0780	0.3406	0.2053	0.1126	0.1220	0.1088	0.1109
Hours	1.5242	0.7419	0.9824	0.9276	0.5592	0.9540	0.1601	0.1427	0.3485
Productivity	0.6259	0.3046	0.8903	0.8258	0.4979	0.9392	1.0760	0.9596	0.9902
Price level	0.8300	0.4039	−0.7451	0.9970	0.6009	−0.8345	1.1717	1.0449	−0.9120
Discount factor	—	—	—	0.0177	0.0107	−0.1306	0.0567	0.0506	−0.9045

of the fixed time preference model, cyclical features are completely invariant to the changes in the average growth rate of money. In the variable time preference cases, small changes begin to emerge for the $\tau = 0.026$ case, and these are a little more pronounced for the $\tau = 0.05$ case, but at the cost of deteriorating cyclical features. Again, variability in the rate of time preference implies a larger role for monetary shocks than in the fixed time preference model, but this role remains much smaller than that of technology shocks. The results in Tables 4 and 5 also have the interesting implication that cyclical features of high inflation economies will be somewhat different in comparison with low inflation economies.

On the whole, the analysis here suggests a larger role for money in economic fluctuations than what is typically predicted by fixed discount factor models. These results are, of course, subject to the caveat that another free parameter has been introduced, and the changes in cyclical features of the more general model do not amount to an unambiguous improvement in terms of matching properties of the data. However, it is not unreasonable to speculate, on the basis of the results above, that different functional forms for the discount function could have different implications for cyclical features and the role of monetary shocks. It is, for example, conceivable that there exists some combination of discount and utility functions, that generates a larger role for money and improves the model's ability to mimic the data considerably. It is also important to emphasize the exploratory nature of this study. The objective here has been, in part, to gain some insights by examining the mechanisms at work in a more general framework, and whether these can be exploited in improving the empirical performance of standard business cycle models.

6. CONCLUSIONS

This paper incorporated an endogenous utility discount factor into an otherwise standard monetary business cycle model. Quantitative experiments conducted in this paper seem to imply that conclusions about the nature and extent of monetary nonneutralities are sensitive to this generalization. In particular, we find that the presence of variable time preference can lead to a larger percentage contribution of monetary shocks in the fluctuations of variables. Furthermore, variable rates of time preference allow this contribution to increase with increases in the average monetary growth rate. This leads to the interesting prediction that in high inflation economies, money growth will be more important in affecting real economic fluctuations.

In view of these results, several natural extensions of this study are of interest. The variable time preference framework would, for example, have very different implications for normative issues such as the role of monetary policy. It would also be interesting to study similar extensions of equilibrium models that focus on other type of monetary nonneutralities, such as the liquidity-effect models developed by Lucas (1990) and Fuerst (1992). These issues are explored by Lahiri (2000).

NOTES

1. Fisher (1930) defines the rate of time preference as “the (percentage) excess of present marginal want of one unit of present goods over the present marginal want for one more unit of future goods.” Variability in this rate would therefore entail variability in rates associated with different levels of *constant* consumption streams. The assumption that the rate of time preference is increasing in utility therefore requires that the individual’s indifference curves between present goods and future goods get steeper as one moves along a 45-deg line from the origin.

2. Lucas and Stokey (1984) further note that it is not necessary for *all* consumers to exhibit this behavior, as long as some consumers are impatient enough to offset the behavior of others. In the context of representative-agent models, of course, this becomes a necessary condition for the representative consumer.

3. These restrictions ensure that a stable steady state distribution for the state variables exists and is unique. Epstein (1983) also shows that, under these conditions, consumption is a normal good in every period and deviations from the fixed time preference setup are not too great. Although these conditions are specified for the case in which the utility function has only one argument, viz consumption, results from Epstein (1983) should go through if consumption and leisure are treated as a composite commodity. Restrictions specified by Epstein (1983) should then be satisfied with respect to this composite commodity. [See, e.g., Mendoza (1991) and Gomme and Greenwood (1995)].

4. Uppercase letters denote aggregate economywide per capita variables that an individual household regards as being outside its sphere of influence; lowercase letters denote variables specific to the household.

5. It is, of course, possible to generate a *positive* impact of inflation on the real interest rate by means of a slight modification of this model. In Stockman (1981), for instance, this is achieved by imposing a cash-in-advance constraint on investment. In terms of the benchmark model discussed earlier, the cash-in-advance constraint would look like

$$c_t + k_{t+1} - (1-\delta)k_t = \frac{\hat{m}_{t-1} + g_t - 1}{\hat{p}_t g_t}$$

Then, instead of equation (25), we would get

$$r(z, K, H) = \frac{[1 - \beta(1-\delta)]g}{\beta^2},$$

so that higher inflation is associated with a higher long run interest rate.

6. We choose to approximate the results using $\sigma = 1.001$. For the log utility case, the equations (A.1)–(A.5) in the Appendix become linearly dependent, since in this case equations (A.7) and (A.9) are identical.

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APPENDIX

Because of the presence of money, equilibrium allocations in this economy are not necessarily Pareto optimal. Consequently, the competitive equilibrium for this economy cannot be computed indirectly by solving a social planner's problem. The approach followed here is essentially a variation of Den Haan and Marcet (1990), which involves a polynomial approximation of the expectations part of the stochastic Euler equations of the household's problem. Before forming this approximation, however, certain market-clearing conditions need to be imposed. Noting that, in equilibrium, aggregate per capita quantities coincide with the choices of the representative household, we make the relevant substitutions in the first-order conditions of the household's problem. Specifically, we impose $h_t = H_t$, $k_t = K_t$, $x_t = X_t$, and $\hat{m}_{t-1} = \hat{m}_t = 1$. Since the cash-in-advance constraint is assumed to be binding in equilibrium, we also have $c_t = C_t = 1/\hat{P}_t$. The economy's equilibrium characterization is then given by the following set of equations:

$$C_t^{-\sigma}\{1 - E_t\tau\beta(u(C_t, 1 - H_t))V(t+1)\} - \lambda_{1t} - \lambda_{2t} = 0, \tag{A.1}$$

$$-B\{1 - E_t\tau\beta(u(C_t, 1 - H_t))V(t+1)\} + (1-\theta)\lambda_{1t}e^{Z_t}K_t^{\theta}H_t^{-\theta}, \tag{A.2}$$

$$-\lambda_{1t}C_t + E_t\beta(u(C_t, 1 - H_t))\frac{(\lambda_{1t+1} + \lambda_{2t+1})C_{t+1}}{g_{t+1}} = 0, \tag{A.3}$$

$$-\lambda_{1t} + E_t\beta(u(C_t, 1 - H_t))\left\{\theta e^{Z_{t+1}}K_t^{\theta-1}H_t^{1-\theta} + 1 - \delta\right\}\lambda_{1t+1} = 0, \tag{A.4}$$

$$C_t + K_{t+1} - (1-\delta)K_t = e^{Z_t}K_t^{\theta}H_t^{1-\theta}. \tag{A.5}$$

Here, λ_{1t} and λ_{2t} are the Lagrangian multipliers associated with the household budget and cash-in-advance constraints, respectively. To solve the model, we need to form an approximation for the terms involving expectations in equations (A.1), (A.2), (A.3), and (A.4). To that end, we let the term $E_t\tau\beta(u(C_t, 1 - H_t))V(t+1)$ be approximated by $\psi(z, g, K; \mu)$, and the terms $E_t\beta(u(C_t, 1 - H_t))\frac{(\lambda_{1t+1} + \lambda_{2t+1})C_{t+1}}{g_{t+1}}$ and $E_t\beta(u(C_t, 1 - H_t))\{\theta e^{Z_{t+1}}K_t^{\theta-1} H_t^{1-\theta} + 1 - \delta\}\lambda_{1t+1}$ by $\xi(z, g, K; \omega)$ and $\nu(z, g, K; \pi)$, respectively. The functions ψ, ξ, and ν are polynomials in z, g, and K, while $\boldsymbol{\mu}$, $\boldsymbol{\omega}$, and $\boldsymbol{\pi}$ represent the respective vectors of polynomial coefficients. As is conventional, we choose the degree of the polynomial by examining how the results change by increasing the degree of the polynomial. That is, if the solution does not change much between an nth- and $(n+1)$th-degree polynomial, then the nth-degree polynomial is considered a good approximation. It turns out that for the model in question a second-degree polynomial is a good one. The procedure for forming the approximation is as follows: Starting with an initial guess for the vectors $\boldsymbol{\mu}$, $\boldsymbol{\omega}$, and $\boldsymbol{\pi}$, say μ_0, ω_0, and π_0, we can solve the first-order conditions above to construct a time series for consumption, hours, and the capital stock, for a given series of monetary and technology shocks. Specifically, we can solve for C_t by dividing equation (A.3) by equation (A.4). Given K_0 and z_0, we can use (A.2) and (A.4) to solve for H_t, and then use (A.5) to solve for K_{t+1}. Once a series has been constructed, it can be used to run three nonlinear least-squares regressions to estimate a new set of coefficients μ_1, ω_1, and π_1. For example, we get an estimate for μ_1 by running a nonlinear regression of the series $\tau\beta(u)V(t+1)$ on the function ψ. [For details on the procedure used for the nonlinear regressions, see Den Haan and Marcet (1990) and Pindyck and Rubinfeld (1987).] The next step is to construct a new series using a linear combination of the coefficients μ_0 and μ_1. The new series is used to run another regression to compute μ_2, and so on, until estimates from successive iterations have converged. However, in this case, we need convergence in all three polynomial coefficients μ, ω, and π.

https://doi.org/10.1017/S1365100501010021 Published online by Cambridge University Press