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Characteristics of the resonant charge transfer in strongly coupled plasmas including quantum and shielding effects

Woo-Pyo Hong[1] **and Young-Dae Jung**[2,3,†]

[1]Department of Electronics Engineering, Catholic University of Daegu, Hayang 38430, South Korea

[2]Department of Applied Physics and Department of Bionanotechnology, Hanyang University, Ansan, Kyunggi-Do 15588, South Korea

[3]Department of Physics, Applied Physics, and Astronomy, Rensselaer Polytechnic Institute, 110 8th Street, Troy, NY 12180-3590, USA

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The influence of quantum tunnelling and shielding on the resonant electron transfer process in strongly coupled plasmas is investigated. The screened atomic states and energy eigenvalues are employed to obtain the resonant electron transfer cross-section in strongly coupled plasmas. It is found that the classical resonant electron transfer cross-section increases with an increase of the ion-sphere radius. However, the energy-dependent quantum tunnelling resonant electron transfer cross-section is shown to decrease with increasing ion-sphere radius. It is demonstrated that an increase of the nuclear charge decreases the screening effect on the electron transfer cross-section while the quantum tunnelling effect enhances the resonant electron transfer cross-section in strongly coupled plasmas. In addition, it is shown that the effect of quantum tunnelling on the resonant electron transfer process decreases when both the collision energy and ion-sphere radius increase. The variation of shielding effect on the resonant electron transfer process in strongly coupled plasmas is also discussed.

1. Introduction

The charge exchange or transfer process between a positive ion and a neutral atom has been of a great interest since it is one of the most fundamental atomic collision processes in many areas of physics such as astrophysics, atomic physics, chemical physics and plasma physics, and also has many applications in plasma diagnostics (Rapp & Francis 1962; Janev, Presnyakov & Shevelko 1985; Shevelko & Vainshtein 1993; Beyer, Kluge & Shevelko 1997; Song & Jung 2003; Jung 2005*a*; Fridman 2008). It is shown that the Debye–Hückel theory describes the physical properties of low density plasmas and also corresponds to a pair correlation approximation. Hence the electron capture processes in weakly coupled plasmas have been extensively investigated using the screened interaction of the collision system described by the Debye–Hückel potential since the average interaction energy

† Email address for correspondence: ydjung@hanyang.ac.kr

between charged particles is usually smaller than the average kinetic energy of a plasma particle (Baimbetov, Nurekenov & Ramazanov 1995). However, it is known that the influence of multiparticle correlation caused by the collective interactions should be taken into account with increasing density in a plasma. Hence, the interaction potential in strongly coupled plasmas would not be properly described by the standard Debye–Hückel theory owing to the strong collective effects of multiparticle interactions. It is shown that the ion-sphere model (Salpeter 1954; Jung 1999; Fujimoto 2004) has also played a crucial role in elucidating the properties of the strongly coupled plasma and is also found to be more suitable than the cutoff potential model. Hence, it would be expected that the charge transfer and exchange processes in strongly coupled plasmas are quite different from those in weakly coupled plasmas. If the electron transfer in the ion-neutral collision has no energy exchange in the collision system; this process is known as the resonant electron transfer process; otherwise is called the non-resonant electron transfer case (Fridman 2008; Fridman & Kennedy 2011). The resonant and non-resonant electron transfer processes have been extensively explored using various classical and the quantum mechanical methods depending on the physical states of the ion-neutral collision system (Janev *et al.* 1985; Fridman 2008; Fridman & Kennedy 2011). In recent years, the quantum tunnelling phenomena have been comprehensively explored in wide areas of modern physics such as in semiconductors and superconductors, as well as in individual atoms on metal surfaces in various nanostructures including single-wall carbon nanotubes, molecular transistors and quantum wires formed in a two-dimensional electron gas (Komnik & Gogolin 2003; Razavy 2003). It is also shown that the quantum tunnelling effect on the resonant electron transfer process enhances the electron transfer cross-section (Fridman 2008; Fridman & Kennedy 2011). However, the resonant electron transfer process including the quantum tunnelling effect in strongly coupled plasmas has not been investigated as yet. Thus, in this paper, we investigate the plasma shielding and quantum tunnelling effects on the charge transfer process in collisions of a positive ion and a neutral atom in strongly coupled plasmas described by the ion-sphere model with the energy eigenvalue and ionization energy obtained by the Rayleigh–Ritz variational technique (Mathews & Walker 1970) since the theoretical atomic spectroscopy is essential to the study of plasma parameters as well as collision dynamics.

This paper is composed as follows: in § 2, we discuss the charge transfer process in strongly coupled plasmas. In § 3, we obtain the energy expectation value and ionization energy by using the Rayleigh–Ritz variational technique including the non-thermal and plasma shielding effects. In addition, we obtain the quantum tunnelling resonant electron transfer cross-section between the neutral atom and the ion in strongly coupled plasmas. In § 4, we discuss the shielding and quantum tunnelling effects on the resonant electron transfer process in strongly coupled plasmas. Finally, the conclusions are given in § 5.

2. Electron transfer process in strongly coupled plasmas

Since the resonant electron transfer has no effect on the electronic energy in the reaction system, the interaction between a positive ion A^+ with nuclear charge $Z_A e$ and a neutral atom B with nuclear charge $Z_B e$ is expressed by $A^+ + B \rightarrow A + B^+$. The potential energy of an electron in the field of free ions can be found in an excellent work of Fridman & Kennedy (2011). The physical concept of Debye–Hückel theory (Kleinstreuer 2010) as a cooperative screening phenomenon is no longer applicable

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in very dense plasmas since the probability of finding other charged particles in a Debye sphere is quite small and almost vanishes in strongly coupled plasmas. It has been shown that the Debye–Hückel model would not be reliable to represent the interaction potential in strongly coupled plasmas. It has also been shown that the ion-sphere model (Salpeter 1954; Salzmann 1998) in strongly coupled plasmas is equivalent to the Wigner–Seitz sphere in condensed matter theory since the ion sphere consists of a single ion and its surrounding negatively charged sphere. The screened Coulomb interaction potential $V_{SC}(r)$ between the electron and the ion with charge Ze in strongly coupled plasmas would be represented by the following ion-sphere model (Jung & Jeong 1996; Jung 2000):

$$V_{SC}(r) = -\frac{Ze^2}{r}\left[1-\frac{r}{2R_I}\left(3-\frac{r^2}{R_I^2}\right)\right]\theta(R_I-r), \tag{2.1}$$

where r is the interparticle distance, $\theta(R_I - r)$ ($= 1$ for $R_I \geqslant r$; $= 0$ for $R_I < r$) is the step function and R_I ($= [3(Z-1)/4\pi n_e]^{1/3}$) is the ion-sphere radius given by the electron density n_e and change number Z since the total charge within the ion sphere would be neutral. As shown in (2.1), it is found that the ion-sphere potential $V_{SC}(R_I)$ and its first derivative $\mathrm{d}V_{SC}(r)/\mathrm{d}r|_{r=R_I}$ vanish at the surface of the ion sphere. An improved version of the ion-sphere model has been proposed both in the bound state and in free electron terms taking into account relativistic and exchange effects (Salzmann 1998). However, the original version of the ion-sphere model has been extensively used to explore various physical processes in high-density and low-temperature plasmas (Fujimoto 2004). Hence, the screened potential energy $V_{IS}(r, R_I)$ of an electron in the field of ions A^+ and B^+ in strongly coupled plasmas based on the ion-sphere model is represented by

$$V_{IS}(r, R_I) = -\frac{e^2}{r}\left[1-\frac{r}{2R_I}\left(3-\frac{r^2}{R_I^2}\right)\right] - \frac{e^2}{|r_{AB}-r|}\left[1-\frac{|r_{AB}-r|}{2R_I}\left(3-\frac{|r_{AB}-r|^2}{R_I^2}\right)\right], \tag{2.2}$$

where r is the distance between the electron and the ion A^+ and r_{AB} is the interatomic distance. Since the ion-sphere radius R_I is usually greater than the interatomic distance r_{AB} in strongly coupled plasmas, the maximum potential energy V_{IS}^{Max} would be obtained by the condition $\partial V_{IS}(r, R_I)/\partial r = 0$ with the constraint of $r < R_I$ as

$$V_{IS}^{Max}(r = r_{AB}/2, R_I) = -\frac{4e^2}{r_{AB}}\left[1-\frac{r_{AB}}{4R_I}\left(3-\frac{r_{AB}^2}{4R_I^2}\right)\right], \tag{2.3}$$

where $V_{IS}^{Max}(r = r_{AB}/2, R_I)$ is the maximum potential value at $r = r_{AB}/2$. It should be understood that the classical analogy of the electron transfer process would be possible if the maximum potential energy V_{IS}^{Max} were smaller than the initial energy E_B as follows:

$$\begin{aligned} E_B(Z_B, r_{AB}, R_I) &= -\frac{I_B(Z_B, R_I)}{n^2} - \frac{e^2}{r_{AB}}\left[1-\frac{r_{AB}}{2R_I}\left(3-\frac{r_{AB}^2}{R_I^2}\right)\right] \\ &\geqslant -\frac{4e^2}{r_{AB}}\left[1-\frac{r_{AB}}{4R_I}\left(3-\frac{r_{AB}^2}{4R_I^2}\right)\right], \end{aligned} \tag{2.4}$$

where $I_B(Z_B, R_I)/n^2$ is the ionization energy of the electron in the neutral atom B with principal quantum number n.

3. Atomic states and transfer cross-section

In strongly coupled plasmas, the radial Schrödinger equation (Jung 2000) of the neutral atom B with the ion-sphere model and the effective screened charge $Z_B - \delta_{nl}(Z_B, R_I)$ is represented by

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R_{nl}(r)}{\mathrm{d}r}\right)-\frac{l(l+1)}{r^2}R_{nl}(r)\right]-\frac{[Z_B-\delta_{nl}(Z_B, R_I)]e^2}{r}R_{nl}(r)=E_{nl}R_{nl}(r), \tag{3.1}$$

where $\hbar$ is the rationalized Planck constant, m is the mass of the electron and $R_{nl}(r)$ and E_{nl} are the screened radial wave function and energy eigenvalue of the nlth state, respectively. The inner screening constant $\delta_{nl}(Z_B, R_I)$ is determined by $\delta_{nl}(Z_B, R_I) = 4\pi \int_0^{n^2a_0/Z_{nl}} \mathrm{d}r\, r^2 n_e(R_I)$, a_0 $(=\hbar^2/me^2)$ is the Bohr radius of the hydrogen atom and Z_{nl} is the effective Z value in the nlth state. For free atoms, the effective Z values for atomic electrons have been introduced by the modified Slater rules (Jung & Gould 1991). Using the 1s variation parameter ξ_{1s} in the ion-sphere model, i.e. the effective Bohr radius in strongly coupled plasmas, the radial part of the screened normalized variational ground state wave function (Jung 2000) $R_{1s}(r, \xi_{1s})$ is assumed to be

$$R_{1s}(r, \xi_{1s}) = \frac{2}{\xi_{1s}^{3/2}}\exp\left(-\frac{r}{\xi_{1s}}\right). \tag{3.2}$$

Using the Rayleigh–Ritz variational technique (Mathews & Walker 1970) with the ansatz $R_{1s}(r, \xi_{1s})$ (3.2) and the energy expectation value $\langle E_{1s}(R_I, \xi_{1s})\rangle$ for the ground state, the variation parameter (Jung 2000) $\xi_{1s}(Z_B, R_I)$ for $a_{Z_B} < R_I$ would be obtained by the minimization condition $\partial\langle E_{1s}(R_I, \xi_{1s})\rangle/\partial\xi_{1s} = 0$:

$$\xi_{1s}(Z_B, R_I)/a_{Z_B} = \frac{1}{1-\delta_{1s}(Z_B, R_I)/Z_B}, \tag{3.3}$$

where $a_{Z_B} \equiv a_0/Z_B$ and the 1s screening constant $\delta_{1s}(Z_B, R_I)$ is found to be

$$\delta_{1s}(Z_B, R_I) = \frac{(Z_B-1)(a_{Z_B}/R_I)^3}{1-3(1-1/Z_B)(a_{Z_B}/R_I)^3}. \tag{3.4}$$

The ionization energy (Jung 2000) $I_B(Z_B, R_I)$ of the electron in the neutral atom B in strongly coupled plasmas is then represented by

$$I_B(Z_B, R_I) = Z_B^2 Ry[1-\xi_{1s}(Z_B, R_I)/a_{Z_B}]^2, \tag{3.5}$$

where Ry $(= me^4/2\hbar^2 \approx 13.6$ eV) is the Rydberg constant. In (3.5), it is shown that the ionization limits would come down owing to the pressure ionization (Fujimoto 2004). After some mathematical manipulations using (2.3), (2.4), and (3.5), the upper limit of the screened interparticle distance $r_{AB}(Z_B, R_I)$ would be obtained by the following relation:

$$r_{AB}(Z_B, R_I) \leqslant \frac{3e^2}{\dfrac{Z_B^2 Ry[1-\xi_{1s}(Z_B, R_I)/a_{Z_B}]^2}{n^2}+\dfrac{3e^2}{2R_I}}. \tag{3.6}$$

As seen in (3.6), the term $3e^2/2R_I$ in the denominator represents the shielding effect on the interparticle distance $r_{AB}(Z_B, R_I)$ in strongly coupled plasmas. Since the maximum screened interparticle distance $r_{AB}^{max}(Z_B, R_I)$ is represented by $r_{AB}^{max}(Z_B, R_I) = 3e^2/[I_B(Z_B, R_I)/n^2 + 3e^2/2R_I]$, the classical expression of the resonant electron transfer cross-section $\sigma_{Transf}^{Class}(Z_B, R_I)$ between the ion A^+ and the neutral atom B in strongly coupled plasmas is then found to be

$$\sigma_{Transf}^{Class}(Z_B, R_I) = \frac{9\pi e^4 n^4 / Z_B^4 Ry^2}{\left[1 + 3\frac{n^2}{Z_B}\left(\frac{a_{Z_B}}{R_I}\right) - 2\left(1 - \frac{1}{Z_B}\right)\left(\frac{a_{Z_B}}{R_I}\right)^3\right]^2}, \tag{3.7}$$

where the terms $3(n^2/Z_B)(a_{Z_B}/R_I) - 2(1 - 1/Z_B)(a_{Z_B}/R_I)^3$ in the denominator represent the shielding effects on the classical resonant electron transfer cross-section. Hence, the classical characteristic function $C_{IS}(Z_B, R_I)$ for the influence of plasma shielding on the classical expression of the resonant electron transfer cross-section in strongly coupled plasmas based on the ion-sphere model is obtained by

$$C_{IS}(Z_B, R_I) = \left[1 + 3\frac{n^2}{Z_B}\left(\frac{a_{Z_B}}{R_I}\right) - 2\left(1 - \frac{1}{Z_B}\right)\left(\frac{a_{Z_B}}{R_I}\right)^3\right]^{-2}. \tag{3.8}$$

As shown in (3.8), it is found that the plasma shielding effect diminishes the classical expression of the resonant electron transfer cross-section in strongly coupled plasmas. When $Z_B \gg 1$, from (3.7) and (3.8), the classical expression of the resonant electron transfer cross-section is found to be $\sigma_{Transf}^{Class} \propto Z_B^4$. In the quantum mechanical charge transfer process, it has been shown that potential barrier penetration or the quantum tunnelling effect (Fridman 2008; Fridman & Kennedy 2011) enhance the resonant electron transfer cross-section for a positive ion and a neutral atom collision case. Using the Wentzel–Kramers–Brillouin (WKB) method (Razavy 2003), the electron tunnelling probability can be represented by the following exponential integral expression: $|T_{Tunnel}^{Q}|^2 = \exp[-(2/\hbar)\int_{z_1}^{z_2} \mathrm{d}z\sqrt{2m(V(z) - E)}]$, where z_1 and z_2 are the turning points of the potential $V(z)$. Using the screened potential energy $V_{IS}(r, R_I)$ (2.2) with (3.3) and (3.5), the quantum tunnelling probability for the resonant electron transfer process between the ion A^+ and the neutral atom B in strongly coupled plasmas would be then obtained by

$$|T_{Tunnel}^{Q}(Z_B, R_I, d)|^2 \approx \exp\left[-\frac{2dZ_B}{\hbar}\sqrt{2mRy[1 - \xi_{1s}(Z_B, R_I)/a_{Z_B}]}\right], \tag{3.9}$$

where d is the width of the potential barrier. Since the maximum width [7] d_{Max} of the potential barrier would be determined when the electron tunnelling frequency exceeds the inverse ion-neutral atom collision time: $I_B(Z_B, R_I)|T_{Tunnel}^{Q}(Z_B, R_I, d)|^2/\hbar > v/d$, where v is the relative collision velocity, the quantum tunnelling resonant electron transfer cross-section can be represented by the geometric relation $\sigma_{transf}^{Q\text{-}Tunnel} \approx \pi d_{Max}^2$. Hence, the quantum tunnelling resonant electron transfer cross-section $\sigma_{transf}^{Q\text{-}Tunnel}(Z_B, R_I, \bar{E})$ between the neutral atom B and the ion A^+ in strongly coupled plasmas is found to be

$$\begin{aligned}\sigma_{transf}^{Q\text{-}Tunnel}(Z_B, R_I, \bar{E}) &= \frac{\pi a_{Z_B}^2}{4\left[1 - 2\left(1 - \frac{1}{Z_B}\right)\left(\frac{a_{Z_B}}{R_I}\right)^3\right]} \\ &\quad \times \left\{\ln\left[\frac{Z_B^2}{\sqrt{8\bar{E}}}\left(1 - 2\left(1 - \frac{1}{Z_B}\right)\left(\frac{a_{Z_B}}{R_I}\right)^3\right)\right]\right\}^2, \end{aligned} \tag{3.10}$$

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where $\bar{E} \equiv (\hbar v/dRy)^2/8[= (\tilde{E}/2\bar{d}^2)(m/\mu)]$, $\tilde{E} \equiv \mu v^2/2Ry$, μ is the reduced mass of the colliding system (A^+, B) and $\bar{d} \equiv d/a_0$. When $Z_B \gg 1$, from (3.10), the quantum tunnelling resonant electron transfer cross-section is found to be $\sigma_{transf}^{Q\text{-}Tunnel} \propto [\ln Z_B^2 + f(\bar{E})]^2$, where $f(\bar{E})$ is the energy-dependent function. Since the quantum tunnelling process is resolved by the relative collision velocity of the system, the quantum characteristic function $Q_{IS}(Z_B, \bar{R}_I, \bar{E})$ for the plasma shielding effect on the quantum tunnelling resonant electron transfer cross-section in strongly coupled plasmas is then determined by

$$Q_{IS}(Z_B, \bar{R}_I, \bar{E}) = \frac{\ln\left[\dfrac{Z_B^2}{\sqrt{8\bar{E}}}\left(1 - 2\left(1 - \dfrac{1}{Z_B}\right)\left(\dfrac{1}{\bar{R}_I}\right)^3\right)\right]}{\left[1 - 2\left(1 - \dfrac{1}{Z_B}\right)\left(\dfrac{1}{\bar{R}_I}\right)^3\right]\ln\left(\dfrac{Z_B^2}{\sqrt{8\bar{E}}}\right)}, \tag{3.11}$$

where $\bar{R}_I (\equiv R_I/a_{Z_B})$ is the scaled ion-sphere radius. Using (3.7) and (3.10), the quantum tunnelling effect on the resonant electron transfer cross-section can also be obtained by the following expression of the tunnelling characteristic function T_{IS}:

$$\begin{aligned} T_{IS}(Z_B, \bar{R}_I, \bar{E}) \; &= \frac{Z_B^2}{36n^4} \frac{\left[1 + 3\dfrac{n^2}{Z_B}\left(\dfrac{a_{Z_B}}{R_I}\right) - 2\left(1 - \dfrac{1}{Z_B}\right)\left(\dfrac{a_{Z_B}}{R_I}\right)^3\right]^2}{1 - 2\left(1 - \dfrac{1}{Z_B}\right)\left(\dfrac{1}{\bar{R}_I}\right)^3} \\ &\times \ln\left[\frac{Z_B^2}{\sqrt{8\bar{E}}}\left(1 - 2\left(1 - \frac{1}{Z_B}\right)\left(\frac{1}{\bar{R}_I}\right)^3\right)\right]. \end{aligned} \tag{3.12}$$

As seen, the influence of quantum tunnelling on the resonant electron transfer process could be explicitly investigated through the logarithmic term in (3.12). Recently, the resonant charge transfer processes in collisions between positive ions have been investigated in strongly coupled plasmas (Jung 2005*b*) as well as in weakly coupled Lorentzian plasmas (Hong & Jung 2014). However, the quantum tunnelling effect has not been included in the investigation of the resonant electron transfer process in strongly coupled plasmas. Hence, (3.10)–(3.12) are reliable to explore the influence of quantum tunnelling on the resonant electron transfer cross-section in strongly coupled plasmas based on the ion-sphere model. In addition, extensive and excellent investigations for the effective interaction potentials have been carried out in dense semiclassical and quantum plasmas (Deutsch 1977; Deutsch, Gombert & Minoo 1978; Ramazanov, Dzhumagulova & Gabdullin 2010; Ramazanov *et al.* 2011; Shukla & Eliasson 2011; Akbari-Moghanjoughi & Shukla 2012; Shukla & Eliasson 2012; Akbari-Moghanjoughi 2013; Dzhumagulova *et al.* 2014; Ramazanov *et al.* 2014). However, quantum tunnelling resonant electron transfer has not been explored as yet in semiclassical or quantum plasmas. Hence, the investigation on the quantum tunnelling and plasma shielding effects on the resonant electron transfer process in dense semiclassical and quantum plasmas will be treated elsewhere. It is important to note that the ion-sphere potential is designed so that potential and its first derivative with respect to r vanish at the ion-sphere radius. In strongly coupled plasmas, plasma screening is better described by this ion-sphere picture, in which

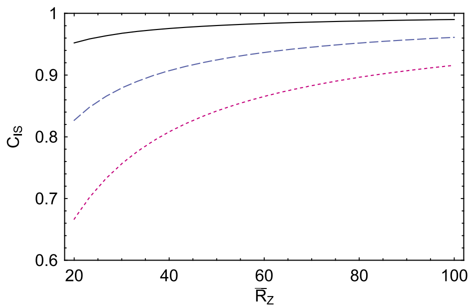

FIGURE 1. The classical characteristic function C_{IS} for the influence of plasma shielding on the classical expression of the resonant electron transfer cross-section in strongly coupled plasmas as a function of the scaled ion-sphere radius $\bar{R}_I$ for $n=1$ and $E_F/\hbar\omega_p=4$, where ω_p is the electron plasma frequency. The solid line is the case of $Z=2$ and $\Gamma r_s\theta=7.32$. The dashed line is the case of $Z=6$ and $\Gamma r_s\theta=65.08$. The dotted line is the case of $Z=8$ and $\Gamma r_s\theta=111.12$.

the stationary hydrogenic ion of total electric charge $Z-1$ is surrounded by $Z-1$ plasma electrons, uniformly distributed throughout the ion-sphere radius. Hence, the ion-sphere model would be reliable when the ion charge Z is greater than unity since the ion-sphere radius $R_I \propto (Z-1)^{1/3}$. In strongly coupled plasmas (Baimbetov, Nurekenov & Ramazanov 1996), the temperature T and number density n are shown to be approximately $(1\text{–}10)\times 10^4$ K and $10^{19}\text{–}10^{20}\ \text{cm}^{-3}$, respectively. Additionally, it has been shown that the physical properties of dense plasmas (Ramazanov *et al.* 2010) could be expressed by the plasma coupling parameter Γ $[=(Ze)^2/ak_BT]$, degeneracy parameter θ $(=k_BT/E_F)$ and density parameter r_s $(=a/a_0)$, where E_F is the Fermi energy and a is the average distance between particles in dense plasmas. Recently, the excellent works (Ramazanov *et al.* 2006; Omarbakiyeva, Ramazanov & Röpke 2009) have provided the useful effective interaction potentials to describe electron–ion, electron–atom and ion–atom interactions in partially ionized dense plasmas taking into account quantum mechanical effects as well as plasma screening effects. In addition, an excellent investigation has provided the electron–atom interaction, including Pauli blocking and plasma screening in partially ionized hydrogen plasmas, using the Beth–Uhlenbeck approach (Omarbakiyeva *et al.* 2010) to the second-virial coefficient.

4. Discussion

In order to specifically investigate the plasma shielding and quantum tunnelling effects on the resonant electron transfer process, we consider the reaction: $A^+ + A \rightarrow A + A^+$, i.e. $A=B$, then $a_{Z_A}=a_{Z_B}\equiv a_Z$. Figure 1 represents the classical characteristic function C_{IS} for the influence of plasma shielding on the classical expression of the resonant electron transfer cross-section as a function of the scaled ion-sphere radius $\bar{R}_I$ for various ion charge number Z in strongly coupled plasmas. As shown in this figure, it is found that the classical characteristic function C_{IS} increases with an increase of the ion-sphere radius $\bar{R}_I$. Hence, we have found that the plasma shielding effect suppresses the classical resonant electron transfer cross-section in strongly coupled plasmas. It is also shown that the classical characteristic function C_{IS} decreases with increasing ion charge number Z. We then understand that the shielding effect on the classical resonant electron transfer cross-section is found

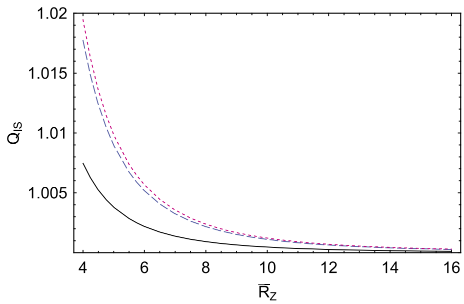

FIGURE 2. The quantum characteristic function Q_{IS} for the shielding effect on the quantum tunnelling resonant electron transfer cross-section in strongly coupled plasmas as a function of the scaled ion-sphere radius $\bar{R}_I$ for $\bar{E} = 0.001$ and $E_F/\hbar\omega_p = 4$. The solid line is the case of $Z = 2$ and $\Gamma r_s \theta = 7.32$. The dashed line is the case of $Z = 6$ and $\Gamma r_s \theta = 65.08$. The dotted line is the case of $Z = 8$ and $\Gamma r_s \theta = 111.12$.

to be more significant at larger ion charge numbers. Hence, we can expect that hydrogen or helium atom collisions would be less appropriate to investigate the plasma shielding effect in strongly coupled plasmas. Figure 2 shows the quantum characteristic function Q_{IS} for the shielding effect on the quantum tunnelling resonant electron transfer cross-section as a function of the scaled ion-sphere radius $\bar{R}_I$ for various ion charge number Z in strongly coupled plasmas. As seen, we find that the quantum characteristic function Q_{IS} decreases with an increase of the ion-sphere radius $\bar{R}_I$, which indicates that the shielding effect increases the quantum resonant electron transfer cross-section in strongly coupled plasmas. It is also shown that the quantum characteristic function Q_{IS} increases with increasing ion charge number Z. Hence, the shielding effect on the quantum resonant electron transfer cross-section is also more significant at larger ion charge numbers. Figure 3 represents the surface plot of the quantum characteristic function Q_{IS} as a function of the scaled ion-sphere radius $\bar{R}_I$ and the scaled collision energy $\bar{E}$, which demonstrates that the shielding effect on the quantum resonant electron transfer process decreases with increasing collision energy as well as ion-sphere radius. On the other hand, the energy dependence of the quantum characteristic function Q_{IS} is found to be more significant at small ion-sphere radii. Figure 4 represents the tunnelling characteristic function T_{IS} for the influence of quantum tunnelling on the resonant electron transfer cross-section as a function of the scaled ion-sphere radius $\bar{R}_I$ for various values of the collision energy $\bar{E}$ in strongly coupled plasmas. We note in figure 4 that the influence of quantum tunnelling strongly enhances the resonant electron transfer cross-section and decreases with an increase of the ion-sphere radius $\bar{R}_I$. It is also found that the quantum tunnelling effect on the resonant electron transfer process decreases with increasing collision energy $\bar{E}$ in strongly coupled plasmas.

5. Conclusions

In this work we have investigated the shielding and quantum tunnelling effects on the resonant electron transfer process in strongly coupled plasmas based on the ion-sphere model. We also obtained the screened atomic states and energy eigenvalues by using the Rayleigh–Ritz variational method for the resonant electron transfer process in strongly coupled plasmas. In addition, we have derived an analytic expression for the resonant electron transfer cross-section in strongly coupled plasmas. In this work,

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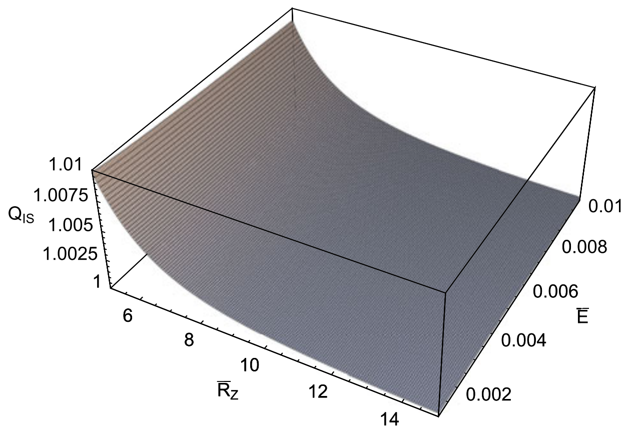

FIGURE 3. The surface plot of the quantum characteristic function Q_{IS} as a function of the scaled ion-sphere radius $\bar{R}_I$ and the scaled collision energy $\bar{E}$ for $Z = 8$, $E_F/\hbar\omega_p = 4$ and $\Gamma r_s \theta = 111.12$.

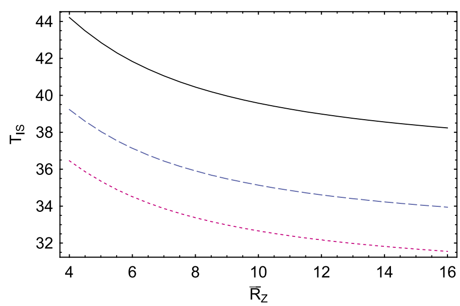

FIGURE 4. The tunnelling characteristic function T_{IS} for the influence of quantum tunnelling on the resonant electron transfer cross-section in strongly coupled plasmas as a function of the scaled ion-sphere radius $\bar{R}_I$ for $n = 1$ and $E_F/\hbar\omega_p = 4$, $Z = 6$, $\Gamma r_s \theta = 65.08$. The solid line is the case of $\bar{E} = 0.001$. The dashed line is the case of $\bar{E} = 0.002$. The dotted line is the case of $\bar{E} = 0.003$.

we have found that the classical resonant electron transfer cross-section increases with an increase of the ion-sphere radius. However, we found that the energy dependent quantum tunnelling resonant electron transfer cross-section decreases with increasing ion-sphere radius. We have shown that the screening effect on the electron transfer cross-section decreases with an increase of the nuclear charge. We have also found that the quantum tunnelling effect enhances the resonant electron transfer cross-section in strongly coupled plasmas while the influence of quantum tunnelling on the resonant electron transfer process decreases with increasing collision energy and ion-sphere radius, as shown in figures 3 and 4. In conclusion, we have found that the shielding and quantum tunnelling effects play crucial roles on the resonant electron transfer process between a positive ion and a neutral atom in strongly coupled plasmas. These results would provide useful information on the plasma screening effect as well as the quantum mechanical tunnelling phenomena on the resonant charge exchange or transfer process in dense plasmas.

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