

NON-CIRCULAR GAS MOTIONS IN THE INNER GALAXY

Robert H. Sanders
Kapteyn Astronomical Institute
Groningen, The Netherlands

Abstract. It is shown that the observed motion of neutral hydrogen in the inner 1000 pc of the Galaxy is, for the most part, consistent with flow on circular streamlines in the potential of the Galactic bulge as derived from the observed distribution of near infrared emission. The implied mass distribution is also consistent with recent kinematic determinations of the stellar mass in the inner few parsecs of the bulge. The non-circular gas motion seen between two and four kpc is most likely due to flow on elliptical streamlines in the presence of a weak bar distortion of the Galactic disk. Circular gas motion in the region of the bulge and elliptical streaming further out is an observed characteristic of flow in barred galaxies and is consistent with our present theoretical understanding of such systems. The implication is that non-circular motions of the molecular clouds in the inner 200 pc have a non-gravitational origin. A possible mechanism for exciting such motions is an accretion event resulting from an encounter of a molecular cloud with a massive black hole. A starburst leading to a high supernovae rate 10^7 years ago in the inner 50 pc is an alternative explanation. Observations of molecular cloud regions in the nuclei of external normal galaxies could distinguish between alternative mechanisms.

1. The mass distribution in the galactic bulge.

Not all of the gas in the inner region of the Galaxy flows on circular streamlines, as would be expected for steady state flow in an axisymmetric system. I want to discuss the possible causes of such non-circular gas motions and the relevance, if any, to past or present activity in the Galactic Nucleus. But first let us consider that part of the gas flow in the inner Galaxy which may be on circular orbits and is therefore a tracer of the gravitational force in the principal plane of the disk.

The distribution of galactic far-infrared sources as seen by IRAS demonstrates in a spectacular way that the Galaxy—like most spiral systems—consists of a flat disk and a central spheroidal component or bulge. The density distribution of the stars comprising the inner part of this bulge was revealed some time ago by the near infrared observations of Becklin and Neugebauer (1968), and this has been extended to larger galacto-centric distances by more recent observations (Matsumoto et al. 1982). If the near-infrared surface brightness distribution is converted to a stellar density distribution using a reasonable mass to IR light ratio of 2 in solar units (i.e., comparable to that of the bulge of M31), then the resulting rotation law in the inner 1000 pc agrees very well with that derived from 21 cm line profiles (Sanders and Lowinger 1972, Oort 1977). This is illustrated in Fig. 1 which shows a general model rotation curve for the Galaxy compared to the observed 21 cm line

rotation curve (Sellwood and Sanders 1988).

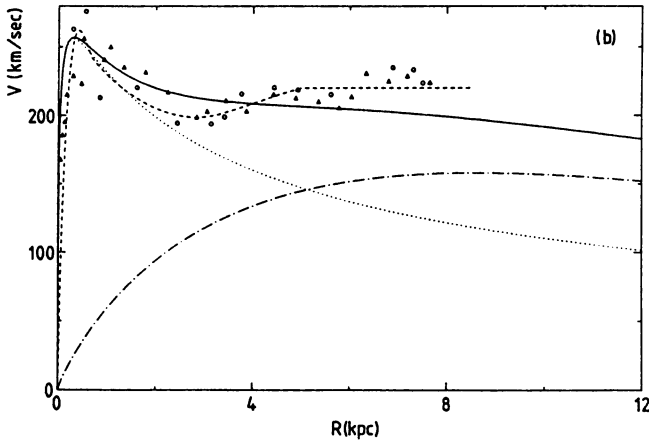


Figure 1. A model rotation curve of the Galaxy (solid line) compared to observational data. The short dashed curve is the polynomial fit to the data proposed by Burton and Gordon (1978). The symbols are taken from Sinha (1978), circles show southern data, triangles northern. All data is corrected to $R_o = 8.5$ kpc and $V_o = 220$ km/s. The other two curves show the rotation curves due to the disk alone (dot-dash line) and to the adopted bulge model (dotted lines). Note that the bulge overwhelmingly dominates inside 1000 pc.

The model consists of two components— an exponential disk and a central bulge with a surface density distribution identical in form to the near infrared intensity distribution in the inner 10° of the Galaxy. Given a bulge $M/L_{IR} = 2$, the form and amplitude of the model curve (solid line) agrees very well with the observed curve (points) in the inner 1 kpc where the bulge dominates the mass distribution. The implied mass distribution in the inner 300 pc is given by

$$M(< r) = 4.25 \times 10^6 r_{pc}^{1.2} M_\odot \tag{1}$$

and the resulting rotation law is

$$V(r) = 134 r_{pc}^{0.1} \text{ km/s.} \tag{2}$$

The agreement between the observed and calculated rotation curves suggests that non-circular gas motions are not an important aspect of gas kinematics inside 1000 pc.

It has been argued by Liszt and Burton (1980) that the gas inside 1300 pc is on elliptical streamlines as would be expected if the gravitational potential in the central region were non-axisymmetric due to the presence of a bar or triaxial bulge. In this case the central peak in the observed rotation curve would not be a measure of the central mass concentration but rather a kinematic aspect of the elliptical streaming velocities near the subcentral point. As pointed out by Genzel and Townes(1987) in their recent review, the Liszt and Burton model would imply that the mass of the bulge is about a factor of two smaller than in the model discussed above (eq. 1) and that consequently the bulge M/L_{IR} is on the order of one. There is however a recent observation which calls this

interpretation into question— and that is the dynamical estimates of the very central mass distribution based upon observations of stellar radial velocities in the inner few parsecs by Rieke and Rieke (1988). It is established that the same $r^{-1.8}$ density distribution observed by Becklin and Neugebauer extends at least into the very inner parsec and perhaps into 0.1 pc. Therefore, in Fig. 2, I have plotted (solid line) the enclosed mass as a function of radius in the inner 10 pc for the bulge model given above (eq. 1). Shown on the same figure are the enclosed mass estimates of Rieke and Rieke based upon an assumed isotropic velocity distribution. The agreement is obvious and gives strong support to the bulge mass distribution estimated from the near IR on the basis of a $M/L_{IR} \approx 2$. The implications are that the central peak in the rotation curve is an indication of the true bulge mass distribution, that the gas motions inside 1000 pc are primarily on circular orbits, and that the gravitational field inside 1000 pc is close to axisymmetric.

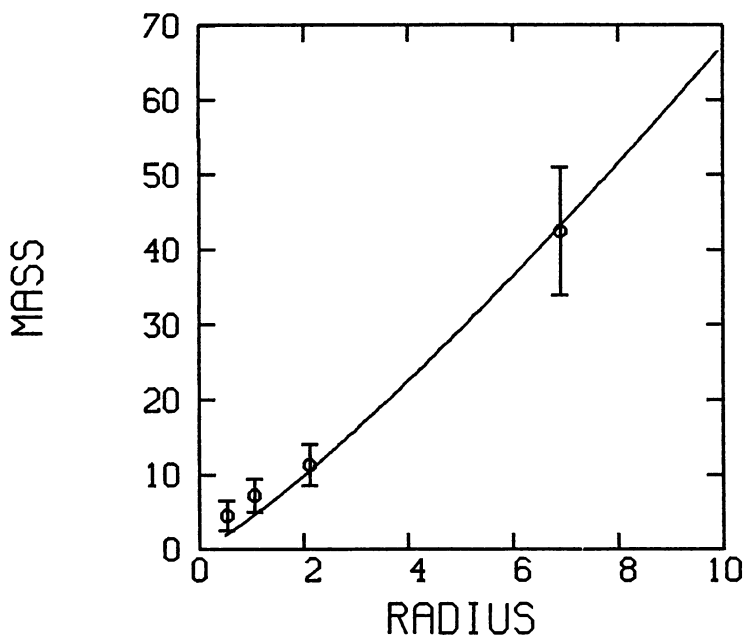


Figure 2. The enclosed mass (solid line) in units of $10^6 M_{\odot}$ as a function of distance from the Galactic Center in parsecs for the bulge model derived from the near infrared intensity (the model producing the rotation curve in Fig. 1). The points show the enclosed mass estimates from Rieke and Rieke (1988) based upon an assumed isotropic stellar velocity distribution.

This argument is not air-tight. The Rieke and Rieke mass estimates are about 30% smaller if the stellar orbits are more nearly circular and the total enclosed bulge mass at larger radii might be smaller if there is a systematic decrease in the mass-to-light ratio, but both of these possibilities appear less likely.

2. Non-circular velocities– the 3 kpc arm.

What about non-circular gas velocities? The most prominent feature in the early 21 cm line studies of the inner Galaxy is the “3 kpc arm” (van Woerden et al. 1957, Rougoor and Oort 1960). This is a distinct continuous feature in 21 cm longitude-velocity contour maps which, by its longitude extent, must lie at a distance of about 3 kpc from the center (if it can be represented as a circular feature) and which appears in absorption in front of Sgr A with a velocity away from the center of 53 km/s. An early speculation was that such a feature could be an expanding ring of gas due to a super-explosion in the Galactic Center, but this possibility now seems quite unlikely in view of the enormous energy ($> 10^{58}$ ergs) and isotropically ejected mass ($> 10^8 M_{\odot}$) required to excite non-circular gas motions at such a large distance from the center (Sanders and Prendergast 1974).

A more physically plausible explanation is provided by gas flow on elliptical stream lines in the potential of a central bar. The idea that the inner Galaxy may be barred to some degree is not new (de Vaucouleurs 1963). Moreover, a rather weak $\cos(2\phi)$ distortion of a disk (corresponding to an axial ratio in the mass distribution of 3/2) can result in apparent expansion motions of about 75 km/s especially near the position of the principal inner resonance (Sanders and Huntley 1976). This is illustrated in Fig. 3 which is a contour map of radial velocity (flow toward or away from the center) based upon a two dimensional hydrodynamical calculation of flow in such a non-axisymmetric potential.

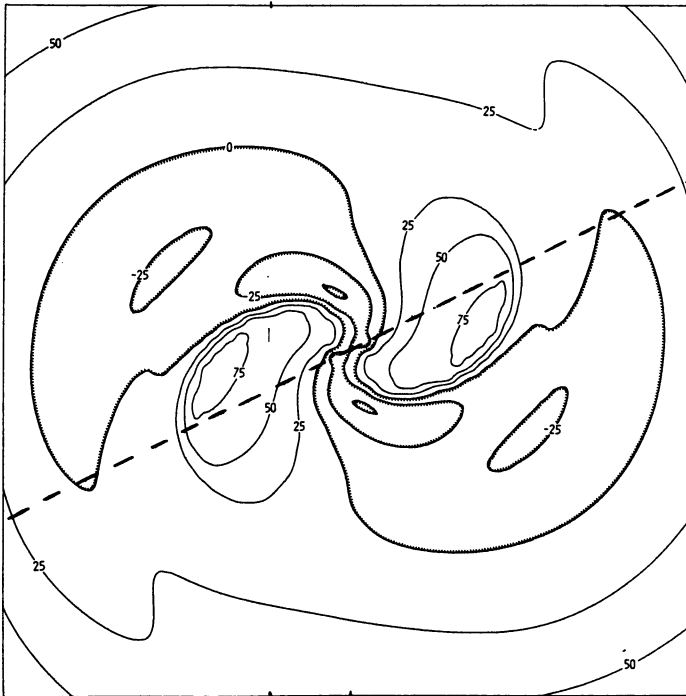


Figure 3. Contours of inflow (negative) and outflow (positive) velocities resulting from elliptical streaming in the potential of a weak bar distortion of a galactic disk (Sanders and Huntley 1976). An observer along the indicated line would detect features such as the 3 kpc arm.

The major axis of the bar is horizontal and the line shows the orientation of the line of sight with respect to the bar which is necessary for the observation of apparent outflow velocities. Mulder (1985), using a more sophisticated hydrodynamical code, has derived a detailed description for a gas flow within a system having a similar non-axisymmetric mass distribution. He then generates synthetic 21 cm line profiles as seen by an observer at various positions in the plane of the disk and finds that distinct features with the longitude-velocity dependence of the 3 kpc arm are easily reproduced by elliptical streaming.

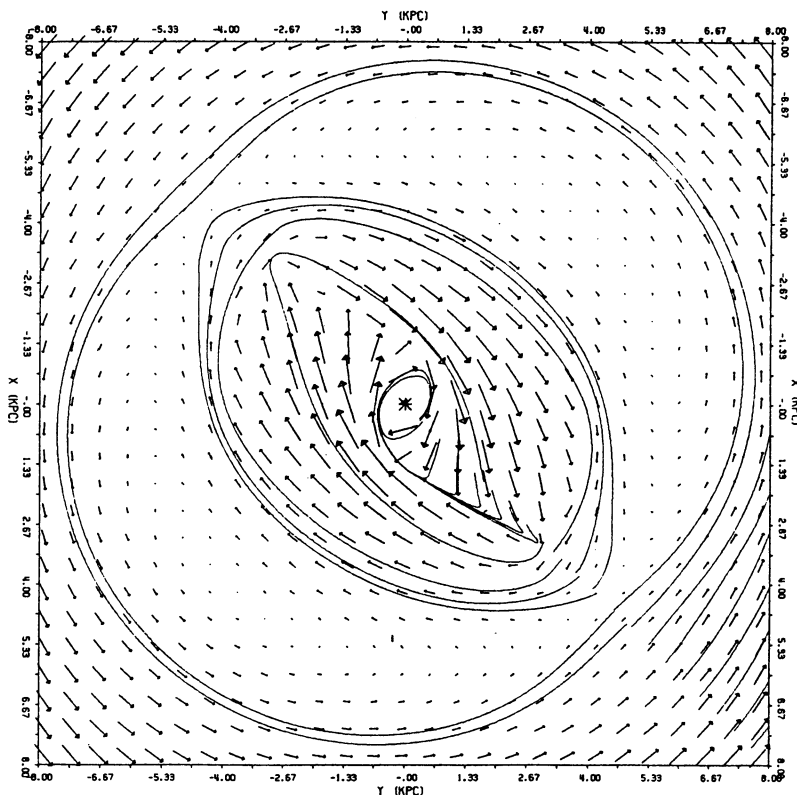


Figure 4. Gas stream lines in the potential of a strong bar (axial ratio 1/3) and central axially symmetric "bulge" component from unpublished calculations by G.D. van Albada. The flow is on highly elliptical streamlines in the region of the bar but is more circular closer to the center where the bulge dominates.

Thus it does appear that the mass distribution of the Galaxy contains a non-axisymmetric component which manifests itself by non-circular gas flow between 2 and 4 kpc. But I have argued above that within 1000 pc of the center the flow is generally circular. Are these two possibilities consistent? Indeed they are, both from an observational and theoretical point-of-view. Observations of the optical emission lines in a strongly barred galaxy NGC 1365 (Teuben et al. 1986) reveal that, in the region of the bar between 2 and 10 kpc, the pattern of radial velocities is consistent with flow on highly elliptical

streamlines oriented parallel to the bar; however, within the region of the bright bulge the flow is more nearly circular (iso-velocity contours are parallel to the visual apparent minor axis as determined from the light distribution further out). There is even some indication that flow streamlines near the center are slightly elongated perpendicular to the bar major axis. This is also an aspect of the gas dynamical simulations of flow in strong barred spiral potentials as is shown in Fig. 4 (G.D. van Albada, unpublished). Here again we see strong elliptical streaming along the bar, but, further in, the gas motion is more circular or even elliptical oriented perpendicular to the bar. The reasons for this have to do with the location of resonances and the fact that the disk is barred but not the bulge— the bulge is a more nearly axisymmetric structure. Thus in the Galaxy the dominant pattern of flow within the inner 1 kpc can be on circular orbits even though further out the streamlines are elliptical. This is consistent with our theoretical and observational understanding of barred galaxies.

3. Non-circular gas motions— the inner 200 pc.

I have argued that the gas flow within 1 kpc is primarily circular. However, within 200 pc of the Galactic Center the system of massive molecular clouds exhibits very large non-circular motions as we see in Fig 5 (Bally et al. 1988).

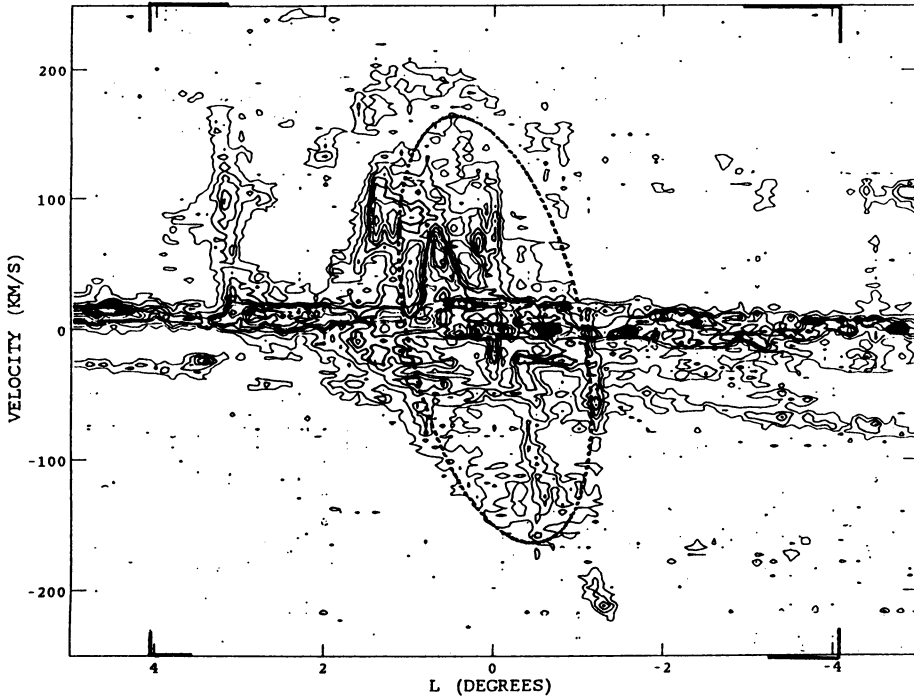


Figure 5. A longitude-velocity contour map of ^{13}CO emission in the inner Galaxy (Bally et al. 1988) averaged over $1.^\circ_2$ in galactic latitude.

This is a longitude-velocity contour map of CO emission in the inner few degrees. Here we see obvious non-circular velocities on the order of 135 to 165 km/s. A 21 cm absorption

feature in front of Sgr A with a velocity of -135 km/s implies an apparent motion of these molecular clouds away from the Galactic Center (expansion).

If, as I have claimed, the bulge is essentially axisymmetric, then the origin of these non-circular motions cannot be gravitational. There is a possible escape from this conclusion which I shall mention below— but for now let us assume that this is true. Then, what are the possible non-gravitational origins of such high velocities. First of all, it is obvious that those clouds with the highest non-circular motions are not just isolated features with a high random velocity but part of a larger systematic structure. This was first noticed by Scoville (1972) and Kaifu et al. (1972) who attempted to fit the longitude-velocity dependence of these high velocity molecular clouds by a kinematic model consisting of an expanding rotating ring. On the basis of more complete CO observations, Bania (1977) defined the characteristics of such a ring: the radius would be about 160 pc, the expansion velocity 150 km/s, and the rotation velocity 65 km/s. The locus of this ring on the longitude-velocity CO map is shown by the dashed line in Fig. 5 which illustrates, as emphasized by Bania, that such a description can be only approximately correct. Liszt and Burton (1980) have stressed that this “ring” is tilted with respect to the Galactic plane and argue that it part of a larger kinematic structure.

If the total mass of this structure is about $10^7 M_{\odot}$ then the kinetic energy in expansion would be about 5×10^{54} ergs. Given the rotation law in the inner region (eq. 2) we can estimate, by conservation of angular momentum, that the average original distance of these clouds from the Galactic Center would be about 50 pc, so roughly a comparable amount of energy must be supplied in overcoming the gravitational potential lifting the ring from 50 to 160 pc.

It is also evident that not all the Galactic Center molecular clouds are part of this structure. There is a separate population of massive clouds (including the Sgr A and Sgr B complexes) within the ring. It has been noted before (and especially emphasized by Oort 1984) that this population of clouds has a highly asymmetric distribution with respect to the Galactic Center— mostly being at positive longitudes. Given the rotation law in the inner region and the implied high differential motion, such an asymmetric distribution must be highly transient; the lifetime of the asymmetry would be about 10 rotation periods or several million years. Note that these clouds generally have radial velocities that are permitted in the sense of galactic rotation; however, the magnitude of these velocities is considerably less than the maximum radial velocity due to circular motion in the bulge gravitational field (eq. 2). This means that the clouds such as Sgr A are either much further away from the Galactic Center than implied by their galactic longitude (and lying nearly along the line of sight to the Galactic Center) as argued by Gusten and Downs (1980), or the clouds are near the distance indicated by their longitude and are on highly non-circular orbits. In view of the close association of the Sgr A cloud with radio continuum features near the Galactic Center, Brown and Liszt (1984) and Bally et al. (1988) consider the second possibility to be more likely. Therefore we would have to conclude that even the second population of clouds exhibits strong non-circular motion although the radial velocities are generally permitted. Bally et al. (1988) stress that the “turbulent motion” of these clouds represents a reservoir of kinetic energy in excess of 10^{54} ergs which may power the non-thermal structures seen near Sgr A.

4. The origin of the non-circular gas motions in the molecular cloud region.

What could be the source of the kinetic energy of the expanding ring and of the turbulent motion? A massive black hole (10^6 to $10^7 M_{\odot}$) at the Galactic Center could flare periodically due to discrete accretion events. Such an object in the central star cluster

would tidally disrupt and swallow a star about once every 10000 years (Hills 1975, Ozernoy 1977, Sanders and van Oosterom 1984). This would give a time-averaged luminosity of about 10^{42} ergs/s, which exceeds the present bolometric luminosity of a point source at the Galactic Center by two orders of magnitude. This has been used as an argument against the existence of a massive black hole at the center (Ozernoy 1977). However, because of the effectiveness of viscous transfer of angular momentum in the accretion disk or torus resulting from such a stellar disruption, the inflow time scale for the disrupted stellar material could only be 10 to 100 years (Rees 1982,1988, Sanders and van Oosterom 1984). Thus, stellar disruption would manifest itself as short bursts or flares having a luminosity of perhaps 10^{44} ergs/s but only 0.1% to 1% of the time. Obviously the total energy of such a flare could not exceed 10% or so of the rest mass energy of a star or 10^{53} ergs. One such event would fail by at least two orders of magnitude to provide the kinetic energy of the expanding molecular ring.

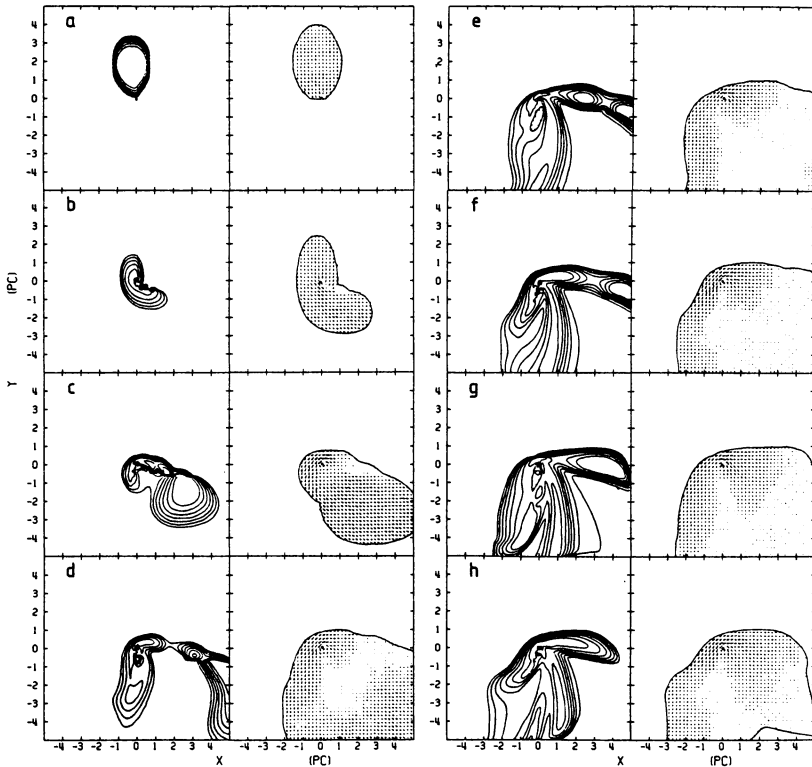


Figure 6. Time sequence of a small cloud falling into a galactic center. The mass of the black hole is $10^7 M_{\odot}$ and the initial velocity of the cloud is 225 km/s. The frame on the left shows logarithmic density contours and the frame on the right the accompanying velocity field. The time interval between successive frames is about 6500 years.

There is a second kind of discrete accretion event that could provide enough energy (Sanders 1981). We have seen that the population of clouds inside the expanding ring could be on highly elongated orbits and thus have a net low angular momentum. Thus we might expect such a cloud to occasionally plunge through the Galactic Center (this might

be happening now to the Sgr A cloud). If cloud motions were completely random and if there were on the order of 10 clouds within 150 pc of the center, each having a radius of 20 pc and a mass of $10^6 M_\odot$, then we might expect a cloud-black hole collision every 10^7 years. In a highly simplified picture of such an encounter, the hole would accrete matter in a cylindrical column through the cloud; the radius of this column would be essentially that of the classical accretion or Bondi radius ($\approx GM_h/v_c^2$ where M_h is the mass of the hole and v_c is the cloud velocity at a large distance from the hole). Hydrodynamical calculations which include density and velocity gradients in the cloud (Bottema and Sanders 1986) show that a hole with a mass of $2.5 \times 10^6 M_\odot$ could capture as much as $5000 M_\odot$ from the cloud. Due to the density or velocity gradients in the cloud the captured gas would have a net angular momentum and form an accretion disk. This disk of gas would then flow into the hole over a timescale of 10^5 years giving rise to a Seyfert luminosity of several times 10^{44} ergs/s. The total energy of such an outburst could be as large as 10^{57} ergs; thus only 10% of this energy need go into gas motions to account for the energy in non-circular motions both of the ring and of the random motion of the massive clouds.

If the size of the cloud is large compared to the Bondi radius as assumed above, then the structure in the gas resulting from such a cloud-black hole encounter is a simple accretion tail which may be curved due to density gradients in the cloud. An interesting variant of this picture occurs when the infalling cloud has a size which is small compared to the accretion radius as we see from the hydrodynamical calculations of Bottema and Sanders (1986) (Fig. 6). Here is a series of frames showing the time evolution of a collision between a small cloud initially at 4 pc from a $10^7 M_\odot$ hole with an initial velocity of 225 km/s and an impact parameter of 0.25 pc. The interval between frames is roughly 6500 years. We see that a rather complex structure develops which certainly resembles the "3 arm spiral" seen in the ionized gas in the inner 2 pc of the Galaxy. Unfortunately the predicted kinematic structure of the spiral does not agree very well with the observed kinematics of the ionized gas comprising the spiral— but the results do imply that complex structure could result from the infall of a single small cloud; the infall of several small clouds (Lo and Claussen 1983, Quinn and Sussman 1985) is not required.

5. Alternatives.

Finally, I should stress that there certainly are alternative explanations for the non-circular gas motions in the central 200 pc. A central explosion can produce a ring of gas which can oscillate about an equilibrium position for a few periods. For the molecular ring at 160 pc the oscillation period would be about three million years. Any event which is short compared to this oscillation period would in effect be a point explosion as far as the gas response is concerned. For example, 10^5 supernovae exploding over a period of one million years within 50 pc of the center would have the same effect on gas motions as an impulsive point explosion of 10^{56} ergs. Thus a starburst leading to a supernova rate of 0.1 to 1 per year 10^7 years ago could supply the kinetic energy of the expanding ring as well as the turbulent energy of the inner clouds. This high supernova rate is apparently seen in the central regions of galaxies like M82 which are presently experiencing starbursts (Kronberg 1985, Unger et al. 1985). There is even a ring of molecular gas in M82 with dimensions of 200 pc to 300 pc as in the central region of the Galaxy. There certainly would appear to be enough gas in the inner 100 pc of the Galaxy to fuel a starburst; the constraint on such a model is that the present level of supernova activity (10^7 years later) should be acceptably low (i.e., no direct evidence for a Galactic Center supernova in the last 30 years).

A second alternative explanation for the non-circular motions is again provided by gravity. I have argued that the bulge is axisymmetric and that the gas flow should be

circular. However, this is only true if we can ignore the self-gravity of the gas. The total mass of the stellar bulge within 300 pc of the center is $4 \times 10^9 M_{\odot}$. Bania (1977) has argued that the mass of molecular gas within 300 pc might be as high as $7 \times 10^8 M_{\odot}$ or almost 20% of the stellar mass. Certainly if the mass of the gas were much higher than this, it could almost be considered as a separate dynamical system. A well-known result of N-body simulations (Athanasoula and Sellwood 1986) is that self-gravitating cold systems (that is, supported primarily by rotation) are unstable to bar formation. If the mass in a cold disk is as large as 1/3 of the mass of the axisymmetric stellar component, a strong bar in the disk could develop over 10 rotation times (3×10^7 years). Thus it is conceivable that the gas in the inner few hundred parsecs could be strongly barred (or even asymmetric) while the stellar system is axisymmetric. This would be a dynamically separate and probably more transient bar than that giving rise to the non-circular gas motions in the regions of the 3 kpc arm, but of course it would also have to be oriented in such a way that we observe apparent outflow.

Due to our special viewing position in the Galactic plane it is probably impossible to conclusively solve the riddle of these non-circular motions in the inner Galaxy. However, the advent of mm wave interferometers will lead to detailed investigations of the distribution and kinematics of the molecular gas in the centers of external normal galaxies. For example, it should now be possible to distinguish between ring-like or bar-like distributions of molecular gas and between systematic outflow (or inflow) and flow on elliptical streamlines in the central few hundred parsecs of nearby galaxies.

Given these several possibilities for exciting the observed non-circular motions in the inner Galaxy, the presence of such motions can in no sense prove or disprove the existence of a massive central object as the source of activity. The definitive answer to this question must be answered by kinematic studies of the inner parsec of the Galaxy. Given the ambiguity in interpreting gas motions, it is the kinematics of the stellar component which may well be most revealing.

I am very grateful to G.D. van Albada for permission to use the results of his unpublished hydrodynamical calculations shown in Figure 4.

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