WELFARE-MAXIMIZING OPERATIONAL MONETARY AND TAX POLICY RULES

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This paper computes welfare-maximizing monetary and tax policy feedback rules in a calibrated dynamic general equilibrium model with sticky prices. The government makes exogenous final good purchases, levies a proportional income tax, and issues nominal one-period bonds. A quadratic approximation method is used to solve the model and to compute household welfare. Optimized policy has a strong anti-inflation stance and implies persistent fluctuations of the tax rate and of public debt. Very simple optimized policy rules, under which the interest rate just responds to inflation and the tax rate just responds to public debt, yield a welfare level very close to that generated by richer rules.

Keywords: Monetary Policy, Fiscal Policy, Welfare

1. INTRODUCTION

There has been much recent work on the effect of *monetary* policy rules on welfare and business cycles [see survey by McCallum (1999)]. *Fiscal* policy rules have received less attention. Existing studies follow two approaches: (i) dynamic extensions of Ramsey (1927) that determine welfare-maximizing time paths of fiscal instruments;[1] (ii) analyses of the macroeconomic effects of simple fiscal feedback rules [e.g., Taylor (2000)].

The Ramsey approach is appealing, as it uses micro-based models and focuses on household welfare as the criterion for evaluating policy. However, that approach faces technical difficulties, as Ramsey problems are generally *not* concave.[2] Furthermore, Ramsey-type studies typically use highly stylized models; also, Ramsey policy rules are often complicated—which may make it difficult to apply them in practice.

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By contrast, most studies on simple feedback policy rules use models that are more realistic, but that are not fully micro-based; ad hoc criteria (such as the implied volatilities of output and inflation) are employed to evaluate policy.

This paper computes welfare-maximizing *operational* feedback rules that link *monetary* and *fiscal* policy to small sets of easily observable macro variables, for a calibrated business cycle model with staggered price setting à la Calvo (1983). The (potential) nonconcavity of the Ramsey problem is inconsequential for the approach here. The model has rigorous micro foundations, but is richer than those used in most applications of the Ramsey approach. The economy features capital, variable labor supply, monopolistic competition in goods markets, and exogenous shocks to productivity and to government purchases. The government levies a proportional income tax and issues nominal unconditional one-period bonds. Monetary policy follows a Taylor-style interest rate rule; the tax rate is set as a function of real public debt, productivity, and government purchases. The steady state tax rate and the ratios of debt and of government purchases to GDP are calibrated to OECD data. I focus on policies characterized by stationary fluctuations of real public debt around its steady state value.

Under staggered price setting (as assumed here), inflation induces inefficient dispersion of prices across firms [e.g., Erceg et al. (2000)]; in an economy in which price stickiness is the only distortion, optimal monetary policy entails full inflation stabilization, as that policy eliminates inefficient cross-firm price dispersion [e.g., Rotemberg and Woodford (1997)]. The economy here has monopolistic competition and tax distortions. It is shown that, nevertheless, optimized policy under sticky prices implies (almost) full inflation stabilization. In all model variants considered here, optimized policy implies persistent fluctuations of the tax rate and sizable and persistent fluctuations of real public debt; productivity shocks are much more important as a source of macroeconomic fluctuations than government purchases shocks. Very simple optimized policy rules—under which the interest rate *just* responds to inflation, and the tax rate *just* responds to public debt—yield a welfare level very close to that generated by rules that stipulate a response to additional variables.

As optimized policy in the baseline sticky-prices model (with nominal public debt) entails strict inflation stabilization, *real* debt returns are (essentially) riskless in that setting; the behavior of the tax rate and of real activity closely resembles that generated by a *flexible-prices* model with *indexed* (real) non-state-contingent debt.

By contrast, a *flex*-prices structure with *nominal* debt implies very different optimized tax behavior than the baseline sticky-prices model. In such a structure, inflation does not cause inefficient price dispersion across firms; when exogenous shocks occur, the government can meet its intertemporal budget constraint by altering the *real* value of the inherited stock of (nominal) public debt via unanticipated inflation changes. As a result, optimized policy in a flex-prices nominal-debt structure entails sizable inflation volatility, but only small movements of the tax rate. Chari et al. (1991) showed that (optimal) monetary-fiscal *Ramsey* policy implies

high inflation volatility in a flex-prices economy with nominal debt; Schmitt-Grohé and Uribe (2004b) and Siu (2004) demonstrated that *Ramsey* policy entails much lower inflation volatility when prices are sticky. The paper here shows that similar predictions (effect of flex vs. sticky prices) hold when policy is described by simple optimized rules.

The model is solved using Sims's (2000) method, which is based on a second-order expansion of the equilibrium conditions. In contrast to the linear, certainty-equivalent approximations that are widely used in macroeconomics, this method allows capturing the effect of risk on agents' decision rules and is thus better suited for welfare analysis. Compared to other nonlinear methods [see Judd (1998)], this technique makes it possible to solve models with a rich structure easily. The approach presented in this paper might thus provide a tractable way of computing optimized policy rules using larger micro-based simulation models (such as those currently developed by, among others, the Fed and IMF).

This study builds on my recent work that computed welfare-maximizing simple monetary policy rules for calibrated New Keynesian models of *open* economies [see Kollmann (2002, 2004a)]; that research abstracted from fiscal policy.[3]

2. THE MODEL

A closed economy with a representative household, firms, and a government (monetary-fiscal authority) is considered.[4] There is a single final good that is produced by combining a continuum of intermediate goods indexed by $s \in [0, 1]$. The final good is produced by perfectly competitive firms; it can be consumed and used for investment. There is monopolistic competition in intermediate goods markets—each intermediate good is produced by a single firm. Intermediate goods firms use capital and labor as inputs. The household owns all firms and the capital stock, which it rents to firms. It also supplies labor. The markets for rental capital and for labor are competitive. The government sets the nominal interest rate, purchases an exogenous quantity of the final good, levies a proportional income tax, and issues nominal non-state-contingent one-period bonds.

2.1. Final Good Production

The final good is produced using the aggregate technology

$$Q_t = \left(\int_0^1 q_t(s)^{(\nu-1)/\nu} ds \right)^{\nu/(\nu-1)}, \text{ with } \nu > 1.$$

Q_t is the date t final good output; $q_t(s)$ is the quantity of the type s intermediate. Let $p_t(s)$ be the price of that good. Cost minimization in final good production gives $q_t(s) = (p_t(s)/P_t)^{-\nu} Q_t$, with $P_t = (\int_0^1 p_t(s)^{1-\nu} ds)^{1/(1-\nu)}$. The price of the final good is P_t (its marginal cost).

2.2. Intermediate Goods Firms

The technology of the firm that produces intermediate good s is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, 0 < \psi < 1.$$

$y_t(s)$ is the firm's output at date t. θ_t is an exogenous productivity parameter (common to all intermediates' producers). $K_t(s)$ [$L_t(s)$] is the capital [labor] used by the firm. Its marginal cost is $MC_t = (1/\theta_t)R_t^\psi W_t^{1-\psi}\psi^{-\psi}(1-\psi)^{\psi-1}$, where R_t [W_t] is the rental rate of capital [wage rate]. The firm's profit is $\pi_t(p_t(s)) = (p_t(s) - MC_t)(p_t(s)/P_t)^{-\nu}Q_t$.

There is staggered price setting, à la Calvo (1983), in the intermediate goods sector: firms in that sector cannot change prices, unless they receive a random "price-change signal". The probability of receiving this signal in any particular period is $1-d$, a constant. Following Erceg et al. (2000), I assume that when a firm does not receive a "price-change signal," its price is automatically increased at Π, the steady state growth factor of the final good price. (In this paper, the term "steady state" refers to the deterministic steady state.) Firms are assumed to meet all demand at posted prices. They maximize the value of their profit stream, net of the income tax paid by the household on profits.

Consider a firm that, at date t, sets a new price, $p_{t,t}$. If no "price-change signal" occurs between t and $t+j$, the price is $p_{t,t}\Pi^j$ at $t+j$. The firm sets $p_{t,t} = \text{Arg Max}_{\mathbf{p}} \sum_{j=0}^\infty d^j E_t\{\rho_{t,t+j}(1-\tau_{t+j})\pi_{t+j}(\mathbf{p}\Pi^j)/P_{t+j}\}$, where τ_{t+j} is the income tax rate at $t+j$; $\rho_{t,t+j}$ is a pricing kernel (for valuing date $t+j$ pay-offs) that equals the household's intertemporal marginal rate of substitution (see below). Let $\Xi_{t,t+j} \equiv \rho_{t,t+j}(P_{t+j})^{\nu-1}Q_{t+j}$. The solution of the decision problem regarding $p_{t,t}$ is

$$p_{t,t} = (\nu/(\nu-1))\left[\sum_{j=0}^\infty (d\Pi^{-\nu})^j E_t \Xi_{t,t+j}(1-\tau_{t+j})MC_{t+j}\right] \Bigg/ \left[\sum_{j=0}^\infty (d\Pi^{1-\nu})^j E_t \Xi_{t,t+j}(1-\tau_{t+j})\right].$$

The final good price P_t evolves according to

$$(P_t)^{1-\nu} = d(P_{t-1}\Pi)^{1-\nu} + (1-d)(p_{t,t})^{1-\nu}.$$

2.3. The Representative Household

Household preferences are described by

$$E_0 \sum_{t=0}^\infty \beta^t U(C_t, L_t), \text{ with } 0 < \beta < 1. \tag{1}$$

C_t [L_t] is consumption [labor effort]. U is a utility function given by $U(C_t, L_t) = \ln(C_t) - L_t$. The household accumulates physical capital, subject to the law of motion

$$K_{t+1} = K_t(1 - \delta) + I_t, \text{ with } 0 < \delta < 1, \tag{2}$$

where I_t is gross investment (δ: depreciation rate of capital). The household holds nominal one-period bonds. Its budget constraint is

$$A_{t+1} + P_t(C_t + I_t) = A_t(1 + i_{t-1}) + W_t L_t + \int_0^1 \pi(p_t(s))\, ds + R_t K_t - T_t. \tag{3}$$

A_t is a stock of nominal one-period bonds that mature in period t; i_{t-1} is the interest rate on these bonds. $T_t \equiv \tau_t(W_t L_t + \int_0^1 \pi(p_t(s))\, ds + R_t K_t - \delta P_t K_t)$ is the tax paid by the household.

The household chooses $\{A_{t+1}, K_{t+1}, C_t, L_t\}_{t=0}^{\infty}$ to maximize (1), subject to (2), (3). The following equations are first-order conditions of this problem:

$$(1 - \tau_t)W_t/P_t = C_t; \quad 1 = (1 + i_t)E_t[\rho_{t,t+1}(P_t/P_{t+1})];$$

$$1 = E_t[\rho_{t,t+1}([R_{t+1}/P_{t+1} - \delta](1 - \tau_{t+1}) + 1)],$$

with $\rho_{t,t+1} = \beta C_t/C_{t+1}$.

2.4. The Government Budget Constraint

The government budget constraint is $P_t G_t + D_t(1 + i_{t-1}) = D_{t+1} + T_t$, where G_t are (exogenous) government final good purchases; D_t is the stock of nominal one-period debt that matures in t.

2.5. Market Clearing Conditions

Markets for intermediate goods clear as intermediate goods firms meet all demand at posted prices. Market clearing in final good, labor, and rental capital markets requires $Y_t = C_t + I_t + G_t$, $L_t = \int_0^1 L_t(s)\, ds$, $K_t = \int_0^1 K_t(s)\, ds$. Bond market clearing requires $A_t = D_t$.

2.6. Policy Rules

Much recent research has focused on monetary policy rules that stipulate a response of the interest rate to inflation [e.g., Taylor (1999a)]. The baseline interest rate rule considered here is:

$$i_t = i + \Gamma_i^{\pi} \widehat{\Pi}_t, \tag{4}$$

with $\widehat{\Pi}_t = (\Pi_t - \Pi)/\Pi$, where $\Pi_t = P_t/P_{t-1}$ is the final good gross inflation rate. i is the steady state nominal interest rate. Throughout the paper, variables without time subscripts denote steady state values, and $\hat{x}_t = (x_t - x)/x$ is the relative deviation of a variable x_t from its steady state value, x. Γ_i^{π} is a policy parameter.

The tax rate is set as a function of real public debt, and of the exogenous variables

$$\tau_t = \tau + \Gamma_\tau^B \, i \, (B_t - B) + \Gamma_\tau^\theta \widehat{\theta}_t + \Gamma_\tau^G \widehat{G}_t, \tag{5}$$

where $B_t \equiv D_t/(P_t Y)$ is real public debt, normalized by steady state real GDP, Y. Γ_τ^B, Γ_τ^θ and Γ_τ^G are policy parameters. Setting Γ_τ^B at a sufficiently high positive value ensures government solvency.

Aiyagari et al. (2002) study optimal (Ramsey) fiscal policy in an infinitely lived economy with a proportional income tax and unconditional real debt;[5] they show that Ramsey policy may entail long-run values of debt and taxes that differ greatly from the values observed in reality. For example, under certain assumptions, the government runs fiscal *surpluses*, until it owns a stock of *assets* whose income covers all subsequent government purchases (at that point the tax rate is set to zero).

To rule out unrealistic long-run behavior of public debt, I impose the restriction that the unconditional mean of real debt has to be close to its steady state value B:[6]

$$|E B_t - B| < 0.01. \tag{6}$$

I set B, τ and the steady state ratio of G to GDP at $B = 0.5$, $\tau = 0.25$ and $G/Y = 0.20$, respectively; these values are in the range of post-WWII fiscal data for OECD economies [see Kollmann (1998)].

Equation (6) is also used for a technical reason: the solution method used here is based on a second-order Taylor expansion of the model around a given steady state (see below); this method is not suitable for the analysis of large changes in state variables. Equation (6) ensures that real debt (and other state variables) stay in the neighborhood of the steady state.[7]

At an initial date $t = 0$, the government makes a commitment to set the parameters Γ_i^π, Γ_τ^B, Γ_τ^θ, and Γ_τ^G at time-invariant values that maximize the conditional expected value of household lifetime utility (1), subject to the laws of motion of the endogenous variables implied by household decisions, and subject to (6). I assume that at $t = 0$ the predetermined state variables equal their steady state values, and that the exogenous variables at $t = 0$ equal the unconditional means of these variables.

Note that the government takes the steady state values of the tax rate and the real debt/GDP ratio as given (perhaps because those values are determined by political/institutional constraints that are beyond its control); hence, the government's only goal is to optimize the "cyclical" characteristics of policy. (The government also takes the steady state gross inflation rate, Π, as given; Π has no real effects, because of the assumed price indexation scheme.)

2.7. Parameters and Solution Method

The model is calibrated to quarterly data. The steady state *real* interest rate r is set at $r = 0.01$, a value that corresponds roughly to the long-run average (quarterly) return on capital. Thus, $\beta = 1/1.01$ is used ($\beta(1 + r) = 1$ holds in steady state).

The steady state price-marginal cost markup factor for intermediate goods is set at $\nu/(\nu - 1) = 1.2$, consistent with the findings of Martins et al. (1996) for OECD countries. The technology parameter ψ is set at $\psi = 0.24$, which entails a 60% steady state labor income/GDP ratio, consistent with OECD data. Aggregate data suggest a quarterly capital depreciation rate of about 2.5%; thus, $\delta = 0.025$ is used.

Empirically, the mean price-change interval is about 4 quarters [Taylor (1999b)]. Thus, I set $d = 0.75$. The steady state growth factor of prices is set at $\Pi = 1$; hence the steady state nominal interest rate is $i = 0.01$. The exogenous variables follow AR(1) processes:

$$\widehat{\theta}_t = \rho^\theta \widehat{\theta}_{t-1} + \varepsilon_t^\theta, \quad 0 \le \rho^\theta < 1; \quad \widehat{G}_t = \rho^G \widehat{G}_{t-1} + \varepsilon_t^G, \quad 0 \le \rho^G < 1. \quad \textbf{(7)}$$

ε_t^θ and ε_t^G are independent white noises with standard deviation σ^θ and σ^G, respectively. I set $\rho^\theta = 0.95$, $\sigma^\theta = 0.01$; these (or very similar) values are widely used in the RBC literature, and are consistent with time series evidence on aggregate productivity [e.g., Prescott (1986)]. Fitting (7) to quarterly government purchases in G7 countries gives estimates of ρ^G and σ^G in the range of 0.95 and 0.01, respectively [see Kollmann (2004b)]; thus, I set $\rho^G = 0.95$, $\sigma^G = 0.01$.

The model is solved using Sims's (2000) algorithm/computer code, which is based on second-order Taylor expansions of the equilibrium conditions around a (deterministic) steady state. I numerically solve the government's optimization problem; attention is restricted to values of the policy parameters [see (4), (5)] for which a unique stationary equilibrium exists.

3. RESULTS

Simulation results are reported in Tables 1 and 2. The variables are quarterly. Y_t is (real) GDP. $\text{Def}_t = (D_{t+1} - D_t)/(P_t Y)$ is the real (secondary) fiscal deficit, normalized by steady state GDP. In the tables, the statistics for Def_t, the interest rate (i_t), the tax rate (τ_t), and real debt (B_t) refer to differences of these variables from steady state values (i_t is a quarterly rate expressed in fractional units), whereas statistics for the remaining variables refer to relative deviations from steady state values. All statistics are expressed in percentage terms.

I express welfare as the permanent relative change in consumption (compared to steady state), ζ, that yields expected $t = 0$ lifetime utility: $(1 - \beta)^{-1} U((1 + \zeta)C, L) = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$. Note that the policy maker maximizes ζ. I decompose ζ into components, denoted ζ^m and ζ^v, that reflect the conditional expected values of consumption and hours $\{E_0 \widehat{C}_t, E_0 \widehat{L}_t\}_{t \ge 0}$ and the conditional variances of consumption $\{V_0(\widehat{C}_t)\}_{t \ge 0}$, respectively.[8]

Columns 1 to 4 in Table 1 show results under sticky prices; columns 1 and 2 pertain to the baseline model (with policy rules (4) and (5)); columns 3 and 4 assume alternative policy rules. Columns 5 and 6 assume flexible prices. Columns 7 and 8 pertain to the first-best allocation, that is, to the solution of a social planning

TABLE 1. Optimized policy rules and first best outcome

	Sticky prices						Flexible prices		First-best	
	Baseline rules		Richer rules	Simpler rules	Nominal debt	Indexed debt				
	θ, G	G	θ, G	θ, G	θ, G	θ, G	θ, G	G		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
Standard deviations (in %)										
Y	6.58	0.98	6.58	6.23	4.92	6.61	5.58	0.38		
C	6.62	1.03	6.62	6.40	4.44	6.64	4.30	0.09		
i	20.65	3.71	20.65	18.58	18.73	20.79	19.02	0.41		
Π	0.00	0.00	0.00	0.01	13.24	—	—	—		
i	0.14	0.02	0.14	0.11	12.95	—	—	—		
G	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20		
τ	1.71	0.69	1.71	1.69	0.13	1.71	—	—		
B	17.21	7.99	17.21	19.53	46.20	16.76	—	—		
Def	0.76	0.35	0.76	0.84	26.10	0.74	—	—		
Correlations with GDP										
i	−0.19	0.06	−0.19	−0.20	−0.87	—	—	—		
G	0.04	0.29	0.04	0.03	0.09	0.02	0.06	0.99		
τ	−0.81	−0.61	−0.81	−0.76	0.77	−0.81	—	—		
Autocorrelations										
Y	0.96	0.93	0.96	0.97	0.95	0.96	0.92	0.94		
i	0.94	0.94	0.94	0.95	0.96	—	—	—		
τ	0.99	0.99	0.99	0.99	0.81	0.99	—	—		
B	0.99	0.99	0.99	0.99	0.98	0.99	—	—		
Means (in %)										
Y	0.10	−0.00	0.10	0.07	0.10	0.10	0.10	0.00		
C	0.05	−0.00	0.05	0.03	0.08	0.05	0.07	0.00		
L	−0.07	−0.00	−0.07	−0.07	0.01	−0.07	0.01	0.00		
K	0.49	0.00	0.49	0.40	0.40	0.50	0.30	0.00		
τ	0.09	0.01	0.09	0.08	0.00	0.09	—	—		
B	1.00	0.06	1.00	1.00	−1.00	1.00	—	—		
Welfare (% equivalent permanent variation in consumption)										
ζ	−0.072	−0.004	−0.072	−0.072	−0.056	−0.072	−0.031	−0.000		
ζ^m	0.101	−0.001	0.101	0.087	0.027	0.103	0.046	−0.000		
ζ^v	−0.173	−0.003	−0.173	−0.160	−0.083	−0.175	−0.078	−0.000		
Policy parameters										
Γ_i^π	8660	8660	8660	−9.53	0.97	—	—	—		
Γ_i^Y	—	—	0.38	—	—	—	—	—		
Γ_i^{Def}	—	—	−0.48	—	—	—	—	—		
Γ_τ^B	9.09	9.09	9.09	8.56	−0.57	9.12	—	—		
Γ_τ^θ	−0.07	−0.07	−0.07	—	0.10	−0.08	—	—		
Γ_τ^G	−0.08	−0.08	−0.08	—	−0.02	0.00	—	—		
Γ_τ^π	—	—	−0.28	—	—	—	—	—		

Notes: Columns labeled "θ, G": simultaneous productivity and government purchases shocks; columns labeled "G": just government purchases shocks. Y: GDP; C: consumption; I: physical investment. Π: gross inflation rate; i: nominal interest rate; G: government purchases; τ: tax rate; B: real public debt; **Def**: real budget surplus; L: hours worked; K: capital stock. ζ, ζ^m, ζ^v: welfare measures. Moments of i, τ, B, **Def** refer to differences from steady state values. Moments of remaining variables: relative deviations from steady state values. All statistics have been expressed in percentage terms.

problem in which household welfare is maximized subject to the economy's resource constraints (taking $\{G_t\}$ as exogenously given). Table 2 shows dynamic responses to shocks.

3.1. Sticky Prices

In the baseline model (sticky prices, nominal debt), the optimized policy parameters are $\Gamma_i^\pi = 8660.77$ and $\Gamma_\tau^B = 9.09$. Thus, optimized monetary policy has a strict anti-inflation stance (an increase in inflation triggers a sharp increase in the nominal interest rate); as a result, the standard deviation of the inflation rate is (essentially) zero.[9]

Under staggered price setting, inflation induces inefficient dispersion of prices across intermediate goods producers [e.g., Erceg et al. (2000)]. In an economy in which price stickiness is the only distortion (so that the flex-prices equilibrium would be efficient), optimal monetary policy fully stabilizes inflation, as that policy eliminates inefficient cross-firm price dispersion [e.g., Rotemberg and Woodford (1997)]. The economy here has other distortions (monopolistic competition, income tax)—nevertheless, optimized policy under sticky prices entails (almost) full inflation stabilization.

Optimized fiscal policy implies that an increase in the stock of debt by an amount equal to (quarterly) steady state GDP (Y) raises the tax rate by 9.09 percentage points (roughly one-third of the steady state tax rate). Experiments with alternative values of Γ_τ^B (holding constant $\Gamma_i^\pi, \Gamma_\tau^\theta, \Gamma_\tau^G$) show that the model has a stationary solution when $\Gamma_\tau^B \geq 1.62$. Values of Γ_τ^B that are smaller than the optimized coefficients (but larger than 1.62) entail violations of the long-run constraint on real debt (6).

Real debt fluctuations are highly persistent (autocorrelation 0.999) and volatile: the standard deviation of real debt (normalized by Y) is 17.2%—which is markedly larger than the standard deviation of (quarterly) GDP and consumption (6.58% and 5.58%, respectively), but smaller than the standard deviations of investment (20.65%) and of the capital stock normalized by Y (49.13%). The tax rate undergoes nonnegligible, countercyclical fluctuations (standard deviation of τ_t: 1.71%; correlation with GDP: -0.81). Because the tax rate is a function of the stock of real debt, it too is highly persistent (autocorrelation 0.998).

The unconditional mean value of the capital stock exceeds its steady state value by 0.49%—which can be viewed as reflecting precautionary saving (in the stochastic economy). Mean consumption is slightly above steady state (by 0.05%), whereas mean hours are below steady state (-0.07%). Conditional welfare is lower than steady state welfare, $\zeta = -0.072\%$. The "mean component" of the welfare measure is positive, $\zeta^m = 0.1015\%$; however, that effect is dominated by the negative effect of consumption variance on welfare: $\zeta^v = -0.173\%$.

These predictions are based on the assumption that the economy is simultaneously subjected to shocks to productivity and to government purchases; see

column 1 in Table 1. Column 2 shows predictions for the case where there are just government purchases shocks. These shocks explain only 16% of the variance of the tax rate, and less than 3% of the variances of consumption and output (which are generated when there are simultaneous productivity and government purchases shocks). G_t shocks also have a markedly smaller effect on welfare than productivity shocks ($\zeta = -0.004\%$ when there are just G_t shocks).

Dynamic responses. Under optimized policy, a positive productivity shock triggers a rise in output, consumption, investment, and hours worked; the price level remains (essentially) constant, and the tax rate falls. Initially, tax revenues rise; real debt falls, with a one-period delay (in the long run, real debt reverts to its preshock level). On impact, a positive shock to government purchases raises output, investment, and hours worked, and it lowers consumption; tax revenues rise, and real debt increases. (See panels (a.1) and (b.1) in Table 2.)

Richer/simpler policy rules. Experiments with richer policy rules (which permit a direct response of the policy instruments to selected additional variables) only yield very small welfare gains, compared to the baseline rules (4), (5).

For example, column 3 (Table 1) assumes that i_t is set as a function of inflation, GDP, and the real deficit, whereas τ_t is a function of real debt, productivity, government purchases and inflation: $i_t = i + \Gamma_i^\pi \widehat{\Pi}_t + \Gamma_i^Y \widehat{Y}_t + \Gamma_i^{\text{Def}} \text{Def}_t$; $\tau_t = \tau + \Gamma_\tau^B i(B_t - B) + \Gamma_\tau^\theta \widehat{\theta}_t + \Gamma_\tau^G \widehat{G}_t + \Gamma_\tau^\pi \widehat{\Pi}_t$. Some prominent critics of the ECB's single-minded pursuit of price stability advocate a "responsiveness" of monetary policy to broader macroeconomic/fiscal conditions [e.g., French president Chirac (2004)]; the interest rate rule shown above permits such "responsiveness." The optimized interest rate rule exhibits a positive [negative] response coefficient for GDP [the deficit], whereas the optimized tax rate rule has a negative response coefficient for inflation. However, the optimized coefficients Γ_i^π, Γ_τ^B, Γ_τ^θ, and Γ_τ^G, as well as predicted behavior and welfare, are essentially the same as under the baseline rules—the welfare gain from using the richer rules [instead of (4) and (5)] corresponds to a permanent consumption increase of merely 0.00001%.

Column 4 assumes that the tax rate just responds to real debt [whereas the interest rate just responds to inflation, as under (4)]. There, the optimized policy parameters are $\Gamma_i^\pi = -9.53$, $\Gamma_\tau^B = 8.56$. Again, predicted statistics are close to those under the baseline specification (e.g., standard deviations of GDP, tax, rate, and inflation 6.23%, 1.69%, and 0.01%, respectively); the welfare loss from using the simpler tax rate rule [instead of (5)] is 0.00076%.[10]

Comparison with first-best economy. A comparison between the distorted sticky-prices economy and the first-best (undistorted) economy (columns 7 and 8) shows that

(i) The *levels* of economic activity and welfare are noticeably lower in the distorted equilibrium—for example, steady state consumption and hours worked are 43% and 34% lower, respectively (than in the first-best economy); the welfare

TABLE 2. Dynamic responses to shocks

	Y	C	I	L	K	P	i	τ	Taxrev.	B	Exogenous variable
(a) % responses to 1% productivity innovation											
(a.1) Sticky prices											θ
$j = 0$	1.62	0.90	7.78	0.81	0.19	0.00	0.04	−0.07	0.34	0.00	1.00
$j = 4$	1.44	1.03	5.76	0.61	0.81	0.00	0.02	−0.15	0.20	−0.96	0.81
$j = 24$	0.74	0.92	0.96	0.13	1.46	0.00	−0.02	−0.23	−0.06	−2.29	0.29
$j = 100$	0.06	0.10	−0.06	0.01	0.19	0.00	−0.00	−0.04	−0.02	−0.38	0.01
(a.2) Flexible prices (nominal debt)											θ
$j = 0$	1.52	0.74	7.85	0.67	0.19	−1.63	−2.97	0.07	0.44	6.37	1.00
$j = 4$	1.30	0.85	5.62	0.41	0.82	−13.98	−2.72	0.02	0.32	11.20	0.81
$j = 24$	0.40	0.60	0.02	−0.20	1.00	−42.48	−1.25	0.00	0.07	4.56	0.29
$j = 100$	0.01	0.02	−0.04	−0.01	0.04	−53.78	0.01	0.00	0.00	0.04	0.01
(a.3) First-best											θ
$j = 0$	2.07	0.66	9.40	1.41	0.23	—	—	—	—	—	1.00
$j = 4$	1.51	0.82	5.45	0.69	0.86	—	—	—	—	—	0.81
$j = 24$	0.37	0.58	−0.23	−0.21	0.97	—	—	—	—	—	0.29
$j = 100$	0.00	0.01	−0.03	−0.01	0.03	—	—	—	—	—	0.01
(b) % responses to 1 % government purchases innovation											
(b.1) Sticky prices											G
$j = 0$	0.35	−0.08	1.59	0.46	0.04	0.00	0.01	−0.02	0.07	0.00	1.00
$j = 4$	0.19	−0.06	0.60	0.22	0.12	0.00	0.00	0.03	0.07	0.49	0.81
$j = 24$	−0.09	−0.12	−0.53	−0.11	−0.06	0.00	0.00	0.10	0.07	1.23	0.29
$j = 100$	−0.02	−0.04	0.01	−0.01	−0.07	0.00	0.00	0.02	0.01	0.20	0.01
(b.2) Flexible prices (nominal debt)											G
$j = 0$	0.19	−0.07	0.36	0.26	0.01	1.26	1.24	0.01	0.05	−2.43	1.00
$j = 4$	0.11	−0.06	−0.02	0.15	0.01	6.03	1.08	0.02	0.04	−4.07	0.81
$j = 24$	0.04	−0.02	−0.01	0.05	0.00	23.94	0.54	0.01	0.02	−1.77	0.29
$j = 100$	0.00	−0.00	0.00	0.00	0.00	44.04	0.04	0.00	0.00	−0.10	0.01
(b.3) First-best											G
$j = 0$	0.12	−0.04	0.20	0.16	0.00	—	—	—	—	—	1.00
$j = 4$	0.10	−0.03	0.11	0.12	0.02	—	—	—	—	—	0.81
$j = 24$	0.03	−0.00	−0.00	0.03	0.02	—	—	—	—	—	0.29
$j = 100$	0.00	−0.00	−0.00	0.00	0.00	—	—	—	—	—	0.01

Notes: j: periods after shock. Columns labeled Y, C, etc. show responses of the corresponding variables. Taxrev.: real tax revenue, normalized by steady state real GDP. (See Table 1 for definitions of other variables.) Responses of capital pertain to end-of-period stocks (K_{j+1}). Responses are generated as follows. At some date T all state variables are set at steady state values. A "baseline" path for endogenous variables is computed by setting all exogenous innovations to zero at $t \geq T$. Then responses to one-time 1% innovations at T are computed; the table reports deviations (multiplied by 100) of these responses from the baseline paths (responses of interest rate (i_j), tax rate (τ_j), Taxrev., and real debt ($B_j \equiv D_j/(P_j Y)$): differences from baseline paths; responses of other variables: relative deviations from baseline paths).

difference is equivalent to a permanent 21.46% consumption loss (not reported in Table 1).[11]

(ii) Responses to shocks are qualitatively similar across the two structures. In the sticky-prices economy, output, hours worked, and investment respond less strongly—on

impact—to productivity shocks, and more strongly to government purchases shocks, than in the first-best economy. See panels (a.3) and (b.3) in Table 2.

3.2. Flexible Prices

Nominal debt.   Optimized policy under *flexible* prices (and nominal debt) differs markedly from optimized policy under sticky prices: with flex-prices, the optimized inflation and debt coefficients are $\Gamma_i^\pi = 0.97$, $\Gamma_\tau^B = -0.57$ (see column 5). In a flex-prices nominal-debt structure, inflation does not cause inefficient price dispersion across firms; when exogenous shocks occur, the government can meet its intertemporal budget constraint by altering the *real* value of the inherited stock of (nominal) public debt via unanticipated inflation changes. As a result, optimized policy entails sizable inflation volatility and much smaller movements of the tax rate than under sticky prices (standard deviations of Π_t and τ_t under flexible prices: 13.2% and 0.1%, respectively). As in the baseline sticky-prices structure, the real value of the stock of debt is highly volatile, and debt and the tax rate are highly serially correlated.

Table 2 [panels (a.2) and (b.2)] shows that, in the flex-prices nominal-debt model variant, the government responds to a positive productivity innovation and the ensuing rise in tax revenues by inducing an unanticipated fall in the price level, and thus a rise in real public debt (thereafter, real debt reverts to its steady state value); a positive shock to government purchases triggers an unanticipated rise in the price level, and thus a fall in real debt.

Chari et al. (1991) showed that (optimal) monetary/fiscal *Ramsey* policy implies high inflation volatility in a flex-prices economy with nominal debt; Schmitt-Grohé and Uribe (2004b) and Siu (2004) demonstrated that *Ramsey* policy entails much lower inflation volatility when prices are sticky. The paper here shows that similar predictions (effect of flex- vs. sticky prices) hold when policy is described by simple optimized rules.

Indexed debt.   Column 6 considers a flex-prices economy with indexed debt. The government cannot use unanticipated inflation changes to meet its solvency condition in that setting (as monetary policy has no real effects there). Welfare ($\zeta = -0.072\%$) is lower than in the flex-prices structure with nominal debt. The optimized fiscal policy rule and the implied behavior of real variables are very similar to those generated in the sticky-prices economy with nominal debt. Intuitively, this is due to the fact that the strict inflation stabilization entailed by optimized policy in the sticky-prices structure implies that (i) all firms set identical prices (as is the case under flexible prices); (ii) nominal bonds are riskless, in real terms (as under indexed debt).[12]

NOTES

1. For example, Aiyagari et al. (2002), Benigno and Woodford (2003), Chari et al. (1991), Correia et al. (2001), Schmitt-Grohé and Uribe (2004b), and Siu (2004) (several of these papers also determine optimal monetary policy).

2. Most papers on Ramsey problems concentrate on the associated first-order conditions, without establishing that second-order conditions are met.

3. Schmitt-Grohé and Uribe (2004a) [SU] also use a second-order approximation to compute optimized *simple* monetary/fiscal rules for a New Keynesian model, but SU focus on a setting with a lump-sum tax (and, contrary to the paper here, SU do not describe the effects of policy on macro aggregates). Second-order approximations are also used by Benigno and Woodford (2003), who *analytically* derive optimal monetary-fiscal policy for a more stylized economy (without capital and with a more restrictive structure of shocks), and by Kim and Kim (2001), who numerically compute optimized tax policy rules for a two-country RBC model.

4. The structure of preferences, technologies, and markets resembles that in Kollmann (2001, 2002, 2004a).

5. The Aiyagari et al. model is more stylized than the structure considered here (it assumes a perfectly competitive, real economy, without capital or productivity shocks).

6. Aiyagari et al. suggest an alternative way of ruling out unrealistic public debt behavior: an exogenous positive lower bound on debt. The numerical algorithm used here cannot handle such a bound. Another approach might be to assume that public debt provides liquidity services to households; see Woodford (1990); if those services are strong enough, then a benevolent government would always select positive debt.

7. When (6) is not imposed, then welfare maximization with respect to policy parameters (see below) selects parameters for which the numerical approximation is very poor and results are nonsensical (astronomical welfare gains, associated with long-run values of debt and taxes that are markedly below steady state values).

8. Note that $U((1+\zeta)C, L) \cong U(C, L) + (1-\beta) \sum_0^\infty \beta^t [E_0(\widehat{C}_t - L\widehat{L}_t) - \frac{1}{2} V_0(\widehat{C}_t)]$. ζ^m, ζ^v are given by $U((1+\zeta^m)C, L) = U(C, L) + (1-\beta) \sum_0^\infty \beta^t [E_0\widehat{C}_t - LE_0\widehat{L}_t]$ and $U((1+\zeta^v)C, L) = U(C, L) - (1-\beta)\frac{1}{2} \sum_0^\infty \beta^t V_0(\widehat{C}_t)$. It appears that $(1+\zeta) = (1+\zeta^m)(1+\zeta^v)$.

9. The optimized parameter Γ_i^π is very large. Welfare is a very flat function of Γ_i^π. Imposing a moderate bound on Γ_i^π (e.g., $|\Gamma_i^\pi| \leq 10$) does not affect the results.

10. Following a suggestion made by a referee, I considered a model variant with a fixed capital stock (detailed results available from author). The key findings go through in that variant (optimized policy entails strict inflation stabilization under sticky prices, very simple rules do well, etc.).

11. The welfare figures (ζ, ζ^m, ζ^v) for the first-best economy in columns 7 and 8 are % equivalent variations in consumption, relative to the steady state of the *first-best* economy.

12. The nominal interest rate cannot be determined uniquely from welfare maximization in the flex-prices indexed-debt structure; thus no predictions for nominal variables (or Γ_i^π) are reported in column 6.

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