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NOTE

A NOTE ON UNCERTAINTY IN SAVINGS DECISIONS: CAN A NAÏVE STRATEGY BE OPTIMAL?

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This paper analyzes the process of decision-making on consumption in a two-period consumption setting, assuming the return on savings is uncertain in the sense of Knight [*Risk, Uncertainty, and Profit.* Boston: Houghton Mifflin (1921)]. The results imply that a naïve strategy to save zero is optimal for a continuum of income values. Under the Permanent Income Hypothesis, consumption equals current income only when current income is equal to permanent income. Indeed Campbell and Mankiw [in Olivier Blanchard and Stanley Fischer (eds.), *NBER Macroeconomics Annual,* pp. 185–214. Cambridge, MA: MIT Press (1989)] assumed that consumers who spend their total income are only following a simple rule of thumb. However, the naïve strategy obtained casts doubt on their interpretation.

Keywords: Consumption, Savings, Uncertainty, Rule-of-Thumb Behavior

1. INTRODUCTION

Since the development of Friedman's (1957) permanent income hypothesis (PIH), economists have agreed that consumption decisions depend on random variables, existing risk or uncertainty. Following Knight (1921), whereas risk is a situation in which a single additive probability measure on the states of nature is available to guide a choice, under uncertainty information is too imprecise to be summarized by a single additive probability measure.

The consumption model often used in the literature is based on a frictionless economy with a single representative agent, whose preferences are represented by constant relative risk aversion (CRRA) utility. Assuming the rational

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expectations hypothesis (REH) and joint lognormality of consumption and interest rate, the Euler equation implies that consumption growth depends on the expected interest rate. However, Campbell and Mankiw (1989) argued that the time series behavior of consumption is better explained when a fraction of consumers follows a simple rule of thumb: consume their current income. The main candidate to explain this result is the lack of credit. Indeed, Flavin (1981) had already stated that the PIH is rejected because consumption is excessively sensitive to income.

Differently from most of the literature, this paper aims to analyze consumption behavior without assuming the REH. Manski (2004) argued that many alternative specifications of preferences and expectations can lead to the same observed choice; however, once the widespread practice is to assume the REH, additional explanations for economic phenomena are discarded. However, there are some exceptions to the use of the REH to explain consumption decisions, as in Miao (2004) and Gomes (2008).

Gomes (2008) used a two-period consumption model and the Choquet expected utility (CEU) approach, concluding that income uncertainty increases savings, a result analogous to precautionary savings. This finding is obtained by assuming that uncertainty is generated by a uniform contraction of an additive measure. Miao (2004) assumed that the set of agents' priors is given by a family of normal distributions. A precautionary motive arises in Miao (2004) as long as income uncertainty and CARA utility are accounted for. He finds that return uncertainty and CRRA utility cause the optimal saving rule to depend on the relative risk aversion parameter, and uncertainty averse agents may save more or less than a consumer who maximizes expected utility.

We extend Gomes's (2008) analysis by investigating uncertainty about return on savings, using the notion of uncertainty and uncertainty aversion introduced by Dow and Werlang (1992). The interest of this issue goes far beyond Manski's advice. Campbell (1987) argued that the excess sensitivity of consumption to income can be better interpreted as insufficient variability of saving—savings moves less than predicted by the PIH—and the potential of uncertainty to generate inertia is well known [Dow and Werlang (1992)].

To preview the main result: uncertainty causes inertia in the sense that there is a continuum of income values in which consumers do not save.[1] It is worth mentioning that we do not impose any credit constraint, and consumers may still choose to spend their entire income. The intuition is straightforward: the agent is too pessimistic to invest, being afraid of a low interest rate, and, at the same time, is too pessimistic to borrow, being afraid of a high interest rate. Finally, the result casts doubt on Campbell and Mankiw's (1989) strategy to identify rule-of-thumb behavior.

The paper proceeds as follows. Section 2 presents useful results from the CEU model and develops the consumption model. Section 3 details the implications for empirical strategies of previous models. The final section summarizes the conclusions.

2. UNCERTAINTY AND CONSUMPTION MODEL

2.1. Uncertainty Aversion

First of all, define Ω as a set of states of nature and Λ as an algebra from its subsets. Thus, (i) $\Omega \in \Lambda$, (ii) $A, B \in \Lambda \Rightarrow A \cup B \in \Lambda$, and (iii) $A \in \Lambda \Rightarrow A^c \in \Lambda$, where A^c is the set of elements of Ω not in A. The elements of Λ are the events. A function $P : \Lambda \to [0, 1]$ is a nonadditive probability if (i) $P(\phi) = 0$, where ϕ is the empty set; (ii) $P(\Omega) = 1$; and (iii) $A, B \in \Lambda$, $P(A) \leq P(B)$ if $A \subset B$. Imposing the additional restriction (iv) $A, B \in \Lambda$, $P(A \cup B) + P(A \cap B) \geq P(A) + P(B)$ leads to a convex nonadditive probability function P. The Choquet expected value of a random variable X is defined as

$$E_P[X] = \int_{\Omega} X \, dP = \int_{-\infty}^{0} [P(X \geq \alpha) - 1] d\alpha + \int_{0}^{\infty} P(X \geq \alpha) \, d\alpha \qquad \textbf{(1)}$$

whenever these integrals exist (in the improper Riemann sense) and are finite.

DEFINITION [Dow and Werlang (1992)]. *Let P be a probability and $A \subset \Omega$ an event. The* uncertainty aversion *of P at A is defined by*

$$\theta(P, A) = 1 - P(A) - P(A^c). \qquad \textbf{(2)}$$

Hence, different P may possess dissimilar degrees of uncertainty aversion, which is proportional to the amount of probability "lost." Indeed, the uncertainty aversion is zero for all events if P is additive. In this case, agents are only risk-averse.

Let X be a random variable; then the following properties will be useful in the next section:[2]

(1) $\forall a \geq 0,\ b \in \Re,\ E_P[aX + b] = aE_P[X] + b$.
(2) If $u : \Re \to \Re$ is a concave function, then $E_P[u(X)]$ is concave.
(3) For $\mu \in \Re$, define $F(\mu) = E_P[H(\mu X)]$, where $H(\cdot)$ is an increasing and differentiable function. Then (i) F is right-differentiable at $\mu = 0$, being $F'_+(0) = H'(0) E_P[X]$; and (ii) F is left-differentiable at $\mu = 0$, being $F'_-(0) = -H'(0) E_P[-X]$.
(4) $-E_P[-X] \geq E_P[X]$.
(5) The following statements are equivalent: (i) P is at least as uncertainty averse as Q; (ii) for all random variables X for which the integrals are finite,

$$-E_P[-X] - E_P[X] \geq -E_Q[-X] - E_Q[X].$$

Last, following Dow and Werlang (1992), P can be built by increasing the uncertainty aversion of any additive probability Q: fix $\theta \in [0, 1]$, and let $P(\Omega) = 1$ and $P(A) = (1 - \theta) Q(A)$ for $A \neq \Omega$. Then, for all $A \neq \Omega$, the uncertainty aversion is constant and identical to the probability "lost" by the uniform contraction $\theta(P, A) = \theta$. It is possible to show a sixth property:

(6) Assuming that $X^{\mathrm{L}} = \inf_{w\in\Omega} X(w) \geq 0$ and $X^{\mathrm{H}} = \sup_{w\in\Omega} X(w) < \infty$, then (i) $E_P[X] = \theta X^{\mathrm{L}} + (1-\theta) E_Q[X]$ and (ii)$-E_P[-X] = \theta X^{\mathrm{H}} + (1-\theta) E_Q[X]$.

It is worth mentioning that uncertainty can be modeled using other approaches, such as the multiple-priors utility model of Gilboa and Schmeidler (1989) or the smooth ambiguity model of Klibanoff et al. (2005). Applications of these approaches in asset pricing models include those of Epstein and Wang (1994), Chen and Epstein (2002), and Ju and Miao (2012).

2.2. The Effect of Uncertainty on Consumption Decisions

Consider a two-period consumption model. In the first period, the consumer has an income w_1 and chooses consumption c_1 and savings s. In the second period, the consumer picks consumption c_2, taking into account income w_2 and financial wealth Rs, where R is the uncertain gross rate of return. Also, assume that the utility function u is C^2 and, as usual, $u' > 0$ and $u'' < 0$. Monotonic preferences imply that budget constraints are binding and the consumer's problem becomes[3]

$$\max_{c_1,c_2,s_1} E_P[u(c_1) + \beta u(c_2)],$$

$$\text{s.t. } c_1 = w_1 - s \text{ and } c_2 = w_2 + sR,$$

where $0 < \beta < 1$ is the intertemporal discount factor and $E_P(\cdot)$ is the expected value on the convex nonadditive probability P. Using the restrictions, the objective function becomes $U(s) = u(w_1 - s) + \beta E_P u(w_2 + sR)$ and the consumer chooses savings. Given properties (1) and (2), the objective function is concave and, consequently, (i) $s > 0$ if the right side derivative of $U(s)$, evaluated at $s = 0$, is greater than zero, $U'_+(0) > 0$; (ii) $s < 0$ if the left side derivative of $U(s)$, evaluated at $s = 0$, is less than zero, $U'_-(0) < 0$; and (iii) $s = 0$ if $U'_+(0) \leq 0 \leq U'_-(0)$.

PROPOSITION 1. *When savings return is uncertain, there can be a continuum of values for current income such that the optimal decision is null savings.*

Proof. For $s \in \Re$ define $f(s) = E_P[u(w_2 + sR)]$. Then, using property (3): (i) f is right-differentiable at $s = 0$ and $f'_+(0) = u'(w_2) E_P[R]$; (ii) f is left-differentiable at $s = 0$ and $f'_-(0) = -u'(w_2) E_P[-R]$. Thus, $s = 0$ if

$$-u'(w_1) + \beta u'(w_2) E_P[R] \leq 0 \leq -u'(w_1) - \beta u'(w_2) E_P[-R],$$

and the difference between the extremes is $\beta u'(w_2)[-E_P(-R) - E_P(R)]$. The term inside the brackets is equal to or greater than zero, by property (4). Q.E.D.

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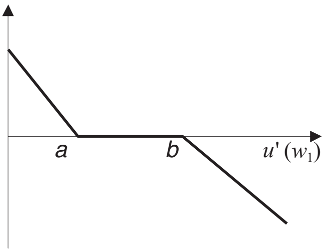

FIGURE 1. Hypothetical saving function.

Therefore, we know that

(1) $s > 0$ if $U'_+(0) = -u'(w_1) + \beta u'(w_2) E_P[R] > 0$;
(2) $s < 0$ if $U'_-(0) = -u'(w_1) - \beta u'(w_2) E_P[-R] < 0$;
(3) $s = 0$ if $-u'(w_1) + \beta u'(w_2) E_P[R] \le 0 \le -u'(w_1) - \beta u'(w_2) E_P[-R]$.

In the first case, consumers decrease consumption in order to increase their current marginal utility. In the second case, the opposite occurs and consumers increase consumption. The third case does not require lending or borrowing.

PROPOSITION 2. *When uncertainty aversion increases, the range of current income in which the optimal saving decision is zero does not decrease.*

Proof. Notice that $s = 0$ when $\beta u'(w_2) E_P[R] \le u'(w_1) \le -\beta u'(w_2) E_P[-R]$, and the difference between the extremes is $\beta u'(w_2) [-E_P(-R) - E_P(R)]$, which is greater than or equal to zero, by property (4). Property (5) implies that $-E_P[-R] - E_P[R] \ge -E_Q[-R] - E_Q[R]$ when P is at least as uncertainty averse as Q. Thus, the range for $u'(w_1)$ such that $s = 0$ does not decrease. Q.E.D.

Figure 1 displays a hypothetical saving function, where $a = \beta u'(w_2) E_P[R]$ and $b = -\beta u'(w_2) E_P[-R_{t+1}]$.

Suppose that P is generated from a uniform contraction of an additive probability, Q, using an uncertainty aversion measure $\theta \in [0, 1]$. Additionally, assume that $R^{\mathrm{L}} = \inf_{R\in\Omega} R \ge 0$ and $R^{\mathrm{H}} = \sup_{R\in\Omega} R < \infty$. As a consequence of property (6),

(1) $s > 0$ if $U'_+(0) = -u'(w_1) + \beta u'(w_2) [\theta R^{\mathrm{L}} + (1-\theta) E_Q(R)] > 0$;
(2) $s < 0$ if $U'_-(0) = -u'(w_1) + \beta u'(w_2) [\theta R^{\mathrm{H}} + (1-\theta) E_Q(R)] < 0$;
(3) $s = 0$ if $\beta u'(w_2) [\theta R^{\mathrm{L}} + (1-\theta) E_Q(R)] \le u'(w_1) \le \beta u'(w_2) [\theta R^{\mathrm{H}} + (1-\theta) E_Q(R)]$.

Notice that the interval for $u'(w_1)$ in which savings is zero is given by $\beta\theta u'(w_2) [R^{\mathrm{H}} - R^{\mathrm{L}}]$, which is increasing with the uncertainty aversion parameter, θ, and the range for R. In summary, as uncertainty and the possibilities for R increase, the possibility that optimal savings is zero is amplified. Additionally, consumers only save when the discounted future marginal utility is greater than the current marginal utility (condition 1). Without uncertainty, savings are greater than zero if $u'(w_1) < \beta E_Q[u'(w_2) R]$. The difference is obvious: by adding uncertainty, savings is greater than zero only if, after an extra weight is given to the lower return, the discounted future marginal utility is still greater than the current marginal utility. On the other hand, consumers borrow resources only if the current marginal utility is greater than discounted future marginal utility, after they give an extra weight to the worst-case scenario: a highest interest rate.

3. RULE-OF-THUMB BEHAVIOR AND UNCERTAINTY

Assuming the REH and the usual CRRA utility, the Euler equation from the consumer problem is given by $\beta E_{t-1}[(C_t/C_{t-1})^{-\gamma} R_t] = 1$, where γ is the coefficient of relative risk aversion. Thus, joint lognormality of asset returns and consumption leads to the Euler equation

$$\Delta \ln C_t = \alpha + \frac{1}{\gamma} r_t + \varepsilon_t, \tag{3}$$

where $r_t = \ln R_t$ and $E_{t-1}(\varepsilon_t) = 0$. However, Campbell and Mankiw (1989) assumed that a fraction of consumers depart from the PIH, presenting rule-of-thumb behavior: they consume their current income. Assuming that these consumers' income holds a fixed proportion to aggregate income, λ, the following relationship between aggregate consumption (C_t) and income (Y_t) follows:

$$\Delta \ln C_t = \lambda \Delta \ln Y_t + (1 - \lambda) \left[\alpha + \frac{1}{\gamma} r_t + \varepsilon_t \right]. \tag{4}$$

Because of an endogeneity problem, the model is estimated by instrumental variables. As a consequence, regressors become the expected growth rate of income and the expected interest rate. Under the PIH, equation (3) is valid and $\lambda = 0$. If $\lambda = 1$, consumption is equal to current income, and the PIH is rejected. Under the PIH, this is an event of measure zero, once consumption is equal to income only if income is equal to permanent income. However, when uncertainty is present, there is a continuum for income such that null savings is optimal.

Using G7 data, Campbell and Mankiw (1989, 1990) concluded that rule-of-thumb behavior was widespread. For instance, in the U.S. economy, about 50% of total income belongs to rule-of-thumb consumers. Some authors argued that rule-of-thumb behavior can be generated by liquidity constraints [Sarantis and Stewart (2003); Vaidyanathan (1993); Brady (2008)].[4] However, if individuals optimally decide not to save, then, because of uncertainty, consumption will be equal to their income. Moreover, λ will be greater than zero, even if consumers do not face liquidity constraints.

4. CONCLUSION

In the model proposed by Campbell and Mankiw (1989), rule-of-thumb consumers spend their entire income and, therefore, save zero. However, we have shown that a naïve strategy such as null savings can be optimal for a range of current income values. This result comes from uncertainty: agents are too pessimistic to invest, being afraid of a low interest rate; at the same time, they do not borrow either, as they are afraid of a high interest rate.

Thus, at least in part, the excess sensitivity of consumption to income can be attributed to uncertainty. Of course, this does not mean that any other explanation, such as credit constraint, is irrelevant. Uncertainty should not be the only reason for the excess sensitivity of consumption to income. Our point is to show how inertia can emerge and lead optimizers' consumers to spend their whole income. In line with the general argument presented in Manski (2004), without assuming the REH, this paper puts forward a new explanation for the excess sensitivity of consumption to income.

NOTES

1. This result was not obtained by Miao (2004). However, because the author assumed that the agent has no income in the second period, the optimal decision is always to save in order to avoid zero consumption in the second period.
2. Proofs are omitted and the reader is referred to Simonsen and Werlang (1991), Dow and Werlang (1992), and Gomes (2008).
3. Inada's conditions are assumed to guarantee an interior solution.
4. Others explanations include buffer stock savings [Carrol (1997)] and self-control problems [Angeletos et al. (2001)], for instance.

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