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Density gradient effects on beam plasma linear instabilities for fast ignition scenario

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Abstract

In the fast ignition scenario for inertial fusion, a relativistic electron beam is supposed to travel from the side of the fusion pellet to its core. One one hand, a relativistic electron beam passing through a plasma is a highly unstable system. On the other hand, the pellet core is denser than its side by four orders of magnitude so that the beam makes its way through a important density gradient. We here investigate the effect of this gradient on the instabilities. It is found that they should develop so early that gradient effects are negligible in the linear phase.

Keywords: Fast ignition scenario; Filamentation Instability; Two-Stream Instability

1. INTRODUCTION

The fast ignition scenario (Tabak *et al.*, 1994; Deutsch *et al.*, 1997) is a very promising approach to inertial fusion because it can significantly reduce the required laser energy (Gus'Kov, 2005). Various approaches such as jet collision or shell impact concepts are now available (Velarde *et al.*, 2005), and the "original" scenario is a two-step process implying two drivers. The deuterium tritium pellet is first pre-compressed by a laser without being ignited. Then, ignition is performed through a petawatt laser shot on the side of the pellet. The interaction generates at the critical density a relativistic electron beam which makes its way to the pellet core (Deutsch, 2004), and ignites the deuterium tritium fuel. At this junction, let us mention that the laser-plasma interaction part of the conventional scenario is debated (Mulser & Bauer, 2004; Mulser & Schneider, 2004) and that particle beams may be required. When the electron beam enters the plasma, it prompts a return current (Hammer & Rostoker, 1970) and the resulting system is eventually a very classical two-stream + plasma one. It has been known for long than two counter streaming beams are very unstable, and many unstable modes are retrieved when performing a linear analysis of such a situation. Let us first mention the two-stream instability which consists in the instability of electrostatic modes with waves vectors aligned with the beams. We then find the filamentation instability, which corresponds to purely transverse modes having their wave vectors normal to the beams. This later instability is sometimes labeled as "Weibel," although the original Weibel instability (Weibel, 1959) has to do with an anisotropic plasma with or without a beam. Finally, one finds that the all wave vector space is unstable and that, interestingly, the fastest growing modes are found for oblique wave vectors as soon as the beam is relativistic (Bret *et al.*, 2005*b*).

The stability analysis we have just refereed too are usually conducted considering infinite and homogenous mediums. However, the fast ignition scenario implies a relativistic electron beam traveling through a very important density gradient since the critical density where the laser deposits its energy is smaller than the core density by four orders or magnitudes. It is therefore important, and this is the aim of this article, to evaluate the effect of such a density gradient upon the various instabilities we just mentioned. Our approach consists in comparing the typical scale length set by the instabilities with the typical scale length set by the density gradient. As we will see, the later is much larger than the former so that a Wentzel-Kramers-Brillouin (WKB) like method is applicable. This paper is structured as follow: we first introduce the main instabilities suffered by the system, comparing their typical scale length with the one set by the density gradient. We then compute the integrated growth rate through the gradient using the WKB approximation and

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calculate the e-folding distance. We find that the e folding distance is so short the fastest instabilities should develop even before the beam "feels" the gradient.

2. MAJOR INSTABILITIES AND SCALE LENGTHS

Let us start considering a beam with density n_b and relativistic velocity V_b entering a plasma of density n_p. The beam prompts a return current with a velocity V_p such as $n_b V_b = n_p V_p$. Ions are considered as a fixed neutralizing background so that the overall system is charge and current neutralized. If the beam density is much smaller than the plasma one, $n_b \ll n_p$ implies that the return current is not relativistic. The main instabilities we are talking about are the two-stream instability (TS), the filamentation (F) instability, and the oblique instability that we denote TSF. This later instability is found for an oblique wave vector and is the fastest growing one for a diluted beam. As far as temperatures are concerned, it has been proved that the growth rates for TS and TSF stay very close to their fluid values as long as temperature are non-relativistic. Unlike TS and TSF, filamentation is very sensitive to temperature, and indeed, it can be suppressed by transverse beam temperature (Silva *et al.*, 2002). The non-relativistic temperature approximation is correct for the plasma because it allows for a 10 keV plasma. As far as the beam is concerned, a 4 MeV electron beam ($\gamma_b = 5$) can perfectly have more than a 0.5 MeV energy spread, which is relativistic. However, because temperature usually tend to reduce the instabilities, a small temperature beam eventually represent the worst case scenario. Accounting for these approximations, the growth rates for the three instabilities read

$$\delta_{TS} = \omega_p \frac{\sqrt{3}}{2^{4/3}} \frac{\alpha^{1/3}}{\gamma_b},$$

$$\delta_{TSF} = \omega_p \frac{\sqrt{3}}{2^{4/3}} \left(\frac{\alpha}{\gamma_b}\right)^{1/3},$$

$$\delta_F = \omega_p \beta \sqrt{\frac{\alpha}{\gamma_b}} \left(1 - \frac{\rho_b}{\sqrt{\alpha\gamma_b}}\right), \tag{1}$$

where γ_b is the beam relativistic factor, $\beta = V_b/c$, ω_p the plasma electronic frequency, $\alpha = n_b/n_p$ and ρ_b the ratio of the beam thermal velocity to the beam velocity. The expression for δ_F is a quite accurate linear fit which cancels the instability for a beam temperature $\rho_b = \sqrt{\alpha\gamma_b}$ (Bret *et al.*, 2005*a*). A typical growth rate map is displayed on Figure 1 in terms of the parallel and perpendicular components of the wave vector.

We can now introduce the scale length set by these instabilities through their respective mode wave vectors. We give their coordinate under the form $(k_\parallel, k_\perp)$, where $k_\parallel$ is the wave vector component parallel to the beam and $k_\perp$ the one normal to the beam,

$$k_{TS} = \left(\frac{\omega_p}{V_b}, 0\right),$$

$$k_{TSF} = \left(\frac{\omega_p}{V_b}, \chi \frac{\omega_p}{V_b}\right),$$

$$k_F = \left(0, \chi \frac{\omega_p}{V_b}\right), \tag{2}$$

where χ is a function of the beam and the plasma temperatures with $\chi > 1$. Because of this, the largest scale length set by the instabilities is $\lambda_I = V_b/\omega_p$, which, for a relativistic beam with $V_b \sim c$, almost coincides with the plasma skin depth.

3. GRADIENT SCALE LENGTH AND WKB APPROXIMATION

We now turn to the determination of the density gradient scale length. To this extent, we consider an exponential gradient such as (Honrubia *et al.*, 2005, 2004)

$$n_p(z) = n_0 \exp(z/\lambda). \tag{3}$$

The density gradient scale length is simply given by $n_p/(\partial n/\partial z) = \lambda$. Let us assume the core density n_c is reached at $z = Z_c$, we find

$$\lambda = \frac{Z_c}{\ln(n_c/n_0)}. \tag{4}$$

We can now compare the density scale length with the instability one. With $n_0 = 10^{21}$ cm^{-3}, $n_c = 10^4 n_0$, $Z_c = 100$ μm and $\gamma_b = 5$, we find

$$\lambda = 10.85\ \mu\text{m},$$

$$\lambda_I = \frac{0.17}{\sqrt{n(z)/n_0}}\ \mu\text{m}. \tag{5}$$

Because $n(z)/n_0$ increases exponentially with z, it is obvious that

$$\lambda_I \ll \lambda, \forall z. \tag{6}$$

We therefore find that the instabilities scale length is much smaller than the gradient one, and all the more than the beam is close to the core. We therefore apply locally the results obtained for a infinite medium, thus making a WKB like approximation.

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4. INTEGRATED GROWTH RATE AND *e* FOLDING DISTANCE

Let us assume a unstable mode is excited from $z = 0$ and propagates to the core with a parallel phase velocity $v_{\varphi z} = \kappa V_b$. When passing through a slice of plasma lying between z and $z + dz$, the mode is amplified by a factor $\exp(\delta t)$. According to the WKB approximation, the growth rate δ is just the local growth rate calculated using Eq. (1) with the local plasma density. The time t is here the time spent in the slice dz, namely $dz/\kappa V_b$. The amplification undergone by the mode when passing through the slice is therefore

$$dA(z) = \exp\left[\delta(z)\frac{dz}{\kappa V_b}\right]. \tag{7}$$

The total amplification of the mode between $z = 0$ and $z = Z$ is the product of these micro amplifications, that is

$$A(Z) = \exp[\Delta(Z)],$$

$$\Delta(Z) = \int_0^Z \delta(z)\frac{dz}{\kappa V_b}. \tag{8}$$

Using Eq. (1), and assuming in a first approximation that only the plasma density varies in the expressions (we will check later that given the rapidity of the instabilities, this is very reasonable), we find for TS and TSF

$$\Delta_{TS}(Z) = \delta_{TS}(0)\frac{6\lambda}{\kappa V_b}\left[\exp\left(\frac{Z}{6\lambda}\right) - 1\right],$$

$$\Delta_{TSF}(Z) = \delta_{TSF}(0)\frac{6\lambda}{\kappa V_b}\left[\exp\left(\frac{Z}{6\lambda}\right) - 1\right]. \tag{9}$$

The case of the filamentation instability will be treated later because its phase velocity is zero. If we try to calculate the e folding distance, we just need to find Z_e such as $\Delta(Z_e) = 1$. An evaluation of the pre-factors $\delta(0)6\lambda/\kappa V_b$ in both expressions shows that the exponential may be developed so that $\Delta(Z_e) = 1$ just yields for TS and TSF

$$Z_e = \frac{\kappa V_b}{\delta(0)}. \tag{10}$$

It is very interesting to notice that this equation does not mention the gradient scale length λ anymore, which means that both TS and TSF grow so quickly that their e folding distance is reached even before they "feel" the gradient.

At this junction, the only remaining unknown quantity is the phase velocity factor κ. We here invite the reader to look at Figure 2. This plot has been produced scanning the wave vectors space. Each time an unstable mode is found, we compute its phase velocity and plot the corresponding point in the phase velocities space. Furthermore, the color of the plot is related to the mode growth rate according to the same color code that is used for Figure 1. One can observe that two-stream modes have a phase velocity which is very close to the beam velocity so that $\kappa \sim 1$ for them. Filamentation modes have no phase velocity and all yield a single point at the origin of the phase space. The plot shows how the gap is bridged between the fast TS convective modes and the absolute F ones. Actually, it can be proved that the phase velocity "cloud" extends along the semi-circle of center $(\frac{1}{2},0)$ and radius $\frac{1}{2}$. As can be checked from the graph, we have $\kappa \sim 0.2$ for the TSF modes. With these values of κ for TS and TSF we can now evaluate the e-folding distance

$$Z_{eTS} = 2.58\ \mu\text{m} = \lambda/4.2,$$

$$Z_{eTSF} = 0.17\ \mu\text{m} = \lambda/63. \tag{11}$$

We can conclude from these numbers that with an e folding distance 63 times smaller than the density gradient scale length λ, it is almost certain that TSF should saturate even before the beam reaches 10 μm.

Regarding the filamentation instability, situation is somewhat different because this instability is absolute. It just

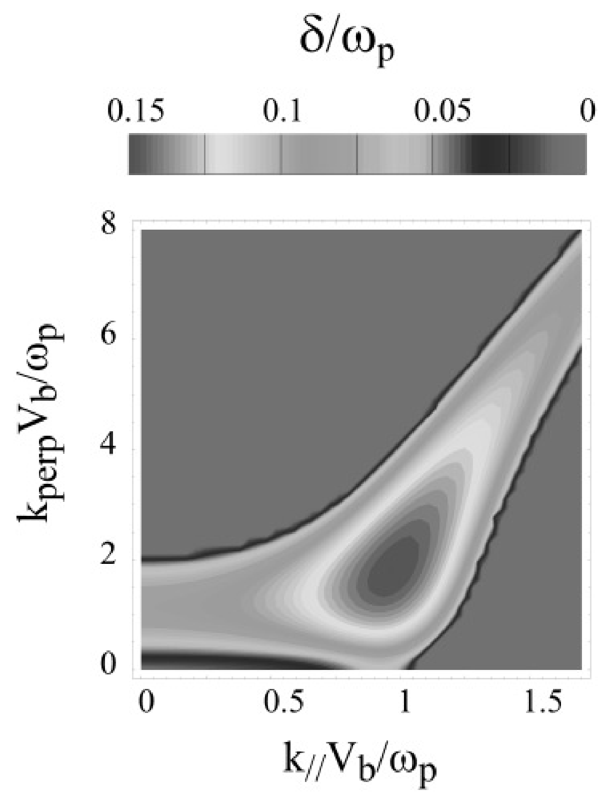

Fig. 1. Typical growth rate map in terms of the parallel and perpendicular wave vector. Thermal velocities are $\sim c/10$, $\alpha = 0.1$ and $\gamma_b = 5$. (Color online.)

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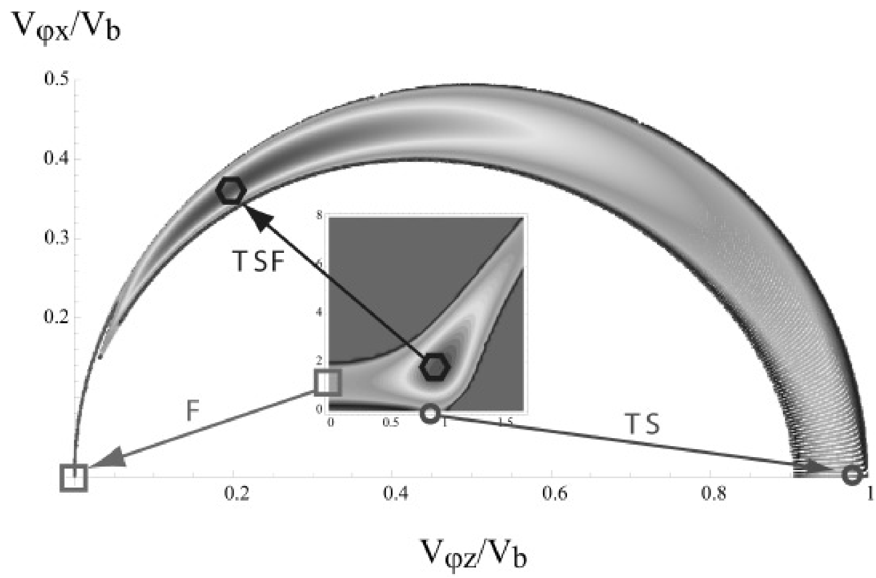

Fig. 2. Portion of the phase velocities space occupied by unstable modes. The color code corresponds to the growth rate and is the same than in Fig. 1. Thermal velocities are $\sim c/10$, $\alpha = 0.1$ and $\gamma_b = 5$. (Color online.)

grows from where it starts, without propagating. The amplification of a filamentation mode excited at a position z is directly given through Eq. (1) as

$$A(z) = \exp\left[\omega_p(z)\beta\sqrt{\frac{\alpha(z)}{\gamma_b}}\left(1 - \frac{\rho_b}{\sqrt{\alpha(z)\gamma_b}}\right)t\right]. \qquad (12)$$

A direct consequence of this expression is that the growth rate turns negative at a certain point of the path because $\alpha(z) = n_b/n_p(z)$ decreases exponentially. It is indeed straightforward to see that filamentation no longer grows for

$$z > Z_F = \lambda \ln\left(\frac{n_b}{n_0}\frac{\gamma_b}{\rho_b^2}\right). \qquad (13)$$

With the typical fast ignition parameters previously used in this article, we find $Z_F = 3.9\lambda = 42$ μm. We thus find that thanks to beam temperature stabilization, the filamentation instability is indeed stabilized before the beam reaches the core. However, we need to be cautious about this result because by the time it reaches 42 μm, it is almost certain that the beam can no longer be considered as an "homogenous, infinite medium" because of the saturation of early instabilities. This point is discussed in the conclusion.

5. OTHER DENSITY GRADIENT EFFECTS

We now discuss two additional gradient effects which can be neglected due to the shortness of the e folding distances for TS and TSF. The first one has to do with a parallel wave vector drift and was discussed by Breizman and Ryutov (1970). Applied to our problem, it implies that if a mode is excited at the beginning of the beam path with a wave vector $\mathbf{k}_0$, the parallel component $k_\parallel$ experiences a drift according to,

$$\frac{\Delta k_\parallel}{k_\parallel(0)} \sim 1 - \exp(z/2\lambda). \qquad (14)$$

If view of the smallness of the e folding distances for TS and TSF compared to λ, we can here neglect this wave vector drift.

But even if the parallel wave vector stays the same when the excited mode propagates, there will be a growth rate drift which must be examined. Let us consider a mode with a wave vector $\mathbf{k}_0$ corresponding initially to the local maximum growth rate $\delta(0)$ of the two-stream instability at $z = 0$ (thin dashed curved on Fig. 3). At a later time, the modes reaches $z_1 > 0$ with almost the same parallel wave vector.

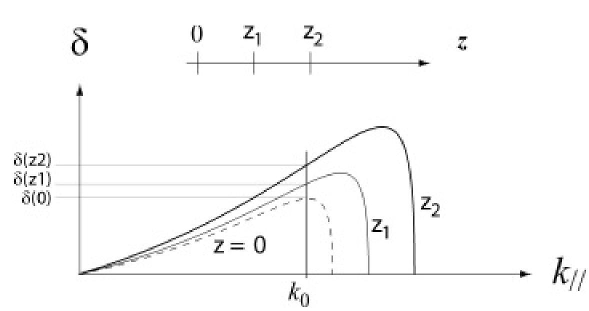

Fig. 3. Schematic representation of the two stream growth rate profile for $0 < z_1 < z_2$.

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But the local maximum two-stream growth rate $\delta_m(z)$ varies like $n_p^{1/6}$ and the local parallel wave vector $k_{m\parallel}(z)$ for which this maximum is obtained varies like $n_p^{1/2}$. The two-stream growth rate profile is therefore given by the thin plain curve, and one sees that the growth rate corresponding to $\mathbf{k}_0$ is no longer the largest local one. Of course, the same trend is observed at $z_2 > z_1$ so that the growth rate for $\mathbf{k}_0$ is not exactly the one given by Eq. (1). To evaluate this effect, we develop the growth rate function around its maximum value (we omit the subscript $\parallel$ of the wave vector for simplicity)

$$\delta = \delta_m - \zeta(k - k_m)^2, \tag{15}$$

where ζ is a parameter which there is no need to provide here because it will turn out to be part of the second order terms of the expression (see Eq. (16)). The growth rate is eventually a function of the plasma density n_p and we now set $n_p = n_p(0) + \delta n_p$ and $k = k(0)$. We can express the growth rate drift using $k_m = \omega_p/V_b$ together with Eq. (1) as,

$$\frac{\delta}{\delta(0)} \sim 1 + \frac{1}{6}\frac{\delta n_p}{n_p(0)} + \mathcal{O}\left(\frac{\delta n_p}{n_p(0)}\right)^2, \tag{16}$$

so that here again, the correction can be neglected for the distances considered and of course, the same reasoning can be conducted for TSF and any convective unstable mode.

6. CONCLUSION

We presented an evaluation of the density gradient effects encountered in the fast ignition scenario upon the linear electromagnetic instabilities. We find that the e folding distances of the convective unstable modes are much smaller than the density gradient scale length λ. The number of e folding for the TSF oblique mode by the time is reaches λ is 63 so that it is almost certain that saturation takes place even before. As far as filamentation is concerned, we find that due to beam temperature stabilization, its growth rate become negative after approximately 4λ. Let us however add that it is possible to study the growth of one single mode apart from the others only as long as the system remains in its linear phase. But once this linear phase is over because of the saturation of some mode, it makes no longer sense to talk about the growth of the slower ones as if they were still pertaining to a linear system. We have here considered an option where the initial beam density $n_b(0)$ is smaller than the local plasma one $n_p(0)$. But $n_b(0)/n_p(0)$ could be close to one (Mason, 2006). In such a case, the WKB approximation is still valid but the hierarchy of unstable modes is changed as filamentation becomes the fastest growing one (Bret & Deutsch, 2005). An accurate analysis of the electron beam initial characteristics in the fast ignition scenario is therefore required to evaluate correctly the output of the linear phase.

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