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Mathematical objects and the object of theology

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Abstract: This article brings mathematical realism and theological realism into conversation. It outlines a realist ontology that characterizes abstract mathematical objects as inaccessible to the senses, non-spatiotemporal, and acausal. Mathematical realists are challenged to explain how we can know such objects. The article reviews some promising responses to this challenge before considering the view that the object of theology also possesses the three characteristic features of abstract objects, and consequently may be known through the same methods that yield knowledge of mathematical objects.

Here I explore the possibility that realism about mathematical objects can provide a model for thinking about realism within theology. I argue that mathematical objects and the central object of Western theological traditions – God – can be usefully thought of as having a number of key features in common, namely those features typically ascribed to abstract objects: lack of spatio-temporal location; inaccessibility to the senses; and acausality.

Although the tradition of thinking about the divine in relation to mathematics goes back to ancient times, it is an approach that has been neglected in recent philosophy of religion (see Koetsier & Bergmans (2005)). This neglect is unfortunate because, as I show here, considering the similarities between mathematical objects and God in the light of recent philosophical work on abstract objects can provide us with a fresh way of thinking about ontology within the context of religion. It also suggests some interesting answers to familiar questions about how we, as beings with spatio-temporal location, can come to know and interact with abstract objects – whether they are mathematical or divine.

Before proceeding further, I need briefly to explain how I am conceptualizing the relationship between the philosophy of mathematics and the philosophy of theology, on the one hand, and the disciplines of which they respectively are philosophies, on the other hand, as well as the relationship between each of these disciplines and the relevant domain of practice. Allow me to illustrate the key relationships by means of a table (see Table 1).

TABLE 1 The relationship between disciplines and domains of practice

Part of analytic philosophy	Academic discipline	Context of application
Philosophy of mathematics	Theoretical and applied mathematics	Scientific practice
Philosophy of theology	Theology	Religious practice

My focus here is on the items within the first column on the left: the philosophy of mathematics and the philosophy of theology. I will have less to say about the disciplines of theoretical and applied mathematics or about the discipline of theology. I will have even less to say about the mathematical or the theological contexts of application within scientific or religious practice.

One more brief qualification is required before proceeding. With respect to the philosophy of theology, my concern is with the philosophical analysis of just one theological object, namely, God. From henceforth, then, I refer to the object of theology in the singular. Moreover, I do not intend to evaluate whether or not the proposition 'God exists' is true or false. More plainly, I am not concerned to answer a question of the form 'Does God exist?'. God is an object of theology just as numbers, sets, and geometrical objects are objects of mathematics; philosophers typically ask not whether such objects exist but rather how such objects should be analysed. The various versions of realism and non-realism available provide different analyses of such objects. Within the theological context, whether or not one adopts a theistic or an atheistic position will not by itself settle the theoretical question of how to analyse objects within a theological ontology.

My interest here is with realism within the philosophy of mathematics and within the philosophy of theology. After some initial remarks on realism within each of these domains, I provide a more detailed account of realism in the philosophy of mathematics. After outlining the contours of mathematical realism, I consider the main challenge the mathematical realist must face to defend her view against critics; namely, the challenge plausibly to explain our knowledge of mathematical objects. Finally, I bring all of this to bear on theological realism by considering the striking affinities between the objects of the mathematical realist and the object of theology.

Realism in mathematics and theology

It is helpful within the philosophy of mathematics, as it is within the philosophy of theology, to treat separately questions of realism about ontology and realism about truth. Realism about ontology in mathematics is a position about the possibility of the objective, mind-independent existence of mathematical objects. Conversely, non-realism about ontology denies the possibility of the objective, mind-independent existence of mathematical objects and seeks to provide an alternative – reductive or eliminativist – characterization of them. Realism about ontology in theology is a position about the possibility of the objective, mind-independent existence of the theological object. Conversely, non-realism about ontology denies the possibility of the objective, mind-independent existence of the theological object and seeks to provide an alternative – reductive or eliminativist – characterization of it.

Realism about truth in mathematics is the view that mathematical statements are objectively true or false independent of the minds of mathematicians and of the languages and conventions of mathematics. Non-realism about truth in mathematics is the view that the truth or falsity of mathematical statements is dependent on the minds of mathematicians and on the languages and conventions of mathematics (although note that the minds of mathematicians may be thought to have created the mathematical objects which mathematical truths could then be said to be dependent upon). Likewise, realism about truth in theology is the view that theological statements are objectively true or false independent of the minds of theologians and of the languages and conventions of theology. Non-realism about truth in theology is the view that the truth-value of theological statements is dependent on the minds of theologians and of the languages and conventions of theology (although, again, note that the minds of theologians may be thought to have created the theological object which theological truths could then be said to be dependent upon).

Having clarified the distinction between realism about ontology and realism about truth, as well as explaining in broad strokes how realism differs from non-realism with respect to each, most of what I will say about realism here will focus on realism about ontology.

Although the twentieth century saw a number of important developments in non-realist accounts of the ontology of mathematics (specifically work on intuitionistic and predicative mathematics by, for example, L. E. J. Brouwer (1912)), debate about whether or not to adopt a realist interpretation of the possibility of the existence of mathematical objects does not seem to be a live one in the mathematical community. Virtually everyone in that community assumes a realist interpretation of ontology, holding that mathematical objects, such as numbers and sets, exist independently of the minds of mathematicians and the conventions of mathematical practice. Non-realist interpretations of mathematics, such as those by Hartry Field (1980) and Charles Chihara (1990), notwithstanding their conceptual weight and technical proficiency, are widely judged to be too unwieldy for practical use; as Kurt Gödel observed, realism conforms well to the practice of mathematics and so it is difficult to jettison.

The twentieth century also saw a number of important developments in non-realist accounts of the ontology of theology, especially work in the post-Wittgensteinian tradition (see the essays in Moore & Scott (2007)). However, as the focus has been on the generation of different versions of non-realism, little effort has been put into the clarification of realism. In fact, much of the ongoing work within analytic philosophy of religion and the philosophy of theology takes a realist interpretation of ontology for granted. The view that if God exists then God does so independently of the minds of theologians and the practices of religious believers – as with the analogue in the mathematical case – is difficult to jettison as it conforms so well to the needs of the practical contexts within which theology is employed. In such practical contexts realism provides a guide to theological practice, just as realism provides a guide to the practice of mathematics in normal contexts of use.

However, merely adopting realism about the object of theology (as the bare claim that if the object exists it does so objectively and is independent of the minds of theologians or the practices of theology or those of religious believers) leaves a host of questions about that object unaddressed; as it would in the mathematical case if the realist about mathematical objects did not provide some account of what the salient characteristics of mathematical objects were. To fill out our understanding of realism in both domains we need to know more about the objects that the realist is a realist about. This is especially important given that, in keeping with the ongoing ascendency of empiricism within analytic philosophy, the dominant construal of realism in contemporary philosophy seems to be more suited to the interpretation of language about objects that are either directly or indirectly, via technology, accessible to our senses.

The moral of the story so far is that 'realism', like 'existence', is not a univocal and non-question-begging notion. Realism is always realism about some object or type of object (or, as explained above, it is about the truth-value of statements about such objects). Like non-realism, realism comes in many varieties. My contention is that the form of realism developed in philosophy of mathematics in response to theorizing about mathematical objects might be fruitfully applied within the theological domain. So, in the following section, I turn to a closer examination of realism within the philosophy of mathematics.

Realism within the philosophy of mathematics

Let us take realism – at least realism about ontology – to be the bare thesis that objects exist objectively in that they are mind independent (one might complicate the discussion further by considering realism about properties or relations, but I shall not do so here). As explained above, this bare thesis in itself tells us nothing and implies nothing about the identity or nature of the postulated objects. It certainly does not tell us that the objects have spatial or temporal location, or that they are accessible to our senses, or that we should expect them to be capable of involvement in physical casual relations.

With respect to mathematical objects, Stewart Shapiro observes:

> The scientific literature contains no reference to the location of numbers or to their causal efficacy in natural phenomenon or to how one could go about creating or destroying a number. There is no mention of experiments to detect the presence of numbers or determine their mathematical properties. Such talk would be patently absurd. (Shapiro (2011), 27)

It would be absurd because, as almost all mathematical realists including Shapiro hold, mathematical objects are abstract objects. In fact, the debate between realists and non-realists in the post-Fregean philosophy of mathematics has concerned whether or not abstract mathematical objects should be interpreted as existing objectively and mind-independently, and whether or not mathematical theories describe such objectively existing, mind-independent objects. While not all mathematical realists agree about the correct analysis of abstract mathematical objects,[1] they are united against the non-realist claim that mathematics lacks all such objects, in other words, they deny that mathematics is 'a subject with no object' (see Burgess & Rosen (1997)).[2]

Discussion of abstract objects presupposes a distinction between objects which are abstract and those which are concrete. While most people find such a distinction intuitively graspable, it is surprisingly hard to say precisely where the distinction between the abstract and the concrete lies. Current consensus seems to be that while there is an important distinction it cannot be sharply drawn. However, it is not unusual to find the contrast between abstract and concrete objects stipulated along the following lines: abstract objects, unlike concrete objects, are inaccessible to the senses, acausal, and neither spatial nor temporal.

Bob Hale considers whether or not these three proposed characteristics could serve as criteria of abstract objects. Regarding the first – inaccessibility to the senses – he notes that: 'Abstract objects cannot be seen, heard or felt; they exude no fragrance or odour. Such remarks have the ring of truism. It is natural to suppose that they may afford at least the basis of the criterion' (Hale (1987), 46). However, Hale quickly rejects this proposal for reasons earlier suggested by Michael Dummett. Namely, making inaccessibility to the senses the criterion of abstract objects would, in effect, relativize them to human sensory faculties (Dummett (1973), 480–481). Moreover, because we lack a clear account of what it is to be inaccessible to the senses, it would not yield a clear distinction between the abstract and the concrete (must, for example, abstract objects be inaccessible only to the un-augmented senses?).

Hale then considers whether this first characteristic of abstract objects is a consequence of the supposed fact that they are not capable of physical causal interaction. If abstract objects are acausal, given that sense perception is a causal process, it would follow a priori that abstract objects must be imperceptible (see Hale (1987), 46–47). This would imply that acausality was the more appropriate criterion of an abstract object. But Hale also rejects this suggestion for a number of reasons. First, he notes that: '[i]f abstract objects are indeed acausal, there ought to be specifiable categorical features in virtue of which that is so, and it is on these features that the distinction should be based' (*ibid.*, 47). Second, whether or not abstract objects can be subject to change is contentious and the question cannot be appropriately closed by definition. Third, 'it is far from evident that abstract objects cannot be involved in bringing about changes, unless perhaps one restricts consideration to *physical* causation and physical change – but this qualification introduces the further problem of defining physical change' (*ibid.*, 47–48). And, fourth, it would beg the question against those holding that abstract objects may be the objects of non-sensory intuition (which would require some kind of non-physical causal mechanism).

Having, for the reasons outlined above, rejected the view that acausality can serve as a criterion to identify abstract objects and distinguish them categorically from concrete ones, Hale proceeds to a discussion of spatiality and temporality, lack of which is the third proposed criterion of abstract objects. Hale notes the attractiveness of this proposal and observes that it makes sense of the common view that abstract objects are acausal (although, strictly, as he observes, it does not entail this view). However, as it is fairly easy to come up with examples of abstract objects which, although non-spatial, are temporal (a game of chess, for example, or, more controversially, a mind), Hale quickly dispatches the proposal that we can use lack of spatiality and temporality as a criterion of abstract objects (see *ibid.*, 48–50).

Following Hale, we can say that while inaccessibility to the senses, acausality, and lack of spatial or temporal location cannot serve as criteria by means of which we could distinguish between abstract and concrete objects, it is nonetheless beyond serious question that each of these characteristics is typical of abstract objects (or, at least, is so regarded). Indeed, few would be prepared to doubt that mathematical objects are inaccessible to the senses, incapable of physical causal interactions, and without spatial or temporal location.

As Stewart Shapiro notes, this characterization of mathematical objects as inaccessible to the senses, acausal, and non-spatio-temporal conveniently generates an account of the necessity of mathematical truths. On this realist construal, the truths of mathematics turn out to be independent of anything contingent about the physical universe or the human mind or the practices of mathematicians. Realism about the ontology of abstract mathematical objects, for example, numbers, functions, and sets, is thus closely aligned, at least in this account, to realism about mathematical truths. Within contemporary philosophy of mathematics, realism about ontology combined with realism about truth is usually referred to as platonism (although, it must be noted that the term 'platonism' here refers to an explicitly post-Fregean position).[3]

While this form of mathematical realism is widely accepted, there are marked differences of opinion about how to explain our knowledge of mathematical truths. Mathematical realists are frequently challenged to provide an account of how our knowledge of mathematical objects is responsive to those objects. How do human beings, located as we are within space-time, have access to objects which lack spatial and temporal location, and which are acausal, necessary, and, we might now add, indestructible? After all, some cognitive access to these objects would seem to be a requirement of our knowledge of mathematical truths.

The epistemological problem for mathematical realism

In its modern form, the epistemological problem for a realist interpretation of abstract mathematical objects was first stated by Paul Benacerraf (1973).[4] The core of Benacerraf's challenge is the claim that a realist interpretation of mathematical objects is logically incompatible with the indisputable fact that we have mathematical knowledge. The problem for the realist, on this view, is that if abstract mathematical objects did exist we would not be in a position to know them because we would not be able to come into contact with them. Following Benacerraf, opponents of mathematical realism typically aver that as humans are physical beings the only way they can come to know objects is by physically interacting with them. If physical interaction were required for the acquisition of knowledge, it would seem to follow that humans cannot come to know non-physical abstract objects.

In its original Benacerrafian form, the epistemological argument against realism in mathematics presupposes that some version of the causal theory of knowledge (CTK) is correct. According to the CTK, in order for a person S to know that p, it is necessary that S be causally related to the fact that p in an appropriate way.

Consider Mark Balaguer's formulation of Benacerraf's argument:

(1) Human beings exist entirely within space-time.
(2) If there exist any abstract mathematical objects, then they exist outside of space-time.

Therefore, by CTK,

(3) If there exist any abstract mathematical objects, then human beings could not attain knowledge of them.

Therefore,

(4) If mathematical platonism is correct, then human beings could not attain mathematical knowledge.
(5) Human beings have mathematical knowledge.

Therefore,

(6) Mathematical platonism is not correct. (Balaguer (1998), 22)

To address this Benacerrafian problem the realist must either reject the CTK (or at least seriously modify it) or explain how human beings are able to come into appropriate causal contact with mathematical objects such that they could know them. For obvious reasons, the first strategy has been by far the most popular among mathematical realists.[5]

Bob Hale takes this challenge seriously and adopts the first strategy in response to it. He claims that the CTK, as it is typically interpreted, is too strong to allow not only for any a priori knowledge but also for certain forms of empirical knowledge, specifically, knowledge of contingent general truths, and hence it cannot plausibly be accepted. He further argues that once the CTK is weakened so as to allow some a priori knowledge along with knowledge of contingent general truths, then it no longer presents a problem to the mathematical realist.

Balaguer also holds that mathematical realists have good grounds for rejecting the CTK. In fact, both Hale and Balaguer independently conclude that the apparent importance of the CTK in the argument against mathematical realism is a red herring. This is because, even if the CTK is rejected, the onus is still on the mathematical realist to explain exactly how human beings within space-time can come to know abstract mathematical objects that are not located within space-time. An alternative non-causal account of the acquisition and justification of mathematical knowledge is still required.

Despite significant divergences in their views, which do not concern us here, Hale and Balaguer have both identified the same limitation inherent within the way the Benacerrafian argument against mathematical realism was framed. Because the Benacerrafian problem took the CTK for granted, it could be avoided simply by rejecting – or substantially modifying the scope of – the CTK. But this was too easy a solution, and both Hale and Balaguer regard this move taken on its own as tantamount to avoiding the more serious epistemological problem – which is to account for the fact that human beings have mathematical knowledge at all (or, in another version, to account for the reliability of our mathematical beliefs). This problem clearly cannot be addressed merely by rejecting the CTK. So they would agree that the premise of the Benacerrafian argument that requires attention if mathematical realism is to be defended is this one:

> (3) If there exist any abstract mathematical objects, then human beings could not attain knowledge of them.

The onus seems to be on mathematical realists to provide an account of how, exactly, human beings could know abstract mathematical objects.

Hale takes up this challenge and provides a positive account of how knowledge of abstract mathematical objects is available to us (see Hale (1987), ch. 6). He defends a version of the traditional view that our knowledge of mathematical truths is non-empirical, in other words that it is a priori (although he avoids commitment to the further view that because it is a priori it is unrevisable). According to Hale, we acquire a priori knowledge about abstract mathematical objects by our recognition of conceptual connections. He argues that:

> the obvious natural account of our knowledge of truths about abstract objects is that some of them are recognised as such by direct reflective discernment of the relevant conceptual linkages, the rest by reasoning from more basic truths already grasped – in short, that our knowledge of them is, typically at least, non-empirical or *a priori* in virtue of being reached by reflective exploitation of conceptual resources. (*ibid.*, 124)

The principal advantage of Hale's account, aside from its elegance and simplicity, is that it explains how we can acquire knowledge of mathematical truths without coming into contact with mathematical objects. The account trades on there being a fairly clear distinction between the way we acquire empirical knowledge and the way we acquire a priori knowledge, and it claims that the CTK is only relevant with respect to the acquisition of the former.[6] In defence of realism about mathematical objects, Hale has provided a plausible account of how we acquire mathematical knowledge. Adopting Balaguer's terminology, this can be aptly described as a form of 'no-contact' epistemology, for it does not require that the knower is in any sort of physical causal relation to the object which is known (Balaguer (1998), 24). In short, on Hale's view, the knower is not required to come into perceptual contact with mathematical objects.

Other mathematical realists have adopted a different approach and have developed epistemologies which do give an important role to the perception of mathematical objects. Some of these theories can be appropriately described as types of 'contact' epistemology because they claim that we do in fact perceive mathematical objects, such as sets, through our regular visual apparatus.[7]

The claim that we acquire knowledge of mathematical objects by means of something like a perception is most readily associated with Kurt Gödel. However, as I shall explain, despite its reliance on the notion of perception, Gödel's position is more akin to Hale's no-contact epistemology than it is to those theories which require physical contact with mathematical objects to account for our knowledge of them. Gödel defended the existence of mathematical objects against those such as Henri Poincaré who, early in the twentieth century, had argued that mathematical objects do not exist independently of the mathematician. In response to Poincaré's view that 'in mathematics the word "exist" . . . means free from contradiction' (Poincaré (1913), 454), Gödel proposed that:

> [c]lasses and concepts may . . . be conceived as real objects . . . existing independently of us and our definitions and constructions. It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. (quoted in Shapiro (2011), 10)

The above passage introduces the hallmark idea of Gödel's distinctive approach to realism in mathematics; namely, the idea that there is an analogy between, on the one hand, mathematical objects and the way we come to know them and, on the other hand, ordinary physical objects and the way we come to know them. This is

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in sharp contrast to Hale's view, considered above, that there is no real comparison between the way truth about mathematical objects is apprehended and the perception of physical objects in the empirical realm. Gödel develops this idea further in the following passage:

> But, despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves on us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them . . . (Gödel (1983 [1964]), 483–484)

In Gödel's view, then, not only are mathematical objects analogous to physical objects, but mathematical intuition is analogous to sense perception.

Gödel's theory is subject to different interpretations. According to one standard interpretation, Gödel's claim is that through mathematical intuition the human mind comes into non-physical contact with the mathematical realm and acquires information about that realm by means of this contact. On this interpretation, Gödel is taken to be proposing that something happens between the human mind which comes to know mathematical truths and the mathematical objects which come to be known. Another way of putting this idea is to say that non-physical information passes from the mathematical objects to the human mind by means of a faculty of mathematical intuition. In a frequently cited passage, Gödel explains:

> It should be noted that mathematical intuition need not be conceived of as a faculty giving an *immediate* knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we *form* our ideas also of those objects on the basis of something else which *is* immediately given. Only the something else here is *not*, or not primarily, the sensations. That something else besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from the sensations or mere combination of sensations, e.g., the idea of object itself. . . . Evidently, the 'given' underlying mathematics is closely related to the abstract elements contained in our empirical ideas. (*ibid.*, 484)

Gödel's interpreters have puzzled over what exactly, in his view, is immediately given to the mind in mathematical intuition. However, despite such exegetical difficulties, it is clear that his theory about mathematical intuition needs to be interpreted in the light of his claim that 'the case for the existence of mathematical objects is an exact parallel of the case for the existence of physical objects' (quoted in Shapiro (2011), 12). It would be a mistake to take this to mean that our theory of mathematical objects and mathematical knowledge should be modelled on a theory of physical objects and our knowledge of them. Rather, Gödel's point seems to be that we draw conclusions about both mathematical objects and physical objects on the basis of successful mathematical or physical theories, respectively. In neither case – the mathematical nor the physical – is the object immediately given in perception (see Brown (1990)). We have no more direct contact with physical objects than we do with mathematical objects and, what is more, we do not require such direct contact in order to acquire knowledge of these objects.[8]

In effect, Gödel, and others who have adopted his approach, rejects the CTK for all forms of knowledge: empirical and a priori. His proposal is that neither mathematical knowledge nor knowledge of the physical world requires direct contact with the objects that are known. If Gödel is right, and direct perception of physical objects does not play a key role in our coming to know them, then that further undermines Benacerraf's epistemological objection to realism about mathematical objects. As we saw above, that objection was based on the idea that mathematical objects could not be known in the same way that physical objects could be known because of the former's inability to be involved in physical causal relations. However, if direct perception doesn't play a key role in the acquisition of mathematical knowledge or the acquisition of knowledge about physical objects, then causation doesn't play a key role either (see Tait (1986), 345–346).[9]

Intuition-based theories of mathematical knowledge, inspired by Gödel, have been developed by a number of more recent thinkers. The thrust of these theories is the attempt to clarify further what mathematical intuition involves. Jerrold Katz, for example, has proposed that mathematical intuition is a process by which we form mental representations of objective abstract mathematical entities. Like Gödel's account, this view requires no physical contact with abstract mathematical objects in order to explain how we come to possess mathematical knowledge. Moreover, it is capable of explaining why what is known through intuition is necessarily true. According to Katz, because mathematical knowledge is non-contingent, what is known could not have been otherwise.[10] There are no empirical variables of which we would need perception to apprise us. By way of illustration, Katz explains that: 'if we construct a sufficiently articulated concept of the number four in intuition, we will be able to see that the concept is a concept of an object that is the sum of two primes' (Katz (1981), 207). Thus, Katz's view seems to have combined elements from both Hale's account and Gödel's.

On each type of view considered above, Gödel's which involved mathematical intuition, Hale's which relied on recognition of conceptual connections, and Katz's which combined both of their positions, our knowledge of abstract objects does not require any physical connection with such objects. Each type of view, then, has responded to the most significant objection to realism about mathematical objects by providing an account of how we can know objects that are not located in space-time when we ourselves are within the spatio-temporal domain. It has not been the purpose of this exposition to defend any one of these types of view in detail; they have been explained in order to display some of the rich resources available for the defence of realism within contemporary philosophy of mathematics. With these arguments in hand, in the following section, attention turns again to realism in theology.

Realism in theology

The arguments considered above pertained to abstract mathematical objects; however, as not all abstract objects belong to the mathematical realm, we should not be surprised to find that these arguments can be deployed elsewhere. This is not the place to attempt a general account of what other kinds of abstract objects there might be;[11] rather here I consider the proposal that the object of theology shares some of the key features of abstract mathematical objects. If this proposal is correct, it suggests that examining theological ontology through the lens of philosophy of mathematics might help us to clarify certain important issues concerning the characterization of the object of theology; issues that tend to be overlooked in the debate between theism and atheism, which keeps attention focused on whether or not that object exists (or can reasonably be believed to exist).

Recall the three widely agreed-upon characteristics of abstract objects, discussed above: inaccessibility to the senses, acausality, and lack of spatial and temporal location. I focus on these three characteristics in what follows, although – for obvious reasons – the related characteristics of eternality, indestructibility, and necessity would be highly relevant to a more detailed argument.

Inaccessibility to the senses

There is a well-known view within theology that God cannot be known through the senses. While the senses might in some circumstances be legitimately used as a stimulus to theological knowledge and to assist in the development of theological concepts (a point which Gödel also accepted with respect to mathematical knowledge), it is widely held that they cannot provide direct knowledge of God. Although the importance of this idea is by no means limited to Christian theology, especially prominent exponents of it can be found in the theologians of the Carmelite tradition. Explicating John of the Cross's teaching on the night of faith and the way to union with God, Edith Stein writes:

> We can only accept what we are told by turning off the light of our natural knowledge. We have to agree with what we hear without having any of the senses elucidate it for us. Therefore faith is a totally dark night for the soul. But it is precisely by these means that it brings her light: a knowledge of perfect certainty that exceeds all other knowledge and science so that one can arrive in perfect contemplation at a correct conception of faith . . . (Stein (2002), 58–59)

According to this theological tradition, then, knowledge of God is not knowledge of an empirical object. Consequently the source of such knowledge cannot be the senses. In fact, Stein emphasizes that knowledge acquired through the senses must be deliberately shunned as it prevents us from acquiring genuine knowledge of God. Moreover, her claim that it is possible to arrive at 'perfect certainty' about such knowledge seems to align it more with mathematical knowledge than with knowledge of the empirical realm.

But just as the mathematical realist was pressed to explain how we can come to know objects that are inaccessible to our senses, so the onus is on the theologian to provide an account of how we can know the object of theology.[12] Adapting Gödel's remarks about mathematical knowledge, perhaps she could say:

> It should be noted that theological intuition need not be conceived of as a faculty giving an *immediate* knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we *form* our ideas also of those objects on the basis of something else which *is* immediately given. Only the something else here is *not*, or not primarily, the sensations. (Gödel (1983 [1964]), 484; I have substituted the term 'theological intuition' for 'mathematical intuition')

Recalling my earlier remarks that Gödel's claims about mathematical intuition need to be interpreted in the light of his view that 'the case for the existence of mathematical objects is an exact parallel of the case for the existence of physical objects' (quoted in Shapiro (2011), 12), we might now expand this argument to cover the case for the existence of the object of theology. Just as it would have been a mistake to take Gödel to mean that our theory of mathematical objects and mathematical knowledge should be based on a theory of physical objects and our knowledge of them, the same can now be said of the theological object and our knowledge of it. Nothing requires us to base our theory of the theological object and theological knowledge on a theory of physical objects and our relations to them. Rather, just as we draw conclusions about both mathematical and physical objects on the basis of successful mathematical or physical theories, respectively, we should draw conclusions about the theological object on the basis of successful theological theories. What counts as a successful theory will, at least in part, be relative to the needs of the context in which the knowledge generated by the theory is employed.

It may be objected that the comparison between mathematical intuition and theological intuition is illegitimate on the grounds that the former often seems to be more widely shared than the latter. Many mathematical truths, especially those of arithmetic, appear to be self-evident; and this is often taken to support the view that at least some of the truths of mathematics are knowable through intuition. Self-evident truths within theology are harder to find than within mathematics, suggesting to some that theological intuition is unlikely to be a source of theological knowledge. One response to this objection is to claim that the scope of mathematical intuition is, for many people, in fact rather limited. The mathematical intuition enjoyed by great mathematicians such as Gödel, Euler, or Gauss would show that most of us only experience a glimmer of mathematical insight through our own mathematical intuitions. The same might be the case in the theological domain, and this would make it much less significant that there seems to be an absence of widely shared intuitions among those who are not theological 'geniuses'.

It would seem then that a theological realist who holds that the object of theology is inaccessible to the senses could find resources within the philosophy of mathematics to contribute to an account of how that object may nonetheless be known by us. I now turn to the next characteristic of abstract mathematical objects – acausality – and briefly consider how that might be relevant within the domain of theological ontology.

Acausality

As explained above, the inability of mathematical objects to enter into causal relations (at least, physical causal relations) generated one of the principal objections to mathematical realism. The realist was challenged to explain the mechanism by which acausal abstract objects relate to the physical world so that we can know them.[13] The question about how abstract mathematical objects non-physically interact with the world of our experience has an analogue in a question that we may well ask about the object of theology and its relation to the empirical realm.

On the face of it there seem to be a host of potential difficulties attendant upon the claim that God is acausal. However, further reflection may reveal that – as in the case of mathematical objects – acausality is not disadvantageous, either for our ability to know the object or for the object's ability to be intimately connected to the world of our experience. I take up some of these issues in the next section, as acausality is a consequence of the more fundamental characteristic of being non-spatio-temporal. However, there is one potential objection to this approach that needs to be quickly addressed before moving on.

It might be objected that God must be capable of involvement in physical causal relations because God is the Creator of everything that exists and creation is a causal process. If this were the case, God could not be non-spatial and non-temporal, thus the grounds for regarding the object of theology as significantly similar to an abstract mathematical object would be seriously undermined. In response, it can be pointed out that what it means for God to create is not well understood. Although talk of causation in this context abounds, we do not have even an approximate understanding of what divine causation involves. Talk of causation in theology seems metaphorical, or analogical, at best. Our experience of causation comes from the spatio-temporal realm of physical objects, and we cannot responsibly extend our understanding of how causation operates in that realm to gain any purchase on the notion of divine causation. So the theological claim that God is the Creator does not entail, as far as we can judge, that it is necessary that God is involved in physical causal relations. This brings me to the third characteristic of abstract objects: lack of spatial and temporal location.

Non-spatio-temporal

Like the objects of mathematics, the object of theology is non-physical (this is why neither God nor mathematical objects are involved in physical causal chains). This understanding of the theological object accords with the standard assertion of classical forms of theism that God is not embodied. It would seem theologically uncontroversial then to claim that God, like the number 3, for example, is not an object with a spatial location. The claim that God is also without a temporal location is more contestable, nonetheless it is widely held. It is a familiar theological stance to hold that God is non-temporal in the sense of being eternal or 'outside' time. In what follows I ignore a number of complications to explore further the view that the object of theology has neither a spatial nor a temporal location.

What might be theologically problematic about the claim that God is not an object with spatial or temporal location? As was the case with the view that abstract mathematical objects are not located within space-time, two problems stand out. The first is the now familiar epistemological problem concerning how we can know objects that are not located within space-time, given that we ourselves are firmly lodged there. The second problem concerns how God might be non-physically related to the spatiotemporal realm.

One might address the epistemological problem, along the lines suggested above, with the claim that there is a non-physical connection between God and human minds mediated through the non-sensory knowledge available through theological intuition (*à la* Stein), but a further response is required to the second problem. Theists typically hold that God is a personal being who can enter into relations with us. How could a God construed as lacking spatial or temporal properties fit this description? Wouldn't such a God be irrelevant to beings such as ourselves who are firmly embedded in the sensory world of space-time? Perhaps a more adequate, albeit still incomplete, response to such worries would be to point out that mathematical objects remain highly relevant to the empirical realm despite the fact that they are abstract. Although they are not objects located within space-time, numbers and other mathematical objects play a vital role in our understanding of our environment and our ability to engage with it.[14] As remarked above, abstract mathematical objects do seem to enter into intimate relations with the world of our experience, despite the fact that they are not causally related to any of the physical objects within that world. Might something analogous be said about the object of theology? This is by no means a fully developed response to the theological concern that we would not be able to enter into personal relations with a God that lacked spatial and temporal properties. It merely suggests that it might be theologically fruitful to consider what has been said within the philosophy of mathematics about the non-physical relation of mathematical objects to the empirical domain.

Such worries aside, there are reasons why it might be theologically desirable to hold that God is not located within space-time and therefore is not physically related to objects that are, such as ourselves. One reason is that a God who was located within space-time would surely be a limited God (that is, limited to some particular space and some particular time), whereas a God without a particular spatial or temporal position could – like the number 3 – be simultaneously

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accessible at all times and places (which would seem to be an obvious theological *desideratum*). A second, and more tentatively advanced, reason is that a God who was an object located within space-time, and possessing the capability to enter into physical causal relations facilitated by that, would seem to face a particularly challenging form of the problem of evil.

Conclusion

Drawing the threads of this discussion together, then, there are good theological reasons for holding that the theological object shares the three principal characteristics of abstract objects that were evident in the case of mathematical objects. Yet one might still assert that the comparison between abstract mathematical objects and the object of theology is illegitimate because the latter possesses a host of other, non-abstract, properties that it would be unthinkable to ascribe to mathematical objects. God is, for example, as acknowledged above, typically referred to as personal. In response to this I suggest that the theological realism outlined above – realism about the object of theology conceived as inaccessible to the senses, acausal, and non-spatiotemporal – is self-consciously minimalistic. It does not attempt to provide a full account of how the theological object may be elaborated upon within different theologies, and it does not need to.

To see this, consider once more the notion of mathematical intuition. As we saw above, according to Jerrold Katz (expanding on Gödel's view), through mathematical intuition we form a mental representation of objective abstract entities and this allows us to know necessary mathematical truths. Adapting this idea to the theological case we might venture to claim that through theological intuition we form a mental representation of the object of theology and what we come to know about this entity is a necessary truth. This necessary truth will be completely formal, lacking all empirical content. It will be the minimalist assertion that the object of theology is something along the lines of a suitably qualified version of Anselm's 'that-than-which-a-greater-cannot-be-conceived'.

This bare theological object, as Anselm seems to have recognized, functions as a conceptual scaffolding upon which more elaborate theological systems can be built. Such systems might be constructed by following the type of process described by Bob Hale in his effort, outlined above, to explain how we acquire non-empirical knowledge. Hale proposed that this was done partly by 'direct reflective discernment of the relevant conceptual linkages' and partly by 'reasoning from more basic truths already grasped' (Hale (1987), 124). So, perhaps this method can be employed to fill out a richer positive content for theology. One necessary task for someone adopting this approach would be to consider to what extent the 'relevant conceptual linkages' would be governed by what Hale refers to as 'principles of abstraction'. Such principles are important in Hale's account of mathematical knowledge because they explain how we can increase our knowledge by considering the logical properties of mathematical objects. Presumably, within theology the principles governing this process of reflection would have a rather different character.

Another advantage of this approach is that it opens the way for a novel account of pluralism within theology. In different branches of mathematics the same abstract objects can play different roles and exhibit different properties, and something analogous may be the case within theology. In one theological system God may play a certain role and exhibit certain properties that are different from God's role and properties in another theological system. Just as algebra and geometry, for example, use mathematical objects in different ways and with different results, so might diverse theological systems use the concept of God in different ways and with different results. This suggests that, within both mathematics and theology, pluralism is likely to be ineliminable and irreducible. The various branches of mathematics cannot be reductively collapsed into each other (although translation may be possible through category theory). We should not be surprised if the same turns out to be the case with respect to different theological systems. Although, according to the view outlined above, different theological systems may be premised upon the same object, they may have each elaborated upon that object in irreducibly different, and – lacking a theological analogue of category theory – possibly incommensurable ways.

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Notes

1. The locus of the disagreement between different camps of mathematical realism is whether mathematical objects are to be construed as entities, such as numbers, functions, or sets, etc., or instead are to be construed as structures. Noting the division between those realists who adhere to object-platonism and those adhering to structuralism, Mark Balaguer points out that, despite this entrenched disagreement, the core of realism in philosophy of mathematics 'is belief in the abstract, that is, the belief that there is something real and objective that exists outside of spacetime and that mathematical theories characterize' (Balaguer (1998), 8). Balaguer argues that the debate between object-platonists (such as Frege and Gödel) and structuralists (such as Shapiro) is in fact misdirected because structuralists still have to address the fact that '*positions* in structures can be taken as mathematical objects' (*ibid.*, 9).
2. The debate in the post-Fregean context should not be confused with the earlier debate in Scholastic philosophy and theology concerning the existence of universals and particulars. The abstract objects under discussion in current philosophy of mathematics are not equivalent to the universals of the mediaeval debate about nominalism.
3. Non-platonic realism about mathematical objects is seldom defended. It involves holding that mathematical theories are not about abstract objects but rather about concrete ones. John Stuart Mill (1843), for example, held such a view.
4. See also Cheyne (2001).
5. For a critical survey of realist responses to Benacerraf see Balaguer (1998), ch. 2; also see Hale (1987), ch. 4.
6. Beall (2001) deploys a similar distinction in his criticism of the causal condition on existential knowledge.
7. See, for example, Maddy (1997). See also Parsons (1979).
8. W. W. Tait remarks: 'I know of no argument against the existence of mathematical objects which does not have a replica in the case of sensible objects', and he adds that '[s]ceptics about mathematical objects should be sceptics about physical objects too' (Tait (1986), 345).
9. Richard Tieszen (1989) pursues a similar line of argument, holding that neither sense perception nor mathematical intuition involves a causal process.
10. In his later work, Katz writes:

    > [I]nvestigation in the natural sciences seeks to *prune down the possible to the actual*, while investigation in the formal sciences seeks to *prune down the supposable to the necessary* . . . Since pruning down the supposable to the necessary requires only reason, formal knowledge is *a priori* knowledge. Since pruning down the possible to the actual requires interaction with natural objects as well as reason, natural knowledge is *a posteriori*. (Katz (1998), 59)

    For a critical discussion of Katz's view, see Cheyne (2001), 149–153.
11. See the discussion of this in Hale (1987).
12. A fuller treatment would include a discussion of the theological tradition of the 'spiritual senses'. See, for example, Hans Urs von Balthasar (1982).
13. A closely related difficulty concerns how abstract mathematical objects are sufficiently related to the physical world to allow us to use them in the formulation of successful predictions about that world.
14. The Quine–Putnam indispensability argument, which is widely held to be one of the principal arguments in support of realism about mathematical ontology, is an elaboration of this point. See, for example, Quine (1983 [1976]) and Putnam (1979).