

# AGENCY COSTS, RISK SHOCKS, AND INTERNATIONAL CYCLES

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We add agency costs into a two-country, two-good international business-cycle model. In our model, changes in the relative price of investment arise endogenously. Despite the fact that technology shocks are uncorrelated across countries, the relative price of investment is positively correlated across countries in our model, much as it is in detrended U.S./Euro-area data. We also find that financial frictions tend to increase the volatility of the terms of trade and the international correlations of consumption, hours worked, output, and investment. We then compare this model to an alternative model that also includes risk shocks. We use credit spread data (for the United States) to calibrate the AR(1) process for risk shocks. We find that risk shocks are too small to significantly impact the model's dynamics.

**Keywords:** Agency Costs, Risk Shocks, International Business Cycles

## 1. INTRODUCTION

The seminal work on international real business cycles by Backus et al. (1992, hereinafter BKK) highlighted three puzzling facts that separate their model from the data. The first of these puzzles has been referred to as the “quantity anomaly.” (In BKK, the cross-country correlation of output is less than the cross-country correlation of consumption, a phenomenon not observed in the data.) The second anomaly concerns cross-country correlations of investment and hours worked. (They are positive in the data but negative in BKK.) The third anomaly pointed out by BKK highlights their model's inability to adequately capture volatility in the terms of trade (TOT) and net exports simultaneously. This phenomenon has been coined as the “price anomaly.”

In the literature, there have been several attempts made to address the puzzles/anomalies listed above.<sup>1</sup> Recent attempts to bring the BKK model closer to the data have been made by incorporating investment-specific technology (IST)

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into the list of exogenous shocks included in the model. Since its conception by Greenwood et al. (1988, hereinafter GHH), the standard method has been to identify IST by the inverse of the relative price of investment (RPI). The motivation for this method is simple. If there exists a perfectly competitive firm converting consumption goods into investment goods via an IST, then the RPI can be used to perfectly identify movements in IST. Thus, by assuming that IST is an exogenous shock, we also assume that the RPI is orthogonal to any forms of economic disturbances. This assumption, however, seems unlikely, since, as Schmitt-Grohe and Uribe (2011) point out, both neutral and investment-specific technologies follow a common stochastic trend in the postwar United States. Likewise, Floetotto et al. (2009) point out that by assuming that IST is an exogenous technology shock, we are assuming that the RPI does not respond to variations in investment demand over the course of the business cycle. Fisher (2009) highlights this problem by emphasizing that if there are any asymmetries separating the investment- and consumption-producing sectors, this can be problematic when identifying IST by the inverse of the RPI. Thus, applying the typical method for identifying IST that has been in place since its conception by GHH may be inappropriate.

The ability of IST shocks to generate realistic business cycles is questionable. Justiniano et al. (2011) demonstrate that in a closed-economy framework, when IST is properly calibrated to match the observed fluctuations in the RPI, IST shocks lose their ability to generate business-cycle volatility. A similar debate has been going on in the international business-cycle literature. Raffo (2010) adds IST shocks to a BKK-style model. He finds that adding IST shocks to the model solves the puzzles listed in our opening paragraph. A significant drawback of his analysis is that the IST shocks in his model are not tied to RPI data. Mandelman et al. (2011) also look at the role of IST shocks in a BKK-style model. They use RPI data for the United States and the “rest of the world” to estimate the stochastic process for IST shocks included in their model. They find that IST shocks are not powerful enough to resolve these puzzles. One of our main objectives is to revisit this debate. Much as Mandelman et al. (2011), we find that IST shocks do little to improve canonical international business-cycle predictions.

Our first central contribution to the literature is related to the debate above. By including agency costs, as in Carlstrom and Fuerst (1997, hereinafter CF), in a two-country, two-good model,<sup>2</sup> the RPI becomes entirely endogenous and fluctuates in response to technology shocks (the only disturbances included in the model). We show that these financial frictions increase the variance of the TOT and cross-country correlations.<sup>3</sup> Our model is able to produce positive cross-country correlations, despite the fact that technology shocks are completely uncorrelated across countries. Furthermore, these encouraging results are obtained in a context where the volatility of the RPI and its cross-country correlations resemble their counterparts in the data. The relative standard deviation of the RPI is about 25% smaller than in U.S. data. Trade in intermediate goods causes a transmission of agency costs across countries, which produces a small positive correlation of the

RPI across countries—much like in our U.S./Euro-area data. Our work suggests that our model with agency costs does better than a model with IST shocks,<sup>4</sup> with the additional advantage that the RPI is actually explained by the model. Below, we provide more details about the model and its inner workings.

Our second central contribution relates to risk shocks. This concept was developed by Christiano et al. (2014), who include risk shocks into an otherwise standard Bernanke et al. (1999) costly state-verification framework. As in the paper by Christiano et al., a positive risk shock in our model increases the cross-sectional dispersion of the entrepreneur's idiosyncratic productivity, which, in turn, increases the uncertainty regarding the probability of entrepreneurial failure. Christiano et al. (2014) find that disturbances to this degree of uncertainty, referred to as "risk shocks," lead to realistic business-cycle dynamics in a closed-economy environment. Our analysis shows that risk shocks have the potential to raise the volatility of the RPI and that of the TOT. However, in our model, risk shocks lead to responses of output, consumption, investment, and hours worked that are of opposite signs across countries. This is not a desirable feature, given the model's natural inability to produce positive cross-country correlations. We use credit spreads between rates on U.S. Baa-rated corporate bonds and 10-year U.S. government bonds to calibrate the bivariate AR(1) process driving the variance of entrepreneurial productivity. The variance of the spreads in our model matches the variance of the spreads in the data. We find that risk shocks are actually not large enough to materially change the statistical moments implied by our benchmark model.

Why use financial frictions to generate endogenous movements in the RPI, both within and across countries? In the simplest stochastic dynamic general-equilibrium models, a representative agent can convert one unit of consumption good into one unit of capital ready for production. This very simple capital accumulation process can be made more realistic by carrying it out in two steps. In the first step, a technology is employed to convert consumption goods into investment goods. Shocks hitting that technology are known as the IST shocks (mentioned above). For example, shocks that lower costs in the investment sector, such as advancements in computer processing or improvements in production methods, are of the IST type. In the second step, investment goods are transformed into productive capital before entering the production process. Shocks that hit this transformation process are known as shocks to the marginal efficiency of investment (or MEI shocks). For example, shocks that affect the dissemination of investment goods, such as new forms of transportation or troubles in the financial sector, are of the MEI type.

One of the most obvious impediments to acquiring capital is the credit needed to purchase investment goods. Disturbances in the credit market, including the recent financial crisis, affect the ability of firms to acquire capital, with consequences for the course of the economy. In their empirical work using U.S. data, Justiniano et al. (2011) identify MEI shocks as the primary source of business-cycle volatility, accounting for 60% of the variation in U.S. gross domestic product (GDP),

while IST shocks account for only a small fraction of that variance. Moreover, Schmitt-Grohé and Uribe (2012) estimate a closed-economy model that includes an extensive list of anticipated and unanticipated shocks. They conclude that MEI shocks are “estimated to explain a significant fraction of variation in output (28%) and investment (63%)” in the United States.<sup>5</sup> Given those empirical results, it is of interest to see how financial frictions change the dynamics of an international business-cycle model. It is worth noting that there is evidence that total factor productivity (TFP) and the RPI are interrelated in the United States [e.g., Schmitt-Grohé and Uribe (2011)]. These two variables are intimately related in our theory, since TFP shocks eventually lead to changes in the RPI.

As highlighted above, our model is capable of reproducing international co-movements in the RPI, despite the fact that innovations in TFP are not correlated across countries. To understand why the RPI is positively correlated across countries, consider an unexpected positive productivity shock in country 1 in period 1. Since this shock is persistent, households desire to increase their capital stock; hence, they demand more investment goods. In light of this, entrepreneurs increase the size of their projects. Given that their net worth is mostly predetermined, they must borrow more. This greater reliance on external funds raises agency costs, and the price of investment goods in terms of the final good shoots up. This tempers country 1’s demand for investment goods, leading to more units of that country’s intermediate good finding their way to the world market. This results in a more important depreciation of the TOT than in the BKK model. A significant drop in the price of country 1’s good prompts country 2 to buy more of it to raise consumption and investment (consumption smoothing). This increase in the demand for investment goods in country 2 pushes agency costs up, which raises the RPI in that country. These initial positive responses of the price of investment in both countries promote positive correlations in our simulations.

The remainder of this paper is organized as follows. Section 2 lays out our model with agency costs. We explain how we select parameter values in Section 3. Section 4 discusses the benchmark model’s implications. Section 5 covers a model with IST shocks and one with risk shocks. Sensitivity analysis is done in Section 6. Section 7 concludes.

## 2. BENCHMARK MODEL

Our model is based on the two-country, two-good international real business-cycle model of Backus et al. (1994). It consists of two countries that trade intermediate goods and financial assets. Each country has a large number of identical households, a large number of perfectly competitive and identical intermediate-goods producers, and a large number of perfectly competitive and identical final-goods producers. Each country produces a single intermediate good. Intermediate goods from both countries are needed to produce final goods. Final goods are not traded internationally and can be either consumed or used as inputs in the production of new physical capital.

We extend the BKK model by adding to each country (i) a financial sector that contains a large number of perfectly competitive and identical capital mutual funds (CMFs) and (ii) a set of entrepreneurs that are producers of physical capital in the economy. As in CF, the idiosyncratic productivity of entrepreneurs is private information, which gives rise to agency costs. These agency costs lead to endogenous changes in the RPI. The next few subsections provide a detailed description of the model.

**2.1. Intermediate-Goods Sector**

Let  $l$  index countries, where  $l \in \{1, 2\}$ . Within each country, there are a large number of identical and perfectly competitive intermediate-goods producers. Hence, we restrict our attention to a representative producer. Each country specializes in the production of an intermediate good. Country 1 (or the home country) produces intermediate good  $a$  (one can think of this as aluminum) and country 2 (or the foreign country) produces good  $b$  (one can think of this as bricks). These goods are traded across countries. We let  $q_{lt}$  ( $q_{lt}^*$ ) denote the price of the intermediate good produced in country  $l$  in terms of the final good in country 1 (country 2).

The production of intermediate goods requires capital and labor, which are assumed to be immobile across countries. The representative producer in country  $l$  rents the economy's entire capital stock<sup>6</sup> available at the beginning of period  $t$  ( $K_{lt}$ ) and hires the labor services supplied by households ( $H_{lt}$ ) to produce intermediate goods according to the production function

$$Y_{lt} = Z_{lt} K_{lt}^{\alpha_k} (H_{lt})^{\alpha_H}, \quad l = \{1, 2\}, \tag{1}$$

where  $0 < \alpha_k, \alpha_H < 1, \alpha_k + \alpha_H = 1$ , and  $Z_{lt}$  denotes an unanticipated stationary productivity shock. The representative intermediate good producer in country 1 maximizes

$$\Pi_{1t} = q_{1t} Y_{1t} - w_{1t} H_{1t} - r_{1t} K_{1t}, \tag{2}$$

subject to (1) for  $l = 1$ . The producer rents capital from both households and entrepreneurs at a rental rate  $r_1$  and hires household labor at the wage rates  $w_1$ . Owing to the constant returns-to-scale production function and perfectly competitive markets, the wage and rental rates of labor and capital are equal to their corresponding marginal product.

**2.2. Households**

Each country is populated by a large number of identical, infinitely lived households. The representative household in country  $l = \{1, 2\}$  has the following expected lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_{lt}^H - \psi H_{lt}^\nu X_{lt})^{1-\gamma}}{1-\gamma}, \quad 0 < \beta < 1, \nu > 1, \psi > 0, \tag{3}$$

where

$$X_{lt} = C_{lt}^{H\eta} X_{lt-1}^{1-\eta}, \quad 0 \leq \eta \leq 1. \tag{4}$$

We denote consumption by  $C_{lt}^H$  and labor supply by  $H_{lt}$ . Whether or not consumption and hours worked by the household are time-separable is determined by  $\eta$ , through the inclusion of  $X_{lt}$  in the household’s utility function. These preferences were first proposed by Jaimovich and Rebelo (2009). They allow us to dial up or down the strength of the wealth effect on the household’s labor supply decision. When  $\eta = 1$  (full wealth effect), these preferences become equivalent to the widely used preferences of King et al. (1988, hereinafter KPR). When  $\eta = 0$  (no wealth effect), they become equivalent to those in GHH.

The representative household in country 1 must satisfy the budget constraint

$$\begin{aligned} C_{lt}^H + P_{lt}I_{lt}^H + q_{1t}\Sigma_{s_{t+1}}M(s^t, s_{t+1})D_1(s^t, s_{t+1}) \\ = w_{lt}H_{lt} + r_{1t}K_{lt}^H + q_{1t}D_1(s^{t-1}, s_t) + PR_{1t}. \end{aligned} \tag{5}$$

The left side of the equation reflects the fact that the household uses its income for three purposes: to consume, to invest, and to insure. Households can increase their capital stock by purchasing  $I_{lt}^H$  units of new investment goods at a unit cost  $P_{lt}$ . Here,  $P_{lt}$  represents the price of new investment goods in country 1 relative to the price of that country’s final good. The price of new investment goods is taken as given by households (just like all other prices). Finally, households trade state-contingent (Arrow–Debreu) securities. We can imagine that households are the ones deciding how many units of intermediate goods to import or export. Accordingly, they will trade contingent claims with households in the other country to cover any trade-balance deficit. Let  $D_1(s^t, s_{t+1})$  be the quantity of contingent claims purchased by households in country 1 after history  $s^t$ , which pays one unit of good  $a$  in period  $t + 1$  when the state of the economy is  $s_{t+1}$ . We denote the price of these contingent claims (in units of good  $a$ ) by  $M(s^t, s_{t+1})$ .

The right side of (5) shows that the representative household income in period  $t$  consists of its labor income  $w_{lt}H_{lt}$ , rental income  $r_{1t}K_{lt}^H$ , payoffs  $D_1(s^{t-1}, s_t)$  from the relevant state-contingent security purchased in the previous period, and the collective profits of all domestic firms. The budget constraint of the representative household in country 2 is analogous.

The representative household capital accumulation equation is

$$K_{lt+1}^H = (1 - \delta)K_{lt}^H + I_{lt}^H, \tag{6}$$

where  $\delta$  is the depreciation rate. The representative household chooses the sequences of consumption, labor, contingent claims, and investment that maximize their lifetime utility (3), subject to (4)–(6), taking all prices as given.

### 2.3. Final-Goods Sector

Each country has a large number (measure one in each country) of identical and perfectly competitive final-goods<sup>7</sup> producers. Hence, we restrict our attention to

a representative final-goods producer. Each country specializes in the production of a nontraded final good, which is used for household and entrepreneurial consumption and as an input in the production of investment goods. Production of the final good requires both domestic and imported intermediate goods. Country 1's production of its final good is given by the following Armington (1969) aggregator:

$$G(a_{1t}, b_{1t}) = \left[ \kappa a_{1t}^{\frac{\sigma-1}{\sigma}} + (1 - \kappa) b_{1t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad 0 < \kappa < 1, \quad \sigma > 0. \quad (7)$$

When  $\kappa > 0.5$ , each country has a bias toward the domestically produced intermediate good. Here  $\sigma$  represents the elasticity of substitution between domestic and foreign goods.

The representative final-goods producer in country 1 chooses quantities  $a_1$  and  $b_1$  to maximize its profits given by  $G(a_{1t}, b_{1t}) - q_{1t}a_{1t} - q_{2t}b_{1t}$ . Similarly, the foreign final-goods producer chooses  $a_2$  and  $b_2$  to maximize  $G(b_{2t}, a_{2t}) - q_{1t}^*a_{2t} - q_{2t}^*b_{2t}$ , where

$$G(b_{2t}, a_{2t}) = \left[ \kappa b_{2t}^{\frac{\sigma-1}{\sigma}} + (1 - \kappa) a_{2t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \quad (8)$$

### 2.4. The Entrepreneurs

Each country is populated by an infinite number of entrepreneurs (measure one in each country). Entrepreneurs are not identical. The efficiency of the process employed by an entrepreneur to transform consumption goods (the final good in this model) into investment goods is specific to each entrepreneur and is determined by the random variable  $\omega$ . The latter is assumed to be independently and identically distributed across entrepreneurs, with a lognormal cumulative distribution function  $\Phi(\omega)$  and density function  $\phi(\omega)$ . An entrepreneur with productivity  $\omega$  transforms (within one period)  $i$  units of the consumption good into  $\omega i$  units of investment goods. The resources required by entrepreneurs to fund the investment projects come from their own net worth  $n$  (internal funds), as well as from funds borrowed (via intraperiod loans) from financial intermediaries (CMFs). Intermediaries obtain funds from households.

To introduce agency costs into our model, we assume that each entrepreneur's random productive potential,  $\omega$ , is private information. If a CMF wishes to observe an entrepreneur's level of productivity, it must incur a monitoring cost  $\mu i$ , which is measured in units of investment goods. Owing to this asymmetric information, there is a moral hazard problem between borrowers/entrepreneurs and their potential lenders. The model is set up such that entrepreneurs will always truthfully report their  $\omega$ .

An individual entrepreneur living in country  $l$  maximizes its expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} (\beta \Gamma)^t c_{lt}^E, \quad 0 < \Gamma < 1, \quad (9)$$

where  $c_{lt}^E$  is the entrepreneur’s level of consumption. The assumption  $0 < \Gamma < 1$  implies that entrepreneurs are relatively more impatient than households. This assumption, which is common in this literature, is made to avoid scenarios where entrepreneurs fully finance their investment projects by dramatically reducing their level of consumption.

Entrepreneurs face a budget constraint. An important component of that constraint is net worth. At the beginning of period  $t$ , an entrepreneur rents out its current capital stock (denoted by  $k_{lt}^E$ ) to local intermediate-goods producers, which generates a rental income  $r_{lt}k_{lt}^E$ . Then, the entrepreneur sells off all of its undepreciated capital stock  $(1 - \delta)k_{lt}^E$  to the local CMF, which pays  $P_{lt}(1 - \delta)k_{lt}^E$  units of consumption goods to the entrepreneur. After all these transactions, an entrepreneur in country  $l = \{1, 2\}$  has total net worth (measured in units of domestic consumption goods) of

$$n_{lt} = r_{lt}k_{lt}^E + P_{lt}(1 - \delta)k_{lt}^E. \tag{10}$$

In the next section, we outline the financial contract between entrepreneurs and the CMF. But for now, it is enough to state that an entrepreneur’s net worth is inversely related to the interest rate charged on its loan. Hence, a rational risk-neutral entrepreneur always chooses to sell off all of its undepreciated capital supply to bolster net worth. The income earned from the production of investment goods, discussed in the next subsection, constitutes the last element required to determine an entrepreneur’s budget.

*Financial contract and investment decision.* Since country  $l$ ’s entrepreneurs are dealing with country  $l$ ’s CMF exclusively (and vice versa), we momentarily omit the country subscript on variables to ease notation. Similarly, since loan contracts are entirely “resolved” within a period, we also omit time subscripts. Entrepreneurs are indexed by  $j$ . For example, entrepreneur  $j$ ’s productivity is denoted by  $\omega_j$ .

Asymmetric information in our model becomes relevant only when an entrepreneur’s net worth is low enough that it must partially rely on external financing from the CMF. In that context, an entrepreneur with net worth  $n_j$  that invests  $i_j$  must borrow  $(i_j - n_j)$  units of consumption goods from the CMF in order to start production. The sequence of events within a period is borrowed from CF:

- (i) Entrepreneur  $j$  and a CMF enter into a contractual agreement, whereby investment  $i_j$  is determined. Given entrepreneurial net worth  $n_j$ , the amount borrowed  $(i_j - n_j)$  is also determined.
- (ii) Productivity  $\omega_j$  is realized. Based on this realization, the entrepreneur decides whether to honor its contract. The latter decision rests on a threshold productivity level  $\bar{\omega}_j$  defined below. More specifically, if  $\omega_j \geq \bar{\omega}_j$ , the entrepreneur repays the CMF. Otherwise, the entrepreneur defaults on its loan, which triggers monitoring by the CMF and confiscation of all of the investment goods just produced by the entrepreneur.<sup>8</sup>
- (iii) Entrepreneurs that did not default make their consumption decision.



Despite the heterogeneity of the entrepreneurial population, asymmetric information implies that the CMF charges a common interest rate  $r^k$  on all funds borrowed by entrepreneurs. Thus, an entrepreneur that borrows  $(i_j - n_j)$  units of consumption goods from the CMF in any given period (and does not default) has to pay back  $(i_j - n_j)(1 + r^k)$  units of investment goods to the CMF in that same period.

As mentioned in point (ii), there exists a productivity threshold  $\bar{\omega}_j$  at which entrepreneur  $j$  is indifferent between defaulting or paying back its loan to the CMF. This indifference point is reached when the quantity of investment goods produced by the entrepreneur is exactly equal to the amount it has to pay back to the CMF. Hence,  $\bar{\omega}_j$  is the value of  $\omega_j$  for which

$$i_j \bar{\omega}_j = (i_j - n_j)(1 + r^k) \Rightarrow \bar{\omega}_j = (1 + r^k)(1 - n_j/i_j). \tag{11}$$

Any productivity realization  $\omega_j$  less than  $\bar{\omega}_j$  leads entrepreneur  $j$  to default.

Given the definition of  $\bar{\omega}$  provided above, we can now calculate entrepreneur  $j$ 's expected net income (before idiosyncratic risk is resolved, i.e. before  $\omega_j$  is observed) associated with the production of investment goods:

$$F_j^E \equiv P \left\{ \int_{\bar{\omega}_j}^{\infty} \omega i_j \phi(\omega) d\omega - [1 - \Phi(\bar{\omega}_j)](1 + r^k)(i_j - n_j) \right\}. \tag{12}$$

The first term in  $\{ \}$  calculates the expected output of investment goods when entrepreneur  $j$  does not default. From this quantity, we subtract the expected amount that must be paid back to the CMF. Recall that an entrepreneur that defaults on its loan sees its entire output confiscated by the CMF, so its net income is zero.

Using (11), expected income can be written as follows:

$$F_j^E = P i_j \underbrace{\left\{ \int_{\bar{\omega}_j}^{\infty} \omega \phi(\omega) d\omega - [1 - \Phi(\bar{\omega}_j)]\bar{\omega}_j \right\}}_{\equiv f(\bar{\omega}_j)} = P i_j f(\bar{\omega}_j). \tag{13}$$

Similarly, the expected income of the CMF in its dealings with entrepreneur  $j$  is given by

$$F_j^{CMF} \equiv P \left\{ \int_0^{\bar{\omega}_j} \omega i_j \phi(\omega) d\omega - \Phi(\bar{\omega}_j) \mu i_j + [1 - \Phi(\bar{\omega}_j)](1 + r^k)(i_j - n_j) \right\}, \tag{14}$$

where income is calculated in units of consumption goods. The first two terms in  $\{ \}$  calculates the expected amount the CMF confiscates when the entrepreneur defaults, net of the expected monitoring costs. The last term accounts for the expected payment made by entrepreneur  $j$  when it does not default.

Using (11), expected income can be written as follows:

$$F_j^{CMF} = P i_j \underbrace{\left\{ \int_0^{\bar{\omega}_j} \omega \phi(\omega) d\omega - \Phi(\bar{\omega}_j)\mu + [1 - \Phi(\bar{\omega}_j)]\bar{\omega}_j \right\}}_{\equiv g(\bar{\omega}_j)} = P i_j g(\bar{\omega}_j). \tag{15}$$

Quantities  $f(\bar{\omega}_j)$  and  $g(\bar{\omega}_j)$  refer to the fraction of expected net capital output going to the entrepreneur and the CMF, respectively. Assuming  $E[\omega] = 1$  (a normalization), one can easily show that

$$f(\bar{\omega}_j) + g(\bar{\omega}_j) = 1 - \Phi(\bar{\omega}_j)\mu. \tag{16}$$

So far, we have set up the general form of the contract, fully identified by the level of consumption goods used in investment production ( $i_j$ ), and the threshold  $\bar{\omega}_j$  with a gross interest rate charged on borrowed funds of

$$1 + r^k = \frac{\bar{\omega}_j}{1 - n_j/i_j}, \tag{17}$$

which is implied by (11).

Following CF, we assume that the lender is unaware of the entrepreneur’s previous history, eliminating any repeated game scenario. Recall our assumption that  $\omega$  is i.i.d. over time. Therefore, each contract is relevant for the current period only. From one of the many potential contracts, there exists an optimal contract that maximizes the expected income to the entrepreneur while leaving the lender indifferent between lending and retaining the necessary funds. Therefore, the optimal contract is the pair  $(i_j, \bar{\omega}_j)$  maximizing  $F_j^E$ , subject to

$$F_j^{CMF} \geq (i_j - n_j). \tag{18}$$

This optimization problem implies

$$i_j = \frac{n_j}{[1 - P g(\bar{\omega}_j)]} \tag{19}$$

and

$$P \left[ 1 - \Phi(\bar{\omega}_j)\mu + \frac{\phi(\bar{\omega}_j)\mu f(\bar{\omega}_j)}{f'(\bar{\omega}_j)} \right] = 1. \tag{20}$$

Since each entrepreneur takes the RPI as given, equation (20) implies that  $\bar{\omega}_j$  is an implicit function of the relative price  $P$ . Since the relative price is the same for all entrepreneurs, we have  $\bar{\omega}_j = \bar{\omega}$  for all  $j$ . Furthermore, given  $\bar{\omega}(P)$ , equation (19) determines the optimal amount of investment by the entrepreneur. This can be written as a function of the entrepreneur’s wealth, and the RPI as

$$i_j = \frac{n_j}{1 - P g(\bar{\omega}(P))} \equiv i(P, n_j). \tag{21}$$

Before turning to the consumption decision of an entrepreneur, we derive the aggregate amount of investment goods produced and supplied within a country. Entrepreneur  $j$ 's choice  $i(P, n_j)$  indicates the quantity of consumption goods the entrepreneur decides to use as input in the production of investment goods. The expected quantity of *investment goods* produced and supplied by entrepreneur  $j$ , net of monitoring costs when  $\omega_j < \bar{\omega}$ , is given by

$$i^s(P, n_j) \equiv i(P, n_j) [1 - \mu\Phi(\bar{\omega}(P))] = \frac{1 - \mu\Phi(\bar{\omega}(P))}{1 - Pg(\bar{\omega}(P))} n_j. \tag{22}$$

Since the price of capital  $P$  is taken parametrically, an entrepreneur's choice of  $i^s(P, n_j)$  is a linear function of its net worth  $n_j$ , which is convenient from the point of view of aggregation. Given our assumption of an infinite number of entrepreneurs (with measure unity), the law of large numbers implies that the above formula can be interpreted as the total amount of investment goods supplied by the population of entrepreneurs when total entrepreneurial net worth turns out to be  $n_j$ . Accordingly, denoting total entrepreneurial net worth by  $N$ , the supply of investment goods in the entire country is given by

$$I^S(P, N) = \frac{1 - \Phi(\bar{\omega}(P))\mu}{1 - Pg(\bar{\omega}(P))} N. \tag{23}$$

Bringing back country and time subscripts, the supply of investment goods in country  $l$  in period  $t$  is given by

$$I_{l,t}^S = \frac{1 - \Phi(\bar{\omega}_{lt})\mu}{1 - P_{lt}g(\bar{\omega}_{lt})} N_{lt}, \tag{24}$$

where  $\bar{\omega}_{lt}$  is a shorthand notation for  $\bar{\omega}(P_{lt})$ .

The model is set up such that an entrepreneur always wishes to fully disinvest its entire net worth to fund investment projects. Accordingly, one can calculate an entrepreneur's gross expected return on internal funds (ERIF) as the ratio of  $P_{lt}f(\bar{\omega}_{lt})i_{lt}$  (the expected income generated by capital production by an entrepreneur) to net worth  $n_{lt}$  (the amount of its own funds the entrepreneur places in capital production). Using equation (21), we write

$$\text{ERIF}_{lt} = \frac{P_{lt}f(\bar{\omega}_{lt})i_{lt}}{n_{lt}} = \frac{P_{lt}f(\bar{\omega}_{lt})}{1 - P_{lt}g(\bar{\omega}_{lt})}, \tag{25}$$

which is always greater than the return to external funds owing to agency costs.

*Budget constraint and consumption decision.* Here, we must take into account the heterogeneity across entrepreneurs. Let us first consider the case of an entrepreneur that draws a period- $t$  level of productivity  $\omega_{jt} < \bar{\omega}_t$ . The entrepreneur defaults on its loan contract and sees the CMF take away its entire output of investment goods  $\omega_{jt}i_{jt}$ . The entrepreneur, thus, has no income left to finance current consumption and no capital goods to carry into the next period.

Consider now the case of an entrepreneur that does not default ( $\omega_{jt} > \bar{\omega}_t$ ). The entrepreneur produces  $\omega_{jt}i_{jt}$  units of investment goods. Some of those,  $(1 + r_t^k)(i_{jt} - n_{jt})$ , go back to the CMF to honor the debt agreement. What is left is allocated between the (i) current-period consumption ( $c_{jt}^E$ ) and (ii) capital carried into the future period ( $k_{jt+1}^E$ ). Evidently, the choice of  $k_{jt+1}^E$  influences next-period net worth  $n_{jt+1}$ , which influences the expected income next period  $F_{jt+1}^E$ .<sup>9</sup> Therefore, when making its decision about  $k_{jt+1}^E$  in period  $t$ , the entrepreneur takes into account the following  $t + 1$  budget constraint:

$$\frac{c_{jt+1}^E}{P_{t+1}} + k_{jt+2}^E \leq i_{jt+1}f(\bar{\omega}_{jt+1}). \tag{26}$$

Updating the formulas for investment (21) and net worth (10) by one period in the future, we can rewrite the above budget constraint as

$$\frac{c_{jt+1}^E}{P_{t+1}} + k_{jt+2}^E \leq [r_{t+1}k_{jt+1}^E + P_{t+1}(1 - \delta)k_{jt+1}^E] \frac{f(\bar{\omega}_{t+1})}{1 - P_{t+1}g(\bar{\omega}_{t+1})}. \tag{27}$$

The solvent entrepreneur’s problem is to maximize expected lifetime utility (9), subject to its sequence of budget constraints. Assuming an interior solution, this problem yields the following Euler equation (now including country subscript  $l$ ):

$$P_{lt} = \beta\Gamma E_t \left\{ [r_{lt+1} + P_{lt+1}(1 - \delta)] \underbrace{\frac{P_{lt+1}f(\bar{\omega}_{lt+1})}{1 - P_{lt+1}g(\bar{\omega}_{lt+1})}}_{ERIF_{lt+1}} \right\}. \tag{28}$$

This Euler equation has a natural interpretation. The left side reflects the cost incurred by the entrepreneur to increase  $k_{jt+1}^E$  by one unit. The right side shows the expected discounted benefit of increasing  $k_{jt+1}^E$ . Such an increase raises the entrepreneur’s net worth next period by an amount given by the term in square brackets. This additional net worth will be entirely used to finance investment projects in  $t + 1$ , which is why it is multiplied by the gross ERIF [displayed in (25)].

To complete this section, we derive the aggregate budget constraint of the entire group of entrepreneurs. Recall that entrepreneur  $j$  has expected net income (in period  $t$ ) given by  $i_{jt}f(\bar{\omega}_{jt})$ . This takes into account (i) repayment of the loan to the CMF when the realized productivity turns out to be greater than  $\bar{\omega}_{lt}$  and (ii) the possibility that the entrepreneur has a level of productivity lower than  $\bar{\omega}_{lt}$ , in which case it does not pay back its loan to the CMF and sees its output of investment goods confiscated by the CMF. Now, let  $I_t$  denote aggregate “per-entrepreneur” investment in the production of investment goods. With an infinite number of entrepreneurs and given the linearity of the expected income formula given above, the law of large numbers implies that we can interpret  $I_t f(\bar{\omega}_t)$  as aggregate “per-entrepreneur” net income in period  $t$ . At the end of the period, the group of entrepreneurs sell  $C_t^E / P_t$  units of capital back to the CMF to fund

their period- $t$  consumption, where  $C_{lt}^E$  denotes “per-entrepreneur” consumption. The remaining units of investment goods are carried into the following period and constitute  $K_{l,t+1}^E$ , the “per-entrepreneur” stock of capital at the beginning of period  $t + 1$ . Then, aggregating over all the entrepreneurs’ budget constraints yields

$$\frac{C_{lt}^E}{P_t} + K_{l,t+1}^E \leq I_t f(\bar{\omega}_t). \tag{29}$$

Using the aggregate equivalent of (21) and aggregate “per-entrepreneur” net worth, we can rewrite the above budget constraint as (now adding back the country subscript  $l$ )

$$\frac{C_{lt}^E}{P_{lt}} + K_{l,t+1}^E \leq [r_{lt} K_{lt}^E + P_{lt}(1 - \delta)K_{lt}^E] \frac{f(\bar{\omega}_{lt})}{1 - P_{lt}g(\bar{\omega}_{lt})}. \tag{30}$$

### 2.5. The Representative Capital Mutual Fund

The final component of our model is the intermediation sector, where there are a large number of identical and perfectly competitive CMFs. All loanable funds in our model ultimately come from households who forward some of their savings to the representative CMF in return for investment goods. The CMF then lends to entrepreneurs these units of consumption goods received from households. We assume that all CMFs are risk-neutral. Is this assumption appropriate, given that CMFs allocate household savings and that households are risk-averse? It turns out that it is. First, note that there is no uncertainty over the length of the contracts between CMFs and entrepreneurs. Second, there are an infinitely large number of entrepreneurs and there is no aggregate uncertainty about the distribution of  $\omega$ . This implies that CMFs face no uncertainty about the total quantity of investment goods they are going to get from the population of entrepreneurs. Hence, CMFs can guarantee households that they will receive  $I^h$  units of investment goods in exchange for  $PI^h$  units of consumption goods. Thus, we can think of households as being risk-neutral in their willingness to loan funds.

We now describe all flows of consumption and investment goods going in and coming out of the financial intermediation sector.<sup>10</sup> We begin by accounting for the flow of consumption goods in country  $l$ . The items in the list below are arranged chronologically:

- Households pay  $P_{lt}I_{lt}^H$  to CMFs to purchase investment goods. The investment goods will be delivered to households later in period  $t$ .
- CMFs pay  $P_{lt}(1 - \delta)K_{lt}^E$  to entrepreneurs when purchasing their undepreciated capital stock.
- CMFs lend  $I_{lt} - N_{lt}$  to entrepreneurs as per their contracts. Aggregating up (21), we can write  $I_{lt} - N_{lt} = P_{lt}g(\bar{\omega}_{lt})I_{lt}$ .
- CMFs sell  $C_{lt}^E$  to entrepreneurs for their period- $t$  consumption.

The flow of consumption goods in country  $l$  implies that

$$\underbrace{P_{lt} I_{lt}^H}_{\text{inflow}} = \underbrace{P_{lt}(1 - \delta)K_{lt}^E + P_{lt}g(\bar{\omega}_{lt})I_{lt} + C_{lt}^E}_{\text{outflow}}. \tag{31}$$

Let us now look at flows of investment goods in country  $l$ , again arranged chronologically:

- CMFs buy  $(1 - \delta)K_{lt}^E$  from entrepreneurs.
- Solvent entrepreneur  $j$  pays back  $(1 + r_t^k)(i_{jt} - n_{jt})$  to CMFs to honor its debt contract. Alternatively, CMFs get  $(\omega_j - \mu) i_j$  from entrepreneur  $j$  when it defaults. In the aggregate, the law of large numbers implies that CMFs receive from the entire group of entrepreneurs  $g(\bar{\omega}_{lt})I_{lt}$  units of investment goods.
- Households receive  $I_{lt}^H$  units of investment goods they paid for earlier in the period.
- Entrepreneurs pay  $C_{lt}^E/P_{lt}$  for the purchase of  $C_{lt}^E$  units of consumption goods.

Therefore, it must be the case that

$$\underbrace{(1 - \delta)K_{lt}^E + g(\bar{\omega}_{lt})I_{lt} + \frac{C_{lt}^E}{P_{lt}}}_{\text{inflow}} = \underbrace{I_{lt}^H}_{\text{outflow}}. \tag{32}$$

Therefore, total profits (denominated in consumption goods) earned by the financial intermediaries in country  $l$  (inflows less outflows) are given by

$$\begin{aligned} \Pi_{lt}^{\text{CMF}} = & \left\{ P_{lt} I_{lt}^H + P_{lt} \left[ (1 - \delta)K_{lt}^E + g(\bar{\omega}_{lt})I_{lt} + \frac{C_{lt}^E}{P_{lt}} \right] \right\} \\ & - \left\{ P_{lt}(1 - \delta)K_{lt}^E + P_{lt}g(\bar{\omega}_{lt})I_{lt} + C_{lt}^E + P_{lt}[I_{lt}^H] \right\}, \end{aligned} \tag{33}$$

which is equal to zero, as implied by (31) and (32).

### 2.6. Shocks

The productivity shocks driving the TFP of intermediate-goods producers are assumed to follow a stationary first-order vector autoregressive process given by

$$\begin{bmatrix} \ln Z_{1t} \\ \ln Z_{2t} \end{bmatrix} = \begin{bmatrix} \rho_z & \rho_s \\ \rho_s & \rho_z \end{bmatrix} \begin{bmatrix} \ln Z_{1t-1} \\ \ln Z_{2t-1} \end{bmatrix} + \epsilon_{z,t}, \tag{34}$$

$$E(\epsilon_{z,t} \epsilon'_{z,t}) = \Sigma_z, \tag{35}$$

where the vector of innovations  $\epsilon_{z,t} = [\epsilon_{Z_{1,t}} \ \epsilon_{Z_{2,t}}]'$  is realized at the beginning of period  $t$  (before any decisions are made). Innovations are normally distributed and are independent over time. Parameters  $\rho_z$  and  $\rho_s$  govern the degrees of persistence

and international spillovers of the shocks, respectively.  $\Sigma_z$  denotes the variance–covariance matrix of  $\epsilon_z$ .

**2.7. Market-Clearing Conditions and Other Variables**

The total amount of intermediate goods produced in period  $t$  in country 1 is allocated as follows:

$$Y_{1t} = a_{1t} + a_{2t}, \tag{36}$$

where  $a_{1t}$  is the number of units of good  $a$  used in the production of country 1’s final good and  $a_{2t}$  is the number of units exported to country 2, where it is used in the production of that country’s final good. In the same way, country 2’s output of good  $b$  is allocated as

$$Y_{2t} = b_{1t} + b_{2t}, \tag{37}$$

where  $b_{2t}$  is the number of units of good  $b$  used in the production of country 2’s final good and  $b_{1t}$  is the number of units exported to country 1, where it is used in the production of that country’s final good.

The variable  $H_{lt}$ , which appears in the representative household’s utility function and the representative intermediate good producer’s production function, represents both total household labor supplied and total demand for household-type labor by intermediate firms in country  $l$ , thus clearing the labor market.

For capital markets to clear, capital supply  $K_{lt}^E + K_{lt}^H$  must equal the total capital demanded by the intermediate firm  $K_{lt}$ :

$$K_{lt} = K_{lt}^E + K_{lt}^H. \tag{38}$$

The representative household’s capital accumulation equation (6) and the overall entrepreneurial budget constraint (29) imply the accumulation equation

$$K_{lt+1}^H + K_{lt+1}^E = (1 - \delta)K_{lt}^H + I_{lt}^H + f(\bar{\omega}_{lt})I_{lt} - \frac{C_{lt}^E}{P_{lt}}. \tag{39}$$

Substituting out  $I_{lt}^H$  using equation (32) yields

$$K_{lt+1}^H + K_{lt+1}^E = (1 - \delta)K_{lt}^H + (1 - \delta)K_{lt}^E + [g(\bar{\omega}_{lt}) + f(\bar{\omega}_{lt})]I_{lt} + \frac{C_{lt}^E}{P_{lt}} - \frac{C_{lt}^H}{P_{lt}}. \tag{40}$$

Using (38) and (16) yields the aggregate capital accumulation equation<sup>11</sup>

$$K_{lt+1} = (1 - \delta)K_{lt} + [1 - \Phi(\bar{\omega}_{lt})\mu]I_{lt}. \tag{41}$$

As for the market clearing condition for final goods, we have

$$C_{1t}^H + C_{1t}^E + I_{1t} = G(a_{1t}, b_{1t}), \quad C_{2t}^H + C_{2t}^E + I_{2t} = G(b_{2t}, a_{2t}). \tag{42}$$

We assume complete markets in state-contingent claims available to the households to diversify country-specific risks. These assets are traded exclusively between domestic and foreign households. Market clearing requires that the following condition holds:

$$D_1(s^t, s_{t+1}) + D_2(s^t, s_{t+1}) = 0 \quad \text{for all } s_{t+1}. \tag{43}$$

When these assets are available, households perfectly diversify country-specific risk, and the equilibrium allocations are such that in all states of the world, the utility gained from an additional unit of good *a* in country 1 is exactly the same as the utility gained in country 2. The same holds for good *b*.

There are some key variables that we have not yet defined. Unless otherwise indicated, all of our consumption statistics are calculated using aggregate consumption measured as  $C_{1t} = C_{1t}^H + C_{1t}^E$ .

A country's TOT are defined as the price of that country's imports relative to that of its exports. When we refer to the TOT, we are implicitly referring to country 1's TOT, which are given by  $q_{2t}/q_{1t}$ . Using the representative final-goods producer's first-order conditions, we find

$$\text{TOT}_t = \frac{q_{2t}}{q_{1t}} = \frac{\partial G_1(a_{1t}, b_{1t})/\partial b_{1t}}{\partial G_1(a_{1t}, b_{1t})/\partial a_{1t}} = \frac{1 - \kappa}{\kappa} \left( \frac{a_{1t}}{b_{1t}} \right)^{\frac{1}{\sigma}}. \tag{44}$$

The GDP in country 1 in units of the final good is  $\text{GDP}_{1t} = q_{1t}Y_{1t}$ , while for country 2, we have  $\text{GDP}_{2t} = q_{2t}^*Y_{2t}$ .

Country 1's ratio of net exports to GDP is calculated as

$$\text{NX}_{1t} = \frac{q_{1t}a_{2t} - q_{2t}b_{1t}}{\text{GDP}_{1t}} = \frac{a_{2t} - \text{TOT}_t b_{1t}}{Y_{1t}}. \tag{45}$$

Following Raffo (2008), we also look at the trade balance at constant prices (prices are set to their steady-state values). For the home country, we have

$$\text{NXQTY}_{1t} = \frac{a_{2t} - T\bar{O}T b_{1t}}{Y_{1t}}, \tag{46}$$

where  $T\bar{O}T$  denotes the steady-state value of TOT.

### 3. BENCHMARK MODEL SOLUTION AND PARAMETER VALUES

We perform a second-order approximation around a symmetric deterministic steady-state equilibrium where both countries' net exports equal zero. The parameter values we select are commonly used in both agency costs and international real business-cycle literature. Just like BKK and their many offspring, we have in mind a model where one country represents the United States, the other represents a Euro area, and the period is one quarter of a year. Table 1 summarizes the parameter values we use in our benchmark model. The model is simulated



**TABLE 1.** Parameter values for benchmark calibration

Parameter	Value
$\beta$	0.985
$\gamma$	2
$\nu$	1.64
$\psi$	2.54658
$\eta$	0.1
$\rho_z$	0.95
$\rho_s$	0
correl( $\varepsilon_1, \varepsilon_2$ )	0
$\sigma_\varepsilon$	$\sqrt{0.000139}$
$\delta$	0.025
$\alpha_K$	1/3
$\alpha_H$	$1 - \alpha_K$
$\sigma$	0.5
$b_1/y_1 = a_2/y_2$	0.15
$\kappa$	0.9698
$\phi(\omega)$	lognormal( $1, \sigma_w^2$ )
$\sigma_w$	0.207
$\Gamma$	0.947
$\mu$	0.25

for 100,000 periods and the first 100 periods are discarded. The simulated data are passed through the Hodrick–Prescott filter (with a smoothing parameter of 1,600) before calculating the statistical moments listed in Tables 2 and 3. We now explain how we assign values to the parameters in our international business-cycle model.

### 3.1. Household Preferences

The value of the representative household's discount factor is  $\beta = 0.985$ . The household's coefficient of risk-aversion is set equal to  $\gamma = 2$ . The values for  $\beta$  and  $\gamma$  are well within the acceptable range found in the literature. The parameter  $\nu$  governs the curvature of the household's preferences. When preferences are of the GHH type ( $\eta = 0$ ), the labor supply elasticity is given by  $1/(\nu - 1)$ . Jaimovich and Rebelo (2009) set  $\nu = 1.4$ , Letendre and Luo (2007) use 1.7, while Johri, Letendre, and Luo (2011) use 3. Raffo (2010) points out that when preferences are of the GHH type,  $\nu = 1.64$  implies the same Frisch elasticity of labor supply (1.5) as is considered by BKK. We set  $\nu = 1.63$  but also consider a higher value in our sensitivity analysis. Parameter  $\eta$  influences the intensity of the wealth effect on household labor supply. GHH preferences are seen more and more in the international macro literature. An early example is Correia et al. (1995) and a recent example is Mandelman et al. (2011). In the spirit of Jaimovich and Rebelo

**TABLE 2.** Business-cycle statistics

	Data	Benchmark	No financial frictions	IST shocks	Risk shocks
SD(GDP)	1.92	1.92	2.02	1.92	1.92
correl(GDP <sub>t</sub> , GDP <sub>t-1</sub> )	0.86	0.76	0.71	0.71	0.76
RelSD(C)	0.75	0.57	0.56	0.55	0.57
RelSD(H)	0.61	0.53	0.57	0.58	0.53
RelSD(I)	3.27	3.54	3.57	3.80	3.54
RelSD(TOT)	1.12 <sup>R</sup> , 1.92	1.74	1.47	1.58	1.75
RelSD(NX)	0.27, 0.30 <sup>R</sup>	0.27	0.27	0.28	0.27
RelSD(RPI)	0.15, <sup>LW</sup> 0.39 <sup>LW</sup>	0.30	—	0.39	0.31
correl(GDP, C)	0.82	0.97	0.98	0.96	0.97
correl(GDP, H)	0.88	0.98	0.98	0.98	0.98
correl(GDP, I)	0.94	0.96	0.97	0.94	0.96
correl(GDP, NX)	-0.51 <sup>R</sup> , -0.37	-0.50	-0.54	-0.55	-0.50
correl(GDP, NXQTY)	-0.41 <sup>R</sup>	-0.15	-0.39	-0.36	-0.16
correl(GDP, TOT)	-0.20, 0.12 <sup>R</sup>	0.45	0.50	0.38	0.45
correl(GDP <sub>1</sub> , GDP <sub>2</sub> )	0.66	0.50	0.43	0.37	0.50
correl(C <sub>1</sub> , C <sub>2</sub> )	0.51	0.21	0.17	0.17	0.21
correl(I <sub>1</sub> , I <sub>2</sub> )	0.53	0.06	-0.04	-0.13	0.06
correl(H <sub>1</sub> , H <sub>2</sub> )	0.33	0.46	0.37	0.29	0.45
correl(RPI <sub>1</sub> , RPI <sub>2</sub> )	0.12 <sup>LW</sup>	0.25	—	0.12	0.25

Notes: SD(x) = standard deviation of x. RelSD(x) = SD(x)/SD(GDP). correl(x, y) = correlation between x and y. Numbers in the “Data” column are from BKK, except for the numbers with a superscript. Moments for the RPI (bearing a superscript LW) are based on our own calculations using data described in the Appendix. Numbers with a superscript R are from Raffo (2008). For both NX and NXQTY, moments are calculated in levels.

(2009), we select a small value for  $\eta$  so that we do not entirely shut down the income effect on labor supply like GHH preferences do. We use  $\eta = 0.10$  but also consider  $\eta = 1$ . Parameter  $\psi$  is set to ensure that the representative household spends 30% of its time working in the steady state.

### 3.2. External Shock Process

Parameter  $\rho_z$  represents the degree of persistence of productivity shocks. For quarterly models, estimated values for this parameter range from 0.906 [Backus et al. (1992)] all the way up to unity [e.g. Baxter and Crucini (1995)]. We pick the mid-point of this range and use  $\rho_z = 0.95$  in our benchmark case. Given that we want to study the model’s ability to produce positive international co-movements in the RPI, we do not allow for any connections between the two countries’ shocks in our benchmark case. Accordingly, we set  $\rho_s = 0$  and the correlation of  $\varepsilon_{Z_1}$  and  $\varepsilon_{Z_2}$  to zero. We consider other values for  $\rho_z$ ,  $\rho_s$ , and  $\text{correl}(\varepsilon_{Z_1}, \varepsilon_{Z_2})$  in our sensitivity analysis. The standard deviation of  $\varepsilon_{Z_1}$  and  $\varepsilon_{Z_2}$  is set to match output volatility in the United States.

**TABLE 3.** Sensitivity analysis

	Large risk shocks	$\eta = 1$	$\nu = 3$	$\sigma = 0.9$	$\rho_z = 0.97$ $\rho_s = 0.025$ $\text{correl}(\varepsilon_1, \varepsilon_2) = 0.29$
SD(GDP)	2.02	1.45	1.57	2.04	2.05
correl(GDP <sub>t</sub> , GDP <sub>t-1</sub> )	0.71	0.76	0.75	0.76	0.75
RelSD(C)	0.59	0.55	0.55	0.51	0.68
RelSD(H)	0.57	0.19	0.30	0.53	0.54
RelSD(I)	3.59	3.63	3.64	3.42	2.89
RelSD(TOT)	1.93	2.01	2.00	1.14	1.34
RelSD(NX)	0.27	0.27	0.28	0.14	0.21
RelSD(RPI)	0.72	0.29	0.31	0.28	0.24
correl(GDP, C)	0.96	0.97	0.97	0.97	0.99
correl(GDP, H)	0.97	0.94	0.99	0.99	0.98
correl(GDP, I)	0.96	0.97	0.97	0.98	0.93
correl(GDP, NX)	-0.54	-0.50	-0.51	-0.53	-0.40
correl(GDP, NXQTY)	-0.33	0.04	0.01	0.08	0.16
correl(GDP, TOT)	0.21	0.47	0.48	0.51	0.36
correl(GDP <sub>1</sub> , GDP <sub>2</sub> )	0.37	0.50	0.48	0.33	0.68
correl(C <sub>1</sub> , C <sub>2</sub> )	0.07	0.17	0.14	0.32	0.67
correl(I <sub>1</sub> , I <sub>2</sub> )	-0.05	0.08	0.06	0.01	0.07
correl(H <sub>1</sub> , H <sub>2</sub> )	0.22	0.72	0.47	0.33	0.66
correl(RPI <sub>1</sub> , RPI <sub>2</sub> )	0.25	0.24	0.21	0.29	0.26

### 3.3. Intermediate Goods

We set the capital share to  $\alpha_K = 1/3$ . We preserve a constant returns-to-scale set up by having  $\alpha_H = 1 - \alpha_K$ .

### 3.4. Final-Goods Production

The production of the final good requires both domestically produced and imported intermediate goods. Recall that  $\sigma$  denotes the elasticity of substitution between domestic and imported intermediate goods and that  $\kappa$  determines home bias. Values of  $\sigma$  employed in the literature vary greatly. For example, BKK, Chari et al. (2002), and Raffo (2008) use 1.5. Heathcote and Perri (2002) use time-series data and derived an estimated elasticity of 0.9. Mandelman et al. (2011) use 0.62. Others, such as Hooper et al. (2000), estimate a short-run trade elasticity in the United States of around 0.6 and less for other G7 countries. Even short-run trade elasticities as low as 0.22, as used in Taylor (1993), have been suggested, inferring that traded goods are highly complementary. Corsetti et al. (2008), Benigno and Thoenissen (2008) and Raffo (2010) all allow for an elasticity of substitution at or

below 0.5.<sup>12</sup> In line with the current research, we use an elasticity of substitution  $\sigma = 0.5$ . We also use Heathcote and Perri's (2002) estimated elasticity in our sensitivity analysis. As for home bias, we set  $\kappa$  such that the share of imports to total intermediate good output (denoted  $im$ ) is always equal to 0.15, as in BKK, among others. With import share  $im$ , and the value assigned to  $\sigma$ , this implies a home bias of

$$\kappa = \left[ 1 + \left( \frac{1 - im}{im} \right)^{(1/\sigma)} \right]^{-1}, \tag{47}$$

which is the same for both countries.

### 3.5. Stochastic Entrepreneurial Productivity

Recall that an entrepreneur is given a productivity level  $\omega$  drawn from a distribution  $\phi(\omega)$ . These idiosyncratic productivities must be nonnegative, and could range in value from 0 to  $\infty$ . The baseline model assumes that the distribution of entrepreneurial productivity is a lognormal distribution with a mean of 1. Accordingly, we use

$$\phi(\omega) = \frac{1}{\omega\sqrt{2\pi\sigma_\omega^2}} e^{-\frac{(\ln(\omega) - \mu_\omega)^2}{2\sigma_\omega^2}}, \quad \Phi(\omega) = \frac{1}{2} + \frac{1}{2} \text{ERF} \left[ \frac{\ln(\omega) - \mu_\omega}{\sqrt{2\sigma_\omega^2}} \right], \tag{48}$$

where  $\mu_\omega$  and  $\sigma_\omega$  refer, respectively, to the mean and standard deviations of the variable's natural logarithm.

When an entrepreneur's realized productivity is below the threshold  $\bar{\omega}$ , it defaults on its contract, which prompts monitoring by the CMF. In the aggregate, the monitoring costs incurred by the CMF equal  $\mu Pi \Phi(\bar{\omega})$ . These losses can be anything from legal fees to lost sales. Given the broad interpretation of what could be included in these losses, the possible range for the monitoring cost  $\mu$  is quite broad. Carlstrom and Fuerst (1997) give a possible range for the value of  $\mu$  from as low as 0.2 to as high as 0.36. For ease of comparison, we adopt the same monitoring cost as them, setting  $\mu$  equal to 0.25.

We choose the standard deviation of entrepreneurial productivity  $\sigma_\omega$  to match the 0.974 quarterly bankruptcy rate, as reported by Fisher (1999). We follow CF and set  $1/\Gamma$  equal to the return on internal investment. Notice that, when there are agency costs, the internal rate of return is always greater than the rate of return on external funds due to the risk premium charged on borrowed funds. If the discount factor  $\Gamma$  is too high, the entrepreneur continues to accumulate capital until the point it is self-financed, and there would be no agency costs in the steady state. If the discount rate is too low, the entrepreneur heavily discounts future consumption by opting for current consumption, in which case the entrepreneur chooses to hold no capital in the steady state. Therefore, in order for there to be a

stable equilibrium, the internal rate of return must equal  $1/\Gamma$  or

$$P_l \frac{f(\bar{\omega})}{1 - P_l g(\bar{\omega})} = \frac{1}{\Gamma}. \quad (49)$$

Accordingly, we set  $\Gamma = 0.947$ . This value implies an entrepreneurial discount factor of  $\beta\Gamma = 0.937$ . The steady-state relative price of capital  $\bar{P}$  is equal to 1.024, with the entrepreneur's share of investment production  $f(\bar{\omega}) = 0.39$ .

## 4. RESULTS: BENCHMARK MODEL

### 4.1. Business-Cycle Statistics

Table 2 contains the statistical moments produced by our benchmark model. The "Data" column reproduces the moments reported in BKK. The column also displays the statistics pertaining to the TOT and net exports reported in Raffo (2008) (indicated with a superscript *R*). Finally, we calculated statistics related to the RPI using the data described in the Appendix. In the "Data" column, the statistics we calculated are identified by a superscript *LW*. We report two numbers for the relative standard deviation of the RPI. Both are calculated using U.S. data. When using quality-adjusted data to calculate the RPI, its relative standard deviation is 0.39. When no quality adjustments are made, the relative standard deviation of the RPI is 0.15. Since we do not have quality-adjusted data for our aggregate of the Euro area, we use unadjusted data to calculate the correlation between the RPI-United States and RPI-Europe. All statistics reported are calculated using Hodrick–Prescott (HP)-filtered data (smoothing parameter equal to 1,600).

Our model matches exactly the standard deviation of GDP, since the calibration of the variance of technology shocks targeted that moment. The first autocorrelation of GDP is large and positive (0.76 vs. 0.86 in the data). The "within-country" moments are broadly consistent with the data. Consumption and hours are less volatile than GDP, while investment is significantly more variable than GDP. Those three variables are very highly correlated with GDP. The entirely endogenous RPI appearing in the benchmark model is a little less variable (relative to GDP) as the quality-adjusted RPI in the United States (0.30 vs. 0.39).

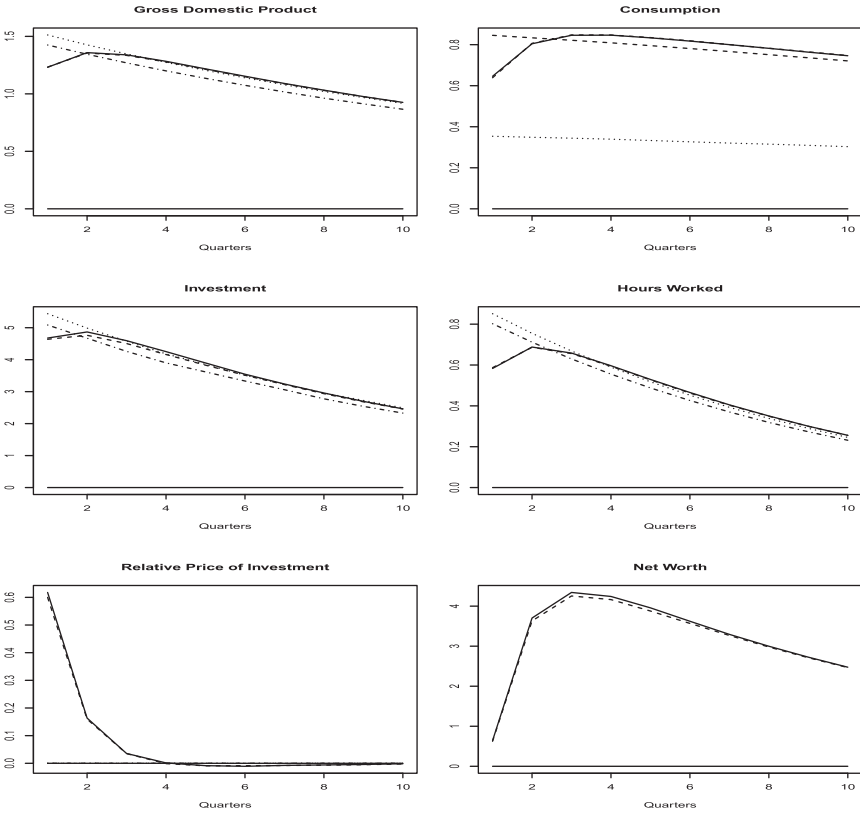
The model is also doing a good job at producing realistic international business cycles. TOT are highly volatile. Their relative standard deviation (1.74) falls in the range of estimates reported in the literature (1.12 in Raffo and 1.92 in BKK). The same thing holds for net exports. The correlation of net exports with GDP (−0.50) is in the range reported in the literature (−0.51 in Raffo and −0.37 in BKK), while net exports at constant prices are not as countercyclical as in the data (Raffo reports a correlation with GDP of −0.41, and in our model, it is −0.15). All cross-country correlations are positive but do not numerically match the corresponding numbers in the data. It is true that the cross-country correlations of investment and consumption are rather low. However, one has to remember that as opposed

to what is commonly done in the literature, productivity shocks are not correlated across countries in our model.

As is well known, the BKK model struggles to replicate some key empirical regularities, most notably, the positive correlation of output, consumption, labor, and investment across countries, as well as the volatility of the TOT. A number of extensions have been proposed to address these shortcomings. A recent example is Raffo (2008), who shows that using GHH preferences enables the model to produce a positive cross-country correlation of labor. A detailed sensitivity analysis focussing on international correlations and the volatility of TOT can be found in Heathcote and Perri (2002). They show that lowering the elasticity of substitution between intermediate goods ( $\sigma$ ) and the degree of spillovers in productivity shocks raises the international correlations while also increasing moderately the variance of the TOT. Our paper contributes to that strand of the international business-cycle literature by showing how financial frictions, à la Carlstrom and Fuerst, help address (to some extent) some of the glaring shortcomings of the canonical BKK model.

We document the effect of financial frictions by comparing our benchmark model to a special case of the model where financial frictions have been removed. The moments implied by the latter model are found in Table 2 under the column titled “No financial frictions.”<sup>13</sup> Notice how the cross-country correlations all increase a little and how the standard deviation of the TOT also increases when financial frictions are added to the model.<sup>14</sup> While these changes are not spectacularly large, notice that they occur in a setup where the standard deviation of the RPI relative to that of output in the model is about three-quarters of its quality-adjusted counterpart in our U.S. data (0.30 vs. 0.39). Hence, the model has the potential to do better.

A central contribution of our paper is to propose a model where the cross-country correlation of the RPI is determined entirely endogenously. As a result of international trade in intermediate goods, the model delivers a small positive cross-country correlation for the RPI, which is close to what is seen in the U.S./Euro-area data. Similar to Dmitriev and Roberts (2012), our model relies on a low wealth effect on labor supply in order to help bolster cross-country correlations. However, unlike Dmitriev and Roberts (2012), our model does not rely on cross-country correlations in neutral technology, nor on high levels of habit formation in consumption in order to generate positive cross-country correlations in investment. Furthermore, with a low elasticity of labor supply ( $\eta = 0.1$ ), along with a low elasticity of substitution between domestic and foreign-produced intermediates ( $\sigma = 0.5$ ), our model is capable of generating positive cross-country correlations in hours worked. If the wealth effect on labor supply, as well as the elasticity of substitution between domestic and foreign-produced intermediates, were increased simultaneously, as is the case under Heathcote and Perri’s (2002) choice of calibration, cross-country correlations in hours worked would reduce significantly. In the next section, we use impulse response functions to explain how the model generates this positive correlation.

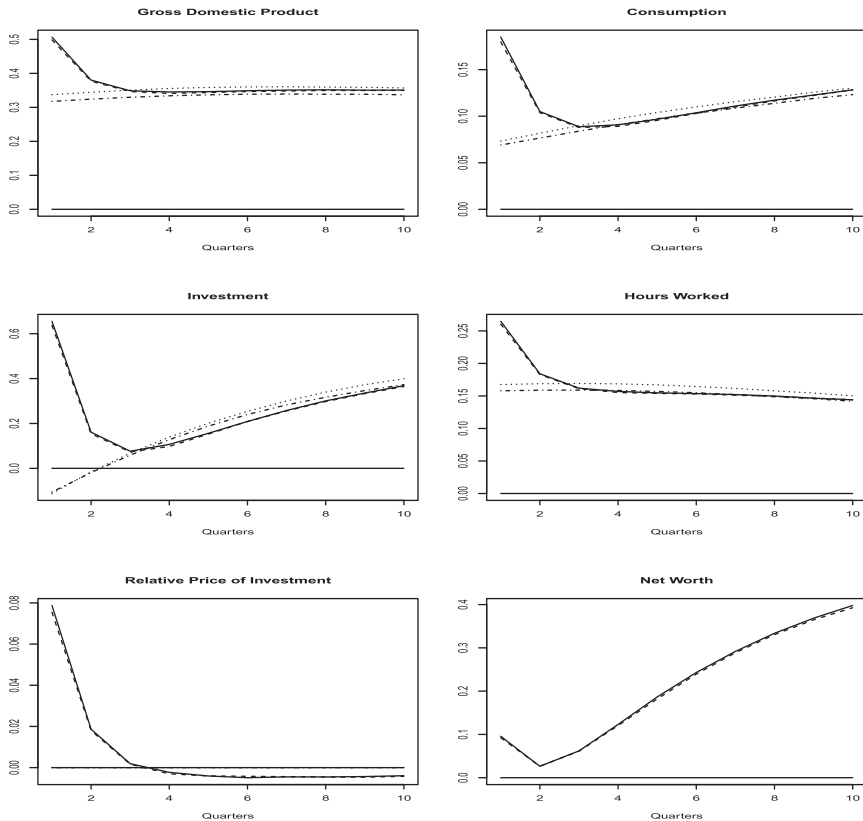


**FIGURE 1.** Home country responses to a TFP shock in the home country. Impulse responses to a one-standard-error innovation to  $\epsilon_t^Z$  for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and the IST model (dot-dashed). Each variable is measured as a percent deviation from their steady state.

### 4.2. Impulse Response Functions

The parameter values used are exactly the same as those used in our benchmark case (see Table 1). In all figures discussed below, the variables are represented in terms of percent deviations from the steady state and have not been filtered in any way. The economy is in the steady state in period 0 and a shock hits one of the countries in period 1.

Figures 1–3 plot the responses to a 1% unexpected increase in the home-country’s total factor productivity ( $Z_1$ ) in period 1. The solid line corresponds to the benchmark model and the dotted line to the no-financial-frictions model (hereinafter NFF). Recall that our stochastic process is such that the first-order autocorrelation of  $Z_1$  is 0.95. Therefore, the total factor productivity in country 1

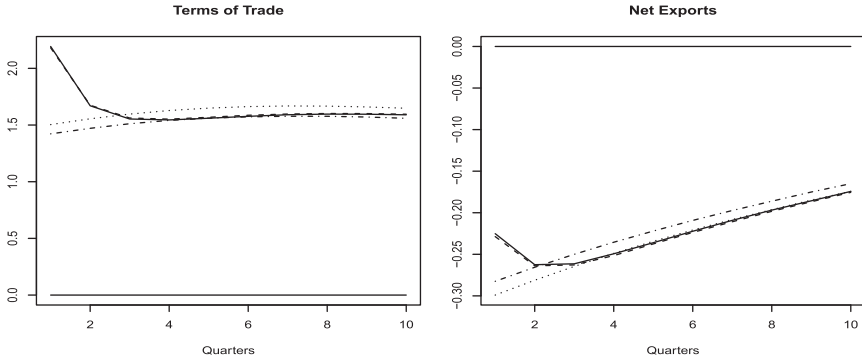


**FIGURE 2.** Foreign country responses to a TFP shock in the home country. Impulse responses to a one-standard-error innovation to  $\epsilon_t^Z$  for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and the IST model (dot-dashed). Each variable is measured as a percent deviation from their steady state.

remains above its steady-state value for several periods (half-life of 14 periods). Output, hours worked, investment, and consumption in both countries increase in response to a shock in country 1 and have a prominent hump shape.<sup>15</sup> The shape of these responses is not surprising in light of the work of Carlstrom and Fuerst (1997).

Figure 1 shows that the response of investment in the home country is slightly weaker when financial frictions are present. Interestingly, the sign of the initial response of investment in the foreign country differs in the two models (Figure 2). We witness a small positive response in our benchmark model and a slightly negative response in the NFF model. Figure 3 shows the responses of the TOT and net exports. The initial response of the home-country TOT is noticeably larger





**FIGURE 3.** Responses to a TFP shock in the home country: Home Country’s TOT and net exports. Impulse responses to a one-standard-error innovation to  $\epsilon_t^Z$  for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and the IST model (dot–dashed). Each variable is measured as a percent deviation from their steady state.

in the benchmark model than in the NFF model. As the discussion below makes clear, the higher volatility of the TOT in the benchmark model and the response of investment are intimately related. We now provide some economic intuition connecting the model to the impulse responses.

Recall that productivity shock  $Z_1$  appears in the production function of intermediate good producers in country 1 and nowhere else. These producers use capital and labor to produce good  $a$ . A positive shock to  $Z_1$  increases the marginal product of capital and labor, which drives up the demand for those factors of production. Hence, wages and rents on labor and capital services increase. Since entrepreneurs supply capital to the intermediate-goods producer, the increased return on capital generates an immediate increase in the entrepreneurs’ net worth [see equation (10)]. Since the bulk of an entrepreneur’s net worth is determined by the value of their capital stock (a state variable), the overall increase in net worth is relatively small in the period of the shock. Figure 1 shows that net worth increases by about 0.5% in period 1, which is much smaller than the peak increase in net worth (about 4% in period 3).

Responding to an increase in the return to physical capital, households desire to acquire more capital goods. The slow response of net worth implies that entrepreneurs who want to increase the size of their projects need more external financing. As a result, agency costs and, hence, the price of investment increase (see Figure 1). Since the return to internal funds (25) is increasing in  $P$ , entrepreneurs respond to the increased demand for capital goods by significantly reducing their consumption (it falls 25% below its level in steady state) in response to the TFP shock, in order to bolster internal funds. Consequently, entrepreneurial net worth increases significantly in period 2. As entrepreneurial wealth increases, entrepreneurs can respond to the increased demand for capital while relying less

on external financing. This drives down both agency costs and the RPI in period 2, making for a short-lived increase in the RPI. One may wonder why  $I_2$  increases in the model with financial frictions, given that the price of investment  $P_2$  increases in response to the shock, whereas that very same price is constant in the model without financial frictions. The answer lies in the response of another relative price: the TOT. Given the increase in  $P_1$ , the home country faces higher costs of producing investment goods in our model than in the NFF model; hence, it desires to invest less. Therefore, country 1 needs fewer units of final goods, which means it requires fewer units of their own intermediate good. Hence, a larger quantity of good  $a$  ends up being supplied to the world markets. This leads to a sharper fall in the price of that good and, hence, a more important increase in country 1's TOT (as made clear in Figure 3). Figures 1 and 3 show a spike in the TOT at the very same time that  $P_1$  spikes.

When the price of good  $a$  plummets, country 2 sees an opportunity to purchase a greater quantity of it, which can then be bundled with its own intermediate good to raise the quantity of final goods produced there. Being consumption smoothers, households in country 2 consume some of that additional quantity of final goods and invest some of it in physical capital to raise future consumption. Raising the amount of final goods in country 2 requires that intermediate producers increase their output of good  $b$  since goods  $a$  and  $b$  are not perfect substitutes. Given that  $Z_2$  does not change, more output of good  $b$  requires more labor and capital. The stronger demand for factors of production raises wage and rental rates. Consequently, the wealth of country 2 entrepreneurs increases but not enough to match the increase in the size of their projects. Thus, they must rely more heavily on external funding, which raises agency costs in country 2, as well as  $P_2$  (see Figure 2).

Overall, our theoretical setup suggests that agency costs can be transmitted from one country to another through trade in goods. This endogenous transmission channel produces a positive cross-country correlation in the RPI.

## 5. ALTERNATIVE MODELS

### 5.1. Model with an Exogenous RPI (IST Model)

In this section, we look at a model where the RPI is exclusively driven by IST shocks. We consider this model for two reasons: first, to provide a direct comparison of a model where the RPI is entirely endogenous (our benchmark model) to a model where it is entirely exogenous and, second, to contribute to the debate about the role played by IST shocks in international business-cycle models.

We add IST shocks (denoted by  $V_{1t}$  and  $V_{2t}$ ) to our model without financial frictions. That involves replacing the resource constraints shown in equation (42) with  $C_{1t}^H + C_{1t}^E + \frac{I_{1t}}{V_{1t}} = G(a_{1t}, b_{1t})$  and  $C_{2t}^H + C_{2t}^E + \frac{I_{2t}}{V_{2t}} = G(b_{2t}, a_{2t})$ , and adding

to the model the bivariate vector autoregressive process

$$\begin{bmatrix} \ln V_{1t} \\ \ln V_{2t} \end{bmatrix} = \begin{bmatrix} \rho_v & 0 \\ 0 & \rho_v \end{bmatrix} \begin{bmatrix} \ln V_{1t-1} \\ \ln V_{2t-1} \end{bmatrix} + \epsilon_t, \quad (50)$$

$$E(\epsilon_t \epsilon_t') = \Sigma, \quad (51)$$

where the vector of innovations is now  $\epsilon_t = [\epsilon_{v1t}, \epsilon_{v2t}]'$ . The bivariate autoregression above is estimated using U.S. and Euro-area data to determine the degree of spillovers and persistence of the HP-filtered RPI in both countries.<sup>16</sup> With estimated values of 0.84 and 0.72 for the persistence of technology in the United States and Euro area, respectively, we chose a value for the persistence parameter  $\rho_v$  of 0.78, the average of the two estimates. Off-diagonal elements were found to be not statistically different from zero. The standard deviations of  $\epsilon_{v1t}$  and  $\epsilon_{v2t}$  and their correlation are calibrated to ensure that the model with IST shocks is consistent with the relative standard deviation of the quality-adjusted RPI (0.39) and the cross-country correlation of RPI reported in Table 2. Finally, since the volatility of GDP increases when we include IST shocks, we lower the standard deviation of TFP shocks to ensure that the model produces a standard deviation of GDP equal to 1.92.<sup>17</sup> All other parameters retain the values reported in Table 1.

Focussing exclusively on the moments produced by the benchmark and IST models (Table 2), one could argue that the overall fit of the IST model is just as good as that of our benchmark model. On the one hand, the benchmark model generates more variance in the TOT and produces cross-country correlations of GDP and investment that are significantly closer to the data. On the other hand, the IST model produces a more countercyclical real trade balance (NXQTY) and a cross-country correlation of hours worked closer to the data. Both models produce a realistic variance in the RPI and a realistic cross-country correlation of RPI. Given that the RPI is entirely endogenous in the benchmark model, we conclude that it dominates the IST model.

A question that has been debated in the international business-cycle literature is the importance of IST shocks. Raffo (2010), using an ad-hoc calibration of the IST shocks process, find that these shocks “resolve” the so-called quantity and price anomalies first underlined by BKK. Mandelman et al. (2011) estimate the process for IST shocks using RPI data and do not find that IST shocks resolve the quantity and price anomalies. The moments reported in Table 2 show that adding IST shocks to our model without financial frictions improves somewhat the volatility of the TOT and their correlation with GDP. However, the cross-country correlations of GDP and investment worsen when we add IST shocks. Overall, it is far from clear that IST shocks help international business-cycle models match the data better. Our benchmark model fits, at least, as well as the IST model and as the superior feature that changes in RPI are actually explained by the model.

### 5.2. Model with Risk Shocks

Christiano et al. (2014) find that risk shocks play a significant role in explaining the U.S. business cycle. Our analysis of risk shocks contributes to the literature by providing a detailed account of the implications risk shocks have in an international environment. Risk shocks are incorporated into our benchmark model by making the variance of the distribution of entrepreneurial productivity ( $\omega$ ) a random variable. A positive risk shock in country  $i$  leads to a dispersion in the cross-sectional distribution of entrepreneurial productivity in that country. This effectively increases the level of uncertainty for lenders, as the range of possible values for  $\omega$  increases.

Risk shocks follow a stationary first-order vector autoregressive process given by

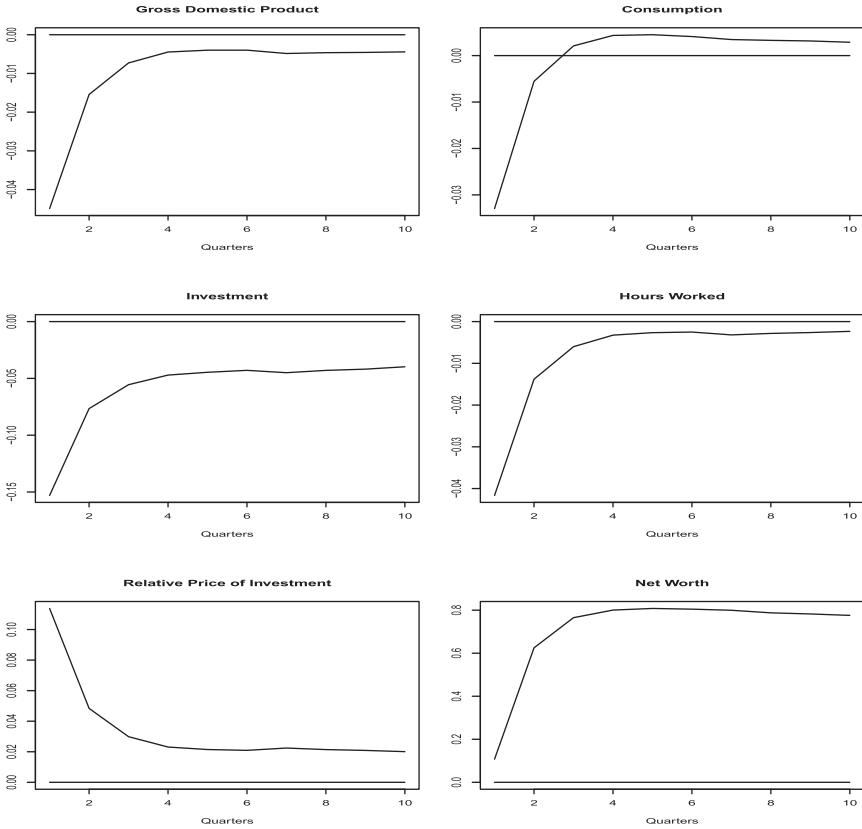
$$\begin{bmatrix} \ln(\sigma_{\omega_1,t}/\sigma_\omega) \\ \ln(\sigma_{\omega_2,t}/\sigma_\omega) \end{bmatrix} = \begin{bmatrix} \rho_\sigma & 0 \\ 0 & \rho_\sigma \end{bmatrix} \begin{bmatrix} \ln(\sigma_{\omega_1,t-1}/\sigma_\omega) \\ \ln(\sigma_{\omega_2,t-1}/\sigma_\omega) \end{bmatrix} + \epsilon_{\sigma_\omega,t}, \tag{52}$$

$$E(\epsilon_{\sigma_\omega,t} \epsilon'_{\sigma_\omega,t}) = \Sigma_\omega, \tag{53}$$

where the vector of innovations  $\epsilon_{\sigma_\omega,t} = [\epsilon_{\sigma_{\omega_1},t} \ \epsilon_{\sigma_{\omega_2},t}]'$  is realized at the beginning of period  $t$  (before any decisions are made). Innovations are normally distributed and are independent over time. Parameter  $\rho_\sigma$  governs the degree of persistence of risk shocks, while  $\sigma_\omega$  denotes the steady-state value for the standard deviation of  $\log(\omega)$ .  $\Sigma_\omega$  denotes the variance–covariance matrix of  $\epsilon_{\sigma_\omega,t}$ .

Risk shocks are calibrated using data on the spread/risk premium between Baa-rated corporate bond rates and 10-year-ahead U.S. government bond rates, measured in levels. We use these data over the period 1985Q1–2015Q1 to calculate a risk premium. Then, we HP-filtered that risk premium series and then calculated its first-order autocorrelation (0.77) and variance (0.5). Our model does not produce much endogenous persistence in the risk premium paid by the entrepreneurs, so we set  $\rho_\sigma = 0.99$ . The variance of  $\epsilon_{\sigma_{\omega_1},t}$  was set to 0.000145 to match the variance of the risk premium series we calculated. We want to study the model’s ability to produce positive international co-movements in the RPI; hence, we do not allow for any correlation, nor spillovers in risk shocks across countries. All other parameters are set as in the benchmark model.

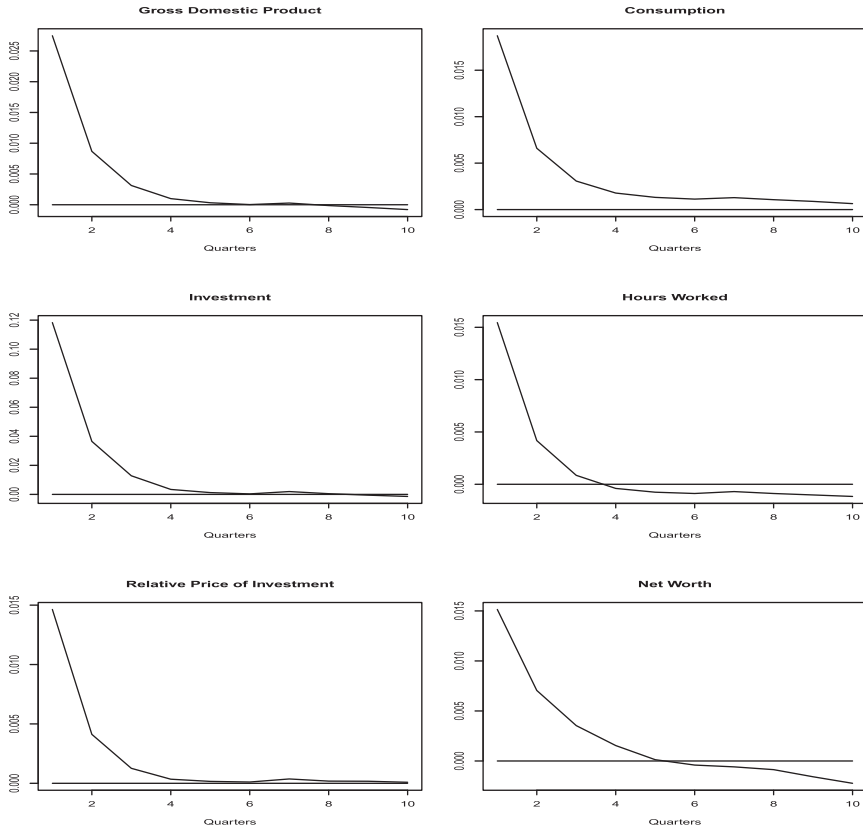
We begin our analysis of risk shocks by looking at impulse response functions. Figures 4–6 plot responses to a 1% unexpected increase in the standard deviation of  $\log(\omega)$  in the home country in period 1. Recall that a positive risk shock leads to a spread in the distribution of entrepreneurial productivity. This leads to a rise in the productivity threshold  $\bar{\omega}$ , separating those that declare bankruptcy from those that choose to continue operating. The increase in this threshold leads to a rise in the proportion of entrepreneurs declaring bankruptcy. With the subsequent rise in agency costs, the RPI increases in the home country (see Figure 4). Consequently, we see a drop in investment, followed by a decline in consumption and output in the home country. As with a technology shock, there is a surge in the



**FIGURE 4.** Home country responses to a risk shock in the home country. Impulse responses to a one-standard-error innovation to  $\epsilon_t^{Risk}$  for the benchmark model. Each variable is measured as a percent deviation from their steady state.

quantity of the home-country’s intermediate goods in the global market, leading to a deterioration of the home-country’s TOT (see Figure 6). This leads to a rise in investment, output, consumption, and hours worked in the foreign country, as they take advantage of the declining price of the home-country’s intermediate goods. The rise in investment in the foreign country gives rise to agency costs, leading to an increase in the RPI. As with a technology shock, trade in intermediate goods propagates a spike in the home-country RPI to the foreign-country RPI. However, unlike technology shocks, risk shocks produce responses of GDP, consumption, hours, and investment that are of opposite signs across countries.

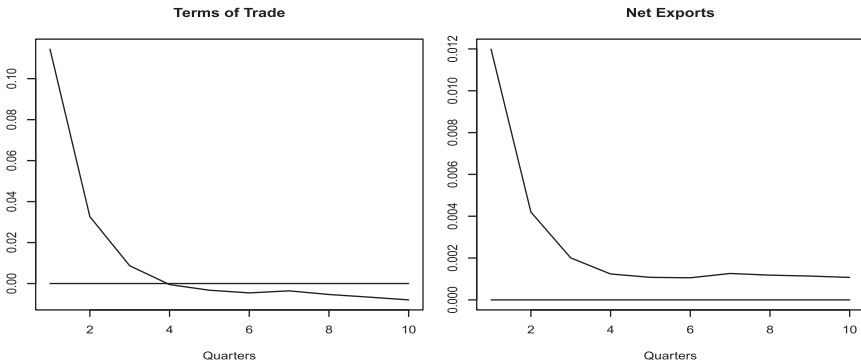
The last column in Table 2 displays the moments generated by the risk-shocks model. It is very clear that risk shocks of the size we include in our model do very little compared to technology shocks. For all practical purposes, the numbers implied by the risk-shocks model are identical to those implied by the benchmark



**FIGURE 5.** Foreign country responses to a risk shock in the home country. Impulse responses to a one-standard-error innovation to  $\epsilon_t^{Risk}$  for the benchmark model. Each variable is measured as a percent deviation from their steady state.

model. In order to see which of the moments are most influenced by risk shocks, we run the model once more after multiplying the variance of risk shocks by 100. The results are reported in the first column of numbers in Table 3. Given our discussion of impulse responses above, it is not surprising to see that risk shocks have their most significant impacts on the variance of the RPI, on the TOT, and on the cross-country correlations, which all become smaller (except that of RPI). Given the high significance placed by Christiano et al. (2014) on risk shocks in generating realistic business cycles, this presents a challenge to the current line of research.

Given the relative importance of risk shocks in the closed-economy model of Christian et al. (2014), the lack of importance of risk shocks in our open-economy model is intriguing. This result, however, appears to be due to our decision to separate the entrepreneurs consumption/savings decisions from that of the household.

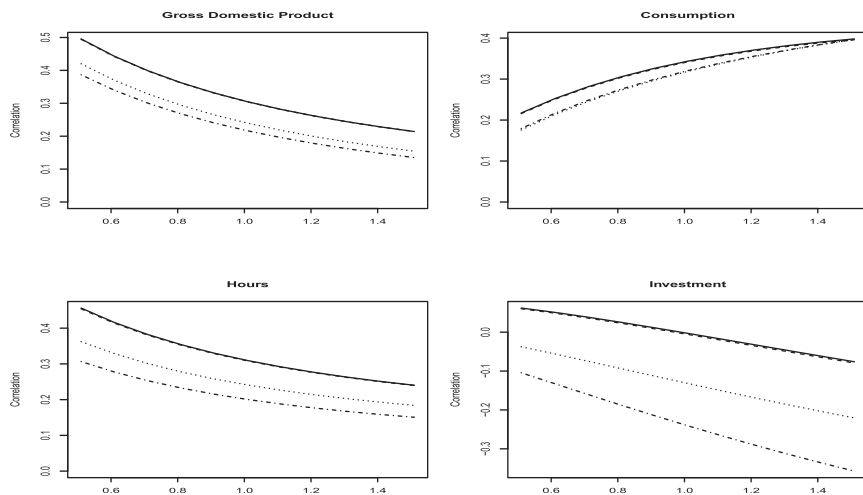


**FIGURE 6.** Responses to a risk shock in the home country: Home Country's TOT and net exports. Impulse responses to a one-standard-error innovation to  $\epsilon_t^{Risk}$  for the risk shock model. Each variable is measured as a percent deviation from their steady state.

As a consequence, a risk-premium shock causes entrepreneurs to forgo consumption in order to bolster their net worth and decrease their reliance on borrowed funds. As entrepreneurs become more self-reliant (due to increased net worth), the risk premium charged on borrowed funds decreases. Thus, the added flexibility of separating the entrepreneurs' consumption decision from the households causes an entrepreneurs net worth to respond quickly to a risk shock. For this reason, the increase in the risk premium charged on borrowed funds is relatively short-lived and contributes very little to the volatility of output, consumption, and investment. When household and entrepreneurial consumption decisions are integrated, as is the case in Christiano et al. (2014), an entrepreneur's net worth increases only gradually. This prolongs the lifespan of the risk shock on the real economy. An interesting result of our decision to separate household consumption from entrepreneurial consumption is the decline in aggregate consumption that results from a positive risk shock. The entrepreneurs' decision to reduce consumption to bolster net worth parallels the decline in consumption observed in the data. This is despite neither sticky prices, nor sticky wages, which are required by Christiano et al. (2014) to match the positive correlation in consumption investment observed in the data.

## 6. SENSITIVITY ANALYSIS

As discussed in Section 4, the addition of financial frictions in the form of agency costs into an otherwise canonical two-good two-country international real business-cycle model has three key implications. First, the endogenous movements in the RPI resulting from the presence of agency costs are empirically plausible. The variability of the RPI is about 75% of its counterpart in U.S. data. The benchmark model produces a small positive cross-country correlation in the RPI,



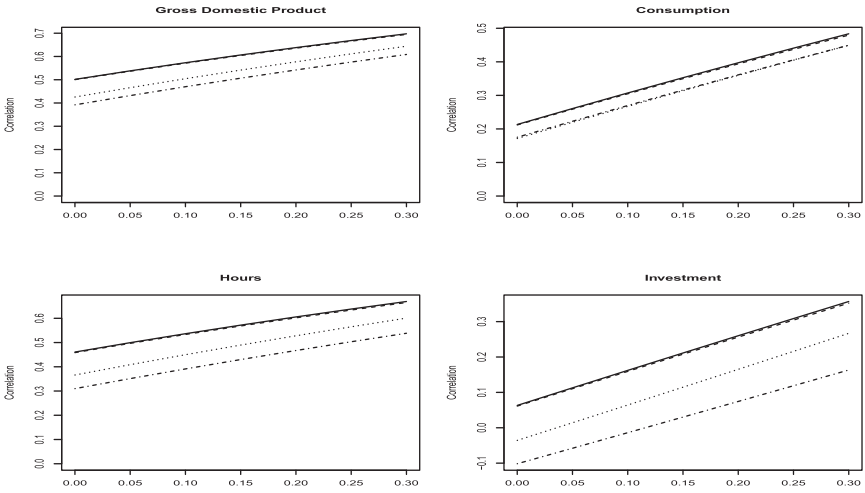
**FIGURE 7.** Cross-country correlation sensitivity to the elasticity of substitution. Cross-country correlations are calculated for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and, finally, the IST model (dot-dashed).

which is just a little higher than what is measured in U.S./Euro-area data. Second, financial frictions lead to higher volatility in the TOT while leaving the variance of net exports constant. Third, financial frictions lead to higher cross-country correlations in output, consumption, investment, and hours worked. The latter two results improve upon the shortcomings of the canonical BKK model.

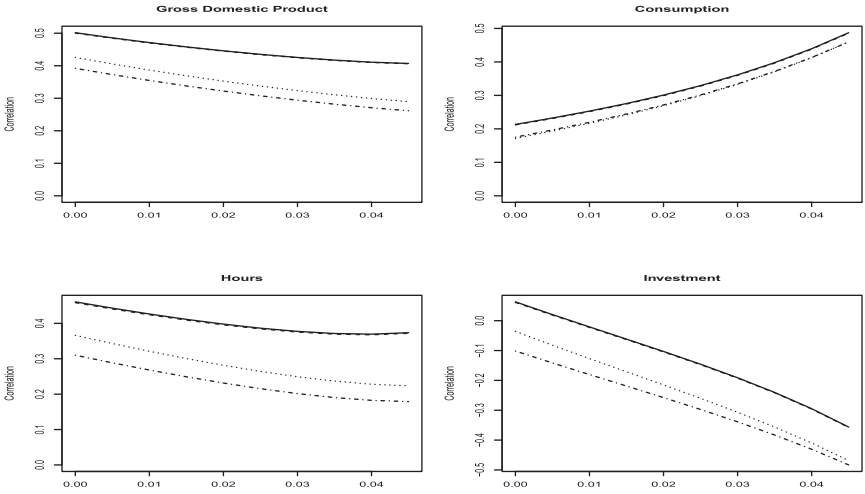
In this section, we analyze the sensitivity of our key results to changes in six parameters. We proceed in two ways. First, we show how the moments shown in Table 2 change when we vary parameters. Second, Figures 7–10 display the sensitivity of cross-country correlations in output, consumption, investment, and hours worked to four key parameters.<sup>18</sup> Each graph contains four lines. The solid line corresponds to the benchmark model, the dashed line corresponds to the model with risk shocks, the dotted line corresponds to the model with no financial frictions, and the dot-dashed line corresponds to the IST model.

The households’ utility function we are using allows us to control the strength of the wealth effect on the households’ labor supply decision. In our benchmark calibration, this wealth effect is modest ( $\eta = 0.1$ ). Table 3 displays the moments implied by the benchmark model when the wealth effect is at its fullest ( $\eta = 1$ ), effectively reverting to a KPR utility function. The volatility of the RPI barely changes. The same holds for its cross-country correlation. TOT are still highly variable. Cross-country correlations of GDP, consumption, and investment change very little. As expected, changing the labor supply wealth elasticity leads to significant changes in the volatility and the cross-country correlation of hours worked.

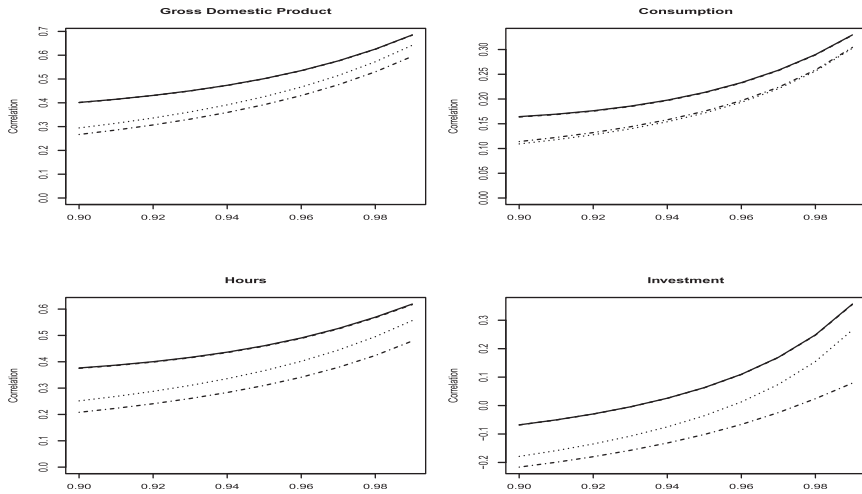




**FIGURE 8.** Cross-country correlation sensitivity to correlated technologies. Cross-country correlations are calculated for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and, finally, the IST model (dot-dashed).



**FIGURE 9.** Cross-country correlation sensitivity to technology spillover. Cross-country correlations are calculated for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and, finally, the IST model (dot-dashed).



**FIGURE 10.** Cross-country correlation sensitivity to technology persistence. Cross-country correlations are calculated for the benchmark model (solid), a model with risk shocks (dashed), a model with financial frictions removed (dotted), and, finally, the IST model (dot-dashed).

Parameter  $\nu$  determines the labor supply elasticity. We test our results by raising  $\nu$  from 1.64 to 3. The moments reported in Table 3 (column  $\nu = 3$ ) are very similar to those observed when raising  $\eta$  to unity.

Next, we increase the elasticity of substitution  $\sigma$  from 0.5 to 0.9 [value estimated by Heathcote and Perri (2002)]. The moments in Table 3 (column  $\sigma = 0.9$ ) show that the volatility of the RPI falls only marginally, while its cross-country correlation increases marginally. Figure 7 shows how cross-country correlations vary with  $\sigma$ . Increasing the elasticity of substitution leads to declines in cross-country correlations for GDP, investment, and hours worked with increase in the cross-country correlation of consumption. Notice how the correlations produced by the benchmark model (solid line) are never smaller than those produced by the other three models.

The final three sensitivity tests concern parameters appearing in the stochastic process for technology shocks. Not surprisingly, when we increase the cross-country correlation of productivity shocks, all cross-country correlations increase, as shown in Figure 8. The relative standard deviation of the RPI basically remains unchanged (0.29 vs. 0.30) when the cross-country correlation of productivity shocks increases from 0 to 0.30. Figure 9 varies the technology spillover parameter  $\rho_s$ , while Figure 10 varies the persistence parameter  $\rho_z$ . In all three figures, the correlations produced by our model with financial frictions are greater than those produced by the other three models. The last column in Table 3 shows the moments produced by the benchmark model when we adopt the stochastic process for technology shocks estimated by Heathcote and Perri (2002). Here,  $\rho_z = 0.97$ ,

$\rho_s = 0.025$ , and the cross-country correlation of technology shocks is 0.29. This combination of changes lowers the volatility of the RPI marginally from 0.30 to 0.24 and increases its cross-country correlation trivially (0.26 vs. 0.25).

Overall, the results reported in Table 3 and shown in Figures 7–10 demonstrate the robustness of our key results to parameter changes.

## 7. CONCLUSION

The effects agency costs have on economic aggregates in a closed economy have been well established [early examples are Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997)]. However, little research has been done on the effects of agency costs in the context of international business-cycle models. We contribute to the international business-cycle literature by adding a financial sector in each country of a two-good two-country BKK style model. Our add-on is inspired by the work of Carlstrom and Fuerst (1997).

Our main finding concerns the dynamics of the RPI. In our benchmark model, its variance as well as its cross-country correlation is comparable to the data. It is important to note that our model produces fluctuations and positive international co-movements in the RPI in an entirely endogenous fashion. Since its conception, IST has been identified exclusively by the inverse of the RPI. Given our benchmark model generates positive co-movement in the RPI across countries endogenously, our model adds to the work of Miyamoto and Nguyen (2013) by proposing an additional way that measured technology (in this case IST) moves together across countries via international trade. We show that this positive international correlation of the RPI is remarkably robust to changes in parameter values. We also contribute to the international business-cycle literature by showing that financial frictions have the ability to increase the variance of the TOT, as well as the international correlations of output, consumption, labor, and investment.

Finally, we also considered risk shocks, à la Christiano et al. (2014). They highlight the relative importance of risk shocks in generating realistic business-cycle volatility in a closed-economy framework. Our research suggests that in an international framework, risk shocks tend to move cross-country correlations away from what we observe in the data. Given the relative importance of risk shocks in a closed-economy model, their inability to drive international business cycles is striking. Further research is required to explore whether risk shocks can move economic aggregates together across countries.

## NOTES

1. See, for example, Baxter and Crucini (1995), Stockman and Tesar (1995), Boileau (2002), Heathcote and Perri (2002), Benigno and Thoenissen (2008), Corsetti et al. (2008), Raffo (2010), and Mandelman et al. (2011), to name a few.

2. We elected to use the CF model over other costly state-verification models, since it allows for endogenous movements in the RPI.

3. Yao (2012) documents the effects of a leverage constraint on international correlations in an extended one-good BKK model.
4. Where IST shocks are tied to RPI data.
5. Schmitt-Grohé and Uribe (2012, p. 2759).
6. Uppercase variables denote aggregate variables, whereas lowercase variables are specific to an individual agent.
7. We use “final goods” and “consumption goods” interchangeably.
8. Alternatively, a fraction of the entrepreneurs’ investment production could serve as collateral, as suggested by Agénor et al. (2014). Exploring how collateralized debt impacts this research is left to future research.
9. The expected income in period  $t + 1$ , denominated in units of  $t + 1$  investment goods, is  $F_{jt+1}^E/P_{t+1} = i_{jt+1}f(\bar{\omega}_{jt+1})$ . This takes into account the (i) repayment of the loan to the CMF when the realized productivity turns out to be greater than  $\bar{\omega}_{jt+1}$  and (ii) the possibility that the entrepreneur has a level of productivity lower than  $\bar{\omega}_{jt+1}$ , in which case, it does not pay back its loan to the CMF and sees its output of investment goods confiscated by the CMF.
10. Recall that CMFs produce neither consumption nor investment goods.
11. The aggregate capital accumulation equation suggests that our model with financial frictions resembles a model with investment-adjustment costs. That is indeed the case. As pointed out by Carlstrom and Fuerst (1997), the financial-frictions framework, as presented in our benchmark model, provides a micro-founded alternative to investment-adjustment costs through the short-term impact financial frictions have on investment production. The results of a model with investment-adjustment costs and without the financial-frictions framework outlined above are available upon request.
12. This lack of consensus on the appropriate value of  $\sigma$  led Bodenstein (2011) to publish an article listing the consequences of varying this elasticity.
13. The only difference between the benchmark model and the no-financial-frictions model is that the latter has no entrepreneurs, nor CMFs. Therefore, future capital can be increased by one unit simply by reducing consumption by one unit. We simulate that model using the very same parameter values as in our benchmark model.
14. Most other moments remain basically unchanged, except the correlation of GDP with net export at constant prices, which increases unfortunately.
15. The observation that GDP responds in a hump-shape manner to a temporary productivity shock goes, at least, as far back as Cogley and Nason (1995).
16. We thank a referee for suggesting this approach.
17. Specifically, we use  $SD(\epsilon_{v1}) = SD(\epsilon_{v2}) = \sqrt{0.000004}$ ,  $SD(\epsilon_{z1}) = SD(\epsilon_{z2}) = \sqrt{0.0000122}$ , and  $\text{correl}(\epsilon_{v1}, \epsilon_{v2}) = 0.12$ .
18. We thank a referee for suggesting this exercise.

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## APPENDIX: DATA

RPI is measured as the price deflator for investment goods divided by the price deflator for consumption goods. The variables considered in our consumption deflator include consumption of nondurables as well as consumer services listed in the National Income and Product Accounts (NIPA) provided by the Bureau of Economic Analysis. The investment deflator considered in this paper is the quality-adjusted price deflator for producer durable equipment calculated by Gordon (1990). Unlike equipment price deflators from the NIPA tables, Gordon's (1990) series is adjusted for changes in equipment quality, such as faster computer processing speeds, or more energy efficient vehicles, both of which expand the production possibility of equipment, and hence the real value of these investment goods. Gordon's (1990) quality-adjusted investment price deflator is reported annually from 1949:1 to 1983, and is extended to span from 1949:1 to 2006:4 using Fisher's (2006) technique, which applies the work of Gordon's time series, as well as Cummins and Violante (2002) (hereinafter GCV), to expand the range of the data set. Annual data are disaggregated into quarterly data via a splice interpolation.

The seasonally adjusted RPI in the Euro area from 1970:1 through to 2007:4 is obtained from the Area Wide Model data set available through the Euro Area Business Cycle Framework. It is calculated by dividing the price deflator for investment goods by the price deflator for consumption goods. Unlike the American counterpart, this time series is not adjusted to allow for variation in the quality of capital goods. Furthermore, this time series calculates the RPI for all investment goods, rather than just equipment. Due to these

shortcomings, a similar time series is calculated for the United States using NIPA table data on the investment price deflator, rather than the equipment price deflator, as done above. This series, spanning the same timespan as the Euro-area RPI, is utilized for the calibration of the cross-country correlation in IST innovations (refer to Section 5.1). These data are made available via Ireland's (2013) academic website.

Data on the Baa-rated corporate bond rates and the 10-year rate on government bonds are obtained from the Federal Reserve Bank of St Louis' FRED database.