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ARTICLE

The effect of an instantaneous dependency rate on the social equitability of hybrid PAYG public pension schemes

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Abstract

The defined convex combination (DCC) pay-as-you-go public pension systems recently introduced in the literature are a form of hybridization between defined benefit (DB) and defined contribution (DC) designed to maintain intergenerational social equitability by reacting to demographic shocks in an optimal way. In this paper, we augment DCC schemes with the assumption that the dependency ratio between pensioners and workers is driven by an exogenously modelled instantaneous stochastic rate of change. This assumption enjoys support from the empirical data and allows explicit solutions for the contribution and replacement rate processes which make transparent the nature of the dynamic evolution of a DCC system, as well as the role of the variables involved. The analysis of intergenerational social equitability measures under the assumption of an instantaneous dependency rate confirms the view expressed in previous literature that neither DB nor DC achieves social fairness, and that DCC plans have the potential to improve on both. We perform a calibration test, and our findings seem to indicate that in ageing economies the DC system might indeed be superior to the DB one in terms of intergenerational fairness.

Key words: Hybrid PAYG pension schemes; instantaneous dependency ratio; social equitability

JEL codes: H75

1. Introduction

In the last few decades, the defined benefit (DB) social security pension paradigm is being slowly abandoned by a number of countries worldwide. In face of the current economic and demographic global developments, DB systems suffer from a double financial vulnerability: one with respect to population ageing, as a consequence of a decline in fertility and increase in life expectancy, which creates an ever increasing number of members, and the other to economic crises, determining unemployment and ultimately system defunding.

A solution to guarantee pensions financial sustainability is implementing structural reforms shifting towards a defined contribution (DC) type architecture. The idea of reforming national pension systems in this direction started be discussed and gain consensus in the mid-1990s. The newly proposed systems seemed to yield several advantages compared to DB. In the first place, having workers make pension payments directly in individual accounts to be annuitized at retirement, avoids by definition the systemic financial stability problems to which DB schemes are exposed. Another advantage of the new system is that it is fair in the sense that two individuals from the same birth cohort making the same payments at the same time can expect the same pension rights related to that payment. Furthermore, the response of DC systems to the change in socio-demographic factors is fully endogenized in the internal rate of return set for the accumulation and decumulation phase, whereas in DB ad hoc actions must be taken, which may impact distributional fairness. Finally, income transparency is granted, as individuals receive pensions only based on their current accounts and, unlike the DB case, there are no within-scheme internal income redistribution rules which may be set and changed in possibly discretionary ways (see Proc. NDC Conference Sandham 2006 for an exhaustive overview). Following these and other considerations, in recent years pension reforms shifting to DC systems have been, and are being, put into effect in a number of European countries: for a discussion see Chłoń-Domińczak *et al.* (2012); a technical analysis is offered in Jabłonowski and Müller (2013).

However, financial stability is not all that must be considered when assessing and formulating a structural pension reform. The impact of changing the rule of collection and distribution of resources on the living standards of contributors and pensioners alike is also important, insofar as pension reforms may possibly generate *future social inequality*. For example, as certain macroeconomic scenarios materialize, future unbalances of standards of living between working generation and retirees may take place. Whether this perspective shift is excessively penalizing for one of the two cohorts in a reformed pension system compared to the current, is certainly something to account for: measures targeted at optimizing the *intergenerational risk sharing* when re-thinking the system should therefore be an important aspect to consider for pension reforms. *Intergenerational social equitability* of a social security system is achieved when sharing rules are put in place in such a way that the living standards of neither cohort unduly benefits or suffers from certain economic or social scenarios at the expense of the other.[1] The nature of longevity risk is such that in DC systems both contributors and retirees bear their own individual risk, which the working generation is better equipped to face by virtue of their higher saving potential. In contrast, in DB schemes the weight of the longevity risk of retirees is shifted to contributors, who are asked to make up for the deterioration of the pensioners living standards. Therefore, risk-sharing between generations is sub-optimal in both cases.

Historically, one of the first theoretical contributions addressing this issue is the one by Musgrave (1981), who introduced the idea that a pension system achieves social equitability if a certain fixed relationship between the replacement and contribution rates is satisfied regardless of future population evolution: namely, that the ratio between the current replacement rates and salaries netted of contributions is maintained constant. Under the Musgrave rule, the contribution and replacement rate are thus not fixed but must dynamically vary in order to respond to demographic changes, and in particular, the social security system must act on the contribution rate on an ongoing basis.

The idea that in order to guarantee social fairness of the pension social security system rules of automatic contribution adjustments may be necessary seem to have already resonated with policy-makers, as the German experience of benefit indexation suggests (Börsch-Supan *et al.*, 2003). More recently, on government request, a policy proposal has been put forth by a panel of experts in Belgium, regarding a new form of intergenerational social contract on pension welfare based on the Musgrave rule (Schokkaert *et al.*, 2018).

In a further theoretical development, a recent study of Devolder and de Valeriola (2019) introduces the idea of 'hybridizing' DB and contribution with the view of attaining a socially fairer system. Such a hybrid scheme is determined by requiring that a certain convex combination, or 'weighted average', of the replacement and contribution rates remains constant through time. Such pension plans are termed *Defined Convex Combination* (DCC) schemes. Just like in the case of the Musgrave rule, this procedure can be seen to require automatic rate adjustments: indeed, in a very exact sense, the convex schemes of Devolder and de Valeriola represent a generalization of the Musgrave system.

We insert in this new literature strain by further sharpening the framework of Devolder and de Valeriola (2019). Our motivation is to address the two main shortcomings of such a study, namely: (i) the unavailability of an explicit expression for the contribution and replacement rate processes; and (ii) the lack of empirical backing of the proposed model specifications, whose time series properties are in conflict with the ones traced in the data (as the study in Section 4 demonstrates).

[1] *Intra*generational equitability instead refers to the commitment of the national security system to deliver the same retirement benefits to individuals from the same birth cohort who have contributed to the system by the same total amount, regardless of their contribution histories.

The main idea of this paper is that both of these issues can be solved by introducing in the old-age dependency ratio – the ratio between the number of pensioners and that of the working-age individuals – a stochastic instantaneous rate of change modelled exogenously. We call this process the *instantaneous dependency rate*. The idea of an instantaneous rate of activity as a fundamental driver of core economic quantities has a long distinguished history in finance, for example in fixed income (short rate models: e.g., Cox *et al.*, 1985; Vasicek, 1993) or equity markets (stochastic volatility models, e.g., Heston, 1993). We employ here a similar structure for a demographic process, the dependency ratio. As it turns out, both of the issues in Devolder and de Valeriola (2019) expressed above are resolved by means of this modelling choice, which at the same time agrees with the empirical data and makes explicit solutions available.

We calibrate the model to the Italian data and repeat and expand, within our model, the analysis in Devolder and de Valeriola (2019) of the social fairness of convex schemes in terms of impact on living standards of contributors and retirees. Our numerical study recovers the sub-optimality, already pointed out by the authors, of both the DB and DC schemes, and confirms that major improvements are brought about by the implementation of Musgrave-type rules. In addition, the parameters of our dependency rate expressly embed various demographic scenarios, corresponding to different optimal risk-sharing profiles. Whether DB or DC is superior ultimately depends on the country demographic trends, as captured in our framework by the dependency rate calibration. In our case study, and we suspect in all cases where countries show population ageing, the DC system is shown to be indeed better than the DB one.

The paper is organized as follows. In Section 2 we review the DCC schemes; in Section 3 we introduce the instantaneous dependency rate and explicitly solve the equations for the replacement and contribution rates; in Section 4 we digress into a statistical analysis of the dependency ratios of Organisation for Economic Co-operation and Development (OECD) countries providing empirical support for a dependency rate; in Section 5 we calibrate a prototypical Ornstein–Uhlenbeck (OU) specification of the dependency rate to the Italian data; in Section 6 we analyse the social fairness of the DCC schemes with a dependency rate in the case analysed, and express some policy implications; we conclude in Section 7.

2. Hybrid convex pension systems

In a pay-as-you-go (PAYG) public pension system, a representative of the working population receives at each $t \geq 0$ a salary $S_t > 0$ and pays to the system a fraction of this salary as a *contribution rate* $\Pi_t > 0$ in order to cover an average present pension $P_t > 0$. If N_t and M_t are respectively the number of workers and pensioners at time t it must hold

$$M_t P_t = N_t \Pi_t S_t. \tag{1}$$

We define further the *dependency ratio* $D_t > 0$ to be the ratio of the retired population to the active workforce:

$$D_t = \frac{M_t}{N_t}. \tag{2}$$

For all $t \geq 0$ we can thus write the following from of the *budget equation*

$$D_t P_t = \Pi_t S_t, \tag{3}$$

The individual average *replacement rate* $\Delta_t > 0$ is defined as the ratio of the average pension paid to the salary i.e.,

$$\Delta_t = \frac{P_t}{S_t}. \tag{4}$$

Substituting in (3) we attain the *equilibrium equation*

$$D_t \Delta_t = \Pi_t. \tag{5}$$

Clearly, the process D_t does not completely define the pension plan unless a second equation determining an allocation rule of the cashflows coming from the workers and to be transferred to the pensioners is defined.

In national PAYG pension systems essentially two allocation rules are considered, DB and DC. A DB system assumes that the replacement rate is constant to the current value at all times $t \geq 0$, i.e.,

$$\Delta_t = \Delta_0, \quad \Pi_t = \frac{D_t}{\Delta_0} \tag{6}$$

Conversely, a DC system assumes that the contribution rate is kept constant in time, i.e., for all $t \geq 0$:

$$\Pi_t = \Pi_0, \quad \Delta_t = \frac{\Pi_0}{D_t}. \tag{7}$$

From these equations we deduce that the ageing of a country, reflected in an increasing value of D_t, in a DB (respectively DC) system is a cost borne by the workers (resp. retirees) alone in the form of an increased contribution rate (resp. a decreased replacement rate). It is self-evident (and will be later more rigorously justified) that neither any of these two attain inter-generational social fairness in that the burden (or advantage) of a demographic shock is carried (or enjoyed) by either one of the cohorts of pensioners or workers.

In a recent paper, Devolder and de Valeriola (2019) have introduced the idea of a new allocation rule for the public resources available through (5), based on the idea of hybridizing DB and DC using a *convex combination* of the two systems.

For a fixed $\alpha \in [0, 1]$ the *Defined Convex Combination* (DCC) scheme is the pension system that gathers contributions and pays pensions in such a way to keep the α-convex combination between Δ_t and Π_t constant, regardless of the variations in D_t. That is we require that for all $t \geq 0$ the condition:

$$\begin{cases} \alpha \Delta_t + (1-\alpha)\Pi_t = C_\alpha \\ C_\alpha = \alpha \Delta_0 + (1-\alpha)\Pi_0 \end{cases} \tag{8}$$

is in force alongside (3). Conditions (3) and (8) fully describe the pension system. It is clear that DC and DB are naturally embedded in this framework, and correspond respectively to the values $\alpha = 0$ and $\alpha = 1$.

The intuition behind the DCC systems is that even neither the DB of DC scheme attain social fairness, equitability could still be achieved by some kind of co-movement in the contribution or replacement rate, able to resolve the shocks in D_t in a 'socially optimal' way. Naturally, in order to do so, the national security system must be statutorily empowered with the ability of dynamically adjusting the rates as to maintain the relationships above.

As we shall see further on, in a DCC scheme such optimality could for example be achieved solving a minimization problem on α of some kind of loss functional applied to the contribution and replacement rates.

Although a novel concept, a non-trivial instance of the DCC pension scheme class was already present in prior literature, and that is the one associated to the *Musgrave rule* (Musgrave, 1981). The Musgrave rule states that social fairness is achieved if the system leaves the ratio of the replacement rate to the salaries net of contribution constant. In the formula

$$\frac{\Delta_t}{1-\Pi_t} = \frac{\Delta_0}{1-\Pi_0} := C_M \tag{9}$$

with $C_M > 0$. We call this system the *Defined Musgrave* scheme (DM). To see that this is of DCC type one just rearranges the above and divides by $1 + C_M$, obtaining

$$\frac{\Delta_t}{1+C_M} + \Pi_t \frac{C_M}{1+C_M} = \frac{C_M}{1+C_M} \tag{10}$$

which is a DCC with $\alpha = (1 + C_M)^{-1}$ and $C_\alpha = 1 - \alpha$.

As it turns out this correspondence can be generalized. Consider a pension system that keeps constant the ratio of the replacement rate to the salary net of a *multiple* $\gamma\Pi_t$ of the contribution rate, with $0 \leq \gamma < 1/\Pi_0$, leading to:

$$\frac{\Delta_t}{1-\gamma\Pi_t} = \frac{\Delta_0}{1-\gamma\Pi_0} := C \tag{11}$$

for some $C > 0$ and all $t \geq 0$. Note that when $\gamma = 0$, 1 we have respectively a DB and a DM pension system. We can rearrange (11) to

$$\Delta_t + \gamma C \Pi_t = C \tag{12}$$

which after dividing by $1 + \gamma C$ can be seen to be a convex combination of the form

$$\alpha\Delta_t + (1-\alpha)\Pi_t = C_\alpha \tag{13}$$

where

$$\alpha = \frac{1-\gamma\Pi_0}{1+\gamma(\Delta_0 - \Pi_0)}, \qquad C_\alpha = \frac{\Delta_0}{1+\gamma(\Delta_0-\Pi_0)}. \tag{14}$$

Observe $0 < \alpha < 1$. Since these transformations are invertible, implementing the constant replacement rate to income net-of-a-multiple-of-contribution ratio rule is fully equivalent to the design of a DCC scheme. In other words, we could say that the DCC plans are effectively a generalization of pension systems based on the Musgrave rule. The role of γ is that of setting the balance of the adjustments made in reaction to D_t more favourably to either pensioners or workers. As we shall see γ is strictly linked to the maximum value that Π_t can assume.

3. The instantaneous dependency rate and explicit solutions

It is evident from the foregoing discussion that once the dependency ratio D_t is specified the DCC architecture is fully determined by the equilibrium and convexity conditions (5)–(8). An appropriate choice of the dependency ratio model is thus the critical element to the design of a DCC scheme that adequately describes a realistic pension system.

Naturally, the positive process D_t is better modelled as an exponential evolution, i.e., modelling is equivalently effected on $E_t = \log D_t$. In Devolder and de Valeriola (2019) the authors assume two alternative versions of D_t. The first is a Brownian motion with drift, i.e., the stochastic differential equation (SDE)

$$dE_t = \mu dt + \sigma dW_t \tag{15}$$

for some Brownian motion W_t and constant drift and volatility μ, $\sigma > 0$. The second is a mean-reverting OU process:

$$dE_t = \kappa(\theta - E_t)dt + \sigma dW_t \tag{16}$$

for a long run mean θ and a mean reversion speed coefficient $\kappa > 0$. According to the authors, these alternative choices are rooted in opposing views regarding mean-reversion in mortality rates expressed in the literature (Luciano and Vigna, 2005; Zeddouk and Devolder, 2018), whose absence (resp. presence) supports model (15) (resp. (16)).

The view of this paper is that models (15) and (16) should not be used to model the dependency ratio. With regards to the theoretical motivation offered for these models, we observe that D_t is not a pure mortality process, but it is impacted by at least three socio-economic and demographic factors: mortality, fertility and employment rates. So a model selection with reference to literature related to only one of such factors is debatable. Second, and most importantly, models (15) and (16) are rejected by the statistical evidence in a large number of countries. We will detail and discuss this thoroughly in Section 4.

This provides motivation for a change of paradigm, and what we propose is the introduction of an instantaneous stochastic rate r_t driving the variation of the dependency ratio. Under such a specification D_t does not suffer from the limitations exposed above. The choice we operate is well-grounded in classic demographic theory, produces tractable solutions for Π_t and Δ_t providing insight on the role of the DCC parameter α, and is adequately supported by the empirical data.

We assume D_t follows a classic exponential population growth model (e.g., Kendall, 1949) with a stochastic growth rate r_t, that is, D_t solves for any possible realizations of a stochastic process r_t the equation

$$\frac{dD_t}{dt} = r_t D_t, \quad D_0 > 0 \tag{17}$$

that is

$$D_t = D_0 \exp\left(\int_0^t r_u du\right). \tag{18}$$

The process r_t, which we shall leave unspecified at the moment, is the *instantaneous dependency rate*, encoding the time evolution of the demographic, economic and social determinants combining to form the dependency ratio. Note that the dependency ratio is increasing in t if and only if $r_t > 0$. This means that a positive process for the rate r_t would be excessively binding for modelling of D_t, since it would not allow its (at least) temporary downturn.

Henceforth we also assume that Π_t and Δ_t are pathwise differentiable. Without loss of generality we can thus set, for Π_0, $\Delta_0 > 0$:

$$\Pi_t = \Pi_0 \exp(\pi_t) \tag{19}$$

$$\Delta_t = \Delta_0 \exp(\delta_t) \tag{20}$$

for some differentiable processes π_t, δ_t, with $\delta_0 = \pi_0 = 0$, chosen such that the equilibrium equation (5) is fulfilled, which together with (18) implicates

$$r_t = \pi_t' - \delta_t'. \tag{21}$$

Furthermore, (8) must be in force, and we know from the previous section that this is equivalent to the expression in (11) being constant. Therefore, differentiating and using (21) we have that

$$\frac{d}{dt}\frac{\Delta_t}{1-\gamma\Pi_t} = \frac{\Delta_t(\delta_t'(1-\gamma\Pi_t)+\pi_t'\gamma\Pi_t)}{(1-\gamma\Pi_t)^2} = \frac{\Delta_t(\delta_t'+\gamma r_t\Pi_t)}{(1-\gamma\Pi_t)^2} = 0 \tag{22}$$

holds if and only if

$$\delta_t = -\int_0^t \gamma\Pi_u r_u du. \tag{23}$$

Replacing (23) in (20) and using again (21), we have that π_t must satisfy pathwise the ODE

$$\begin{cases} \pi_t' = (1-\gamma\Pi_0 e^{\pi_t})r_t, \\ \pi_0 = 0. \end{cases} \tag{24}$$

Remembering $\pi_t = \log(\Pi_t/\Pi_0)$ we arrive at the relation:

$$\Pi_t' = \Pi_t(1-\gamma\Pi_t)r_t, \quad \Pi_0 > 0. \tag{25}$$

Equation (25) is a familiar logistic equation which naturally arises in population evolution models, and nicely complements (17). Its solution is:

$$\Pi_t = \frac{D_t\Pi_0}{\gamma D_t\Pi_0 + D_0(1-\gamma\Pi_0)} \tag{26}$$

for $0 \le \gamma \le 1/\Pi_0$, as it can be shown by a standard separation of variables argument, which for the reader's convenience we recall in the Appendix. Therefore:

$$\Delta_t = \frac{\Pi_0}{\gamma D_t\Pi_0 + D_0(1-\gamma\Pi_0)}. \tag{27}$$

For $\gamma = 0$, we have $\Delta_t = \Delta_0$ from (27) which leads to the DB scheme. For $\gamma = 1/\Pi_0$ it is $\Pi_t = \Pi_0$ from (26) and thus we recover a DC scheme, something which cannot be obtained from (11) or (13). For $\gamma = 1$ the two equations above also clearly imply the DM scheme.

We see that the stochastic intensity rate r_t is also the stochastic growth factor of the contribution rate: to an increase in the dependency ratio, that is, ageing of the population, must correspond to an increase in the contribution rate to maintain the equilibrium. The faster the population ages, the steeper this revision must be.

Also the role of γ is of interest. In a logistic equation the quantity γ^{-1} is the *capacity* of the dynamical system, that is, the maximum value[2] of the solution. In other words, for any chosen γ, its inverse γ^{-1} is the maximum value of contribution which is needed to maintain the system in equilibrium: remarkably this value is <1 with certainty (implying <100% of the salary required to be paid as a pension contribution) if and only if $1 \le \gamma \le 1/\Pi_0$. This means that if instead $0 < \gamma < 1$ then we have that $\Pi_t > 1$ with positive probability, that is with positive probability the pension system could potentially face collapse,[3] which is something one might want to remember. We do observe that this property is not a byproduct of the specification (18), but rather a characteristic of the DCC system as a whole, as a numerical example readily shows.[4]

[2]In a deterministic system with constant positive growth rate this is also the asymptotic value of the solution, but since r_t can fluctuate negatively or positively this is not the case here.

[3]In practice, this would happen long before Π_t even gets close to 1.

[4]Taking $\alpha = 0.9$, $\Delta_0 = 0.8$, $\Pi_0 = 0.15$, and $D_1 = 1.5$ results in $\Pi_1 = 1.07$ after using the equations in Section 2.

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The conclusion is that the parameter γ – which in the constructive definition of a DCC scheme is the multiple of the contribution rate such that the ratio of the dependency rate to the salary netted of such a multiple is constant – retains the following socio-economic interpretation. It is *the inverse of the maximum contribution rate that workers will ever need to pay into the pension system*. When $\gamma^{-1} = 100\% = \gamma$, we are in a DM system, and the maximum contribution is the whole salary. In the case $\gamma^{-1} = \Pi_0$ we are in a DC scheme and such a maximum is the given fixed rate of contribution. As another example, if $\gamma = 2$ then $\gamma^{-1} = 50\%$, and the workers will never have to contribute to pensions with more than the half of their salaries. The parameter γ could be explicitly or implicitly set by the national authorities in accordance with the country's economic outlook.

In the next section we provide statistical backing for DCC plans with an instantaneous dependency rate.

4. Statistical evidence of an instantaneous dependency rate

In this section we conduct a brief statistical study motivating the reasons for going beyond the modelling of the dependency ratio as the logarithm of an Ito diffusion, and providing supporting evidence for the choice of an instantaneous dependency rate. When looking at the annual time series of the dependency ratio, if either one of (15) or (16) were a plausible model, the time series analysis would support either some sort of stationarity, or a drifting random walk behaviour. As we shall see both of these hypotheses are strongly rejected by the statistical analysis. In contrast, assuming a mean reverting SDE for the rate r_t in the model (18) finds empirical support.

Let $\{d_t^C\}_{t_{min} \le t \le t_{max}}$ be the available data set for the generic country C and set $x_t^C = \log d_t^C$. We begin by assuming the true generating data process of d_t^C to be of the form (18): since the data visualization shows curves with very low convexity/concavity at the given annual frequency, we can use backward differences with $\Delta t = 1$ and write

$$x_t^C - x_{t-1}^C \sim \frac{d}{dt} \log D_t = r_t. \tag{28}$$

Hence, the realizations of the process r_t can be thought as being approximated by the series

$$y_t^C = \Delta x_t^C := x_t^C - x_{t-1}^C. \tag{29}$$

Therefore, in order to justify the introduction of a mean-reverting instantaneous dependency rate r_t to model D_t we must provide adequate evidence of stationarity of the series y_t^C, or, equivalently, show that the series x_t^C are integrated of order one.

In order to test this hypothesis we first visualize the data, study the correlations and select reasonable models. We then check for stationarity combining two popular statistical tests: the augmented Fuller–Dickey (ADF) test (Dickey and Fuller, 1979) and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test (Kwiatkowski *et al*., 1992). These tests are complementary to each other, in the sense that the null hypothesis of the former is the presence of a unit root in the regression model, whereas the null of the second is stationarity.

We used the OECD official dataset[5] for the old-age dependency ratio,[6] comprising a set of 36 countries, with records ranging from 1950 to 2014. Figures 1 and 2 present the time series x_t^C and y_t^C for selected countries in the sample. The series x_t^C exhibits a marked growth trend, and the drifting random walk model or a stationary process reverting to a constant are already suspect. In contrast, it does seem plausible to postulate an autoregressive model for y_t^C.

The correlograms of the two set of series (two examples are shown in Figures 3 and 4) suggest strong, slowly decaying autocorrelation function (ACF) function for the series x_t^C for all the countries considered,

[5]Available at the website OECD.stats. Germany includes former GDR. Israel data are supplied under the responsibility of the Israeli authorities and are used without prejudice to the status of the Golan Heights, East Jerusalem and the West Bank, under the terms of the International Law. Some 2014 missing data have been linearly interpolated.

[6]Defined as the ratio of the number of individuals of age 65 or more to the individuals of age 14–65.

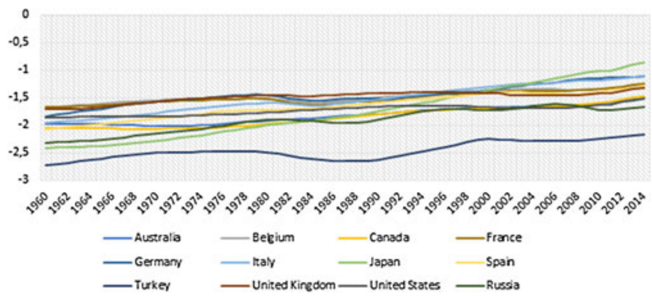

Figure 1. Time series of the logarithmic dependency ratios x_t^C for various countries.

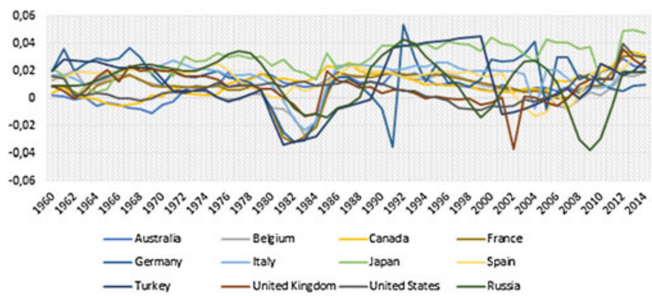

Figure 2. Time series of the differenced logarithmic dependency ratios y_t^C.

which is at odds with stationarity, and ultimately model (16). After differencing, we notice that in most cases the slow decay of the ACF is resolved, while the partial autocorrelation function (PACF) is still non-negligible for the first one or two lags (see table in Appendix B); in some cases the correlations change sign so that a moving average component can be conjectured. Non-zero PACF for y_t^C at lag one for all C is in contrast to the random walk with drift hypotheses for x_t^C, that is, with model (15). We reject such model on this basis. Overall, the correlogram plots suggest an autoregressive model[7] with dependency up to the first two lags for y_t^C, and therefore an ARIMA$(p, 1, q)$ model for x_t^C, $p = 1, 2$, $q \geq 0$.

The ADF test for a time series x_t is concerned with finding the statistical significance of δ in the autoregression model

$$\Delta x_t = \alpha + \beta t + \delta x_{t-1} + \sum_{i=1}^{k} \delta_i \Delta x_{t-i} + \epsilon_t \tag{30}$$

where ϵ_t is white noise and k is an optional lag applied to account for possible higher order correlations (when $k = 0$ the sum is absent). The null hypothesis is $\delta = 0$ which is equivalent to a unit root being present in (30). The regressors α and β may or may not be equal to zero, which must be assessed from the qualitative character of the series under inspection: the ADF test thus tests the random walk hypotheses, with optional drift and trend. The ADF statistics DF_τ^k is a negative number, and its critical values at a given level of confidence depend on the sample size. From the study of the correlograms of the OECD data we argue that an AR(1) model is adequate for y_t^C in all but ten countries, for which an AR(2) model is better suited. Therefore we set $k = 1$ for this latter group, and $k = 0$ in all the others.

The KPSS test for stationarity decomposes the process x_t as a sum of a deterministic part, a random walk u_t and white noise ϵ_t:

$$x_t = \beta t + u_t + \epsilon_t \tag{31}$$

[7]Given the low frequency of data (annual), heteroscedasticity has been not accounted for. In any case, we did look for autocorrelation in the squared residuals and as expected it does appear to be negligible.

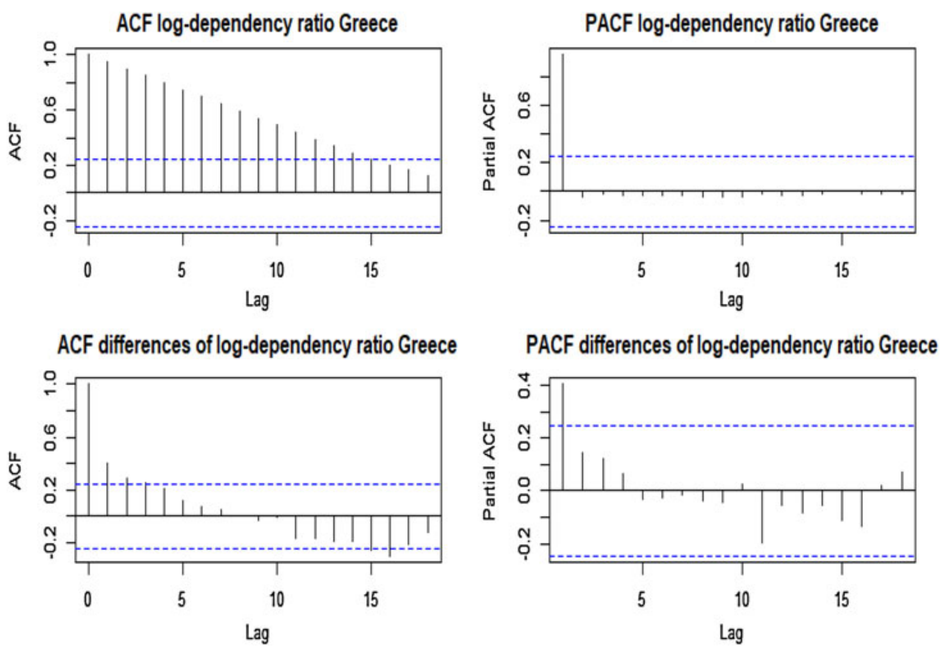

Figure 3. Correlograms of x_t^{EL} and y_t^{EL}.

with $u_t = u_{t-1} + \eta_t$, $\eta_t \sim \mathcal{N}(0, \sigma^2)$; the null hypothesis is thus $\sigma^2 = 0$, and the case $\beta \neq 0$ incorporates a trend. An additional parameter k appears in the formulation of the estimator of the test, whose recommended choice in Kwiatkowski *et al.* (1992) –which we follow– is fixed as the entire part of $4(N/100)^{1/4}$ where N is the length of the series. To our knowledge exact p-values for the test statistic K_τ are not easily obtained, but bounds are available.

Both the ADF and KPSS tests are known to have low power, so the failure to reject the null by either test must be accompanied by confirmatory evidence (rejection) from the other one in order to have a good degree of certainty that the null can be indeed considered true.

Preliminarily, we analyse model (16); in order to do so we apply the ADF and KPSS tests with $\alpha \neq 0$, $\beta = 0$ to the series x_t^C. In the KPSS test the null hypothesis of stationarity is rejected at a 5% significance level for 35 of the 36 countries (97.2%); for the ADF test the presence of a unit-root is rejected at the same confidence level for only 5 out of 36 countries (13.8%). This is overwhelming evidence that model (16) is not supported by the data. Insisting in this direction, we then asked ourselves whether Figure 1 could be seen as the realization of a trend-stationary processes, so that a deterministic linear trend could be incorporated in (16) to produce a reliable model, e.g., by replacing the constant θ with a linearly growing mean reversion level $\theta(t) = t\theta$. This tantamount to testing the KPSS and ADF regressions with α, $\beta \neq 0$: the result is that H_0 is rejected at a 5% significance for 32 out of the 36 countries (88.8%) in the KPSS test, and for 4 out of 36 countries (11.1%) for the ADF test. This indicates that there also appears to be no obvious way of extending model (16) as to include the observed linear drift in the log-dependency ratio.

We then run the ADF and KPSS tests on the set y_t^C using the specification $\alpha > 0$ and $\beta = 0$ suggested by the data visualization. The frequencies of the p-values of the ADF test and the statistic K_τ of the KPSS test are reported respectively in Figures 5 and 6. The country-by-country summary is given in Appendix B. For the ADF p-values we observe a skewed frequency with a clustering around small values (the median is 0.0899), from which we can conclude that the observed data are consistently in contrast to the null hypotheses of non-stationarity, which is thus unlikely to be true. By the same token, there is a high frequency of low values of the KPSS test statistics, the median being 0.2768; exactly 66.6% of the values are lower than the critical value 0.347 of K_τ for a 10% significance level,

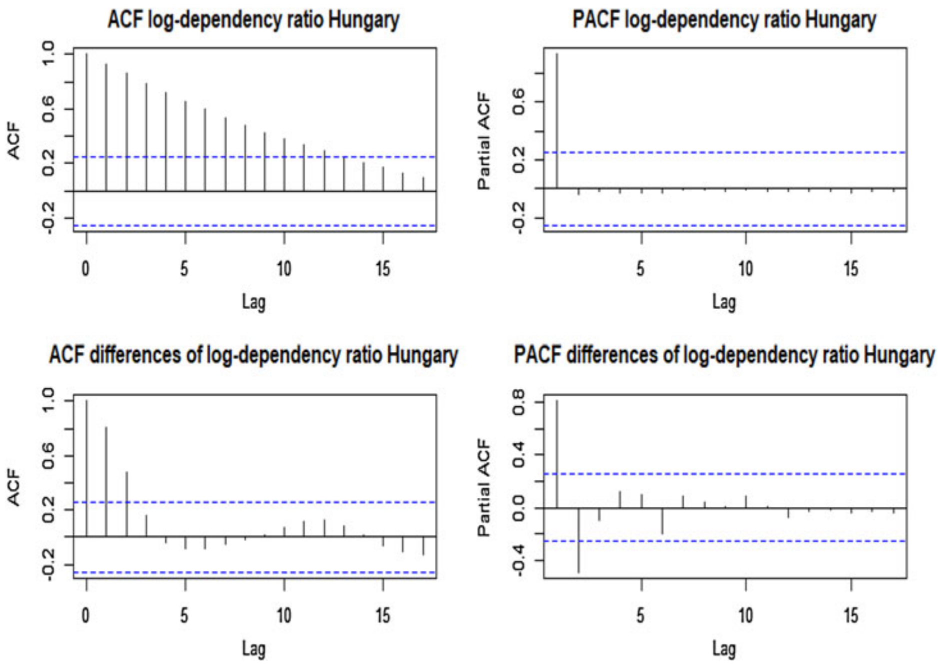

Figure 4. Correlograms of x_t^{HU} and y_t^{HU}.

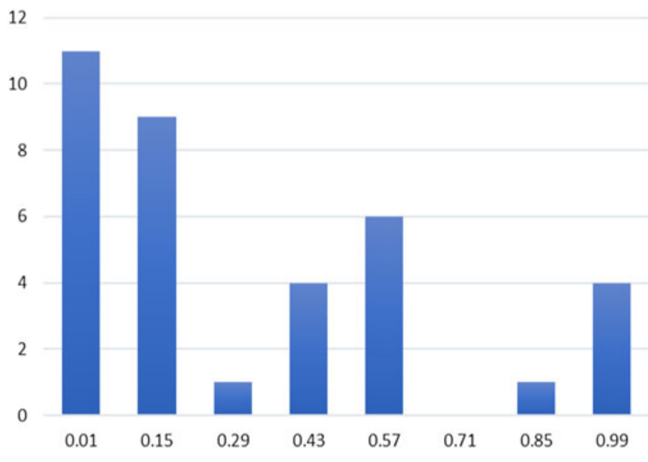

Figure 5. Frequency histogram of the p-values of the ADF test.

which is no basis for the rejection of H_0. Taken jointly, the two test results show that the hypothesis of stationarity of the true data generating process of y_t^C is compatible with the historically observed data.

In conclusion, this short cross-country study finds evidence that the differenced log-dependency ratio can be modelled with a simple autoregressive process. This provides a first statistical basis for an instantaneous dependency rate based on a mean-reverting continuous time process (essentially of OU type, see the next section).

5. Model specification and calibration

In this section we instantiate our model and fit it to the Italian data as made available by the OECD in the period 1955–2014. The residuals analyses of the times series y_t^{IT} fitted using ordinary least squares

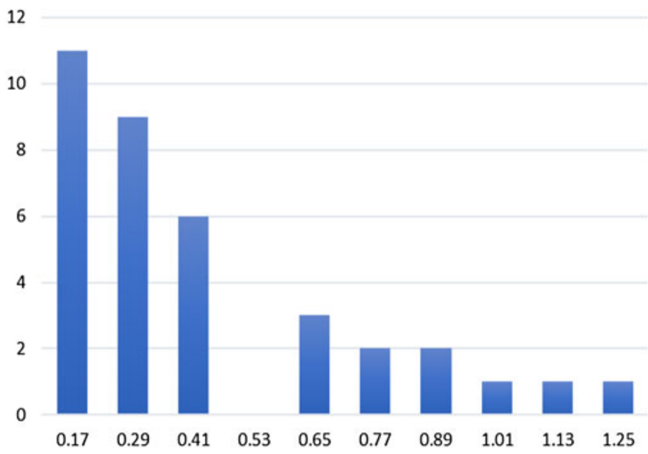

Figure 6. Frequency histogram of the KPSS test statistics K_τ.

on an AR(1) model is rather satisfactory, as shown in Figure 7, which means a simple autoregression of order one is a reasonable model for the data. We therefore can assume r_t to be the continuous counterpart of an AR(1) regression, i.e., a stationary mean-reverting OU process driven by the SDE:

$$dr_t = \kappa(\theta - r_t) + \eta W_t \tag{32}$$

for some standard one-dimensional Brownian motion W_t. The parameters retain the following interpretation: the mean reversion parameter θ is the drift at which the dependency ratio is expected to grow; κ is the speed of mean reversion towards this drift, and the volatility of the rate η expresses the degree of reliability attached to the estimate of the future dependency ratio.

The solution of r_t started at $s < t$ is

$$r_t = r_s e^{-\kappa(t-s)} + \theta(1 - e^{-\kappa(t-s)}) + \eta \int_s^t e^{-\kappa(t-u)} dW_u \tag{33}$$

where the last term indicates the stochastic integral with respect to the Brownian motion W_t.

We recall that r_t is normally distributed with time $s < t$ conditional mean and variance:

$$\mathbb{E}_s[r_t] = r_s e^{-\kappa(t-s)} + \theta(1 - e^{-\kappa(t-s)}) \tag{34}$$

$$\mathrm{Var}_s(r_t) = \frac{\eta^2}{2\kappa}(1 - e^{-2\kappa(t-s)}) \tag{35}$$

where $r_0 \sim N(\theta, \sigma^2/2\kappa)$ since we start the process from the stationary distribution. As observed, since r_t can be either positive or negative, D_t can both increase or decrease: when $\theta > 0$ (ageing countries) D_t will have a tendency to rise, whereas if $\theta < 0$ (rejuvenating countries) will have a tendency to decrease.

We can calibrate r_t by matching the coefficients of the three terms in (33) with the corresponding constant, autoregressive and noise coefficients in the AR(1) model

$$y_t = c + \phi y_{t-1} + \epsilon_t \tag{36}$$

with $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. The coefficients of the fit of (36) to the differentiated series y_t^{IT} are

$$\phi = 0.72456 \tag{37}$$

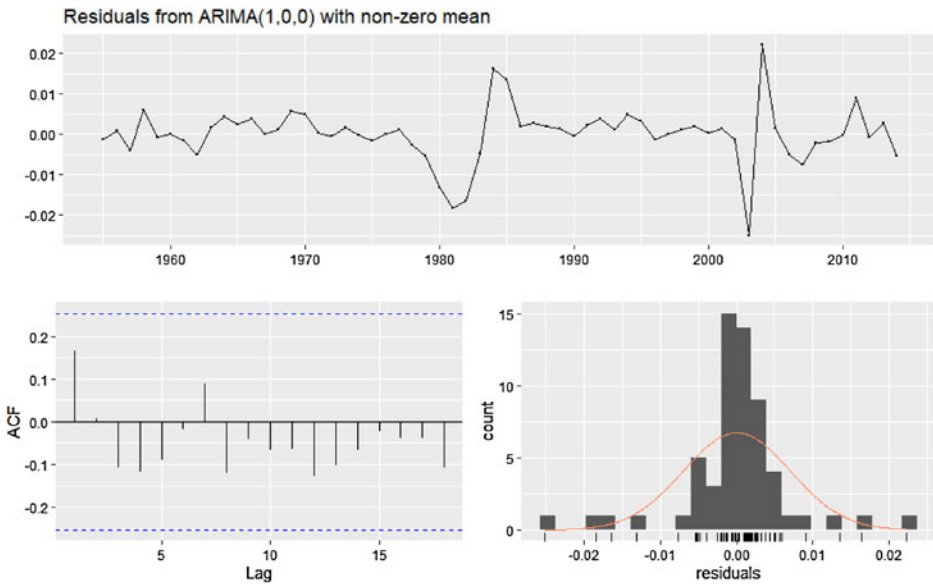

Figure 7. Residual time series, ACF and distribution, from the fit of model (36) to y_t^{IT}. The p-value of the Ljung–Box test is 0.6481.

$$c = 0.00424 \tag{38}$$

$$\sigma^2 = 4.838 \times 10^{-5}. \tag{39}$$

Coefficient matching of (36) and (33) when $s = t - 1$ and solving for the OU parameters yields:

$$\theta = \frac{c}{1 - \phi} \tag{40}$$

$$\kappa = -\log \phi \tag{41}$$

$$\eta = \sigma \sqrt{\frac{-2 \log \phi}{1 - \phi^2}}. \tag{42}$$

Substituting the numerical values in the above we finally have

$$\theta = 0.01536 \tag{43}$$

$$\kappa = 0.32214 \tag{44}$$

$$\eta = 0.00810. \tag{45}$$

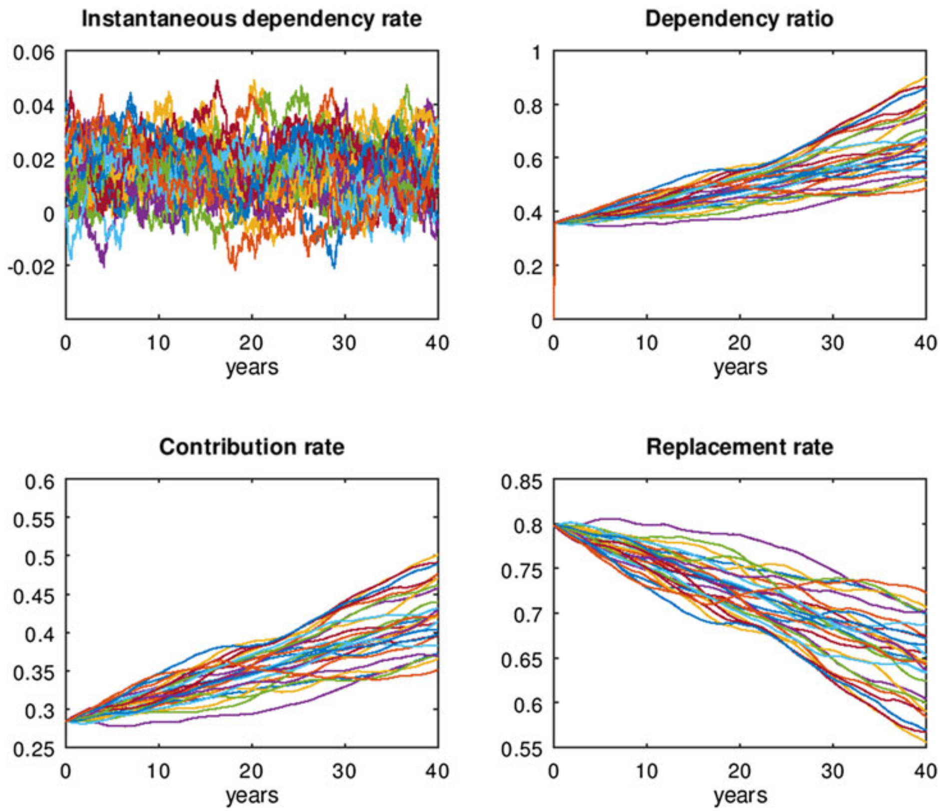

Figure 8. Sample paths of r_t (top left panel), D_t (top right), Π_t (bottom left) and Δ_t (bottom right). The D_t paths trend upwards (although not monotonically) because the population gets older: contributions must increase and replacement rates drop.

We simulate D_t, Π_t and Δ_t for the DM pension system ($\gamma = 1$) in the calibrated model based on a simple Euler scheme for r_t combined with equations (26) and (27). The time horizon is 40 years. We use as initial data the 2019 dependency ratio $D_0 = 35.6\%$ in the website dati.istat.it by the Italian Institute of Statistics (ISTAT), and a replacement rate $\Delta_0 = 79.8\%$ as estimated by the Italian Accountant General.[8] The two imply an initial contribution rate Π_0 of 28.4%.

Sample trajectories of the three processes are shown in Figure 8. The positive mean reversion level θ is what generates the clearly visible upward pull of the dependency ratio. As the plots show, as the population gets older and the dependency ratio increases, in order to maintain constant the ratio of the replacement rate to salaries-net-of-contribution, the national pension system revises the contribution rates upwards and lets the replacement rate decrease. Finally we observe that consistently with what already remarked, the paths of D_t, although upward trending, are not monotonically increasing. This is a realistic feature: for example, the Italian dependency ratio shows a clear downturn in the 1980s and in the early 2000s.

6. Social equitability in DCC schemes. DB or DC?

In Devolder and de Valeriola (2019) the authors suggest that intergenerational fairness is attained when the present (or target) relationship between the living standards of pensioners and workers is

[8]'Le tendenze di medio-lungo periodo del sistema pensionistico e socio-sanitario'. Ministero dell'Economia e delle Finanze, Dipartimento della Ragioneria generale dello Stato, 2019 Annual Report. Average of employees and self-employed. Rates are netted of taxes.

maintained through time. The living standards of the two groups can be for instance taken to be respectively the replacement rate and the salary net of contribution. The key idea is thus that the variability of some kind of 'spread' between the two should be minimized in order to attain social equitability. This idea is consistent with the original approach in Musgrave (1981). As explained in Section 2 the formulation of one such minimization problem is made possible precisely by the introduction of the DCC pension systems and it is one of the main motivation for considering hybrid DB and DC type schemes.

Two natural variables quantifying the divergence (or similarity) of the living standards are given by the *difference* and the *ratio* of the replacement rate to the salary net of contributions, i.e.,:

$$d_t = \Delta_t - (1 - \Pi_t) = \Delta_t + \Pi_t - 1 \tag{46}$$

and

$$m_t = \frac{\Delta_t}{1 - \Pi_t}. \tag{47}$$

As suggested, the second is exactly the quantity first analysed by Musgrave.

We introduce then six different measures of social fairness by applying three different dispersion statistics to both of the variables above: namely variance, interquartile range and median absolute variation. We denoted these respectively SD^x, IQR^x, MAD^x, where $x = m, d$ according to which one of (47) and (46) is used. Social fairness is thus achieved, for each of the measures under consideration, at the minimizer:

$$\gamma_M^x = \underset{\gamma \in [0, 1/\Pi_0]}{\text{argmin}} \; M^x(\gamma) \tag{48}$$

where $x = m, d$ and $M \in \{\text{SD, IQR, MAD}\}$.

Let us observe that such minimizers can be found analytically, and the minimum in all cases is zero. Indeed in a DCC scheme for all M we have

$$M^d = M^d(\Delta_t + \Pi_t) = 0 \tag{49}$$

if and only if $\alpha = 1/2$, since by definition of DCC scheme $2(\Delta_t/2 + \Pi_t/2)$ must be constant in t. Inverting the first equation in (14) one gets the equivalent value of γ_M^d:

$$\gamma_M^d = \frac{1}{\Delta_0 + \Pi_0}. \tag{50}$$

In the case when the ratio of the living standards is considered, the minimizers of all the three measures is trivially $\gamma_M^m = 1$ because this choice generates the DM scheme which is defined by the property that the ratio of the replacement rate to the salary net of contribution is constant.

Using the calibrated model and the numerical methods of Section 5, we ran a Monte Carlo simulation of 10,000 sample paths of Π_t and Δ_t at an horizon of $t = 40$ years, for values of γ ranging from $\gamma = 0$ (DB) to $\gamma = 1/\Pi_0$ (DC). We computed all of the six social fairness measures: the results are the blue lines shown in Figure 9. The shapes are those of a pasting between two concave functions, one decreasing and one increasing, with a kink in the minima at the points γ_M^d and γ_M^m.

The general outlook from the results of the time series calibration presented confirms the consideration expressed in Devolder and de Valeriola (2019) stating that social fairness is best attained using an allocation rule of DCC type rather than relying on pure DB or DC architectures. As highlighted above, which value of γ is optimal depends on the chosen living standard spread measure.

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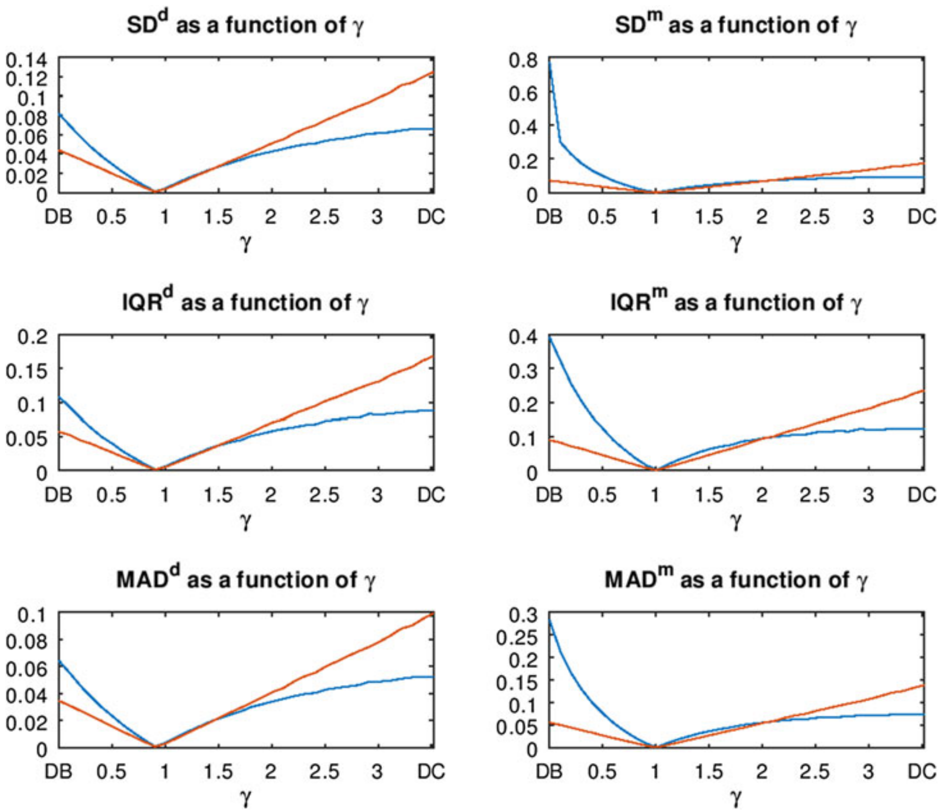

Figure 9. Social fairness measures in the calibration to the Italian data. Note the concave shapes and the minima $\gamma_M^d = 0.9241$ and $\gamma_M^m = 1$. The blue line is the result of the full calibration, the red line is the simulation outcome when altering the mean reversion level to $\theta' = 0$. The endpoints relative positions, corresponding to the DB scheme (left endpoint) and the DC scheme (right endpoint) are inverted in the two cases.

Another important element of analysis, having policy implications, is the relative intergenerational fairness of DB and DC pension systems. When comparing among them the two systems we notice that in all six cases the values of the equitability measures are lower for DC, implicating that such a system is socially fairer. In other words, the calibration to the Italian data of a DCC structure seem to suggest that the choice of a DC public pension system (as it has been recently put in place in the country), for such a country is not only preferable to DB in terms of financial sustainability, but *also in terms of social equitability*.

However, we should not be tempted to draw undue general conclusion on the social equitability of various DCC schemes from this specific case study. Of course, different parameter estimation procedures may give raise to different results, even when they refer to the same country. For example, our analysis is fully based on the historically observed data: however the model could be also calibrated to the projections of the National Institute of Statistics, which predicts – as in many other European countries – the dependency ratio increase to slow down once the baby boom generation effect on pensions ceases.

To illustrate how the scenario may change under different model parameter estimations, we re-run the simulations of the Italian case using the same random numbers drawing and κ, η, but we assume a different future evolution of the dependency ratio is estimated, lower than historical one, which implies $\theta' = 0$. The new sample paths for the rates are shown in Figure 10: r_t is now oscillating around

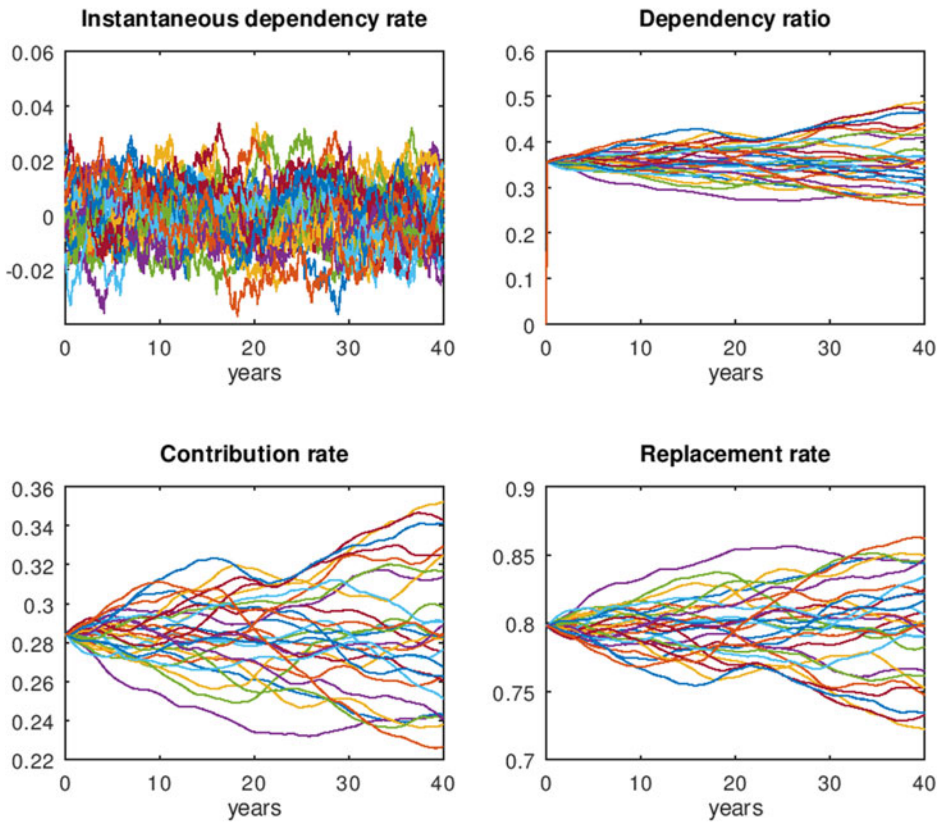

Figure 10. Sample paths of r_t, D_t, Π_t and Δ_t when $\theta = 0$. There is equal probability of r_t assuming positive or negative values: the dependency ratio has no tendency to increase, and thus contribution and replacement rate fluctuate freely.

a lower mean reversion and thus becomes negative with higher probability, determining less steep paths for D_t and thus less extreme scenarios for Π_t and Δ_t. The corresponding results for the measures M^x are the red lines in Figure 9: what we obtain now is that among DB and DC it is DB to show the lower values of M^x.

The mean reversion level θ is therefore a critical parameter: it is the annualized expected value of the dependency ratio growth. When such a value is low, the dependency ratio rises, reflecting a high longevity risk, low fertility, and/or a shrinking economy. In such a case DB contributors are inevitably overburdened with such risk factors being transferred to them from the older generations, and this tips the balance of the living standards in favour of the current pensioners. In this scenario, a DC system may then be preferred.

However, when θ is low or negative, implicating a low or decreasing dependency ratio, we might be observing a period of low longevity risk or booming economy. In such a case, workers could expect to be able to adequately support with their contributions current and future retirees living standards within a DB architecture, without excessively penalizing their own. In fact, they *must* do so, in order to maintain the intergenerational risk sharing equilibrium.

A DCC system, through its automatic adjustment mechanisms, is precisely one that permits to address a dynamic macroeconomic evolution between the two scenarios described above, such as the one that has already historically materialized during the 1960s boom (demographic and economic) and the 1980s recession.

7. Conclusions

In this paper we have revisited the recent introduction by Devolder and de Valeriola (2019) of DCC pension plans by supplementing it with an instantaneous dependency rate driving the log-dependency ratio. Such a model choice has at least two benefits. First, it resolves the observed inconsistency of the dependency ratio specification in Devolder and de Valeriola (2019) with the empirical data. Second, it allows the determination of an explicit logistic solution for the contribution rate which makes transparent the role of the constant γ and the rate r_t in the DCC architecture, as respectively the capacity and growth of the contribution rate.

We have calibrated the DCC model to the Italian dependency ratios and obtained consistent and realistic demographic and pension time evolutions. Social equity measures have been analysed as a function of the convexity parameter, obtaining curves with a global minimum and DB and DC as end-points similar to those in Devolder and de Valeriola (2019), confirming the sub-optimality of both of these plans in terms of social fairness.

Inserting in the debate of the relative equitability of DB and DC systems, the introduction of an instantaneous rate of variation in the dependency ratio leads to the conclusion that: (a) whether DB or DC is superior in terms of intergenerational risk-sharing depends heavily on the demographic and economic trends; (b) an analysis based on past data shows that in countries with an ageing population a DC system might be indeed preferable.

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Appendix A: The contribution rate process

Dividing both members of (24) by $1-\gamma\Pi_t$, assumed nonzero, we have

$$\frac{\Pi_t'}{\Pi_t(1-\gamma\Pi_t)} = r_t, \tag{A.1}$$

so that we can directly integrate both members separately in t. The first member yields, using the substitution $y = \Pi_u$ and the fraction decomposition of $1/y(1-\gamma y)$:

$$\begin{aligned}\int_0^t \frac{\Pi'_u}{\Pi_u(1-\gamma\Pi_u)}du &= \int_{\Pi_0}^{\Pi_t} \frac{dy}{y(1-\gamma y)} = \int_{\Pi_0}^{\Pi_t}\frac{dy}{y} + \gamma\int_{\Pi_0}^{\Pi_t}\frac{dy}{(1-\gamma y)} \\ &= \log\left|\frac{\Pi_t}{1-\gamma\Pi_t}\right| - \log\left|\frac{\Pi_0}{1-\gamma\Pi_0}\right| \end{aligned} \tag{A.2}$$

Substituting in (A.1), integrating the right-hand side and exponentiating we have

$$\frac{\Pi_t}{1-\gamma\Pi_t} = \frac{\Pi_0}{1-\gamma\Pi_0}e^{\int_0^t r_u du} \tag{A.3}$$

which after rearranging becomes

$$\Pi_t\left(1 + \frac{\gamma\Pi_0}{1-\gamma\Pi_0}e^{\int_0^t r_u du}\right) = \frac{\Pi_0}{1-\gamma\Pi_0}e^{\int_0^t r_u du}. \tag{A.4}$$

Simplifying

$$\Pi_t\left(1-\gamma\Pi_0 + \gamma\Pi_0 e^{\int_0^t r_u du}\right) = \Pi_0 e^{\int_0^t r_u du} \tag{A.5}$$

from which (26) easily follows using (18). That $\Pi_t < \gamma^{-1}$ is now clear.

Appendix B: Tests statistics

Following are the country-by-country ADF and KPSS test statistics from the study in Section 4; p-values are only available for the ADF test. The value k is the number of lags applied to the ADF test according to equation (30). The frequencies corresponding to these data are shown in Figures 5 and 6 and support the assumption of an integrated stationary process for x_t^C, without unit roots. The PACF of the first difference series y_t^C is statistically significant, rejecting the random walk hypothesis for x_t^C.

Country	ADF			KPSS	PACF
	k	DF_τ^k	p-value	K_τ	y_t^C, lag 1
Australia	0	−1.5656	0.4946	0.6000	0.852
Austria	1	−3.7650	0.0100	0.2933	0.865
Belgium	0	−2.0739	0.3004	0.1339	0.829
Canada	0	−1.3973	0.5545	0.6838	0.836
Chile	0	−0.4222	0.8963	0.7340	0.918
Czech Republic	0	−1.5208	0.5112	0.2056	0.873
Denmark	0	−0.5935	0.8365	0.3001	0.94
Estonia	1	−2.8443	0.0624	0.2583	0.876
Finland	0	−0.1457	0.9361	0.2066	0.735
France	1	−3.9295	0.0100	0.1652	0.853
Germany	0	−4.3196	0.0100	0.1923	0.495
Greece	0	−5.0827	0.0100	0.1688	0.403
Hungary	1	−3.8895	0.0100	0.2743	0.807
Iceland	0	−2.8413	0.0629	0.1052	0.705

(Continued)

Appendix B: (*Continued.*)

Country	ADF			KPSS	PACF
	k	DF_{τ}^{k}	p-value	K_{τ}	y_t^C, lag 1
Ireland	1	−2.6686	0.0893	0.0963	0.851
Israel	1	−3.0603	0.0386	0.5458	0.34
Italy	0	−2.8868	0.0559	0.0659	0.737
Japan	0	−2.9620	0.0469	1.2201	0.723
Korea	0	−3.7094	0.0100	1.0706	0.563
Luxembourg	0	−2.4721	0.1483	0.5897	0.804
Mexico	0	−3.8108	0.0100	0.8927	0.471
Netherlands	0	−0.2735	0.9189	0.3561	0.863
New Zealand	0	−2.4514	0.1562	0.7877	0.762
Norway	0	−1.3618	0.5665	0.7720	0.927
Poland	0	−1.5332	0.5065	0.3237	0.827
Portugal	1	−5.4027	0.0100	0.0552	0.465
Slovenia	0	−1.7551	0.4209	0.2808	0.843
Spain	0	−1.9878	0.3333	0.1535	0.817
Sweden	0	0.3182	0.9767	0.2793	0.829
Switzerland	1	−3.1055	0.0347	0.1570	0.845
Turkey	1	−2.6602	0.0906	0.1255	0.9
United Kingdom	0	−3.7104	0.0100	0.3068	0.602
United States	0	−1.6068	0.4793	0.2359	0.848
Brazil	0	−7.3859	0.0100	0.3890	−0.272
Colombia	0	−2.0447	0.3142	0.2547	0.684
Russia	1	−5.8868	0.0100	0.1215	0.872

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