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Harmonic generation by the propagation of two-colour laser beams in an underdense plasma

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Abstract. An analytical theory is developed for studying the phenomenon of generation of efficient odd and even high harmonics by the propagation of two-colour linearly polarized laser beams in a homogeneous underdense plasma. The wave equation governing the evolution of the amplitude of various harmonics driven by the current density at corresponding frequencies is set up. The ratio of the fundamental frequencies of the two laser beams is considered to be an arbitrary integer. A numerical evaluation of amplitudes of the third, fourth and fifth harmonics has been presented. It is seen that the third harmonic amplitude generated by the two-colour system is enhanced in comparison to that obtained by a single laser beam. The detuning distance for the former is also increased in comparison to the latter case.

1. Introduction

Laser pulses generated from terawatt laser systems based on the chirp pulse amplification principle can be focused at intensities exceeding 10^{19} W cm^{-2} (Chu et al. 2004). When such high intensity laser pulses interact with a plasma, the plasma electrons quiver with the frequency of the laser field, attaining velocities comparable to the velocity of light. Consequently, relativistic as well as ponderomotive nonlinearities come into play (Gibbon 2005). This nonlinear interaction of laser beams with plasma leads to many interesting phenomena, including laser wakefield acceleration (Tajima and Dawson 1979; Esarey et al. 1996; Jha et al. 2005), inertial confinement fusion (Wilks et al. 1992; Tabak et al. 1994; Deutsche et al. 1996; Regan et al. 1999), terahertz radiation generation (Jha et al. 2011), relativistic self-focussing of laser beams (Max et al. 1974; Sprangle et al. 1987; Jha et al. 2006), self-phase modulation (Antonsen and Mora 1992) and harmonic radiation generation (McPherson et al. 1987; Huillier and Balcou 1992; Liu et al. 1993).

The excitation of coherent radiation at harmonics of the fundamental frequency of the laser is of much practical importance. It provides a source of coherent high-frequency radiation extending up to the X-ray regime (Solem et al. 1989; Amendt et al. 1991; Norreys et al. 1996). Intense isolated attosecond laser pulses can also be synthesized by harmonic generation (Steingrube et al. 2011). It has been theoretically seen that odd harmonics of the laser frequency can be generated by the interaction of linearly polarized laser beams with homogeneous plasma (Mori et al. 1993). In addition, even harmonics can be obtained in the presence of density gradients in plasma (Esarey et al. 1993). Linearly polarized laser beams can generate second harmonic radiation in magnetized plasma (Jha et al. 2007).

For practical applications of harmonic radiation, its conversion efficiency needs to be enhanced. Conversion efficiency enhancement has been obtained by introducing a density ramp (Mori 1994) or by applying various quasi-phase matching and phase-matching schemes (Rax and Fish 1992). A recent demonstration of the enhancement of amplitude of high harmonics using two-colour laser beams of frequencies ω and its second harmonic 2ω in plasma plumes has been reported (Ganeev et al. 2009). Similarly, two-colour pump laser beams propagating in helium gas jet have been used for odd and even harmonic efficiency enhancement (Mauritsson et al. 2006). The enhancement of ion yield and harmonic generation has been experimentally observed for two-colour laser beams of frequencies ω and 3ω propagating in argon and neon gases (Watanabe et al. 1994). The generation of an XUV supercontinuum (Yao et al. 2010), isolated attosecond pulses (Siedschlag et al. 2005) and terahertz radiation (Penano et al. 2010) has also been studied theoretically by using two-colour linearly polarized laser pulses.

The aim of the present work is to take up an analytical study of enhanced harmonic (both even and odd) generation by two-colour linearly polarized laser beams propagating in a homogeneous underdense plasma. The fundamental frequency of one of the laser beams is a second or higher multiple of the fundamental frequency of the other laser. The study proceeds by considering plasma to be cold, so that thermal motion of electrons can be neglected compared with their collective motion driven by the laser field. Ions are too heavy to move significantly with the evolution of the laser beam; hence, they are considered to be static. A one-dimensional study has been undertaken in the mildly relativistic regime ($a \ll 1$, where $a = eA/mc^2$ is the laser strength parameter), using the perturbative technique. The organization of the paper is as follows. In Sec. 2, the configuration of the two-colour laser system is defined and the wave equation for the vector potential driven by the current density at various harmonic frequencies is set up. In Sec. 3, the generation of radiation at various harmonics has been studied. Summary and conclusions are presented in Sec. 4.

2. Formulation

Consider two intense, linearly polarized laser beams having fundamental frequencies ω and $m\omega$, co-propagating along the z direction in homogeneous plasma. The plane of polarization of both beams is considered to be along the x direction. The normalized vector potentials representing the two laser beams and their harmonics are given by

$$\mathbf{a} = \sum_{j=1}^{N} a_j \cos(k_j z - j\omega t)\hat{x}, \tag{1}$$

$$\mathbf{a}' = \sum_{j=1}^{N} a'_l \cos(k_l z - l\omega t)\hat{x}, \tag{2}$$

where $l = mj$ and the frequency multiplication factor $m \geqslant 2$. The wave equation governing the propagation of the laser pulse through plasma is given by

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{a}_t = -\frac{4\pi e}{m_e c^3}\mathbf{J}, \tag{3}$$

where $\mathbf{a}_t(=\mathbf{a}+\mathbf{a}')$ is the total normalized vector potential and $\mathbf{J}$ $(= -n_e e\mathbf{v}$, n_e and $\mathbf{v}$ are the plasma electron density and velocity, respectively) is the plasma current density.

The equations governing the relativistic interaction between the electromagnetic field and plasma electrons are the Lorentz force, continuity and Poisson's equations, respectively, given by

$$\frac{\partial(\gamma\mathbf{v})}{\partial t} = c\frac{\partial \mathbf{a}_t}{\partial t} + c^2\nabla\phi - c\mathbf{v}\times(\nabla\times\mathbf{a}_t) - (\mathbf{v}.\nabla)(\gamma\mathbf{v}), \tag{4}$$

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e\mathbf{v}) = 0 \tag{5}$$

and

$$\nabla^2\phi = \frac{\omega_p^2}{c^2}\left(\frac{n_e}{n_0} - 1\right) \tag{6}$$

where $\gamma[=(1-v^2/c^2)^{-1/2}]$ is the relativistic factor, n_0 is the ambient plasma electron density, $\omega_p[=(4\pi n_0 e^2/m_e)^{1/2}]$ is the plasma frequency and ϕ is the scalar potential normalized by $e/m_e c^2$. Considering the mildly relativistic regime ($a_t \ll 1$), all quantities can be expanded in terms of the normalized vector potential by using the perturbative technique. Also, the amplitudes of the harmonic frequencies $j \geqslant 2$ present in each of the two laser beams are at least an order of magnitude smaller than the amplitude of their respective fundamental frequency. Thus, the perturbative expansion of (4) up to the third order of the normalized vector potential can be solved to give the transverse velocity as

$$\begin{aligned} v_x^{(1)} + v_x^{(3)} &= a_1 c\left\{1 - \frac{3}{8}\left(a_1^2 + 2a'^2_m\right)\right\}\cos\theta \\ &\quad + a'_m c\left\{1 - \frac{3}{8}\left(a'^2_m + 2a_1^2\right)\right\}\cos\theta' - \frac{3}{8}a_1 a'_m c \\ &\quad \times\{a'_m(\cos\psi_+ + \cos\psi_-) + a_1(\cos\kappa_+ + \cos\kappa_-)\} \\ &\quad - \frac{1}{8}c\left\{a_1^3\cos 3\theta + a'^3_m\cos 3\theta'\right\}, \end{aligned} \tag{7}$$

where $\theta = k_1 z - \omega t$, $\theta' = k_m z - m\omega t$, $\psi_\pm = (2k_m \pm k_1)z - (2m \pm 1)\omega t$ and $\kappa_\pm = (k_m \pm 2k_1)z - (m \pm 2)\omega t$. The second-order perturbative expansion of (4)–(6) may be combined, to give the lowest- (second-) order equation, governing the density perturbation as

$$\frac{\partial^2 n^{(2)}}{\partial t^2} + \omega_p^2 n^{(2)} = \frac{1}{2}\frac{\partial^2 (a_t)^2}{\partial z^2}, \tag{8}$$

where $n^{(2)}$ is the second-order plasma electron density normalized by n_0 and

$$(a_t)^2 = a_1^2 c^2\cos^2\theta + a'^2_m c^2\cos^2\theta' + 2a_1 a'_m\cos\theta\cos\theta'.$$

Equation (8) is solved to give the second-order density perturbation as

$$\begin{aligned} n^{(2)} &= \frac{a_1^2 c^2 k_1^2}{(4\omega^2 - \omega_p^2)}\cos 2\theta + \frac{a'^2_m c^2 k_m^2}{(4m^2\omega^2 - \omega_p^2)}\cos 2\theta' \\ &\quad - \frac{a_1 a'_m k_+^2 c^2}{2\Omega_+}\cos\{k_+ z - (m+1)\omega t\} \\ &\quad - \frac{a_1 a'_m k_-^2 c^2}{2\Omega_-}\cos\{k_- z - (m-1)\omega t\}, \end{aligned} \tag{9}$$

where $k_\pm = k_m \pm k$ and $\Omega_\pm = \omega_p^2 - (m \pm 1)^2\omega^2$.

Equation (9) shows that for various values of m, the plasma electron density oscillates at even as well as odd harmonics of the laser frequency ω. This suggests the possibility of generation of higher odd and even harmonics in plasma by two-colour linearly polarized laser fields.

The perturbed plasma electron velocity (Eq. (7)) and density (Eq. (9)) can be used to obtain the perturbed

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transverse current density $[= -e(n_0 v_x^{(3)} + n_e^{(2)} v_x^{(1)})]$ as

$$J_x^{(3)} = \frac{n_0 ec}{2}\left[\frac{3}{4}a_1\left\{a_1^2 + 2a_m'^2 + \frac{2c^2}{3}\left(\frac{2a_1^2k_1^2}{(\omega_p^2 - 4\omega^2)} + \frac{a_m'^2k_+^2}{\Omega_+} + \frac{a_m'^2k_-^2}{\Omega_-}\right)\right\}\cos\theta + a_1^3\left\{\frac{1}{4} + \frac{c^2k_1^2}{(\omega_p^2 - 4\omega^2)}\right\}\right.$$
$$\times\cos 3\theta + \frac{3}{4}a_m'\left\{2a_1^2 + a_m'^2 + \frac{2c^2}{3}\left(\frac{2a_m'^2k_m^2}{(\omega_p^2 - 4m^2\omega^2)} + \frac{a_1^2k_+^2}{\Omega_+} + \frac{a_1^2k_-^2}{\Omega_-}\right)\right\}\cos\theta'$$
$$+ a_m'^3\left\{\frac{1}{4} + \frac{c^2k_m^2}{(\omega_p^2 - 4m^2\omega^2)}\right\}\cos 3\theta' + a_1 a_m'^2\left\{\left(\frac{3}{4} + \frac{c^2k_m^2}{(\omega_p^2 - 4m^2\omega^2)}\right)(\cos\psi_+ + \cos\psi_-)\right.$$
$$\left.+ \frac{c^2}{2}\left(\frac{k_+^2}{\Omega_+}\cos\psi_+ + \frac{k_-^2}{\Omega_-}\cos\psi_-\right)\right\} + a_1^2 a_m'\left\{\left(\frac{3}{4} + \frac{c^2k_1^2}{(\omega_p^2 - 4\omega^2)}\right)(\cos\kappa_+ + \cos\kappa_-)\right.$$
$$\left.\left.+ \frac{c^2}{2}\left(\frac{k_+^2}{\Omega_+}\cos\kappa_+ + \frac{k_-^2}{\Omega_-}\cos\kappa_-\right)\right\}\right]. \qquad (10)$$

Equation (10) can be substituted into the wave equation (3) to obtain amplitudes of various harmonic frequencies. If $m = 2$, some of the harmonic frequencies at which the current density oscillates are 3ω, 4ω and 5ω, thus pointing towards the possibility of enhancement of these harmonic frequencies. These fast (harmonic) oscillations arise due to nonlinear relativistic effect as well as coupling of the second-order density perturbation with the quiver velocity of plasma electrons. The enhancement of a given harmonic can be measured in terms of the ratio of the amplitude of the harmonic frequencies with respect to the total electric field amplitude of the fundamental frequencies of the two lasers as

$$\alpha_j = \frac{E_j}{E_1 + E_m'} = \frac{j\,|a_j|}{a_1 + m a_m'}. \qquad (11)$$

3. Harmonic radiation generation

In order to study the generation of the third harmonic frequency, we consider $m = 2$. Equation (10) is substituted into the wave equation (3) and the third harmonic terms on both sides are equated. Assuming that the distance over which $\partial a_3(z)/\partial z$ changes appreciably is large compared with the wavelength $(\frac{\partial^2 a_3(z)}{\partial z^2} \ll k_3 \frac{\partial a_3(z)}{\partial z})$ and using the third harmonic dispersion relation $c^2k_3^2 = 9\omega^2 - \omega_p^2$, the evolution of the third harmonic amplitude is given as

$$\frac{\partial a_3}{\partial z} = \frac{3ia_1^3\omega_p^4}{64c^2\omega^2k_3}\exp(i\Delta k_3' z) - \frac{45ia_1a_2'^2\omega_p^4}{256c^2\omega^2k_3}\exp(i\Delta k_3 z), \qquad (12)$$

where $\Delta k_3' = 3k_1 - k_3$, $\Delta k_3 = 2k_2 - k_1 - k_3$, $k_1 = (\omega^2 - \omega_p^2)^{1/2}/c$ and $k_2 = (4\omega^2 - \omega_p^2)^{1/2}/c$. The first term on the right-hand side of (12) is due to a single beam (of frequencyω) while the second term arises on account of mixing of the two colour beams. Integrating (12) with the assumption that the fundamental amplitudes (a_1 and a_2') of each of the two lasers do not evolve significantly with z, the amplitude of the third harmonic evolves as

$$a_3 = \frac{3a_1^3\omega_p^4}{64c^2\omega^2k_3\Delta k_3'}\exp(i\Delta k_3' z) - \frac{45a_1a_2'^2\omega_p^4}{256c^2\omega^2k_3\Delta k_3}$$
$$\times\exp(i\Delta k_3 z) + \text{constant}. \qquad (13)$$

Equation (13) shows that the amplitude of the third harmonic radiation is a superposition of two oscillations having different amplitudes and wave numbers. When the second laser beam is switched off, (13) reduces to the amplitude due to a single laser beam (Esarey et al. 1993). Comparing the amplitudes of the two oscillations for $\lambda = 0.8$ μm, $\omega_p/\omega = 0.1$ and $a_2' = a_1/2$, it is seen that the ratio of the amplitudes of the second and first terms is $\approx 15|\Delta k_3'|/4|\Delta k_3| \approx 7.5$. Therefore, the contribution of the second term to the intensity of the third harmonic will be at least one order larger than that of the first term. This shows that the intensity of harmonic radiation generated by the synthesis of two-colour pulses is higher than that generated by a single pulse. Retaining the predominant (second) term in (13), the modulus of the amplitude of the third harmonic may be obtained as

$$|a_3| = \frac{45a_1a_2'^2\omega_p^4}{128c^2\omega^2k_3\,|\Delta k_3|}|\sin(\Delta k_3 z/2)|. \qquad (14)$$

The ratio of the amplitude of the third harmonic to the sum of fundamental amplitudes (Eq. (11)) is given as

$$\alpha_3 = \frac{135a_1a_2'^2\omega_p^4}{128c^2\omega^2k_3\,|\Delta k_3|\,(a_1 + 2a_2')}|\sin(\Delta k_3 z/2)|. \qquad (15)$$

Equation (15) shows that α_3 is periodic in z. The minimum value of z for whichα_3 is maximum is given by

$$z = L_D = \pi/\Delta k_3,$$

where L_D is the detuning distance. For interaction length $z < L_D$ the third harmonic amplitude increases with z whereas for $z > L_D$ it reduces again. It is important to note that the detuning distance (L_D) for the third harmonic using a two-colour laser system is larger than

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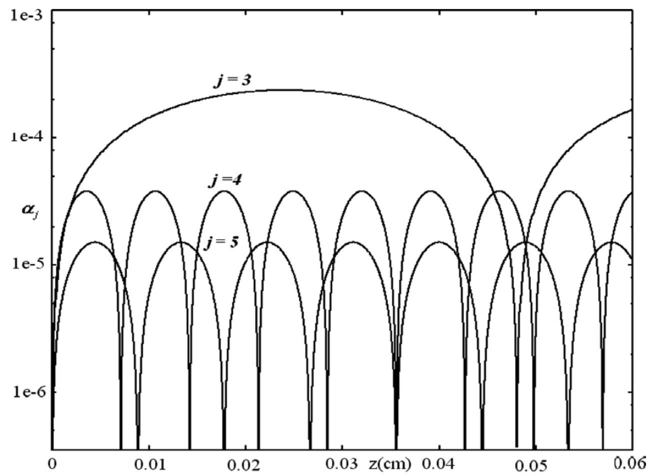

Figure 1. Variation of normalized amplitudes of the third, fourth and fifth harmonic radiation with the propagation distance z (cm), for $\lambda = 0.8$ μm, $\omega_p/\omega = 0.1$, $a_1 = 0.3$ and $a_2' = 0.15$.

that for the third harmonic of a single laser beam (L_D'), since $L_D/L_D' = |\Delta k_3'|/|\Delta k_3| \approx 8$. This implies that higher energy conversion from fundamental to the third harmonic radiation can be achieved by using two-colour laser pulses.

Similarly, the wave equation for the fourth and fifth harmonics of ω can be set up and the respective evolution equations for their amplitudes may be obtained as

$$\frac{\partial(a_4 + a_2')}{\partial z} = \frac{55ia_1^2 a_2' \omega_p^4}{576c^2\omega^2 k_4} \exp(i\Delta k_4 z) \qquad (16)$$

and

$$\frac{\partial a_5}{\partial z} = \frac{139ia_1 a_2'^2 \omega_p^4}{2304\omega^2 c^2 k_5} \exp(i\Delta k_5 z), \qquad (17)$$

where $\Delta k_4 = k_2 + 2k_1 - k_4$ and $\Delta k_5 = 2k_2 + k_1 - k_5$. Therefore, the modulus of amplitudes of the fourth and fifth harmonic frequencies is respectively given by

$$|a_4 + a_2'| = \frac{55a_1^2 a_2' \omega_p^4}{288c^2\omega^2 k_4 |\Delta k_4|} \sin(\Delta k_4 z/2) \qquad (18)$$

and

$$|a_5| = \frac{139a_1 a_2'^2 \omega_p^4}{1152c^2\omega^2 k_5 |\Delta k_5|} \sin(\Delta k_5 z/2). \qquad (19)$$

It may be noted that the current density oscillating at 4ω drives the amplitude of the fourth harmonic of the first laser as well as the second harmonic of the second laser beam. The respective amplitudes of the fourth and fifth harmonic radiation normalized by the fundamental amplitudes are given by

$$\alpha_4 = \frac{220a_1^2 a_2' \omega_p^4}{288c^2\omega^2 k_4 |\Delta k_4| (a_1 + 2a_2')} |\sin(\Delta k_4 z/2)| \qquad (20)$$

and

$$\alpha_5 = \frac{695a_1 a_2'^2 \omega_p^4}{1152c^2\omega^2 k_5 |\Delta k_5| (a_1 + 2a_2')} |\sin(\Delta k_5 z/2)|, \qquad (21)$$

where α_4 and α_5 are also periodic in z, attaining their maximum values at $z = \pi/\Delta k_4$ and $z = \pi/\Delta k_5$ respectively.

The plot of α_j for $j = 3$, 4 and 5 versus propagation distance z is shown in Fig. 1, for the parameters $\lambda = 0.8$ μm, $\omega_p/\omega = 0.1$, $a_1 = 0.3$ and $a_2' = 0.15$. It is seen that the maximum amplitude reduces as the harmonic number increases. The corresponding intensities of the third, fourth and fifth harmonic radiation at saturation, (I_j[W cm^{-2}] $= a_j^2/7.32 \times 10^{-19} \lambda^2[\mu$m]) are found to be 4.2259×10^{10}, 1.1217×10^9 and 1.7488×10^8 W cm^{-2}, respectively.

If larger values of the frequency multiplication factor m are considered, the generation of higher harmonics would be possible. This is evident from the fact that the maximum frequency at which the current density (Eq. (10)) oscillates is $(2m + 1)$. For example, if $m = 10$ is considered, the current density oscillates at 12ω, 19ω and 21ω, leading to the possibility of enhancement of these frequencies.

4. Summary and conclusion

In the present paper, we have analytically studied odd and even harmonic generation using two-colour, linearly polarized, laser beams propagating in a homogeneous underdense plasma. The frequency of the second laser beam is considered to be a second or higher multiple of the first. Perturbative technique has been used to obtain the nonlinear plasma electron current density. The current density is substituted into the one-dimensional wave equation and similar harmonic terms are equated on both sides to obtain the evolution of the amplitude of a given harmonic radiation. Equations for the evolution of amplitudes of the third, fourth and fifth harmonic frequencies have been obtained. While deriving these equations, the respective dispersion relations have been used.

It is observed that in contrast to harmonic generation using a single laser beam in plasma, both even and odd harmonics can be generated by using two-colour, linearly polarized laser beams. The third harmonic intensity, generated by using two-colour laser fields, is one order of magnitude larger than that obtained by a single beam. It is seen that the detuning distance (wave number mismatch) between the fundamental radiation field and the generated third harmonic considerably enhances (reduces) by using two-colour laser fields, due to which the energy conversion from the fundamental to harmonic radiation becomes more efficient. The present analysis also points towards the possibility of higher harmonic generation which can be obtained by considering larger values of the frequency multiplication factor m.

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