THE DETERMINATION OF THE ENERGY DISTRIBUTION OF RELATIVISTIC ELECTRONS BY THE FREQUENCY DISTRIBUTION OF THEIR "SYNCHROTRON RADIATION"

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Relativistic electrons moving in a homogeneous magnetic field H emit "synchrotron radiation" with a frequency distribution which may be written in the following form:

$$I(\nu) = A(H)\int_{\tau_1}^{\tau_2} P(\nu\tau)\varphi(\tau)d\tau \ , \qquad (1)$$

where $\tau = \tau(\mathcal{E}) = \tau_0 H^{-1}\mathcal{E}^{-2}$ and $\mathcal{E} = E/\mathrm{mc}^2 = 1/(1-\beta^2)^{1/2}$. The energy distribution $f(\mathcal{E})$ of the emitting electrons is involved in this manner: $\varphi(\tau) = \tau^{-3/2}\psi(\tau)$ and $\psi(\tau) = \psi[\tau(\mathcal{E})] = f(\mathcal{E})$. Finally $A(H) = A_0 H^{1/2}$ and the emission probability $P(\nu\tau)$ is given by [1, 2]:

$$P(\nu\tau) = \nu\tau\int_{\nu\tau}^{\infty} K_{5/3}(\eta)d\eta \ , \qquad (2)$$

where the K_n are the modified Hankel functions. $\tau_1 = \tau_1(\nu, H)$ and $\tau_2 = \tau_2(\nu, H)$ are to be determined as functions of the limiting values for using the approximations made in (2) for $P(\nu\tau)$.

In case of radio-astronomical observations of galactic or extragalactic sources (e.g., Crab nebula or M 87) it is possible to show that the integral in (1) may be extended from 0 to ∞ without noticeable error. Then we can write:

$$I(\nu) = A(H)\int_{0}^{\infty} P(\nu\tau)\varphi(\tau)d\tau \ . \qquad (1')$$

This possibility results (*a*) from the discrete behavior of the emission probability for classical energies, and (*b*) from the smallness of the fraction of electrons with such energies that the quantum corrections become efficacious.

(1′) then is an integral equation to determinate the energy distribution $f(\mathcal{E})$ contained in $\varphi(\tau)$. The solution of this equation can be written in the form

$$\varphi(\tau) = \int \tilde{P}(\nu\tau)J(\nu)d\nu \ , \qquad (3)$$

where $J(\nu) = I(\nu)/A(H)$ and $\tilde{P}(\nu\tau)$ is the solving kernel. Since the kernel P depends only on the product of the two variables and the solving methods for this case are well known, we get the solution in the form

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$$\varphi(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tau^{-(1/2+iz)} J_N(-z)\rho_N(-z)\frac{dz}{\theta_N(-z)} \,. \tag{4}$$

Here the functions with index N are the N-transformed:

$$\begin{aligned} J_N(-z) &= \frac{1}{\sqrt{2\pi}}\int_0^{\infty} \nu^{-(1/2+iz)} J(\nu)d\nu \,, \\ \rho_N(-z) &= \frac{1}{\sqrt{2\pi}}\int_1^{\infty} s^{-(3/2+iz)} ds \,, \\ \theta_N(-z) &= \frac{1}{\sqrt{2\pi}}\int_0^{\infty} t^{-(3/2+iz)}\left[\int_0^t P(\nu\tau)d(\nu\tau)\right]dt \,. \end{aligned} \tag{5}$$

Further, we can show that for $P(\nu\tau)$, having the asymptotic behavior

$$P(\nu\tau) = \begin{cases} \text{const } H(\nu\tau)^{1/3} & \nu_0 < \nu \ll \nu_\varepsilon = \dfrac{1}{\tau} \\ \text{const } H\sqrt{\nu\tau}\, e^{-\nu\tau} & \dfrac{1}{\tau} = \nu_\varepsilon < \nu \,, \end{cases}$$

one can use without noticeable error in the full range of energies and frequencies the approximate form

$$P(\nu\tau) = q(\nu\tau)^{\alpha} e^{-\nu\tau} \tag{6}$$

with suitably chosen values for the constants q and α.

The solution of our problem then reduces to a Laplace transform:

$$J(\nu) = \int_0^{\infty} \varphi(\tau)\tau^{\alpha} e^{-\nu\tau} d\tau = \mathcal{L}\{\Phi(\tau), \nu\}, \tag{7}$$

$$J(\nu) = I(\nu)/qA(H)\nu^{\alpha} \,,$$

$$\Phi(\tau) = \varphi(\tau)\tau^{\alpha} \,,$$

$$\varphi(\tau) = \tau^{-\alpha}\mathcal{L}^{-1}\{J(\nu), \tau\} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} J(\nu)e^{\nu\tau}d\nu \,. \tag{8}$$

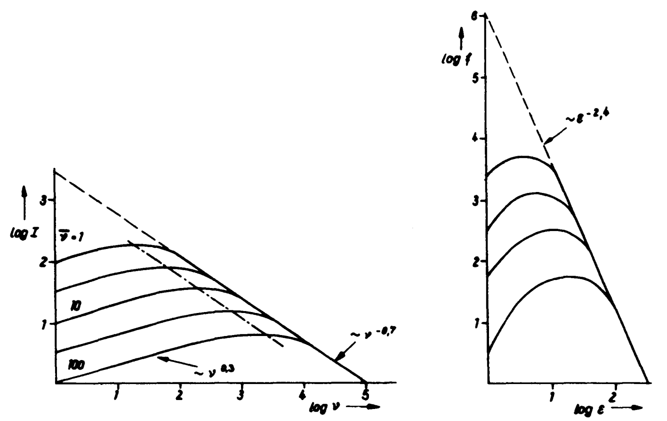

FIG. 1. Determination of the energy distribution of relativistic electrons by the frequency distribution of their synchrotron radiation using the Laplace transform.

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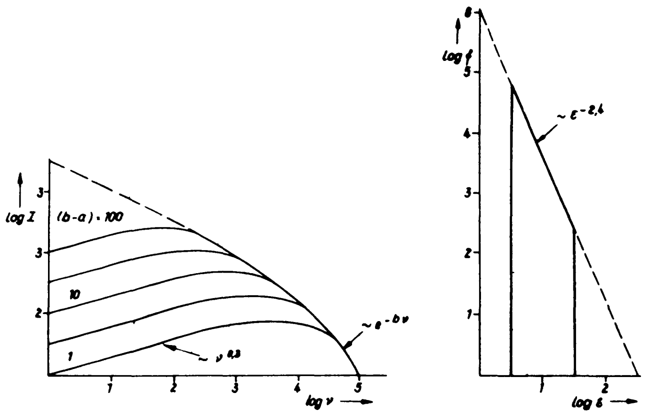

FIG. 2. Determination of the energy distribution of relativistic electrons by the frequency distribution of their synchrotron radiation using the Laplace transform.

Laplace transforms thus easily yield electron energy spectra from synchrotron radiation spectra, as needed in drawing conclusions about physical conditions in radio sources from radio observations. In the following table are listed some examples.

$J(\nu)=\mathcal{L}\{\Phi(\tau),\nu\}$	$I(\nu) = J(\nu)qA(H)\nu^{0.3}$	$\Phi(\tau)=g\varepsilon^{2.4}f(\varepsilon)$	$f(\varepsilon)=g^{-1}\varepsilon^{-2.4}\Phi(\tau)$
$\frac{1}{\nu+\bar{\nu}}$	$\frac{qA\nu^{0.3}}{\nu+\bar{\nu}}$	$e^{-\bar{\nu}\tau}$	$g\varepsilon^{-2.4}e^{-\bar{\nu}g/\varepsilon^2}$
$\frac{\Gamma(m)}{(\nu+\bar{\nu})^m}$, Re $m>0$	$\frac{\Gamma(m)qA\nu^{0.3}}{(\nu+\bar{\nu})^m}$	$\tau^{m-1}e^{-\bar{\nu}\tau}$	$g^{m-2}\varepsilon^{-(2m-1)-2.4}e^{-\bar{\nu}g/\varepsilon^2}$
$\frac{e^{-a\nu}-e^{-b\nu}}{\nu}$ $(a<b)$	$qA\frac{e^{-a\nu}-e^{-b\nu}}{\nu^{0.7}}$	0, $0<\tau<a$ 1, $a<\tau<b$ 0, $b<\tau$	0, $(g/a)^{1/2}<\varepsilon$ $g^{-1}\varepsilon^{-2.4}$, $(g/b)^{1/2}<\varepsilon<(g/a)^{1/2}$ 0, $0<\varepsilon<(g/b)^{1/2}$
$\frac{e^{-a\nu}-e^{-b\nu}}{\nu^3}$ $(a<b)$	$qA\frac{e^{-a\nu}-e^{-b\nu}}{\nu^{2.7}}$	0, $0<\tau<a$ $\frac{1}{2}(\tau-a)^2$, $a<\tau<b$ $\tau(b-a)+\frac{1}{2}(a^2-b^2)$, $b<\tau$	0, $(g/a)^{1/2}<\varepsilon$ $\frac{1}{2}g^{-1}\left(\frac{g}{\varepsilon^2}-a^2\right)\varepsilon^{-2.4}$, $(g/b)^{1/2}<\varepsilon<(g/a)^{1/2}$ $\left\{\frac{g}{\varepsilon^2}(b-a)+\frac{1}{2}(a^2-b^2)\right\}g^{-1}\varepsilon^{-2.4}$, $0<\varepsilon<(g/b)^{1/2}$

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