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A DYNAMIC MODEL OF TAXATION, CORRUPTION, AND PUBLIC INVESTMENT IN THE DYNASTIC CYCLE: THE CASE OF IMPERIAL CHINA

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This paper develops a stochastic growth model that reproduces the main stylized facts of Imperial China's dynastic cycle—in particular, the time path of taxation, public spending, and corruption and their attendant impacts on production and income distribution. In this model, the emperor uses part of his tax income to finance the building of public capital and administrative institutions. This "institutional capital" enhances the productivity of the economy and limits extortion by the county magistrates. The dynastic cycle is driven by random shocks to the authority of the emperor and his central administration, which change the efficiency of institutional capital.

Keywords: Dynastic Cycle, Institutional Capital, Appropriation Function, Corruption

1. INTRODUCTION

From emperor Qin Shihuangdi to Puyi, China, or most of its territory, was governed by a single dynasty for 1,709 years out of 2,132. Most of these dynasties lasted for long periods of time: 289 years for the Tang and 276 years for the Ming, for instance. China's imperial dynasties follow a familiar pattern, called the dynastic cycle: It starts with rehabilitation of the economy and restoration of efficient government, followed by a period of stability, then a gradual decline during which corruption becomes rampant and ended with peasant uprisings leading to a breakdown of the administration.

Usher (1989) and Chu and Lee (1994) develop a convincing explanation of the dynastic cycle based on a model of endogenous population cycles in a Malthusian context. They separate people in ancient China into three groups: peasants, bandits,

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and rulers. Peasants grow and harvest crops and pay taxes, rulers collect taxes and hunt for bandits, and bandits rob and steal food and clash with peasants and rulers. Population growth leads to a gradual fall in income per head, which induces more and more peasants to become bandits. The decrease in the surplus over bare subsistence reduces the amount of collected taxes and thus the ability of the ruler to fight bandits. Eventually, warfare generalizes and causes dynastic collapse and sharp population decline.

Some authors enrich this model with a government sector. For instance, Turchin (2003) notes that during the first part of a dynasty, the growth of the economy allows the ruling class to collect taxes easily and improve its welfare. However, the increase in population density and the fall in income per capita, with the refusal by the elite to adjust its consumption downward, lead to the ruin of the economy and then to famine and political unrest. Another contribution comes from establishing causality between climatic changes in China, the productivity of its land, and consequently its social stability. Climatic changes in the nomadic territories surrounding China also induce invasions and migrations with destabilizing effects [Zhang et al. (2007); Pederson et al. (2012)].

We present in this paper a new explanation of the dynastic cycle, which complements the previous ones. We are particularly interested in investigating the evolution of taxation, public spending, and corruption over the cycle and their effects on production and income distribution.

In Imperial China, taxes were used to pay the cost of maintaining waterways, managing water control works for irrigation, building granary reserves, and all kinds of projects that helped to promote material security and economic growth. They were collected in each county by a magistrate. This official, whenever he could, extracted more income from the taxpayers than was legally allowed, for himself and for his supporters, the local gentry, for instance. The state's revenue increased at the beginning of a dynasty, and then it peaked and finally contracted. Corruption and extortion followed the opposite pattern. The loss of income by the state led to a gradual and sustained reduction in its supply of public goods. This and worsening corruption were the main culprits in the socioeconomic instability of China at the end of a dynasty.

The model developed in this paper can reproduce these stylized facts. It is based on the following assumptions: The emperor uses part of his tax income to finance public capital and the administrative institutions of China, which are summed up here by the term of institutional capital. The first function of this capital is to enhance the productivity of the economy. Its second function is to limit extortions by the county magistrates, and thus to increase the share of the emperor and the farmers in total output. The efficiency of institutional capital in this last respect depends on the capacity of the emperor and the central administration to control and manage this capital, which we call the authority of the emperor.

An important contribution of this paper is that the impulse of the dynastic cycle comes from random change in the authority of the emperor. This takes place after a big event (the unanticipated death of an emperor or an important minister, a palace coup, a big rebellion, a military defeat, etc.); this event is followed by a change in the composition of the central administration, the quality of its members, and their ability to solve their collective action problems in the best interest of the empire. The first emperor of a dynasty, who was the leader of a widespread rebellion or a powerful kingdom, has to have a high level of authority to have reached this position. He can appropriate a share of output, which is sufficient to put the economy on a path of accumulation of institutional capital. The rise in capital leads to an increase in production and in the income of the emperor, which allows the accumulation of still more capital. However, over time, emperors become less able or committed at selecting and controlling a good central administration, and their authority decreases. Then, even if the stock of institutional capital has become high, the share of the emperor in total output is lower, and the share extorted by the county magistrates is higher. The emperor, having lower income, invests less, and the stock of institutional capital decreases and with it the output of the economy. This leads to further decreases in the income of the emperor, and to more extortion by the magistrates.[1]

We can find examples of dynastic cycles other than those in China, such as in the Ottoman or the Mughal Empire. Hence, even though we were inspired by the history of Imperial China, we expect our model to have some relevance to the analysis of other dynasties. For these reasons we adopted a specification as general and simple as possible. Taxation and corruption are modeled by an appropriation game between the emperor, the county magistrates, and the farmers. The changes in the authority of the emperor and his central administration and the end of the dynasty are modeled by a very simple Markov process: we do not try to analyze the many reasons for these events, our focus being their effects on the dynamics of institutional capital, taxation, and corruption.

There are a few papers that are related to our. Sng (2010) elaborates on the principal–agent relationship between the emperor and the county magistrates. His model establishes that the tax rate decreases and the extortion rate increases when monitoring of the magistrates weakens (for instance, because the size of China increases, as happened in the first half of the Qing dynasty). Sng and Miroguchi (2012) extend this analysis to a comparison between China and Japan before 1850.

The reminder of this paper is organized as follows: The second section gives historical evidence on taxation, extortion, and the supply of public goods in Imperial China. The third section presents a model of production, taxation, and extortion in a deterministic environment. The fourth section introduces stochastic shocks into the model in the forms of changes in the authority of the emperor and his central administration and of a revolution ending the dynasty. Simulations of this stochastic model show that it can reproduce the stylized facts of the dynastic cycle.

2. HISTORICAL BACKGROUND

2.1. Taxation, Extortion, and Public Capital in Imperial China

Much information on these matters is available for the Qing dynasty (1644–1912).[2] Qing society comprised two main classes, the gentry and the commoners. A gentry member was one who held an official educational degree. Such a degree was a prerequisite to entering government service, and it could be obtained either by passing at least the first level of the civil service examinations or by purchase. Many members of the gentry owned sizeable landholdings. Although only a small fraction of them ended up in officialdom, the gentry were granted social and legal privileges by the state to distinguish them from the commoners and they were committed to the management of local affairs.

The most important administrative unit in China was the county. There were about 1,500 counties in Qing (and Ming) China. Each was headed by a magistrate, who was fully accountable to his superiors for all affairs concerning the administration of his jurisdiction, especially the collection of land and miscellaneous taxes, which constituted the bulk of the state's tax revenue.

Each county was assigned a tax quota according to its cultivated acreage and land quality. It was the responsibility of the county magistrate to ensure that taxes owed by residents in his county were collected fairly in accordance to actual ownership. The magistrate needed the support of the local gentry to collect taxes. To obtain it he had to give them a share of tax revenues, and to avoid taxing their land too heavily. Otherwise, members of the gentry would accuse him of extorting illegal surcharges and ruin his career—this being possible because, in a thoroughly bureaucratic system, an official is assumed to be honest so long as complaints are not heard, real honesty being less important than peace and quiet. This created enormous opportunities for the magistrate and his underlings to profit from the tax collection process. Clerks and runners assigned to receive tax payments used various pretexts to commit extortion. Then they shared their profits with the magistrate, who would forward some of what he received to higher officials in the form of gifts [Rowe (2009); Sng (2010); Wang (1936)].

Farmers had many ways to resist taxation and extortion: they could hide part of their output, or sell it on the black market; they could bribe petty officials and members of the gentry, or cheat the magistrate. For instance, Wang (1936) explains that influential members of the gentry could gather a number of poor peasant families together and represent them as being one solid clan, owning a large estate and therefore to be taxed only with caution. Poor peasants could by this device escape paying some of the many surcharges, while the scholars who helped them collected part of the difference between the light tax on important people and the heavy tax on common peasants.

According to the law on avoidance, field officials were routinely rotated, with tours of duty no longer than three years [Rowe (2009)]. Thus, the county magistrate had no incentive to invest his own money in local development. In contrast, the emperor cared not only for his own future, but also for the future of his dynasty. Hence, it was in his interest to invest in public goods and services, even when the return on this investment was in the very long run (an example being the huge investment in the building of the Great Wall).

Investments by the emperor improved the productivity of agriculture, but they also limited extortion by the country magistrates. In their comparison between

Qing China and Tokugawa Japan, Sng and Moriguchi (2012) identify four kinds of investments by the sovereign: (a) the standardization of money, weights, and measures; (b) building and maintaining roads; (c) the provision of public services (e.g., firefighting, water management, the building of granaries); (d) environmental management (e.g., limiting deforestation, erosion, and the destructive effects of floods). All these investments had productive effects. The usual forms of extortion involved manipulation of the exchange rate between silver and copper coins and of the weights and measures used by tax collectors. Bad communication made the control of officials more difficult: Sng (2010) notes that extortion increased with distance from Beijing. Thus, investments *a* and *b* limited extortions by the county magistrates. However, investments *c* and *d* could facilitate these extortions if the control power of the emperor was already weak: Sng and Moriguchi (2012) explain that these investments were much higher in Japan than in China, because in the first country the shogun and the daimyo had enough control on the officials to prevent them from using these investments for their own profit.

Taxes were light during most of the Qing dynasty. They roughly amounted to five to ten percent of agricultural output during the eighteenth century. Sng (2010) notes that the state's tax revenue during the Qing dynasty increased for the second half of the seventeenth century, peaked in the 1720s, and then contracted steadily. The loss of income by the state led to a gradual and sustained reduction in its supply of public goods after 1730. Corruption and extortion followed almost the opposite pattern. Major corruption scandals took place infrequently during the reigns of the three first Qing emperors (1644–1735). By the Qianlong's reign (1736–1795), they became a regular occurrence. High corruption in China consumed 22% of China's agricultural output in 1873 according to Ni and Van (2006). The increasing corruption and weakening of the state led to many rebellions in China during the nineteenth century. These facts are consistent with the available data on real income per capita in China: real wages decreased from 1738 until the Taiping Rebellion in the mid-nineteenth century, when they reached a very low level [Allen et al. (2009)]. The later years of the Ming dynasty (1368–1644) were also characterized by excessive taxation and corruption that provoked uprisings all over China.

2.2. The Authority of the Emperor

The central administration in Imperial China, which was in charge of controlling and managing the county magistrates, was made up of an incumbent ruling elite of advisors, ministers, military officers, eunuchs, families of influential consorts, etc., which acted under the supervision of the emperor. We can identify in Chinese history periods when the emperor and the incumbent elite were able to control and be obeyed by local administrations and farmers, collect a reasonable amount of taxes (financing their investment and consumption), and limit extortion by the county officials, and other periods when this control was weak. There are many examples of "weak" emperors: for example, Qianlong (1711–1799), who entrusted the day-to-day governance of the country to the very corrupted Heshen for the last

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twenty-four years of his reign, or Wanli (1563–1620), who for the last twenty years of his reign refused to attend meetings, receive his ministers, or make necessary appointments.

We define the authority of the emperor by the quality of the members of the ruling elite and his ability to induce them to act in his best interest. The immediate cause of a change in authority can in general be related to the occurrence of a big event: the death of the emperor and the appointment of a new emperor; the death of a powerful minister; a change in the ability or the commitment of the emperor; a palace coup or a big rebellion; the rise of conflicts between members of the elite or the resolution of former conflicts. An example of change in the composition of the ruling elite is Yongle (1360–1424), who reversed the policy of the previous emperor, Hongwu (1328–1398), of excluding eunuchs from politics, and deliberately used them to counterbalance the power of officials.

3. A DETERMINISTIC MODEL OF THE IMPERIAL ECONOMY

A model is first developed in a deterministic environment. We will successively present the role of institutional capital in production and appropriation, before explaining how the emperor sets the optimal capital accumulation path.

3.1. Production

We consider an administrative county in China in a given period (there will be no ambiguity from not identifying this period in the notation of this and the next subsection). The county has a stock of public capital, $K > 0$, resulting from past investments by the emperor. As public capital here has a very broad meaning, which includes infrastructure, but also administrative and judicial institutions, we call it institutional capital and interpret it as an index of the quality of institutions.

The mass of farmers is constant and normalized to one.[3] They produce a unique good. Each farmer owns the same area of arable land and has an endowment of one unit of labor, which he devotes entirely to production. His output, Y, depends on these two fixed factors. Moreover, institutional capital raises the productivity of the farmer.[4] We have

$$Y = AK^{\alpha}, \text{ with } A > 0 \text{ and } 0 < \alpha < 1. \tag{1}$$

3.2. Appropriation

The tax system of Imperial China was complex and changing. Its distortions, the extortions by the country magistrate, and the resistance of the farmers took many forms. This is why we prefer to formalize the distribution of the output of the farmers between the different agents using the general and versatile specification of appropriation functions [Skaperdas (1992); Grossman and Kim (1995); Hirshleifer (1995)].

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In the simplest model in this literature, an agent with endowment Y faces the threat of a predator who spends an amount Q to appropriate a share of Y. The agent spends an amount Q' to defend his property. As a result of these two actions, the shares kept by the agent and appropriated by the predator, respectively, are $p = 1/(1+\lambda Q/Q')$ and $1-p$, where $\lambda > 0$ measures the effectiveness of offense over defense. As the resources devoted to appropriation are lost, the consumptions of the agent and the predator, respectively, are $pY - Q'$ and $(1-p)Y - Q$. One can easily establish that the Nash equilibrium of this game is such that the agent and the predator spend the same amounts $Q = Q' = \lambda Y/(1+\lambda)^2$. Thus, their respective shares of Y are $p = 1/(1+\lambda)$ and $1 - p = \lambda/(1+\lambda)$. Their consumption is $Y/(1+\lambda)^2$ and $\lambda^2 Y/(1+\lambda)^2$.

In our model, the appropriation game is played at the county level, first between the farmers and the magistrate. The farmers allocate a quantity Q_{F} of their production to defend it from expropriation. The county magistrate collects taxes on behalf of the emperor, who spends Q_{E} to finance the administration and influence the gentry. The magistrate also spends Q_{M} of his own money to obtain help and support from the gentry, the administration, and more generally anybody influential in the county. These two expenditures decrease the share p_{F} of production that the farmers can keep for themselves. This share is given by the appropriation function

$$p_{\mathrm{F}} = \frac{\lambda_{\mathrm{F}} Q_{\mathrm{F}}}{\lambda_{\mathrm{F}} Q_{\mathrm{F}} + \lambda_{\mathrm{M}} Q_{\mathrm{M}} + \lambda_{\mathrm{E}} Q_{\mathrm{E}}}, \quad \text{with} \quad \lambda_{\mathrm{F}}, \lambda_{\mathrm{M}}, \lambda_{\mathrm{E}} > 0, \tag{2}$$

and the income of the farmers is

$$Y_{\mathrm{F}} = p_{\mathrm{F}} Y - Q_{\mathrm{F}}. \tag{3}$$

The county magistrate collects from the farmers the output $(1 - p_{\mathrm{F}})Y$, which has to be allocated between taxes transferred to the emperor and extortion or corruption money, kept by the magistrate. The spending Q_{M} by the magistrate allows him to defend the share he keeps. The spending Q_{E} by the emperor strengthens his ability to increase the share transferred to him. Finally, the share r of the magistrate is given by a second appropriation function,

$$r = \frac{\lambda_{\mathrm{E}} Q_{\mathrm{E}}}{\lambda_{\mathrm{M}} Q_{\mathrm{M}} + \lambda_{\mathrm{E}} Q_{\mathrm{E}}}. \tag{4}$$

We deduce from (2) and (4) the expressions for the shares of the output of the farmers appropriated by the county magistrate (extortion) and the emperor (taxes), and of their incomes, which respectively are

$$p_{\mathrm{M}} = \frac{\lambda_{\mathrm{M}} Q_{\mathrm{M}}}{\lambda_{\mathrm{F}} Q_{\mathrm{F}} + \lambda_{\mathrm{M}} Q_{\mathrm{M}} + \lambda_{\mathrm{E}} Q_{\mathrm{E}}}, \quad Y_{\mathrm{M}} = p_{\mathrm{M}} Y - Q_{\mathrm{M}}, \tag{5}$$

$$p_{\mathrm{E}} = \frac{\lambda_{\mathrm{E}} Q_{\mathrm{E}}}{\lambda_{\mathrm{F}} Q_{\mathrm{F}} + \lambda_{\mathrm{M}} Q_{\mathrm{M}} + \lambda_{\mathrm{E}} Q_{\mathrm{E}}}, \quad Y_{\mathrm{E}} = p_{\mathrm{E}} Y - Q_{\mathrm{E}}. \tag{6}$$

As in the basic model, λ_M/λ_F and λ_E/λ_F in (2) can be interpreted as measures of the effectiveness of offense (by the magistrate and the emperor) against defense (by the farmers). Similarly, λ_E/λ_M in (4) can be interpreted as a measure of the effectiveness of offense (by the emperor) against defense (by the magistrate). Actually, the share of agent i $(= F, M, E)$, p_i, increases with his appropriation spending, Q_i, and with λ_i. We can interpret this parameter λ_i as an index of his appropriation power.

We note that the model makes a clear-cut distinction between the two types of expenditure by the emperor. On one hand, his investment increases the stock of institutional capital, which enhances the productivity of the farmers, and, as we will see later, their and the emperor's appropriation power. On the other hand, the appropriation cost, Q_E, is a flow, which increases the appropriation share of the emperor.

The farmers, the county magistrate, and the emperor set the value of, respectively, Q_F, Q_M, and Q_E, all non-negative. They play a noncooperative game and we look for its Nash equilibrium.

Let us define the income and the appropriation costs of the three agents measured in shares of the output of the farmers by

$$y_i = Y_i/Y, \quad q_i = Q_i/Y, \quad i = F, M, E. \tag{7}$$

The farmers set the value of $q_F \geq 0$, which maximizes their income:

$$y_F = \lambda_F q_F / \left(\lambda_F q_F + \lambda_M q_M + \lambda_E q_E\right) - q_F, \tag{8}$$

with $q_M \geq 0$ and $q_E \geq 0$ given. If $q_M = q_E = 0$, this problem has no solution. Otherwise, $y_F(q_F)$ is a concave function, which reaches a non-negative and unique maximum for $q_F \in [0\ 1[$. This maximum can be reached either at a positive value of q_F and be the unique root of $y'_F(q_F)$, or at zero. More precisely, the reaction function of the farmers is given by:

$$\left(\lambda_M q_M + \lambda_E q_E\right) / \left(\lambda_F q_F + \lambda_M q_M + \lambda_E q_E\right)^2 = 1/\lambda_F, \text{ if } \quad \lambda_M q_M + \lambda_E q_E \leq \lambda_F;$$
$$q_F = 0, \text{ if } \quad \lambda_F \leq \lambda_M q_M + \lambda_E q_E. \tag{9}$$

The reaction functions of the magistrate and the emperor have the same expression, after λ_F and q_F have been interchanged with λ_M and q_M, or with λ_E and q_E, respectively.

We have the following proposition:

PROPOSITION 1. *The appropriation game has a unique Nash equilibrium. If the appropriation powers of the three agents are not too dispersed around their harmonic mean* λ_H *and satisfy* $\lambda_F, \lambda_M, \lambda_E > 2\lambda_H/3 = 2/(1/\lambda_F + 1/\lambda_M + 1/\lambda_E)$, *the solution of the game is given by*

$$q_i = \frac{2}{3}\frac{\lambda_H}{\lambda_i}\left(1 - \frac{2}{3}\frac{\lambda_H}{\lambda_i}\right), \; p_i = 1 - \frac{2}{3}\frac{\lambda_H}{\lambda_i}, \; y_i = \left(1 - \frac{2}{3}\frac{\lambda_H}{\lambda_i}\right)^2, \; i = F, M, E, \quad (10)$$

and the income of each agent is positive. Otherwise, the income is zero for one of the three agents and positive for the two others.

Proof. Consider the case where the Nash equilibrium is such that $q_F, q_M, q_E > 0$. Then, by adding the three reaction functions, we get

$$\lambda_F q_F + \lambda_M q_M + \lambda_E q_E = 2/\left(1/\lambda_F + 1/\lambda_M + 1/\lambda_E\right) = 2\lambda_H/3. \quad (11)$$

We substitute this expression into the reaction functions and get the first part of (10). We deduce the other parts from the expressions of the output shares and incomes, (2), (3), (5), (6), and (7). Finally, we compute the conditions under which $q_i > 0$ and see that under these conditions we have $y_i > 0$.

We saw that two appropriation costs cannot be simultaneously zero in a Nash equilibrium. However, one of them may be zero and then the output share and the income of the concerned agent are also zero. We can easily compute the conditions on the parameters to obtain this case, and show that the incomes of the two other agents are positive. ■

In the terminology of Acemoglu and Robinson (2008), the appropriation game is part of the economic institutions of Imperial China. It determines the resource spent by the farmers to keep as much as they can of their output, and by the emperor and the magistrate to collect taxes and corruption money. λ_i/λ_H represents the de facto political power of agent i.

We assume that besides its productive effects, investment in institutional capital, that is, in the quality of institutions, K, increases the emperor's and the farmers' de facto political power and decreases that of the county magistrate.[5] However, good institutions can help the emperor only as far as he can use them efficiently. If the emperor and the incumbent ruling elite are competent and can solve their collective action problem, they will be able to manage the institutional capital and be better obeyed by the local administration. We call this ability the authority of the emperor, denoted by ϕ.

An emperor with a given level of authority progressively adjusts the de facto political power of all agents to the levels he wishes by investing in institutional capital. Thus, the persistence of political power comes from the persistence of institutional capital, as we will explain in the two next subsections. In Section 4, where authority follows a stochastic process, the persistence of de facto political power will also come from the persistence of authority.[6]

We adopt a simple specification that the appropriation power λ_i of agent i is related to the stock of institutional capital K and the authority of the emperor ϕ as follows: $\lambda_i^{-1} = a_i \phi K^\gamma + b_i$, with $i = F, M, E$, $\gamma > 0$. We deduce that

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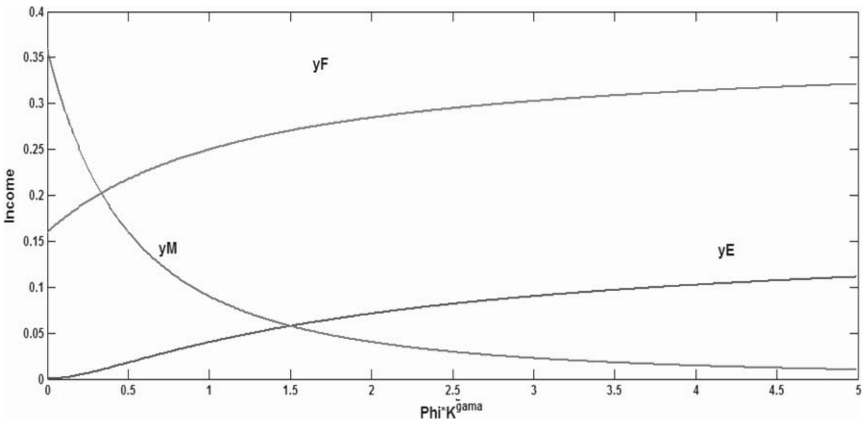

FIGURE 1. Incomes, in shares of output, of the emperor, the chief officials, and the farmers, as functions of ϕK^{γ}.

$\lambda_H^{-1} = (\sum_i a_i \phi K^{\gamma} + \sum_i b_i)/3$. We do not restrict the generality of this specification by setting the normalization conditions $\sum_i a_i = \sum_i b_i = 2$. We deduce from Proposition 1 that the income measured as the share of output of agent i is $y_i = [1 - (a_i \phi K^{\gamma} + b_i)/(\phi K^{\gamma} + 1)]^2$.

We impose the following conditions on these income functions: when ϕK^{γ} increases from zero to infinity, then (i) the income of the emperor increases from zero to a value less than one; (ii) the income of the county magistrate decreases from a value less than one to zero; (iii) the income of the farmers increases from a positive value to a value less than one. Then, we can show that the incomes of the three agents depend only on the two parameters $0 < b_{\mathrm{M}} < a_{\mathrm{E}} < 1$, and are

$$y_{\mathrm{F}} = \left(\frac{a_{\mathrm{E}} \phi K^{\gamma} + b_{\mathrm{M}}}{\phi K^{\gamma} + 1} \right)^2, \qquad \textbf{(12)}$$

$$y_{\mathrm{M}} = \left(\frac{1 - b_{\mathrm{M}}}{\phi K^{\gamma} + 1} \right)^2, \qquad \textbf{(13)}$$

$$y_{\mathrm{E}} = \left[\frac{(1 - a_{\mathrm{E}})\, \phi K^{\gamma}}{\phi K^{\gamma} + 1} \right]^2. \qquad \textbf{(14)}$$

We easily check that the de facto political power of the emperor and the farmers, $\lambda_{\mathrm{E}}/\lambda_{\mathrm{H}}$ and $\lambda_{\mathrm{F}}/\lambda_{\mathrm{H}}$, increases, and the de facto political power of the magistrate, $\lambda_{\mathrm{M}}/\lambda_{\mathrm{H}}$, decreases with ϕK^{γ}.

Fig. 1 plots the graphs of the incomes of the three agents, measured in shares of output, as functions of ϕK^{γ} for the value of the parameters $a_{\mathrm{E}} = 0.6$, $b_{\mathrm{M}} = 0.4$.

3.3. The Accumulation of Institutional Capital

The income of the emperor in period t, which is the product of his appropriated share and the total output of the farmers, is allocated between his consumption, C_{Et}, and institutional investment, I_t. We deduce from (1) and (14) that

$$Y_E(K_t) = y_E(K_t)\, Y(K_t) = \left[\frac{(1-a_E)\,\phi_t K_t^{\gamma}}{\phi_t K_t^{\gamma}+1}\right]^2 A K_t^{\alpha} = C_{Et} + I_t. \tag{15}$$

Under the assumption that the curvature of the appropriated share, $-K_t y_E''(K_t)/y_E'(K_t) = 1-2\gamma+\gamma\phi K_t^{\gamma}/(\phi K_t^{\gamma}+1) \geq 1-2\gamma$, is not too low, that is, $1-2\gamma > \alpha$, we can show that $Y_E(K_t)$ is increasing and concave in K_t, and satisfies Inada's conditions $Y_E'(0) = \infty$ and $Y_E'(\infty) = 0$.

Institutional investment and capital are related by the following accounting identity, where δ represents the capital depreciation rate:

$$I_t = K_{t+1} - (1-\delta)\, K_t, \text{ with } 0 < \delta < 1. \tag{16}$$

The emperor cares for the consequences of his actions for the whole future of his dynasty and not only during his lifetime. We model this attitude as intergenerational altruism à la Barro. The emperor sets his investment plan over his lifetime, which maximizes the sum of the discounted utilities of his and his successors' consumption, assuming that his successors will behave the same:

$$\underset{K_t}{Max} \sum_{t=0}^{\infty} \beta^t \left\{\left[\frac{(1-a_E)\,\phi_t K_t^{\gamma}}{\phi_t K_t^{\gamma}+1}\right]^2 A K_t^{\alpha} - K_{t+1} + (1-\delta)\, K_t\right\}^{1-\sigma} / (1-\sigma), \tag{17}$$

with $0 < \beta < 1$, $0 < \sigma < 1$, and $K_0 > 0$ given.

This program is formally identical to the Ramsey model. We assume here that the authority, ϕ_t, of the emperors follows a deterministic path, which is perfectly forecast at time zero. We will consider in the next section the case when ϕ_t follows a stochastic process.

The first-order condition of the maximization program of the emperor is

$$\begin{aligned}
&\left\{\left[\frac{(1-a_E)\,\phi_{t-1} K_{t-1}^{\gamma}}{\phi_{t-1} K_{t-1}^{\gamma}+1}\right]^2 A K_{t-1}^{\alpha} - K_t + (1-\delta)\, K_{t-1}\right\}^{-\sigma} \\
&= \beta \left\{\left[\frac{(1-a_E)\,\phi_t K_t^{\gamma}}{\phi_t K_t^{\gamma}+1}\right]^2 A K_t^{\alpha} - K_{t+1} + (1-\delta)\, K_t\right\}^{-\sigma} \\
&\quad \times \left\{\left(\alpha + \frac{2\gamma}{\phi_t K_t^{\gamma}+1}\right)\left[\frac{(1-a_E)\,\phi_t K_t^{\gamma}}{\phi_t K_t^{\gamma}+1}\right]^2 A K_t^{\alpha-1} + 1 - \delta\right\}.
\end{aligned} \tag{18}$$

The steady state value of the stock of institutional capital, K^*, is given by

$$\left(\alpha + \frac{2\gamma}{\phi K^{*\gamma} + 1}\right)\left[\frac{(1 - a_E)\,\phi K^{*\gamma}}{\phi K^{*\gamma} + 1}\right]^2 AK^{*\alpha - 1} = 1/\beta - 1 + \delta. \qquad \textbf{(19)}$$

Thus, the marginal effect of institutional capital on the income of the emperor is equal to the sum of his discount rate and of the depreciation rate. As $Y_E(K)$, is concave, the left-hand side of (19) decreases with K^*. It increases with ϕ if we add the new assumption that $\gamma < \alpha$. Consequently, the steady state value of the stock of institutional capital, K^*, increases with the authority of the emperor, ϕ.

The following proposition gives the speed of convergence of the stock of institutional capital to its steady state value:

PROPOSITION 2. *The characteristic roots of the linear approximation of (18) in the neighborhood of its steady state are the roots* Λ_1 *and* Λ_2 *of the polynomial*

$$F_1(\Lambda) - F_2(\Lambda), \qquad \textbf{(20)}$$

with

$$F_1(\Lambda) \equiv (\Lambda - 1/\beta)(\Lambda - 1)$$

and

$$F_2(\Lambda) \equiv (\beta/\sigma)(1/\beta - 1 + \delta) \times \frac{C^*}{K^*}\left\{1 - \alpha - \frac{2\gamma}{\phi K^{*\gamma} + 1}\left[1 - \frac{\gamma\phi K^{*\gamma}}{\alpha(\phi K^{*\gamma} + 1) + 2\gamma}\right]\right\}\Lambda,$$

where C^* *is the steady state value of the consumption of the emperor. We have* $0 < \Lambda_1 < 1 < 1/\beta < \Lambda_2$, *and* Λ_1 *decreases with the authority of the emperor,* ϕ.

Proof. See the Appendix. ■

Proposition 2 establishes that (18) is saddlepoint stable. There is a unique path for the stock of institutional capital, K_t, starting at K_0 and converging to its steady state value, K^*. Λ_1 is the rate of convergence of K_t. In the neighborhood of K^*, we have $K_{t+1} - K^* \approx \Lambda_1(K_t - K^*)$, which means that the difference between the stock of institutional capital and its steady state value is reduced by the proportion Λ_1 in each period.

3.4. Numerical Simulations

We can illustrate the results of the previous section by simulations. We keep the values $a_E = 0.6$, $b_M = 0.4$, set the length of a period to one year, and assume that $\alpha = 0.5$, $1/\beta - 1 = 0.05$, $\delta = 0.05$, $\gamma = 0.2$, and $\sigma = 0.8$. We choose a reference steady state such that $\phi K^{*\gamma} = 2.5$. Then the income shares of the emperor, the county magistrate, and the farmers are $y_E = 0.082$, $y_M = 0.029$, and $y_F = 0.295$, respectively. This reference state represents a situation where the authority of the emperor and the stock of institutional capital are high. Consequently, the share

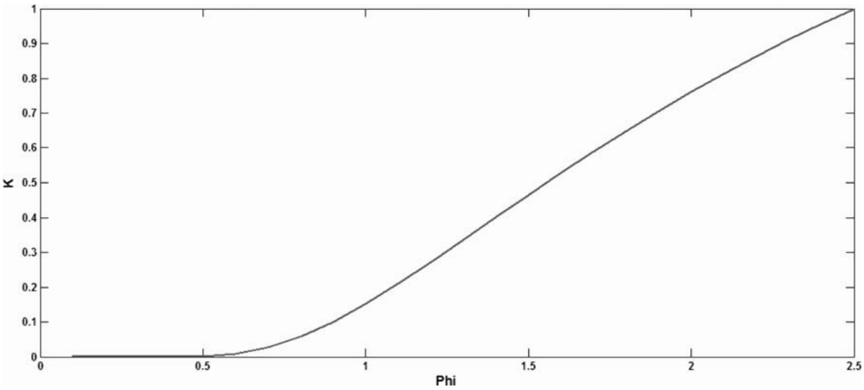

FIGURE 2. Steady state value of the stock of institutional capital as a function of the authority of the emperor.

appropriated by the county magistrate is low. We complete the calibration of the model by norming the reference stock of institutional capital to $K^* = 1$. Then we have $\phi = 2.5$, and with (19), $Y^* = A = 1.994$.

We use (19) to compute the steady state value of the stock of institutional capital when the authority of the emperor, ϕ, decreases from 2.5 to 0.1. The result is plotted in Fig. 2. We can see that the stock of public capital is an increasing function of the authority of the emperor.

We simulate the dynamic model, (15) and (18). Starting from the reference steady state, a shock lowers the authority of the emperor from $\phi = 2.5$ to $\phi = 1.43$.[7] This move will be interpreted in the next section as a decrease in authority from high to average.

In the first simulation, plotted in Fig. 3, the shock lasts one period; that is, the decrease in the value of ϕ occurs only in period 1. In this period, the loss of authority of the emperor induces a decrease in his income, Y_E, from 0.163 to 0.110. As the emperor wants to smooth the variations of his consumption over time, he decreases his consumption in period 1 by a much smaller amount (from 0.113 to 0.108) and he lowers his investment by a large amount (from 0.050 to 0.003). In period 2, the authority of the emperor is restored to its reference value, but the stock of institutional capital, K, is smaller (0.953 instead of 1). Thus, K comes back to its reference value over time at the geometric rate $\Lambda_1 = 0.948$.

In the second simulation, plotted in Fig. 4, the shock is permanent; that is, ϕ is maintained at its depressed value from period 1. In this period, the loss of authority of the emperor induces the same decrease in his income as in the previous simulation, from 0.163 to 0.110. Thus, the emperor must lower his consumption and his investment, and the stock of institutional capital in period 2 becomes equal to 0.968 (instead of 1 in period 1). Consequently, the income of the emperor in period 2 falls to 0.108. Both Y_E and K continue to decrease over time to their new steady state values, respectively equal to 0.062 and 0.422. The rate of convergence

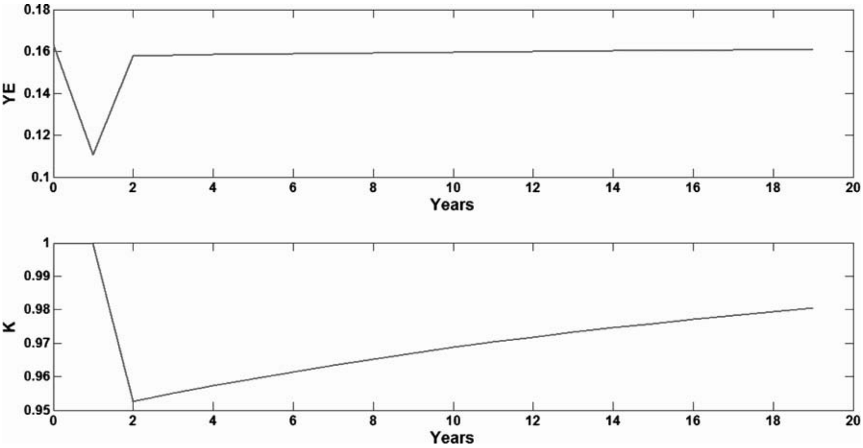

FIGURE 3. Temporary decrease in the authority of the emperor.

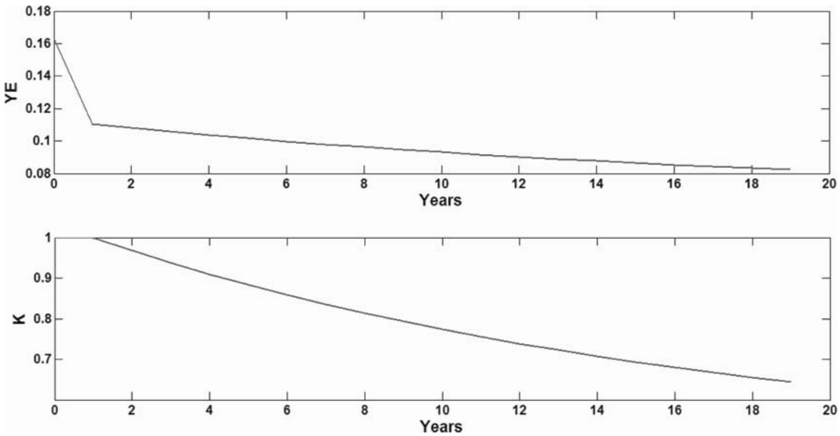

FIGURE 4. Permanent decrease in the authority of the emperor.

is equal to its value in the neighborhood of the reference steady state ($\Lambda_1 = 0.948$) at the beginning of this dynamic process, and to its value in the neighborhood of the new steady state ($\Lambda_1 = 0.958$) at the end. The slowness of the convergence of institutional capital to its new long-run level induces persistence in the quality of the economic institutions and in the de facto political powers after a sudden change in the authority of the emperor.

4. THE DYNASTIC CYCLE

In the last section, the authority of the emperor and his incumbent elite was assumed to be deterministic with perfectly known time path. In this section, their authority will follow a stochastic process. The two first subsections will present the specification of this process and the calibration of its parameters. Subsection 4.3 will compute and discuss the solution of the model.

4.1. The Stochastic Shocks and Their Consequences for the Authority of the Emperor and the End of the Dynasty

We saw in Section 2.2 that the authority of the emperor is persistent, but can also change suddenly after the occurrence of a big event. We formalize these features as follows.

First, we assume that the position of the incumbent ruling elite is threatened by other people of influence, the challenging elite: in each period, the incumbent ruling elite have a probability $p \in \;]0, 1[$ of being defeated and losing power to the challenging elite.

Second, we assume that there are three levels of authority of the emperor, high, average, and low: $\phi_1 > \phi_2 > \phi_3$. When the incumbent elite is defeated and is replaced by the challenging elite, we assume that the new level of authority of the emperor will be ϕ_i with probability $\omega_i > 0$, with ω_i fixed and independent of the previous level of authority.

Note that if the probability of defeat of the incumbent elite, p, is low, then the authority of the emperor will remain the same for a long period, which agrees with what we observe in Chinese history.[8]

More formally, consider a period in state i when the emperor has authority ϕ_i. The next period will be in the same state either if the incumbent ruling elite remains in power, or if it is defeated and the emperor and the new elite keep the same authority. This will occur with probability $\pi_{ii} = 1-p+p\omega_i$. The next period will be in state $j \neq i$ if the incumbent elite loses power and the emperor and the new elite obtain authority ϕ_j. This will occur with probability $\pi_{ij} = p\omega_j$. The π_{ij} are the elements of the transition matrix Π, and the states follow a Markov chain.

Let us define ω as the row vector with generic element ω_i, Ω as the 3×3 matrix with all rows equal to ω, and I as the 3×3 identity matrix. We have $\Pi = (1-p)I + p\Omega$. The matrices Ω and $I - \Omega$ are idempotent. We deduce that $\Pi^t - \Omega = (1-p)^t(I-\Omega) \xrightarrow[t\to\infty]{} 0$. So we see that the Markov chain of the states of authority of the emperor is ergodic and that its steady state distribution is the vector ω. Historically, all dynasties start with an emperor with high authority. The probabilities of the three levels of authority after t periods are given by the elements of the first row of Π^t. The differences between these probabilities and their steady state values decrease by the proportion 1-p in each period[9,10]

Wintrobe (2009) claims that the main predictor of a revolution is the weakness of the state. We formalize this idea in a simple and general way by assuming that a dynasty cannot end if the emperor and the incumbent elite have high or average authority. An emperor with low authority, which leads to depressed values of the stock of institutional capital and of the income of the farmers, as we can see in Table 1, may face a rebellion led by a charismatic leader and promising future

Table 1. The steady state values of the main variables for the three levels of authority of the emperor (ϕ)

ϕ	2.5	1.43	0.9
K^*	1	0.422	0.100
Y^*	1.994	1.296	0.630
y_E*	8.16%	4.78%	2.10%
y_M*	2.94%	7.40%	14.65%
y_F*	29.47%	25.94%	22.32%
Y_E*	0.163	0.062	0.013
Y_M*	0.059	0.096	0.092
Y_F*	0.588	0.336	0.141

emperor, which will end the dynasty with a constant probability q per period. Thus, the utility of the emperor becomes zero, as if he kept his throne and consumed zero.

The dynastic cycle results from the optimal response by the emperors to the current and expected values of these shocks. The program of institutional capital accumulation (17) can be rewritten in this stochastic context as

$$V_i(K) = \max_{K_i'} \left\{ \left\{ \left[\frac{(1-a_E)\,\phi_i K^{\gamma}}{\phi_i K^{\gamma}+1} \right]^2 A K^{\alpha} - K_i' + (1-\delta) K \right\}^{1-\sigma} \Big/ (1-\sigma) + \beta (1-q_i) \sum_{j=1}^{3} \pi_{ij} V_j\left(K_i'\right) \right\}, \tag{21}$$

with $i = 1, 2, 3$ and $q_1 = q_2 = 0$, $q_3 = q$.

$V_i(K)$ is the welfare of the emperor in state i when the stock of institutional capital is K. The solution of this problem is given by the policy functions $K_i' = f_i(K)$, $i = 1, 2, 3$, which determine the stock of institutional capital in the next period.

4.2. The Calibration of the Stochastic Model

We consider all the dynasties of China since the Qin dynasty, and keep for the calibration only those that controlled all or a major part of the country, and that lasted for a significant period of time: Western Han (206 B.C.–9 A.D.), Eastern Han (25–220), Tang (618–907), Song (960–1274), Yuan (1279–1368), Ming (1368–1644), and Qing (1644–1911)[11]. The data on the lives of the emperors of these dynasties suggest that they had high, average, and low authority respectively, 25%, 25%, and 50% of the time. So we set $\omega = [\,1/4 \;\; 1/4 \;\; 1/2\,]$. We also set the probability of a change in authority at $p = 0.05$, which implies expected durations of the period with an emperor with high, average, or low authority respectively

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equal to 26.7, 26.7, and 40 years. These values are consistent with the data.[12] Finally, we set the probability for an emperor with low authority to lose power and end the dynasty at $q = 0.65\%$. The stochastic simulations of the model will show that this value leads to a median of the distribution of the length of a dynasty of 250 years, which is consistent with the historical data.

The calibration of the values of parameter ϕ for emperors with high, average, and low authority is more arbitrary, because there are no historical data on which we could base our choice. After a few trials we have retained the values $\phi_1 = 2.5$, $\phi_3 = 0.9$, and we have set $\phi_2 = 1.43$, which is also equal to the long-run mean authority of the emperor ($\phi_2 = \sum_{i=1}^{3} \omega_i \phi_i$). The steady state values for the deterministic model of Section 3 of the stock of institutional capital, the output of the farmers, the shares of the income of the three agents in this output, and their incomes are given in Table 1.

4.3. Simulation of the Model

We explain in the Appendix how we numerically compute the policy functions, $K_i' = f_i(K)$, $i = 1, 2, 3$, that are solutions of the functional equation (21). These functions express how the stock of institutional capital in period $t+1$ depends on the stock of capital and on the authority of the emperor in period t: $K_{i,t+1} = f_i(K_t)$. We plotted the graphs of these functions in the plane (K_t, K_{t+1}), and we noticed that they cut the 45° line in a unique point and from above (graphs available on request). The coordinates of the intersection points give the steady state values of the stock of institutional capital, which would be stable in the absence of a switch in the authority of the emperor. They are equal to $K_1^{**} = 1.177$, $K_2^{**} = 0.433$, and $K_3^{**} = 0.083$ in the case when the dynasty is eternal, that is, when $q = 0$. If we compare these steady state values to those of the deterministic model, which are given in Table 1, we notice that $K_1^{**} > K_1^*$, $K_2^{**} > K_2^*$, and $K_3^{**} < K_3^*$. An emperor with high authority knows that on the average his authority will be lower in the future. So he targets a stock of institutional capital, which in the long run will be higher than if he would keep the same level of authority forever. Symmetrically, an emperor with low authority targets a stock of institutional capital, which will be lower than if all his successors would have as little authority as he. Finally, an average emperor whose authority is equal to the mean value of its distribution targets a stock of institutional capital higher than if he would keep this same level of authority forever. So he exhibits precautionary investment behavior.

When the probability that an emperor with low authority may lose his throne and end the dynasty is positive and equal to $q = 0.65\%$, the steady state values of the stock of institutional capital become smaller: $K_1^{***} = 1.138$, $K_2^{***} = 0.419$, and $K_3^{***} = 0.065$. The forward-looking emperor accumulates less because his dynasty may end.[13]

We use the policy functions to run a large number of stochastic simulations of the model, all starting with an emperor with high authority and a small stock of institutional capital (approximately equal to this stock at the end of the dynasty).

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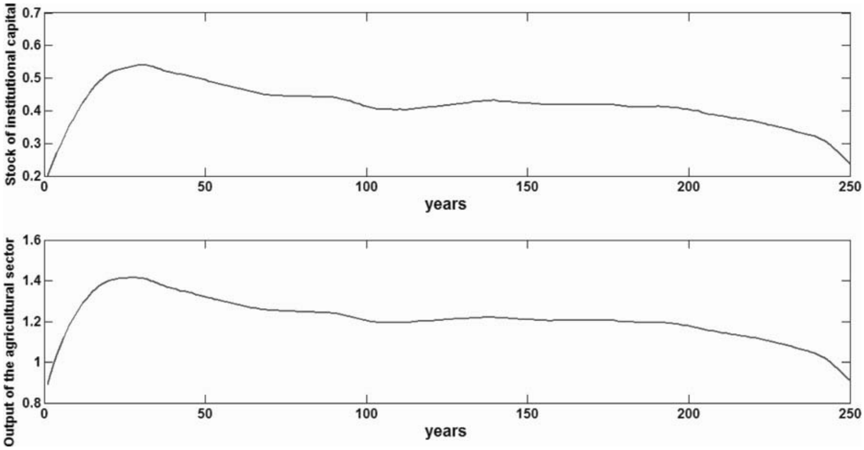

FIGURE 5. Average paths of the stock of institutional capital and the output of the agricultural sector for dynasties that last for 250 years.

The duration of a dynasty is a random variable with a distribution determined by the model. We deduce from its stochastic simulations the distribution of this duration, especially its median, which is equal to 250 years. We recall that the Chinese dynasties lasted from 89 to 314 years, with their duration approximately distributed around a value of 250 years.

If dynasties lasted forever in our model (that is, if $q = 0$), the stock of institutional capital, K_t, would follow a stochastic process starting at the given initial value, K_0, and converging to a stationary process. Thus, K_t would eventually randomly fluctuate around a constant mean with constant variance. However, dynasties do not last forever in our simulations (because $q > 0$). We consider, in the rest of this subsection, the N_T dynasties in our simulations, which last for T years (disappear in the $T+1$st year), and compute the mean values of the main economic variables for each of these years. We focus on dynasties for which $T =$ 250 years (thus $N_T = 238$), but we also examine dynasties that last for longer and shorter periods of time.

We show in Fig. 5 the mean paths over time of the stock of institutional capital and of the output of the agricultural sector. We can see that the stock of institutional capital increases for 32 years, reaches a peak equal to 0.54, decreases for 60 more years, and stabilizes at a value equal to about 0.42, the same as the steady state value of the stock of institutional capital for the emperor with average authority. Then, after year 200, this stock decreases until it reaches about 0.23. The same graph drawn for another series of dynasties, which last for 140 years each, is about the same for the first 90 years. After that, the stock of institutional capital continues to decrease until it reaches the value 0.23.

Actually, for a simulated dynasty, which lasts for $T > 140$ years, the path of the institutional capital, K_t, is influenced by its initial value for $t \leq 90$, and by the

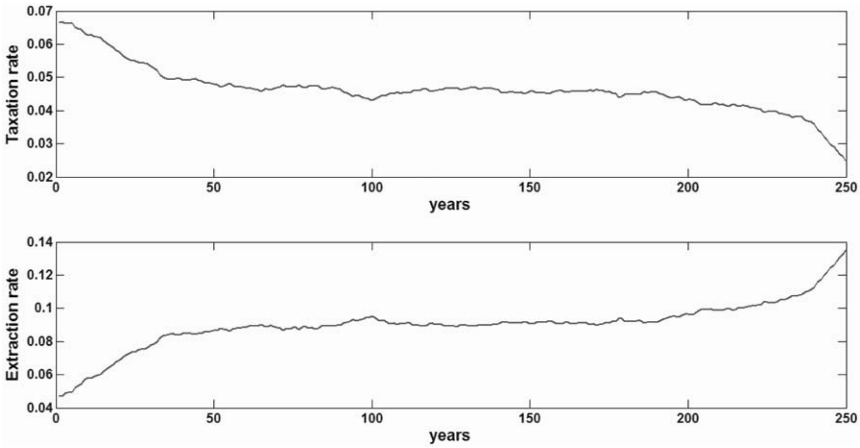

FIGURE 6. Average paths of the taxation rate and the extraction rate for dynasties that last for 250 years.

terminal date, T, for $t \geq T - 50$. For the period $90 < t \leq T - 50$, the simulated value of K_t almost follows a stationary process with the expected value 0.42, and constant variance. As the output of the agricultural sector is $Y = AK^{\alpha}$, its graph in Fig. 5 has the same shape as the graph of the stock of institutional capital.

We draw in Fig. 6 the mean path over time of the taxation rate, y_{E}, and the extraction rate, y_{M}, that is, the appropriated shares of the emperor and the county magistrate. The taxation rate decreases for about 65 years from 6.7% to about 4.5%. The extraction rate increases over the same span of time from 4.6% to about 9%. We recall that in the first 32 years the stock of institutional capital increases, which should improve the appropriation rate of the emperor and lower it for the county magistrate. However, the average authority of emperors has decreased over the same span of time, which has the opposite effect. This last effect is the most powerful, as the graphs show. After year 65 the taxation and the extraction rates stabilize, respectively, at about 4.5% and 9% until year 190. These two values are near those for the emperor with average authority in the steady state. Afterward, the taxation rate decreases sharply and the extraction rate increases. The same graphs drawn for a series of dynasties, which last for 140 years each, mainly differ by the brevity of the stabilization of both rates from year 65 to year 80.

We draw in Fig. 7 the mean of the tax income of the emperor, Y_{E}, and the extraction income of the county magistrate, Y_{M}, over time. The tax income increases from 0.068 to 0.105 in a little less than 20 years. Then it decreases to 0.08 in year 65. It stabilizes around 0.075, a little above its steady state value for the emperor with average authority. Finally, it decreases sharply after year 190. The extraction income increases from 0.05 to 0.12 in year 35. Then it stabilizes at about 0.11, which is a little above its steady state value when the emperor has average authority, until year 240, and increases thereafter in spite of the decrease in total output.

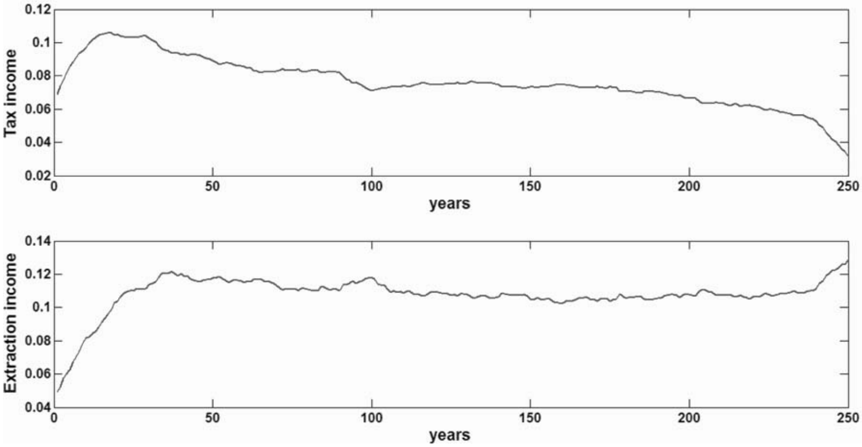

FIGURE 7. Average paths of the tax income and the extraction income for dynasties that last for 250 years.

These results and those of a series of other simulations for dynasties of different durations confirm that for dynasties lasting for more than 140 years, the first 65–75 years represent a transition from an initial state, determined by a high-authority emperor and the stock of institutional capital K_0, to a stationary stochastic process. The last 40–50 years represent a second transition from this process to the destruction of the dynasty.

For *each* stochastic simulation, the economic variables fluctuate. We could have plotted the estimates of their second moments (e.g., the correlation between the tax and the extraction rates, or the first-order autocorrelation of agricultural output) for dynasties lasting T years, computed by averaging over their N_T simulations. The results of these computations are available on request.

The graphs in Figs. 5–7 are consistent with the historical evidence that we have for the Qing dynasty (1644–1912), which was presented in Subsection 2.1. Allen et al. (2009) compute several series of real wage incomes in China, an indicator of GDP per capita, and obtain that they decreased steadily from 1738 to 1860 (the peak of the Taiping Rebellion). Sng (2010) computes the real tax income (measured in rice units). He obtains that it increased sharply until 1680, then stabilized until 1720, and finally decreased steadily until 1850. He also claims that the tax-to-GDP ratio decreased steadily from 9.6% in 1685 to 2.3% in 1848. He further notes that the amount of grain reserves stored in Beijing's state granaries, an indicator of the stock of institutional capital, reached a peak around 1730 and fell steadily from there. As pointed out in Subsection 2.1, the contraction of tax income led to a gradual and sustained reduction in the supply of state-provided public goods in the second half of the 18th century. The time paths of corruption and extortion followed almost the opposite pattern to that of taxation.

5. CONCLUSION

We have developed a model of the dynastic cycle with three main agents: the farmers who produce, the emperor who invests, and the county magistrate who appropriates a share of the output of the farmers and allocates it between the emperor (taxes) and himself (extortion). The investment in public capital by the emperor enhances the production of the farmers. The income shares of the farmers and the emperor increase, and the share of the magistrate decreases, with the stock of public capital, which also represents the quality of institutions, and with the authority of the emperor. The driving force behind the model is the stochastic process followed by this authority: it randomly changes at rare times and keeps the same value between them.

Then the model is simulated and gives results consistent with what can be observed over the history of the imperial dynasties of China. Production increases at the beginning of a dynasty, then bounces back, and stabilizes at an intermediary level. It decreases sharply in the final years of the dynasty. The tax income of the emperor follows a similar pattern. The income extracted by the county magistrates for themselves and their supporters, that is, corruption, starts from a low value, increases, and then stabilizes at a higher level than the tax income. It increases still more in the final years of the dynasty.

NOTES

1. The importance that we give to changes in the authority of the emperor to explain the dynamic cycle is consistent with empirical results from Jones and Olkien (2005). These authors identify all national leaders worldwide from 1945 to 2000 for whom growth data are available in the Penn World Table. They also identify the circumstances under which the leaders came and went from power. Using the leader transitions when the leader's rule ended by death due to natural causes or an accident, they find robust evidence that growth patterns change in a sustained fashion across the leadership transition. The effects of the quality of individual leaders on growth appear especially strong in autocracies.

2. See, e.g., Wang (1936), Fairbank (1986), Fairbank and Goldman (1998), Ni and Van (2006), Rowe (2009), Crossley (2010), Sng (2010), Rosenthal and Wong (2011), and Chan (2013). Information on the previous dynasties is less quantitative and precise. It suggests, however, that the stylized facts that we identify for the Qing dynasty are widely valid for the previous dynasties, at least since the Song and often since the Han dynasty.

3. This assumption helps to differentiate our analysis from those based on a population cycle, quoted in the Introduction.

4. We assume that private investments by farmers are quite limited and do not introduce them into the model.

5. We explained in the previous section that institutional capital is a bundle of goods that the sovereign combines to obtain the best productive and appropriative effects. In an interesting paper, Gonzalez (2002) assumes that capital is either productive or appropriative. Under this assumption, the emperor would have to balance investments that increase output against those that improve his output share. We do not elaborate on this allocation problem in this paper.

6. Acemoglu and Robinson (2008) oppose the elite to the citizens. Both invest in their own political power. Moreover, the citizens have a positive probability of solving their collective action problem and exercising additional de facto political power.

7. Our simulations are run with the software Dynare [Adjemian et al. (2011)].

8. Acemoglu (2007, 2010) develops interesting models with two elites competing for political power. The probability for the incumbent elite to lose power to the benefit of the challenging elite

depends on the resources spent by each of these groups to defend its position or conquer the position of its adversary, with a specification reminiscent of the appropriation functions in Subsection 3.2. In this paper we do not endogenize the probability of a change of the elite in power.

9. In a seminal article, Hamilton (1989) notes that many time series, if observed for a sufficiently long period, undergo episodes in which their behaviour seems to change quite dramatically. Such changes can result from events such as wars, financial panics or significant changes in government policies. Hence, he assumes that the time series process is influenced by an unobserved discrete random variable, s_t, which identifies the state or regime that the process was at date t. Finally, he models s_t by a Markov chain. Hamilton puts no constraint on the structure of the transition matrix of the Markov chain. In a previous version of this paper we also put no constraint on the structure of matrix Π, and just calibrated its coefficients to reproduce the transitions observed in the same dynasties as in subsection 4.2. The results we obtained were about the same as those presented in subsection 4.3.

10. Acemoglu (2008) bases his analysis of the rise, followed by the decline, of oligarchic societies on the assumption that the entrepreneurial skill over time of an agent is persistent and has a Markov structure, like the authority of the emperor in our paper.

11. The data used in this subsection are from Paludan (1998).

12. As these durations are long, we expect the dynamic paths of the stock of institutional capital after a change in the authority of the emperor from high to average to be somewhat similar to what is plotted in the second graph of Fig. 4, for a period equal on the average to 26.7 years. Conversely, an emperor with high authority who follows an emperor with average authority will need time to accumulate institutional capital and build his appropriative power, compared with the case when he succeeds an emperor with high authority.

13. See McGuire and Olson (1996) for an interesting discussion of this result.

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APPENDIX

A.1. PROOF OF PROPOSITION 2

The linear approximation of (18) in the neighborhood of the steady state is

$$dK_{t+1} - (1 + 1/\beta)\, dK_t + (1/\beta)\, dK_{t-1} - (\beta/\sigma)\,(1/\beta - 1 + \delta) \\ \times \frac{C*}{K*}\left\{1 - \alpha - \frac{2\gamma}{\phi K*^{\gamma} + \varphi_1}\left[1 - \frac{\gamma\phi K*^{\gamma}}{\alpha\,(\phi K*^{\gamma} + 1) + 2\gamma}\right]\right\} dK_t = 0. \tag{A.1}$$

Its characteristic polynomial is $F_1(\Lambda) - F_2(\Lambda)$, with $F_1(\Lambda)$ and $F_2(\Lambda)$ given in (20). If ϕ increases, $K*$ and $\phi K*^{\gamma}$ increase too. We deduce from (15), (16), and (19) that $C*/K* = (1/\beta - 1 + \delta)/[\alpha + 2\gamma/(\phi K*^{\gamma} + 1)] - \delta$. Thus, the slope of $F_2(\Lambda)$ increases and Λ_1 decreases with ϕ. ■

A.2. NUMERICAL METHOD USED TO SOLVE THE FUNCTIONAL EQUATION (21)

The first-order condition for the maximization in the right-hand side of this equation is

$$\sum_{j=1}^{3} \pi_{ij} \frac{dV_j\left(K_i'\right)}{dK_i'} = \left\{ \left[\frac{(1-a_E)\,\phi_i K^{\gamma}}{\phi_i K^{\gamma}+1} \right]^2 AK^{\alpha} - K_i' + (1-\delta)\,K \right\}^{-\sigma} / \left[\beta\,(1-q_i)\right]. \tag{A.2}$$

Differentiate (21) with respect to K and apply the envelope theorem:

$$\frac{dV_i\,(K)}{dK} = \left\{ \left[\frac{(1-a_E)\,\phi_i K^{\gamma}}{\phi_i K^{\gamma}+1} \right]^2 AK^{\alpha} - K_i' + (1-\delta)\,K \right\}^{-\sigma} \times \left\{ \left(\alpha + \frac{2\gamma}{\phi_i K^{\gamma}+1} \right) \left[\frac{(1-a_E)\,\phi_i K^{\gamma}}{\phi_i K^{\gamma}+1} \right]^2 AK^{\alpha-1} + (1-\delta) \right\}. \tag{A.3}$$

Take the expectations and move forward one period:

$$\sum_{j=1}^{3} \pi_{ij} \frac{dV_j\left(K_i'\right)}{dK_i'} = \sum_{j=1}^{3} \pi_{ij} \left\{ \left[\frac{(1-a_E)\,\phi_j K_i'^{\gamma}}{\phi_j K_i'^{\gamma}+1} \right]^2 AK_i'^{\alpha} - f_j\left(K_i'\right) + (1-\delta)\,K_i' \right\}^{-\sigma} \times \left\{ \left(\alpha + \frac{2\gamma}{\phi_j K_i'^{\gamma}+1} \right) \left[\frac{(1-a_E)\,\phi_j K_i'^{\gamma}}{\phi_j K_i'^{\gamma}+1} \right]^2 AK_i'^{\alpha-1} + (1-\delta) \right\}. \tag{A.4}$$

Finally, the policy functions are the solution of the functional equation

$$\left\{ \left[\frac{(1-a_E)\,\phi_i K^{\gamma}}{\phi_i K^{\gamma}+1} \right]^2 AK^{\alpha} - f_i\,(K) + (1-\delta)\,K \right\}^{-\sigma} / \left[\beta\,(1-q_i)\right] = \sum_{j=1}^{3} \pi_{ij} \left\{ \left[\frac{(1-a_E)\,\phi_j K\prime\gamma_i}{\phi_j K\prime\gamma_i+1} \right]^2 AK_i'^{\alpha} - f_j\,(K_i\prime) + (1-\delta)\,K_i' \right\}^{-\sigma} \times \left\{ \left(\alpha + \frac{2\gamma}{\phi_j K_i'^{\gamma}+1} \right) \left[\frac{(1-a_E)\,\phi_j K_i'^{\gamma}}{\phi_j K_i'^{\gamma}+1} \right]^2 AK_i'^{\alpha-1} + (1-\delta) \right\}. \tag{A.5}$$

Let $\bar{K}$ be the root of the equation $[\frac{(1-a_E)\phi_1 K^{\gamma}}{\phi_1 K^{\gamma}+1}]^2 AK^{\alpha} - \delta K = 0$. As $C_i \geq 0$, we have

$$K_i' \leq \left[\frac{(1-a_{\mathrm{E}})\,\phi_i K^{\gamma}}{\phi_i K^{\gamma}+1} \right]^2 AK^{\alpha} + (1-\delta)\,K \leq \left[\frac{(1-a_{\mathrm{E}})\,\phi_1 K^{\gamma}}{\phi_1 K^{\gamma}+1} \right]^2 AK^{\alpha} + (1-\delta)\,K.$$

If $K \leq \bar{K}$, then $K_i' \leq [\frac{(1-a_E)\phi_1 \bar{K}^\gamma}{\phi_1 \bar{K}^\gamma+1}]^2 A\bar{K}^\alpha + (1-\delta)\bar{K} = \bar{K}$. If $K > \bar{K}$, then $K_i' \leq [\frac{(1-a_E)\phi_1 K^\gamma}{\phi_1 K^\gamma+1}]^2 AK^\alpha + (1-\delta)K < K$. Thus, if we start a simulation with the stock of institutional capital $K_0 \in [\,0\ \bar{K}\,]$, this stock will remain in this interval for all the other periods. (A.5) will be solved on this interval.

We approximate the expected expression in the right-hand side of (A.5) by a sum of $n = 150$ Chebyshev polynomials: $\sum_{l=1}^{n} c_{jl}\Phi_l(K_i')$. Then, this equation becomes

$$\left\{\left[\frac{(1-a_E)\,\phi_i K^\gamma}{\phi_i K^\gamma + 1}\right]^2 AK^\alpha - f_i\,(K) + (1-\delta)\,K\right\} \times \left\{\beta\,(1-q_i)\sum_{j=1}^{3}\pi_{ij}\sum_{l=1}^{n} c_{jl}\Phi_l\,[f_i\,(K)]\right\}^{1/\sigma} \approx 1. \tag{A.6}$$

We also have

$$\left\{\left[\frac{(1-a_{\mathrm{E}})\,\phi_i K^\gamma}{\phi_i K^\gamma + 1}\right]^2 AK^\alpha - f_i\,(K) + (1-\delta)\,K\right\}^{-\sigma} \times \left\{\left(\alpha + \frac{2\gamma}{\phi_i K^\gamma + 1}\right)\left[\frac{(1-a_{\mathrm{E}})\,\phi_i K^\gamma}{\phi_i K^\gamma + 1}\right]^2 AK^{\alpha-1} + (1-\delta)\right\} \approx \sum_{l=1}^{n} c_{il}\Phi_l\,(K),\ i = 1, 2, 3. \tag{A.7}$$

We want (A.6) and (A.7) to be satisfied at the n Chebyshev nodes K_m. We use the following iterative scheme. For a given value of the parameters c_{jl}, $j = 1, \ldots, 3$, and $l = 1, \ldots, n$, the system of nonlinear equations (A.6) is used to compute the $f_i(K_m)$ by a Newton–Raphson algorithm. Then, for a given value of the $f_i(K_m)$, $i = 1, \ldots, 3$, and $m = 1, \ldots, n$, we use the system of linear equations (A.7) to compute the c_{il}. The iterative procedure is repeated until the norm of the difference between successive values of the c_{il} falls below a prescribed tolerance.

Finally, the value of the policy functions $f_i(K)$ is known at the n collocation nodes K_m. Its value for other values of K is computed by a Chebyshev interpolation. The Chebyshev nodes, polynomials, and interpolations are computed by programs in the Matlab library developed by Miranda and Fackler (2002).