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Energy exchange between the lattice and electrons in a metal under femtosecond laser irradiation

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Abstract

The energy transfer rate between the lattice and electrons in strongly nonequilibrium electron-phonon system of crystalline aluminum created by ultrashort femtosecond laser pulse is calculated in the frame of two-temperature model for a wide range of electron temperature. It is shown that the energy, transmitted from electrons to the lattice per unit volume of the crystal per unut time strongly increases when taking into account the umklapp processes in the electron-phonon scattering.

Keywords: Electron-lattice energy transfer; Femtosecond laser pulse; Umklapp processes

1. INTRODUCTION

Interaction of ultrashort laser pulses with solids leads to a strong increase in the temperature of electrons, absorbing the laser radiation (Anisimov *et al.*, 1974). Electrons in this case are not in the thermodynamic equilibrium with the lattice. Strongly nonequilibrium electron-ion system as a consequence of the short laser pulse duration can be produced in semiconductors (Stampfli & Bennemann, 1990; Ashitkov *et al.*, 2002), as well as in metals (Anisimov *et al.*, 1977; Schoenlein *et al.*, 1992; Fal'kovskii & Mishchenko, 1997), and also in plasmas (Alouani Bibi *et al.*, 2004*a*, 2004*b*). Heat balance in such electron-phonon system depends to a great extent on the energy transfer rate between the electrons and phonons. The energy relaxation between the electrons and crystalline lattice was considered by Kaganov *et al.* (1957), Allen (1987), and Rethfeld *et al.* (2002). Since the electron-electron and phonon-phonon relaxation times are significantly shorter than the electron-phonon relaxation time, electrons and the lattice can be characterized by their own temperatures T_e and T_i. When considering the electron-phonon scattering in the above cited works, it is proposed that it takes place within one Brillouine zone. An important role of the umklapp electron-phonon processes during the photon absorption in metals was emphasized by Lugovskoy and Bray (1999). But at this stage of the laser-metal interaction, the electron-phonon scattering in their work was considered in quasielastic approximation. In our work, we consider the influence of the umklapp processes in the electron-phonon interaction onto the electron-lattice energy transfer rate.

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2. MODEL

In a similar way as in the classical work by Kaganov *et al.* (1957), we consider the local pseudopotential interaction between the electron and crystalline lattice

$$U(\mathbf{r}) = \sum_{\mathbf{a}} v(\mathbf{r} - \mathbf{a} - \mathbf{u}(\mathbf{a})). \tag{1}$$

Here $\mathbf{a}$ is the vectors of an ideal crystalline lattice (face-centered cubic (fcc) for crystalline aluminum), $\mathbf{u}(\mathbf{a})$ is the ion displacement from the site $\mathbf{a}$ due to the lattice vibration. For a small deviations from the equilibrium positions

$$U(\mathbf{r}) = \sum_{\mathbf{a}} v(\mathbf{r} - \mathbf{a}) - \sum_{\mathbf{a}} \mathbf{u}(\mathbf{a}) \nabla v(\mathbf{r} - \mathbf{a}). \tag{2}$$

The perturbation potential, due to the ion oscillations,

$$U'(\mathbf{r}) = -\sum_{\mathbf{a}} \mathbf{u}(\mathbf{a}) \nabla v(\mathbf{r} - \mathbf{a}). \tag{3}$$

By expanding the ion displacements in terms of the normal coordinates $Q(\mathbf{q}, \nu)$ with the unit polarization vector of the corresponding plane wave $\boldsymbol{\xi}(\mathbf{q}, \nu)$, where $\mathbf{q}$ is the phonon wave vector, lying within the Brillouine zone, and $\nu = 1,2,3$ corresponds to three acoustic phonons of the fcc lattice, we obtain

$$\mathbf{u}(\mathbf{a}) = \frac{1}{\sqrt{N_c M}} \sum_{\mathbf{q},\nu} \boldsymbol{\xi}(\mathbf{q},\nu)(Q(\mathbf{q},\nu)e^{i\mathbf{qa}} + Q^*(\mathbf{q},\nu)e^{-i\mathbf{qa}}). \quad (4)$$

Here M is the single atom mass in a monatomic crystal, N_c is a number of unit cells. Then

$$U'(\mathbf{r}) = U_1(\mathbf{r}) + U_1^*(\mathbf{r}), \quad (5)$$

where

$$U_1(\mathbf{r}) = -\frac{1}{\sqrt{N_c M}} \sum_{\mathbf{q},\nu} \sum_{\mathbf{a}} \xi(\mathbf{q},\nu) Q(\mathbf{q},\nu) e^{i\mathbf{qa}} \nabla v(\mathbf{r}-\mathbf{a}). \quad (6)$$

We choose the wave functions of the electron with momentum $\mathbf{k}$ as

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{kr}}, \quad (7)$$

with the energy eigenvalues $\varepsilon(\mathbf{k}) = \hbar^2 k^2/2m$ (V is the crystal volume, m is the electron effective mass, which we consider to be equal to the mass of a free electron). The phonon states are denoted as Φ. Pseudopotential function $v(\mathbf{r})$ is expressed in the form

$$v(\mathbf{r}) = -\frac{Ze^2}{r} \exp(-r/\lambda), \quad (8)$$

corresponding to the screened Coulomb interaction between the electron and ion with the charge Z and the screening length λ. Then the matrix element for the potential energy perturbation U_1 for two electron-phonon states, $|\mathbf{k},\Phi\rangle$ and $|\mathbf{k}',\Phi'\rangle$, equals

$$\langle \mathbf{k}',\Phi'|U_1|\mathbf{k},\Phi\rangle = \frac{i}{V}\sqrt{\frac{N_c}{M}} \sum_{\mathbf{q},\nu} \boldsymbol{\xi}(\mathbf{q},\nu)\cdot(\mathbf{q}+\mathbf{g}) \times \sqrt{\frac{\hbar N_\nu(\mathbf{q})}{2\omega_\nu(\mathbf{q})}} \frac{4\pi Ze^2}{(\mathbf{q}+\mathbf{g})^2+\lambda^{-2}} \delta_{\mathbf{k}-\mathbf{k}'-\mathbf{q}-\mathbf{g},0}. \quad (9)$$

Analogously, for U_1^*

$$\langle \mathbf{k}',\Phi'|U_1^*|\mathbf{k},\Phi\rangle = \frac{i}{V}\sqrt{\frac{N_c}{M}} \sum_{\mathbf{q},\nu} \boldsymbol{\xi}(\mathbf{q},\nu)\cdot(\mathbf{q}+\mathbf{g}) \times \sqrt{\frac{\hbar(N_\nu(\mathbf{q})+1)}{2\omega_\nu(\mathbf{q})}} \frac{4\pi Ze^2}{(\mathbf{q}+\mathbf{g})^2+\lambda^{-2}} \delta_{\mathbf{k}-\mathbf{k}'-\mathbf{q}-\mathbf{g},0}. \quad (10)$$

Here $\mathbf{g}$ is the reciprocal lattice vector.

Energy transfer from the electrons to lattice per unit volume and unit time can be presented as a sum of energies, transmitted to phonons of different branches:

$$\frac{dE}{dt} = \sum_{\mathbf{g},\nu} Q_\nu(\mathbf{g}) = \sum_{\mathbf{g},\nu} \int_{BZ} \dot{N}_\nu(\mathbf{q},\mathbf{g})\hbar\omega_\nu(\mathbf{q}) \frac{d\mathbf{q}}{(2\pi)^3}. \quad (11)$$

Integration in (11) is performed within the Brillouine zone (BZ) and

$$\dot{N}_\nu(\mathbf{q},\mathbf{g}) = \int w_\nu(\mathbf{k},\mathbf{k}',\mathbf{g}) \times [(N_\nu(\mathbf{q})+1)f(\mathbf{k}')(1-f(\mathbf{k})) - N_\nu(\mathbf{q})f(\mathbf{k})(1-f(\mathbf{k})')] \times \delta(\varepsilon(\mathbf{k})+\hbar\omega_\nu(\mathbf{q})-\varepsilon(\mathbf{k}')) \frac{2d\mathbf{k}'}{(2\pi)^3}, \quad (12)$$

is the change in number of phonons of ν-branch per unit volume and unit time due to the process $\mathbf{k}' \to \mathbf{k}-\mathbf{q}-\mathbf{g}$. Assuming, as in the works by Kaganov *et al.* (1957) and Allen (1987), that electron-electron and phonon-phonon collisions lead to the fast equilibrium establishment separately in the electron and phonon subsystems with the temperatures T_e and T_i respectively, we have for the equilibrium distribution functions $f(\mathbf{k})$ for electrons and $N_\nu(\mathbf{q})$ for phonons:

$$f(\mathbf{k}) = \frac{1}{\exp\left(\frac{\varepsilon(\mathbf{k})-\mu}{T_e}\right)+1}, \quad (13)$$

$$N_\nu(\mathbf{q}) = \frac{1}{\exp\left(\frac{\hbar\omega_\nu(\mathbf{q})}{T_i}\right)-1}. \quad (14)$$

Here μ is the electron chemical potential.

For the phonons in the fcc lattice with one atom per unit cell, having only acoustical modes, we suppose the Debye dispersion law, $\omega_\nu(\mathbf{q}) = s_\nu q$, where $s_1 = s_L$ is the speed of longitudinal acoustical phonons, $s_{2,3} = s_T$ is the speed of transverse acoustical phonons. For aluminum at $T_e = T_i = 0$ K sound speeds are connected by the relation $s_L = \gamma s_T$ with $\gamma \simeq 2$.

The quantity $w_\nu(\mathbf{k},\mathbf{k}',\mathbf{g})$ in (12) is equal to

$$w_\nu(\mathbf{k},\mathbf{k}',\mathbf{g}) = \frac{\pi}{\rho V \omega_\nu(\mathbf{q})} \left(\frac{4\pi n e^2 \boldsymbol{\xi}\cdot(\mathbf{q}+\mathbf{g})}{(\mathbf{q}+\mathbf{g})^2+\lambda^{-2}}\right)^2, \quad (15)$$

where n is the atom number density, $n = 4/a^3$, with a being the size of the cubic cell of the fcc crystal, ρ is the mass density ($\rho = Mn$).

Taking into account only eight nearest-neighbor to the Brillouine zone $\mathbf{g} = 0$ reciprocal lattice vectors and introducing in the Debye approach longitudinal (L) phonons with $\nu = 1$ and transversal (T) phonons with $\nu = 2,3$, we obtain:

$$\frac{dE}{dt} = Q_L(0) + 8Q_L(\mathbf{g}_1) + 16Q_T(\mathbf{g}_1). \quad (16)$$

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Here $\mathbf{g}_1 = 2\pi/a(-1;1;1)$ is the nearest-neighbor reciprocal lattice vector. Taking into account (13) and (14), we can write

$$(N_\nu(\mathbf{q})+1)f(\mathbf{k}')(1-f(\mathbf{k})) - N_\nu(\mathbf{q})f(\mathbf{k})(1-f(\mathbf{k}')) = \frac{\exp\left(\frac{\hbar\omega_\nu}{T_i}\right) - \exp\left(\frac{\hbar\omega_\nu}{T_e}\right)}{\exp\left(\frac{\hbar\omega_\nu}{T_i}\right)-1} \cdot \frac{\exp\left(\frac{\varepsilon'-\mu}{T_e}\right)}{\exp\left(\frac{\varepsilon'-\mu}{T_e}\right)+1} \times \frac{\exp\left(-\frac{\hbar\omega_\nu}{T_e}\right)}{\exp\left(\frac{\varepsilon'-\mu-\hbar\omega_\nu}{T_e}\right)+1}. \quad (17)$$

The Debye approach corresponds to the change of the Brillouine zone of fcc lattice to the Debye sphere of radius $k_D = (24\pi^2)^{1/3}/a$. The sound speed s_0 at $T_e = 0$ K, averaged as

$$\frac{3}{s_0^3} = \frac{1}{s_{L0}^3} + \frac{2}{s_{T0}^3} = \frac{2\gamma^3+1}{s_{L0}^3}, \quad (18)$$

is equal to $s_0 = s_{L0}(3/(2\gamma^3+1))^{1/3}$ and defines the Debye temperature Θ_{D0} at $T_e = 0$ K

$$\Theta_{D0} = \hbar s_{L0}\left(\frac{3}{2\gamma^3+1}\right)^{1/3} k_D \quad (19)$$

For aluminum, longitudinal phonon frequencies $\omega_L(q)$ can be well found from the "jellium" model. Taking into account the screened electron-ion interaction, we have

$$\omega_{L0}(q) = \lambda_0 q\sqrt{\frac{4\pi Z^2 n e^2}{3M}}. \quad (20)$$

Here λ_0 is the screening length at $T_e = 0$ K. Then the Debye temperature at $T_e = 0$ K is given by:

$$\Theta_{D0} = Z^{5/6}\left(\frac{2\pi^4}{2\gamma^3+1}\right)^{1/3}(n^{1/3}a_B)^2\varepsilon_{at}\sqrt{\frac{m}{M}} \quad (21)$$

($a_B = \hbar^2/me^2$ is the Bohr radius, $\varepsilon_{at} = e^2/a_B = 27.2$ eV is the atomic unit of energy). For aluminum with $Z = 3$ it gives $\Theta_{D0} = 350$ K (experimental value corresponds to $\Theta_{D0} = 380 - 400$ K (Girifalco, 1973; Landau & Lifshits, 1980)). As it is shown by Medvedev and Petrov (1999), when the electron temperature increases up to $T_e \sim T_F$ (T_F is the Fermi temperature), longitudinal phonon frequencies increase up to several times, whereas transverse phonon frequencies do not depend practically on the electron temperature. Dependence of the longitudinal vibrational frequencies on the electron temperature is determined by the increase of screening length:

$$\omega_L(q) = s_{L0}\frac{\lambda(T_e)}{\lambda_0}q. \quad (22)$$

At the same time independent on the electron temperature transverse phonon frequencies are

$$\omega_T(q) = \frac{s_{L0}}{\gamma}q. \quad (23)$$

We calculate the screening length $\lambda(T_e)$ in Thomas-Fermi approach as

$$\lambda^{-2}(T_e) = 2\sqrt{\frac{2}{\pi}}\frac{e^2}{T_e}\left(\frac{\sqrt{mT_e}}{\hbar}\right)^3 I_{-1/2}\left(\frac{\mu(n,T_e)}{T_e}\right). \quad (24)$$

The electron chemical potential is equal to

$$\mu = \mu(n,T_e) = T_e I_{1/2}^{-1}\left(Zn\pi\sqrt{2\pi}\left(\frac{\hbar}{\sqrt{mT_e}}\right)^3\right). \quad (25)$$

Here $I_{1/2}^{-1}(x)$ is a function, inverse to the Fermi integral

$$I_{1/2}(x) = \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{t^{1/2}\,dt}{\exp(t-x)+1}, \quad (26)$$

so that $I_{1/2}^{-1}(I_{1/2}(x)) = x$.

At $T_e = 0$ K the screening length is

$$\frac{\lambda_0}{a_B} = \frac{1}{2}\left(\frac{\pi}{3Z}\right)^{1/6}(n^{1/3}a_B)^{-1/2}. \quad (27)$$

Then the quantities in (16) are:

$$Q_L(0) = A(n,T_e) \times \int_0^1 \frac{x^3}{G(0)}\left(\frac{\hbar\omega_L}{T_e} - \ln\frac{\exp\left(\frac{\varepsilon_0-\mu}{T_e}\right)+1}{\exp\left(\frac{\varepsilon_0-\mu-\hbar\omega_L}{T_e}\right)+1}\right) \times \frac{\exp\left(-\frac{\hbar\omega_L}{T_e}\right) - \exp\left(-\frac{\hbar\omega_L}{T_i}\right)}{\left(1-\exp\left(-\frac{\hbar\omega_L}{T_e}\right)\right)\left(1-\exp\left(-\frac{\hbar\omega_L}{T_i}\right)\right)}\,dx. \quad (28)$$

Only longitudinal phonons contribute to the electron-phonon collisions when $\mathbf{g} = 0$ (Eq. (28)).

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$$Q_L(g_1) = \frac{1}{2} A(n,T_e) \int_0^{\pi} \sin\theta d\theta \int_0^1 \frac{x^2}{G(g_1)} \times \left(\frac{\hbar\omega_L}{T_e} - \ln \frac{\exp\left(\frac{\varepsilon_L - \mu}{T_e}\right) + 1}{\exp\left(\frac{\varepsilon_L - \mu - \hbar\omega_L}{T_e}\right) + 1} \right) \times \frac{\exp\left(-\frac{\hbar\omega_L}{T_e}\right) - \exp\left(-\frac{\hbar\omega_L}{T_i}\right)}{\left(1 - \exp\left(-\frac{\hbar\omega_L}{T_e}\right)\right)\left(1 - \exp\left(-\frac{\hbar\omega_L}{T_i}\right)\right)} \times \frac{\left(x + \frac{g_1}{k_D}\cos\theta\right)^2 dx}{\sqrt{x^2 + 2x\frac{g_1}{k_d}\cos\theta + \left(\frac{g_1}{k_D}\right)^2}}, \quad (29)$$

$$Q_T(g_1) = \frac{1}{2} A(n,T_e) \left(\frac{g_1}{k_D}\right)^2 \times \int_0^{\pi} \sin^3\theta d\theta \int_0^1 \frac{x^2}{G(g_1)} \times \left(\frac{\hbar\omega_T}{T_e} - \ln \frac{\exp\left(\frac{\varepsilon_T - \mu}{T_e}\right) + 1}{\exp\left(\frac{\varepsilon_T - \mu - \hbar\omega_T}{T_e}\right) + 1} \right) \times \frac{\exp\left(-\frac{\hbar\omega_T}{T_e}\right) - \exp\left(-\frac{\hbar\omega_T}{T_i}\right)}{\left(1 - \exp\left(-\frac{\hbar\omega_T}{T_e}\right)\right)\left(1 - \exp\left(-\frac{\hbar\omega_T}{T_i}\right)\right)} \times \frac{dx}{\sqrt{x^2 + 2x\frac{g_1}{k_d}\cos\theta + \left(\frac{g_1}{k_D}\right)^2}}. \quad (30)$$

For collisions with $\mathbf{g} \neq 0$ (eqs. (29) and (30)) transverse phonons as well as longitudinal phonons are taken into account. Equations (28)–(30) contain the following functions of the modulus of the phonon wave number q normalized by the Debye wave number ($x = q/q_D$):

longitudinal phonon frequencies at the electron temperature T_e

$$\omega_L = \beta \frac{\lambda}{\lambda_0} \frac{\Theta_{D0}}{\hbar} x, \quad (31)$$

transverse phonon frequencies

$$\omega_T = \frac{\beta}{\gamma} \frac{\Theta_{D0}}{\hbar} x. \quad (32)$$

In addition,

$$\varepsilon_0 = \frac{\mu_0}{4}\left(\frac{2}{Z}\right)^{2/3}\left(x + \beta\left(\frac{Z}{2}\right)^{2/3}\frac{\lambda}{\lambda_0}\frac{\Theta_{D0}}{\mu_0}\right)^2, \quad (33)$$

$$\varepsilon_L = \frac{\mu_0}{4}\left(\frac{2}{Z}\right)^{2/3}\left(x^2 + 2x\frac{g_1}{k_D}\cos\theta + \left(\frac{g_1}{k_D}\right)^2\right) \times \left(1 + \beta\left(\frac{Z}{2}\right)^{2/3}\frac{\lambda}{\lambda_0}\frac{\Theta_{D0}}{\mu_0}\frac{x}{x^2 + 2x\frac{g_1}{k_D}\cos\theta + \left(\frac{g_1}{k_D}\right)^2}\right)^2, \quad (34)$$

$$\varepsilon_T = \frac{\mu_0}{4}\left(\frac{2}{Z}\right)^{2/3}\left(x^2 + 2x\frac{g_1}{k_D}\cos\theta + \left(\frac{g_1}{k_D}\right)^2\right) \times \left(1 + \frac{\beta}{\gamma}\left(\frac{Z}{2}\right)^{2/3}\frac{\Theta_{D0}}{\mu_0}\frac{x}{x^2 + 2x\frac{g_1}{k_D}\cos\theta + \left(\frac{g_1}{k_D}\right)^2}\right)^2, \quad (35)$$

$$G(g_1) = \left(1 + \frac{(6\pi^2)^{2/3}\sqrt{2\pi}}{4}\sqrt{\frac{\varepsilon_{at}}{T_e}}\frac{(n^{1/3}a_B)^2}{I_{-1/2}\left(\frac{\mu}{T_e}\right)} \times \left(x^2 + 2x\frac{g_1}{k_D}\cos\theta + \left(\frac{g_1}{k_D}\right)^2\right)\right)^2. \quad (36)$$

The coefficient $A(n,T_e)$ in Eqs. (28)–(30), having the dimension of the energy per unit volume per unit time, is equal to

$$A(n,T_e) = \pi\left(\frac{3\pi^2 Z}{2}\right)^{2/3}\frac{m}{M}(n^{1/3}a_B)^5\left(\frac{\lambda}{\lambda_0}\right)^4\frac{\omega_{at}}{a_B^3}T_e. \quad (37)$$

Here $\omega_{at} = \varepsilon_{at}/\hbar = 4.1 * 10^{16} s^{-1}$ is the atomic frequency unit, $\beta = ((2\gamma^3 + 1)/3)^{1/3}$, $\mu_0 = (\varepsilon_{at}/2)(3\pi^2 Z)^{2/3}(n^{1/3}a_B)^2$ is the electron chemical potential at $T_e = 0$ K.

The change of the screening length can be performed as

$$\left(\frac{\lambda}{\lambda_0}\right)^2 = (3\pi^2)^{1/3}\sqrt{\frac{2}{\pi}}\frac{(Zn)^{1/3}a_B}{I_{-1/2}\left(\frac{\mu}{T_e}\right)}\sqrt{\frac{\varepsilon_{at}}{T_e}}. \quad (38)$$

The Debye temperature and the electron chemical potential at $T = 0$ K ratio in Eqs. (33)–(35) is equal to

$$\frac{\Theta_{D0}}{\mu_0} = \frac{2}{3\beta}(4Z)^{1/6}\sqrt{\frac{m}{M}}. \quad (39)$$

3. RESULTS

In Fig. 1 the results of the calculation of the energy transfer from the electrons to the crystalline lattice per unit volume

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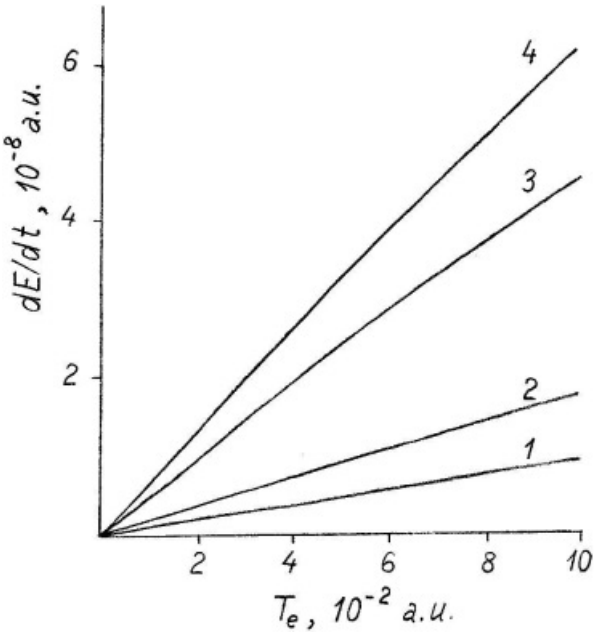

Fig. 1. Energy transferred from electrons to the lattice in aluminum crystal per unit volume per unit time as a function of the electron temperature. Lattice temperature is chosen to be $T_i = 300$ K. 1—in the absence of the umklapp processes in the electron-phonon scattering when considering only longitudinal phonons and reciprocal lattice vector $\mathbf{g} = 0$; 2—contribution of longitudinal phonons with $\mathbf{g} \neq 0$; 3—contribution of the transversal phonons (for them $\mathbf{g} \neq 0$) into the electron-lattice energy relaxation; 4—electron-lattice energy transfer when all umklapp processes are taken into account.

and unit time, as a function of the electron temperature for aluminum are presented within a wide range of electron temperatures. All quantities under consideration are measured in atomic units: $v_{at} = a_B^{-3}$, $t_{at} = \hbar^3/me^4 = 2.4 * 10^{-17} s$. The lattice temperature in Figure 1 is taken to be $T_i = 300$ K. Curve 1 shows the energy transfer in the absence of the umklapp processes ($\mathbf{g} = \mathbf{0}$) in the electron-phonon scattering. In this case only the longitudinal phonons take part in the energy transfer. Even the longitudinal phonons with $\mathbf{g} \neq \mathbf{0}$ only in addition to the longitudinal phonons with $\mathbf{g} = \mathbf{0}$ nearly redouble the energy transfer rate when taking into account umklapp processes (curve 2). Curve 3 shows the contribution of the transverse phonons to the electron-lattice energy transfer. It takes place only for $\mathbf{g} \neq \mathbf{0}$ and exceeds the contribution of longitudinal phonons (curve 2) even when the number of the transverse vibrational modes is greater than the number of longitudinal modes. Curve 4 is obtained when all contributions with $\mathbf{g} = \mathbf{0}$ and $\mathbf{g} \neq \mathbf{0}$, including longitudinal and transverse phonons, are taken into account and presents in our approximation, the total energy transfer from the electrons to the lattice per unit volume and unit time when the lattice is at room temperature. Total energy transfer is several times greater than that in the case when only longitudinal phonons with $\mathbf{g} = \mathbf{0}$ (curve 1) are taken into account.

Then we can obtain the electron-lattice energy transfer rate α, defined by the equation

$$\frac{dE}{dt} = \alpha(T_e - T_i). \tag{40}$$

Energy transfer rate α as a function of the electron temperature is presented in Figure 2. Curve 1 corresponds to the energy transfer in the absence of the umklapp processes and curve 2 takes into account the electron-phonon scattering with $\mathbf{g} \neq \mathbf{0}$.

When the energy transfer between the electrons and crystalline lattice is known, the system of equations governing the change of electron and lattice temperatures can be written. Neglecting the spatial change of crystal during the short time of the electron-lattice energy exchange, for the electron temperatures not exceeding the Fermi temperature, this system of equations can be written as

$$\frac{\pi^2}{2} nZ \frac{T_e}{\mu_0} \frac{dT_e}{dt} = -\alpha(T_e - T_i). \tag{41}$$

$$3n \frac{dT_i}{dt} = \alpha(T_e - T_i). \tag{42}$$

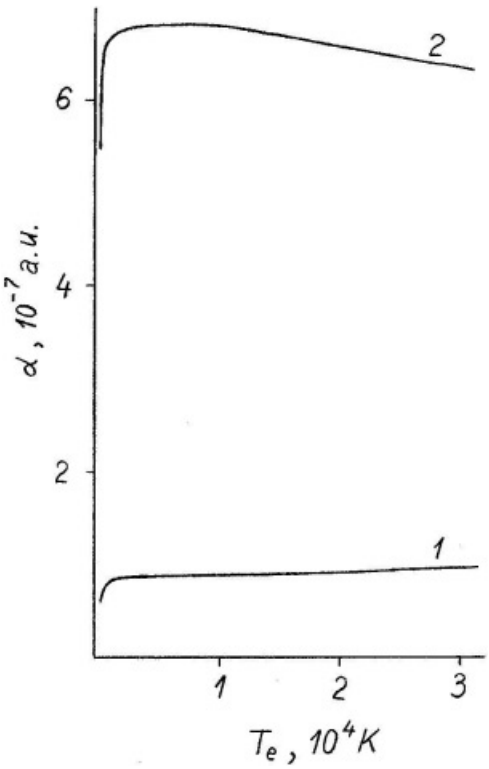

Fig. 2. Electron-lattice energy transfer rate α as a function of the electron temperature. 1—electron-phonon scattering with $\mathbf{g} = 0$ only; 2—the umklapp processes are taken into account.

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Introducing the designation

$$\beta = \frac{\pi^2}{4}\frac{Z}{\mu_0}, \tag{43}$$

these equations can be written in the form

$$2\beta T_e \frac{dT_e}{dt} = -\frac{\alpha}{n}(T_e - T_i). \tag{44}$$

$$3\frac{dT_i}{dt} = \frac{\alpha}{n}(T_e - T_i). \tag{45}$$

As it is shown in Figure 2, α can be considered as a constant value. Combining Eqs. (44) and (45) we obtain the energy integral

$$\beta T_e^2 + 3T_i = \epsilon = \beta T_{e0}^2 + 3T_{i0}. \tag{46}$$

Here ϵ is the thermal part of initial crystal energy after the laser pulse, when the electron temperature is equal to T_{e0} and the lattice temperature stays at the room value, $T_{i0} = 300$ K. Substituting T_i from (46) to (44) and integrating equation obtained, we can write temporal dependence of the electron temperature T_e and lattice temperature T_i in the form of equations:

$$\left|\frac{2\beta T_e + 3 - \xi}{2\beta T_{e0} + 3 - \xi}\right|^{1-(3/\xi)} \times \left(\frac{2\beta T_e + 3 + \xi}{2\beta T_{e0} + 3 + \xi}\right)^{1+(3/\xi)} = e^{-(\alpha/3n)t}. \tag{47}$$

$$\left|\frac{2\sqrt{\beta(\epsilon - 3T_i)} + 3 - \xi}{2\beta T_{e0} + 3 - \xi}\right|^{1-(3/\xi)} \times \left(\frac{2\sqrt{\beta(\epsilon - 3T_i)} + 3 + \xi}{2\beta T_{e0} + 3 + \xi}\right)^{1+(3/\xi)} = e^{-(\alpha/3n)t}. \tag{48}$$

Here we denote $\xi = \sqrt{(9 + 4\beta\epsilon)}$. Now we can obtain from Eqs. (47) and (48) the time τ needed to heat the crystalline lattice from the initial room temperature T_{i0} to the melting temperature T_m (for aluminum T_m = 933 K). This time interval τ as a function of the initial electron temperature just after the laser pulse, T_{e0}, is presented in Figure 3 for the case when umklapp processes in the electron-phonon scattering are excluded (curve 1) as well as for the case when they are taken into account (curve 2). While taking into consideration umklapp processes, τ become significantly shorter.

Assuming in Eqs. (47) and (48), $T_e = T_i = T_m$ at $t \to \infty$, we obtain the threshold value of the initial electron temperature T_{e*} for the heating the lattice up to the melting temperature T_m:

$$T_{e*} = T_m\sqrt{1 + \frac{12(T_m - T_{i0})\mu_0}{\pi^2 T_m^2 Z}} \tag{49}$$

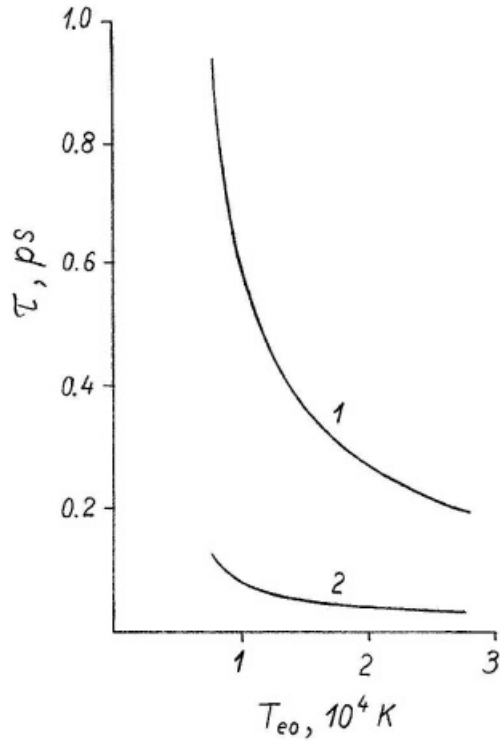

Fig. 3. Time interval τ for heating the lattice from the room temperature to the melting temperature. 1—in the absence of the umklapp processes; 2—the umklapp processes are taken into account.

For aluminum T_{e*} is equal to 0.01885 a.u. = 5950 K.

To estimate a threshold value of laser fluence J_*, which is necessary to heat the lattice up to the melting temperature, suppose that the energy of laser pulse is absorbed by electrons within a thin layer h. Thus we can write the equation:

$$(1 - R)J_* = nhZ\frac{\pi^2}{4}\mu_0\left(\frac{T_{e*}}{\mu_0}\right)^2. \tag{50}$$

Here $R \simeq 0.3$ is a reflectivity of aluminum if the laser wavelength is equal to 620 nm. Assuming $h = 10$ nm, we obtain $J_* = 5$ mJ/cm^2.

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