J. Plasma Phys. (2024), *vol*. 90, 905900111 © The Author(s), 2024.
Published by Cambridge University Press
doi:10.1017/S0022377824000084

Transformation of a plasma boundary curvature into electrical impulses moving along a plasma surface

O.M. Gradov †

Kurnakov Institute of General and Inorganic Chemistry, Russian Academy of Sciences, Leninsky pr. 31, Moscow, 119991, Russia

(Received 11 September 2023; revised 7 January 2024; accepted 8 January 2024)

The self-consistent propagation of electrical impulses and of the accompanying distortions of the electron surface in the framework of a cold plasma model with a sharp boundary has been described with help of a derived system of two equations. The method of 'shallow water theory' has been applied for the case of bounded plasma and deriving an equation with which to link the spatial and temporal structures and evolution of the boundary curvature and the surface charge. Under certain conditions, such perturbations can propagate along the boundary without changing their shape for a long distance. An approximate analytical solution has been found, and numerical calculations have been performed. Mutual connections between basic parameters of the considered perturbations (velocity components, electrostatic field, etc.) have been presented.

Keywords: surface charge, impulse, nonlinear signal, electrostatic potential, boundary curvature, plasma, plasma nonlinear phenomena, plasma waves, plasma dynamics

1. Introduction

Usually, waves on a plasma surface studied in the simplest model of plasma with a sharp boundary are, as a rule, investigated to explain the behaviour of such quantities as perturbation of the density, velocity of particles, as well as their temperature, interactions of various types, etc. (see Boardman 1982; Stenflo 1996; Ma & Hirose 2009) in the considered processes. Meanwhile, some motions of charged particles along the boundary are connected with a change in the shape of the latter, also in a linear approximation. Such phenomena correspond to the behaviour of the nonlinear surface charge (Gradov 2020, 2021) and reveal the influence of its motion on the shape of the plasma surface. The equation describing such interaction was derived as a result of the generalization of the 'shallow water theory' (Stoker 1957) for plasma conditions, and together with the equation for the nonlinear surface charge parameters (Gradov 2020, 2021) on the curved surface of electrons forms a closed system to describe the interrelated dynamics of these processes. A detailed analysis makes it possible to identify the conditions and parameters of the process when excitation of a nonlinear surface charge pulse takes place in the form of an electrical signal, which is able to propagate along the boundary changing the shape

† Email address for correspondence: lutt.plm@igic.ras.ru

of the latter. Thus, a local curvature of the plasma boundary can be created because of an electrical pulse along this boundary. But for the appearance of such a phenomenon, it is necessary that the velocity of electrons along the surface at some selected point of the plasma boundary changes in time according to a law that corresponds to the system of equations used in the present work. A similar pulse can be formed, for example, with the help of a probe, applying a voltage that changes in time according to the required law. The approximate analytical expressions are required to be formulated for the aim to have an idea of the type of necessary behaviour in time of the process parameters at this point in space. Simultaneously, there is the reverse form of this effect when an electrical pulse is excited by forming a curvature of the plasma surface that can propagate along this surface. To realize this phenomenon, it is necessary to accelerate the electrons at the plasma boundary along its normal, which is changed in time at a chosen point in space in accordance with the solutions of the system of equations presented in the work. Such a possibility can be realized, for example, using a laser beam that propagates in a direction perpendicular to the plasma boundary. But in this experiment, the electromagnetic front of the beam should depend on the time according to the law described by the solutions of the system of equations used in the work. Finally, each pulse of electromagnetic radiation is able to create a localized curvature of the electron boundary, which will propagate along the plasma boundary in the form of a solitary electrically charged signal. Consequently, pulsed or appropriately modulated electromagnetic radiation in the case of normal incidence on the flat boundary of the plasma is capable, under certain conditions, of generating pulsed emission of an electrical signal along the plasma boundary. Both of these phenomena can be of great practical importance for solving diagnostic problems as for a variety of applications that use plasma phenomena in their workflow.

2. Basic equations and their solutions

The description of nonlinear waves of surface charge is given in Gradov (2020, 2021) based on using the theory of the potential (Jeffreys & Swirles 1999) in a model of cold plasma with a sharp boundary occupying the region $x > 0$ along the $0X$ axis, and stationary ions with density N and charge e. The plane boundary placed at the point $x = 0$ on the $0X$ axis can be distorted for electrons. Then their surface is described by the equation $x = x_0(t, y, z)$. For the simple case of complete homogeneity along the $0Y$ axis, the dependence of the parameters on the y coordinate can be excluded. The environment surrounding the plasma can be characterized by a dielectric having permittivity ε_d. Under these conditions, it is possible to consider the peculiarities of the occurrence of various nonlinear phenomena on the plasma surface for the selected initial and boundary conditions specified at the origin. The motion of electrons with density n, charge $-e$ and mass m is described by the equation for velocity v as a function of time t and coordinate z, which is represented in dimensionless form in Gradov (2020, 2021, 2022) as the following relation:

$$\frac{\partial u_0}{\partial \tau} + \frac{\partial}{\partial \eta}\left(\frac{\partial u_0}{\partial \eta} \cdot u\right) + u = 0, \tag{2.1}$$

$$\left.\begin{aligned} u_0 &= \frac{\partial u}{\partial \tau} + a^2 \frac{\partial u^2}{\partial \eta} + a_2 \frac{\partial}{\partial \eta}\left\{u^2\left(\frac{\partial \Lambda}{\partial \eta}\right)^2\right\}; \quad a_2 = a_1(1 + a_1); \\ a_1 &= \frac{\ln(\sqrt{2}+1)}{\sqrt{2}\pi} \approx 0.2; \quad a^2 = \frac{1 + a_1^2}{2}. \end{aligned}\right\} \tag{2.2}$$

https://doi.org/10.1017/S0022377824000084 Published online by Cambridge University Press

Here the dimensionless coordinates and variables $\Lambda = k_1 x_0$; $k_1 = \omega_S/s$; $\omega_S = \omega_L/(1+\varepsilon_d)^{-1/2}$; $\omega_L = (Ne^2/m\varepsilon_0)^{1/2}$; $\eta = zk_1$; $s =$ const.; and $\tau = \omega_S t$, $u = v/s$ were introduced with using ε_0 as the notation of the vacuum permittivity. As shown in Gradov (2020), the constant a_1 arises when the value of the electrostatic potential near the boundary is presented like an expansion on a small parameter $(x - x_0)$ as a result of the integration in the corresponding term of this expansion determined by the potential theory of Jeffreys & Swirles (1999). The use of the dimensional constant s is associated with the conversion of all equations into a dimensionless form, and its value with the velocity dimension is determined by the velocity at which the points of the profile of the nonlinear wave under consideration move, as shown by the solutions obtained below.

During derivation of (1) in Gradov (2021), the velocity of electrons was presented in the form $\boldsymbol{v} = \nabla \Psi$ for the case when their transportation is driven by the potential Ψ, which permits one to transform the equation of the motion and leads to the relation

$$\frac{\partial \Psi_0}{\partial t} + a^2 \frac{\partial \Psi_0}{\partial z}^2 = \frac{e}{m}\Phi(z,t); \tag{2.3}$$

$$\Psi_0(z,t) = \Psi(x = x_0(z,t) + 0, z,t). \tag{2.4}$$

Equation (2.3) was derived with the help of the proposition that inside plasma $x > x_0(z,t) + 0$ quasi-neutrality takes place, $N = n$, which provides the validity of the relation $\Delta\Psi = 0$ there in accordance with the equation of the continuity (see Gradov 2021). Equation (2.1) can be used to find the velocity of nonlinear motion as well as the associated density of the surface charge for a given form $\Lambda = \Lambda(\tau,\eta)$ of the distortion of the boundary. Examples of the solution obtained in this case for some special cases of physically justified types of the function $\Lambda(\tau,\eta)$ are given in Gradov (2020, 2021).

On the other hand, the nonlinear motion of electrons can also lead to distortion of their equilibrium surface at the plasma boundary. In order to find an exact self-consistent solution, it is necessary to obtain another new equation describing the relationship of the nonlinear surface charge with the curvature of the plasma surface. Such an equation can be obtained on the basis of a generalization of the nonlinear theory of 'shallow water' in a liquid (see Stoker 1957) to a plasma. Using this approach, it is possible to derive this equation by integrating the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{v}) = 0 \tag{2.5}$$

over the region of homogeneous electron density of plasma in the range $x_0(t,z) < x < \infty$. It should be borne in mind that the change in the density of $n(t,x,z)$ near the boundary is associated with its movement along the axis $0X$, when a jump in density from 0 to N can occur at a fixed point, both over time and with a change of z in the surface of the plasma. Therefore, the function $\partial n/\partial t,z$ behaves like a δ-function of the argument $x - x_0(t,z)(n = N\delta[k_1(x - x_0)])$ and can be written

$$\frac{\partial n(t,z)}{\partial t, z} = \lim_{x\to x_0}\left\{\frac{\partial x_0(t,z)}{\partial t, z}\frac{\partial n_e(t,x,z)}{\partial x}\right\}. \tag{2.6}$$

Given that for a nonlinear surface charge there is a relation of $\Delta\psi = 0$ in the region $x_0(t,z) < x < \infty$ (see Gradov 2020, 2021, 2022), the second term in (2.5) can be represented as $\nabla \cdot (n_e \boldsymbol{v}) = \nabla n_e \cdot \nabla \psi$. Using (2.6), after integrating the values in (2.5)

https://doi.org/10.1017/S0022377824000084 Published online by Cambridge University Press

over this region of the z coordinate, the following equation can be obtained:

$$\frac{\partial x_0}{\partial t} + \left(a_1 + \frac{\partial x_0}{\partial z}\right)\frac{\partial \psi_0}{\partial z} = 0. \tag{2.7}$$

The following formula is also used here:

$$\left.\frac{\partial \psi}{\partial x}\right|_{x=x_0+0} = a_1 \frac{\partial \psi_0}{\partial z}, \tag{2.8}$$

As obtained and used in works (Gradov 2020, 2021, 2022) concerning nonlinear surface charge. The relation (2.8) is a consequence of the potential theory (Jeffreys & Swirles 1999) used to describe the nonlinear surface charge.

As a result of performing the necessary calculations, the following differential equation can be derived, which describes the connection between the distortion value of the boundary $\Lambda(\tau,\eta)$ and the velocity $u(\tau,\eta)$ of electron motion:

$$\frac{\partial \Lambda}{\partial \tau} + \left(a_1 + \frac{\partial \Lambda}{\partial \eta}\right) u = 0. \tag{2.9}$$

Equation (2.9) makes it possible to study the behaviour of the curvature of the surface of electrons at the plasma boundary depending on the speed of their motion along this boundary. At the same time, the system (2.1), (2.9) gives the possibility of searching for such solutions that describe pulses of a certain shape, both boundary distortion and nonlinear surface charge parameters, capable of propagating over long distances without changing their properties. It also follows from (2.9) that the curvature of the surface $\Lambda(\tau,\eta)$ does not change in time if there is no movement of electrons along the boundary.

The obtained dependences on time and spatial coordinates for the velocities of electrons and the shape of curvature of the boundary make it possible to determine the spatially temporal structure of other characteristics of the process, such as, for example, the electrostatic potential $\varphi_e(\xi)$ and the surface charge density $n_S(\xi)$, which is determined by the following formula (Gradov 2020, 2021):

$$n_S(z,t) = \lim_{\delta\to 0} \int_{x_0-\delta}^{x_0+\delta} \mathrm{d}x\, n(x,z,t). \tag{2.10}$$

The connection between this density and magnitudes of the electrostatic potential at the plasma boundary $\Phi(z,t) = \varphi_e(x = x_0(z,t), z)$ is described by the following equation (Gradov 2020) derived with help of the definition (2.10):

$$n_S(z,t) = \frac{a_1(1+\varepsilon_d)\varepsilon_0}{e}\frac{\mathrm{d}\Phi}{\mathrm{d}z}. \tag{2.11}$$

As follows from the theory of the potential (Jeffreys & Swirles 1999), the space distribution of the electrostatic potential φ_e of the surface charge is defined in whole space by its magnitude at the boundary $\Phi(z,t)$ for the case when the half-space plasma has a

https://doi.org/10.1017/S0022377824000084 Published online by Cambridge University Press

boundary in the form of the infinite surface $x = x_0(z)$ consistent with Gradov (2020):

$$\varphi_e(r,t) = \pm\frac{1}{\pi}\int_{-\infty}^{\infty} \mathrm{d}z' \frac{[x - x_0(z',t)]\theta(z',t)\Phi(z',t)}{(z-z')^2 + [x - x_0(z',t)]^2}, \tag{2.12}$$

$$\theta(z,t) = 1 + \left(\frac{\partial x_0}{\partial z}\right)^2. \tag{2.13}$$

Here the assumption was used that inside plasma $x > x_0(z,t) + 0$ the quasi-neutrality ($N = n$) is supported by an equilibrium of forces confining the plasma which transforms the Poisson equation to a Laplace one. It gives the possibility of using the theory of the potential for describing a space structure of a considered function as was done above for the electrostatic potential. In (2.12) the sign '+' should be taken for any point with coordinates $\{x,z\}$ inside the plasma ($x > x_0(z,t)$) and the sign '−' is used for points of observations outside plasma ($x < x_0(z,t)$). Therefore, the spatial distribution of the velocity potential $\Psi(z,t)$ is described by the formula of the potential theory in the same way as was done using formula (2.12) for the electrostatic potential, i.e. through its own value at the plasma boundary. The corresponding expression can be easily written by analogy with (2.12).

The description of the function $\Psi_0(z,t)$ obtained as a result of the solution of the initial system (2.1), (2.3), (2.9) makes it possible to establish the spatial–temporal behaviour of the electrostatic potential at the plasma boundary $\Phi(z,t)$ with using (2.12). Formula (2.11) provides the same possibility of obtaining similar information for the surface charge density $n_S(z,t)$. Finally, with the help of expressions (2.11), (2.12), it is possible to determine all the main characteristics of the process under consideration, if the functions $\Psi_0(z,t)$ and $x_0(z,t)$ are known, the description of which must be obtained as a result of solving the original system of (2.1), (2.3), (2.9). The approximate expressions obtained in this paper for these functions can be useful in estimating all the main parameters of the process. It should be noted that the adopted simplifying assumptions significantly narrow the range of conditions under which the phenomena under consideration may occur. For example, the approach of a cold plasma imposes certain restrictions on the temperature of electrons and the parameters of the wave process under study. This means, first of all, a requirement for a relationship between the Debye radius of the plasma and the characteristic size of the nonlinear wave. The smaller the ratio of these quantities, the weaker the effect of the thermal motion of electrons on the phenomena under consideration.

The study of the nonlinear wave process is based primarily on the assumption that the desired values $u(\tau,\eta)$ and $\Lambda(\tau,\eta)$ depend on the variables τ and η in the form of a combination of $\xi = \eta - \Omega\tau$ ($\Omega = \text{const.}$), that is, the solution has the form of a wave. As can be seen from the definition of ξ, the velocity at which the profile points of the nonlinear wave in question move is equal to the value Ωs. In this case, the original system of (2.1), (2.3), (2.9) can be written in the following form:

$$\Omega\frac{\mathrm{d}u_0}{\mathrm{d}\xi} - \frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\mathrm{d}u_0}{\mathrm{d}\xi}\cdot u\right) + u = 0; \tag{2.14}$$

$$u_0 = \frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\Omega u - a^2u^2 - a_2u^2\left(\frac{\mathrm{d}\Lambda}{\mathrm{d}\xi}\right)^2\right\}; \quad \frac{\mathrm{d}\Lambda}{\mathrm{d}\xi} = \frac{a_1^2 u(\xi)}{\Omega - u(\xi)}. \tag{2.15a,b}$$

The numerical solution (2.15) for some selected characteristics of the problem is shown in figure 1. The impulse form of the desired quantities is provided by the selection of the

https://doi.org/10.1017/S0022377824000084 Published online by Cambridge University Press

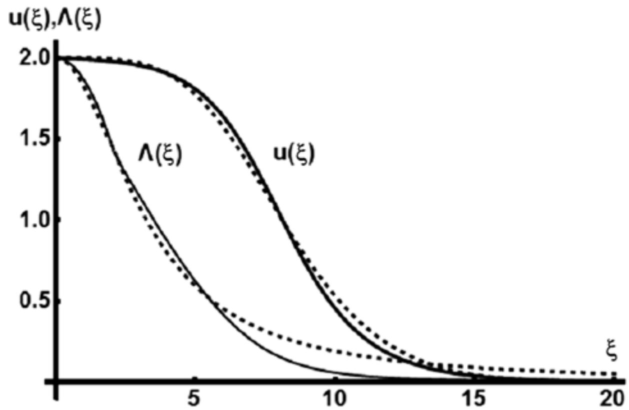

FIGURE 1. The spatial dependence of electron velocity and the boundary distortion caused by the surface charge for some chosen set of parameters of nonlinear perturbations: $b_1 = 1$, $b_2 = 0.5$, $b_3 = 0.5$, $b_4 = 0.5$; $\Omega = 2.2$, $u(\xi = 0) = 2$; $\Lambda(\xi = 0) = 2$; $u'(\xi = 0) = 0$; $u''(\xi = 0) = 0$.

values of the parameters of the initial conditions by their mutual agreement. In order to have an approximate idea of the form of the solution in an analytical form and to obtain comparative estimates, the following functions can be used, which are not an exact solution of the system of (2.15) but sufficiently accurately convey the shape of solitary exact curves:

$$u_1(\xi) = b_1 Sech(b_2\xi^2); \quad \Lambda_1(\xi) = \frac{b_3}{1 + b_4\xi^2}. \tag{2.16a,b}$$

Functions (2.16) are represented in figure 1 by dotted curves.

From the nonlinear surface charge theory (Gradov 2020, 2021), it is known that the velocity of electrons $u_x(\tau, \eta)$ along the $0X$ axis at the boundary $x = x_0(\tau, \eta)$ is associated with a similar parameter of motion along the $u(\tau,\eta)$ boundary by the ratio $u_x(\tau, \eta) = a_1 u(\tau, \eta)$. Therefore, if at some selected point $\eta = \eta_0$ the electrons are given the velocity $u_x(\tau, \eta = \eta_0) = a_1 u(\eta_0 - \Omega\tau)$, then a nonlinear wave of curvature of the electron boundary will begin to propagate along the boundary having the form $\Lambda(\xi)$ satisfying, like the function $\Lambda(\xi)$, the system of (2.15). This wave is a moving electric charge and may be of interest for solving some applied problems. It should be noted that the function $u_1(\eta_0 - \Omega\tau)$ described by formula (2.16) can be used approximately as a boundary condition, which greatly simplifies the problem.

Thus, by creating, for example, with the help of a modulated or impulse laser beam the deviation of the surface of electrons from the original position, it is possible, under certain conditions, to initiate solitary pulses of an electrical signal that are able to propagate along the surface together with the curvature of the boundary. This allows one not only to redirect the force effect to places inaccessible to direct impact, but also to send control signals and information there. Such a scheme is especially relevant for plasma objects of an extended shape such as an ellipsoid, in which large areas of the surface fully meet the conditions of the problem presented in this paper.

The reverse process is also possible, when the curvature of the boundary is generated by electrical impulses along the boundary. Here, pulsating electrostatic fields having, according to (2.12), a large localization area near the boundary, which can not only accelerate charged particles, but also create electromagnetic radiation due to their nonlinear properties, may be of possible interest.

https://doi.org/10.1017/S0022377824000084 Published online by Cambridge University Press

3. Summary and conclusion

Within the framework of a simple model of cold plasma with a sharp boundary, a system of two equations was obtained describing the self-consistent propagation of the distortion of the electron surface at the plasma boundary together with electrical impulses along the boundary. This self-consistent process is possible if certain conditions arising from the solution of the system (2.5) are met when propagation along the boundary is carried out over a long distance without distorting the waveform. Taking into account electromagnetic effects, performed similarly to the calculations presented in Yu & Zhelyazkov (1978) and Brodin & Stenflo (2014), can help to identify new aspects of this phenomenon, for example, the description of radiation along the normal to the boundary in the form of outflowing waves (Tamir & Oliner 1963). But this requires more research. A closed system of equations derived in the present work includes a description of the shape of the surface of electrons, which may help to study a wide range of phenomena associated with the occurrence of a nonlinear surface charge. There is every reason to believe that its widespread use will make it possible to discover and study many interesting and important effects connected with plasma boundaries. For example, special features of the surface charge could help in understanding many interesting nonlinear phenomena in the case when this theory would be generalized in the future to use also phase-mixing (Karmakar *et al.* 2018; Pramanik & Maity 2018), prospective numerical schemes (Verma 2018) as well as quantum effects (Shahmansouri, Aboltaman & Misra 2018). The described approach can be also useful for the design and production of new functional materials and processes (Ye *et al.* 2013). Also, influences of transversal electromagnetic phenomena on nonlinear surface waves have to be taken into account (Yu & Zhelyazkov 1978; Vladimirov, Yu & Tsytovich 1994; Shukla 1999) in some cases of an essential practical interest. Moreover, as was shown in Lee, Jung & Jung (2018) and Chandler-Wilde & Zhang (1998), they can play sometimes a noticeable role in processes across the interface between various physical and biological substances.

Acknowledgements

The work was carried out within the State Assignment on Fundamental Research to the Kurnakov Institute of General and Inorganic Chemistry of Russian Academy of Sciences.

Editor Edward Thomas, Jr. thanks the referees for their advice in evaluating this article.

Funding

There is no funding to declare for this research study.

Declaration of interest

The author reports no conflict of interest.

Availability of data and materials

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

REFERENCES

BOARDMAN, A.D. (Ed.) 1982 *Electromagnetic Surface Modes*. Wiley.

BRODIN, G. & STENFLO, L. 2014 Large amplitude electron plasma oscillations. *Phys. Lett.* A **378**, 1632–1635.

CHANDLER-WILDE, S.N. & ZHANG, B. 1998 Electromagnetic scattering by an inhomogeneous conducting or dielectric layer on a perfectly conducting plate. *Proc. R. Soc. Lond.* A **454**, 519–542.

GRADOV, O.M. 2020 Nonlinear behavior of a surface charge on the curved plasma boundary with a moving cavity. *Phys. Lett.* A **384**, 126566.

GRADOV, O.M. 2021 Self-consistent plasma boundary distortions during the interaction of a normally incident electromagnetic beam and a nonlinear surface charge. *Chin. J. Phys*. **72**, 360–365.

GRADOV, O.M. 2022 Self-consistent deformation of a plasma cylinder boundary under influences of a nonlinear surface charge. *Chin. J. Phys*. **77**, 707–711.

JEFFREYS, H. & SWIRLES, B. 1999 *Methods of Mathematical Physics*. Cambridge University Press.

KARMAKAR, M., MAITY, C., CHAKRABARTI, N. & SENGUPTA, S. 2018 Phase-mixing of large amplitude electron oscillations in a cold inhomogeneous plasma. *Phys. Plasmas* **25**, 022102.

LEE, M.-J., JUNG, G. & JUNG, Y.-D. 2018 Branching and resonant characteristics of surface plasma waves in a semi-bounded quantum plasma including spin-current effects. *Phys. Plasmas* **25**, 052106.

MA, J.Z.G. & HIROSE, A. 2009 Parallel propagation of ion solitons in magnetic flux tubes. *Phys. Scr*. **79**, 045502.

PRAMANIK, S. & MAITY, C. 2018 Langmuir wave phase-mixing in warm electron-positron-dusty plasmas. *Phys. Lett.* A **382**, 1020–1023.

SHAHMANSOURI, M., ABOLTAMAN, R. & MISRA, A.P. 2018 Surface plasmons in a semi-bounded massless Dirac plasma. *Phys. Lett.* A **382**, 2133–2136.

SHUKLA, P.K. 1999 Nonlinear plasma science. *Phys. Scr*. **T82**, 4.

STENFLO, L. 1996 Theory of nonlinear plasma surface waves. *Phys. Scr*. **T63**, 59–62.

STOKER, J.J. 1957 *Water Waves. The Mathematical Theory with Applications*. Interscience Publication.

TAMIR, T. & OLINER, A.A. 1963 The spectrum of electromagnetic waves guided by a plasma layer. *Proc. IEEE* **51** (2), 317–332.

VERMA, P.S. 2018 A novel numerical scheme for nonlinear electron plasma oscillations. *Phys. Plasmas* **25**, 032107.

VLADIMIROV, S.V., YU, M.Y. & TSYTOVICH, V.N. 1994 Recent advances in the theory of nonlinear surface-waves. *Phys. Rep*. **241**, 1–63.

YE, L., ZHANG, Y., ZHANG, X., HU, T., JI, R., DING, B. & JIANG, B. 2013 Sol–gel preparation of $SiO_2/TiO_2/SiO_2$–TiO_2 broadband antireflective coating for solar cell cover glass. *Sol. Energy Mater. Sol. Cells* **111**, 160–164.

YU, M.Y. & ZHELYAZKOV, I. 1978 Solitary surface waves. *J. Plasma Phys*. **20**, 183–188.

https://doi.org/10.1017/S0022377824000084 Published online by Cambridge University Press