those microconditions obtaining at the time in question, given the information you have, is equal to the probability of any particular other one of them obtaining at the time in question."

Surely there must be a clearer way to convey the meaning of this sentence.

Such flaws notwithstanding, the book is a sustained and penetrating look into the heart of a paradox, with proposals for solution which will no doubt generate further discussion and progress.

R. E. Kastner, University of Maryland College Park

REFERENCES

Albert, David (1992), *Quantum Mechanics and Experience.* Cambridge: Harvard University Press.

Arntzenius, Frank (2001), "Time Reversal Operations, Representations of the Lorentz Group, and the Direction of Time", preprint.

Ghirardi, G. C., A. Rimini, and T. Weber (1986), "Unified Dynamics for Microscopic and Macroscopic System", *Physical Review* D 34: 470–491.

Horwich, Paul (1988), *Asymmetries in Time.* Cambridge: MIT Press.

Lewis, David (1979), "Counterfactual Dependence and Time's Arrow", *Nous* 13: 455–476.

Price, Huw (1996), *Time's Arrow and Archimedes' Point.* Oxford: Oxford University Press.

Zeh, Dieter (1989), *The Physical Basis of the Direction of Time.* Heidelberg: Springer-Verlag.

Mayberry, J. P., ***The Foundations of Mathematics in the Theory of Sets. Encyclopedia of Mathematics and Its Applications Ser., Vol. 82***. Cambridge: Cambridge University Press (2000), xx + 429 pp., index, cloth $80.00 (cloth).

Mayberry gives a powerfully informed, tough-minded argument that mathematics absolutely needs a foundation and has one established in practice: the axiomatic method as interpreted in Cantorian set theory. The last third of the book suggests we might eventually abandon Cantor in favor of what Mayberry calls Euclidean set theory. Euclidean set theory also yields a novel approach to non-standard models of arithmetic, related to work by Edward Nelson and Jan Mycielski (391).

Mayberry begins with the notion of a number, which he takes in the sense of Aristotle's *arithmos,* a collection of specific things. This sense survives in English today. Mayberry gives as example "Lieutenant Lightoller was among the number of survivors of the Titanic" (99). He cites Aristotle often, to good effect, and with real sensitivity to the texts, especially on the variety of ways in which things are said "to be," and on the nature of proof. And he discusses Newton's deliberate gesture changing the meaning of number, against Barrow's resistance (191–192).

Then comes the question of finitude, in the ancient meaning of "finite" as "definite, determinate, bounded." Which *arithmoi* are "finite"? Which are definite, determinate, bounded? For Aristotle and Euclid a determinate whole is greater than its proper parts. Mayberry calls this the Euclidean finite. Cantor rejected it, saying there is a determinate whole of the natural numbers, though it is the same size as many of its proper parts. Mayberry calls this the idea of the Cantorian finite, axiomatized in Cantorian set theory.

This theory is essentially Zermelo-Fraenkel set theory (ZF), with some technical differences regarding induction, and with the crucial and carefully observed difference that it is not a formal theory. It is "expressed in a language whose fundamental vocabulary must be understood *prior* to laying down the axioms" and the axioms are "axioms, properly so called. They are fundamental propositions that, although true, neither require, nor admit of, proof" (8). The axiomatic method uses formal axioms, as when we today speak of "axioms" for Euclidean or hyperbolic or elliptic plane geometry. These are neither true nor false. They are to be interpreted in models constructed in set theory. Mayberry also describes both first and second order formal ZF set theory. Those are technical tools to study aspects of actual set theory. He does insist, against Bourbaki among others (99, 229), that the theorems of mathematics are not conventionally, or hypothetically, or supposedly true. They are proved true.

Mayberry's axiomatic method includes "Brouwer's principle": Conventional logic applies only to finite domains. At first Mayberry takes this as Cantorian finite. Conventional logic cannot quantify over all members of any proper class—in particular, not over the whole universe of sets—as, indeed, logic books routinely say the domain of a model is a set. Mayberry notes that he differs here from some category theorists, and both Gödel and Kreisel (248–249). But he argues we need it to avoid several specific kinds of nonsense. Certainly Brouwer never intended anything like the Cantorian finite, nor does Mayberry say he did. The book is admirably scrupulous in historical attributions as it argues for and against views from Plato, Aristotle, and Kant through Frege, Wittgenstein and Dummett.

Then the book turns to Euclidean set theory. The key axiom says the whole is always greater than the part: No set can be placed in 1–1 correspondence with any proper subset. The successor function maps the natural numbers 1–1 onto the non-zero naturals, so there is no set of all natural numbers. Certain predicates do pick out simply infinite systems, in Dedekind's terms. Roughly, their extensions model the Peano axioms. But their extensions are not sets. These simply infinite systems are not all the same size, each can be extended, and none has enough "numbers" to count the members of each set in the universe. Notice the Euclidean axiom does not say sets are finite in the conventional sense today—it only says no set S has a set of pairs f putting it in bijection with any proper subset. From a Cantorian standpoint this is the start of a theory of nonstandard models of arithmetic. From a Euclidean standpoint it is "the beginning of wisdom" (385).

On categorical foundations, Mayberry only warns against the same abuses as for set theory. Since the language of foundations must be antecedently clear, it cannot use sophisticated ideas such as "topos" (9), just as it cannot use "model of set theory." Categorical foundations must speak, for example, of functions and composition, just as set theoretic foundations speak of sets and membership. We also cannot use formal topos theory as a foundation (206), just as we cannot use formal set theory. The question is: Can we take, say, the axioms for the category of sets to be self-evident truths expressed in terms we already understand before we lay them down? I think so. Mayberry does not say. Of course, it is notoriously difficult to distinguish self-evidence from familiarity.

At least since Carnap, many philosophers believe the question of foundations cannot be about truth. It can only be about consistency, power, and parsimony. Mayberry uses Aristotle's acumen plus current proof theory and set theory to argue it is about truth and that is what makes it matter.

Colin McLarty, Case Western Reserve University