THE JOURNAL OF NAVIGATION (2005), **58**, 337–340. 
doi:10.1017/S0373463305003280 Printed in the United Kingdom

# A Comment on Navigation Instruction

Michael A. Earle

(Email: michael.earle@sympatico.ca)

1. INTRODUCTION. Use of a sphere to represent the Earth in the teaching of navigation has the advantage that the mathematics is relatively simple and has sufficient accuracy for most practical purposes. It also allows relatively simple instructional material to be developed for plane and Mercator sailing. Unfortunately, and as remarked by Williams [1], bad practice based on an error of principle has pervaded some navigation texts for some time. This error has passed into various instructional settings where it has gone un-noticed or un-remarked. The error lies in the un-rigorous use of meridional parts for the spheroidal earth together with latitude differences for the spherical earth in the teaching of plane and Mercator sailing. This is exemplified in [2, example1, p 585]. It is often argued that the resulting error in calculated distance or mid-latitude caused by this un-rigorous treatment is small and within what practical course keeping allows. However, permitting the discrepancy to remain unexplained is a disservice to students of navigation. Students should be given a clear explanation of the differences between the fictitious though useful spherical model of the earth and the spheroidal earth as presently described by WGS84. Teaching of plane and Mercator sailings should be based upon either the spherical model or upon the spheroidal model but not on parts taken from both.

2. GENERAL. Often, the navigator needs to calculate the distance and azimuth between two points or he needs to determine co-ordinates of an arrival point given the distance to be travelled at constant azimuth. To solve these problems on the sphere, he needs to determine the difference in latitude (*DLAT*) and the associated difference in meridional parts (*DMP*). These are then inserted into appropriate expressions for distance (*DIST*), mid-latitude (via Cosine(*MIDLAT*)) and difference in longitude (*DLON*) as required.

These expressions are omitted here for brevity but can be found in [1]. For solutions on the spheroid, the difference in latitude parts *DLP*, which takes account of the ellipticity, must be determined and used in place of *DLAT* while *DMP* on the spheroid is also computed from a formula that takes account of the ellipticity. Furthermore, finding arrival latitude on the spheroid from *DLP* for a given distance travelled at constant azimuth requires a simple iterative process. The units used are also different in that the sphere invokes units of nautical miles whereas the spheroid invokes units of geodetic miles. Thus when elements of sphere and spheroid are mixed a dimensional inconsistency also exists. The nautical mile is exactly1852 metres and equal to the span of one minute of latitude on the sphere whereas the geodetic mile is slightly larger at 1855·3248 metres and equal to the span of one minute of longitude at the equator on the WGS84 spheroid.

3. SPHERICAL EARTH. If a spherical model of the earth is chosen as the basis for instruction, then one that has 1 minute of arc on any great circle equal to 1 nautical mile is appropriate. The distance along a meridian between two parallels is then

$$DLAT = L_{a2} - L_{a1} \tag{1a}$$

where latitudes $L_{a1}$, $L_{a2}$ are expressed in minutes. Also for this model, earth radius $a = 10800/\pi$ or 3437·7468 nautical miles.

It is a frequent practice that the table of meridional parts like that found in [2, Table 5] produced from the equation [2, pp 4–5], which applies to the WGS 72 spheroid, has been erroneously used to establish *DMP* for the sphere. On setting the ellipticity *e* to zero in that formula, the correct expression for meridional parts on the sphere and *DMP* is established i.e.

$$M = a ln\left[tan\left(45 + \frac{|L_a|}{2}\right)\right] \tag{1b}$$

Then

$$DMP = M(L_{a2}) - M(L_{a1}) \tag{1c}$$

This formula in which $L_a$ is in degrees can now be employed directly with great ease using a calculator thus removing the need for tables and the interpolation thereof, so that on applying Eqs. 1a, 1b, and 1c, the aforementioned error of principle is removed.

4. SPHEROIDAL EARTH. The earth is presently described by the WGS84 ellipsoid, which defines the mean surface as one produced from an ellipse of ellipticity $\varepsilon = 0{\cdot}08181919$ rotated about the polar axis. This model accounts for the difference between equatorial and polar radii, which are:

Equatorial radius $a = 6378137$ metres

Polar radius $b = 6356752{\cdot}3142$ metres.

For this model, a natural unit of distance is the span of one minute of *longitude* at the equator, which is equal to 1855·3248 metres and also known as the geodetic or geographic mile. Consequently the equatorial radius is given by $a = 10800/\pi = 3437{\cdot}7468$ *geodetic* miles.

On the spheroid, meridional distance is no longer the straightforward difference in latitude *DLAT* as for the sphere, since every degree interval is of a slightly different length. The difference in latitude parts or *DLP*, is cognate to *DLAT* on the sphere and is calculated from

$$DLP = L(\psi_2) - L(\psi_1) \tag{2}$$

Where $L(\psi)$ is the distance from the equator to the geodetic latitude $\psi$ (in radians) and is found from

$$L(\psi) = a(1-\varepsilon^2)\int_0^{\psi} \frac{d\psi}{(1-\varepsilon^2 sin^2\psi)^{\frac{3}{2}}} \tag{3}$$

https://doi.org/10.1017/S0373463305003280 Published online by Cambridge University Press

Meridional parts for the WGS 72 spheroid found from [2, Table 5] are not applicable to the present WGS84 spheroid. Using a calculator, it is a simple matter to re-calculate meridional parts corresponding to each latitude $L$a from Eq. 4 and with the WGS84 value for $\varepsilon$.

$$M = a\left\{ ln\left[\left(tan\left(45^\circ + \frac{|L_a|}{2}\right)\right)\left(\frac{1-\varepsilon sin(|L_a|)}{1+\varepsilon sin(|L_a|)}\right)^{\frac{\varepsilon}{2}}\right]\right\} \tag{4}$$

Difference in meridional parts $DMP$ is then found from

$$DMP = M(L_{a2}) - M(L_{a1}) \tag{5}$$

Many courses seem to specifically avoid calculus expressions such as Eq. 3 and to those who feel diffident towards calculus, the exact formula for meridional distance $L$ from Eq. 3 may seem daunting, even though it responds well to computer numerical evaluation [1, p 23] or to the features of specialized mathematical software. Fortunately, because the denominator under the integral Eq. 3 is well behaved and departs very little from unity, a very good series approximation for meridional distance has been established. Snyder[3] has provided a harmonic series expansion for the integral of Eq. 3, which even when confined to the first few terms, provides a solution for meridional distance. The first two terms of his result are

$$L(\psi) = a\left\{\left(1 - \frac{1}{4}\varepsilon^2 - \frac{3}{64}\varepsilon^4\right)\psi - \left(\frac{3}{8}\varepsilon^2 + \frac{3}{32}\varepsilon^4\right) sin(2\psi)\right\} \tag{6}$$

Numerical testing has shown that terms in $\varepsilon^4$ can also be discarded with a very small penalty and that with further slight modification, results in the following compact two-term replacement for Eq. 3.

$$L = 59{\cdot}9L_a - 8{\cdot}65 sin(2L_a) \tag{7}$$

This expression is conveniently handled by a hand held calculator and has an error less than 15 p.p.m for $0 < L_a < 90^\circ$ i.e. within 0·0015%. In this formula, $L$ is the distance from the equator to a latitude $L_a$ in degrees that corresponds to the geodetic latitude $\psi$ in radians. Thus for consistency in computing plane or Mercator sailings on the spheroid, the expressions Eqs. 2, 4, 5 and 7 will provide self-consistent results in geographic miles. To convert to nautical miles multiply by 1·0018.

## 5. CONCLUDING REMARKS.

Navigational calculations for Rhumb line courses have traditionally been done using an earth assumed to be spherical. However the process has been made mildly inaccurate due to an error of principle occurring in some texts in which solutions have been arrived at using elements taken from both the spherical and spheroidal models. Consequently, instructional material drawn from these texts and examples therein may serve to deny students of navigation a rigorous understanding of underlying principles.

It is possible that the error of principle discussed here has emerged from a practice designed to avoid coping with the spheroidal earth and the associated calculus expression of Eq. 3 and has been further justified on the basis that the resulting errors are small. On using a modern scientific calculator to evaluate Eqs. 4 and 7, sufficiently accurate and self-consistent results can be obtained and any instructional objections to the spheroidal model can be removed. However, solutions based on the spherical earth model are obtained with less effort than for the spheroid and provided that definitions are used consistently, acceptable accuracy suitable for the instructional environment can be obtained.

## REFERENCES

[1] *Geometry of Navigation* by Roy Williams. Pub. Horwood Publishing Ltd. ISBN 1-898563-46-2.
[2] *The American Practical Navigator* (Bowditch) Pub.No.9, Vol. II.1981.
[3] *Map Projections – A Working Manual* by John P. Snyder. Pub.U.S.Geological Survey Professional Paper No.1395.