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Gradient-based optimization method for interference suppression of linear arrays by the amplitude-only and phase-only control

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Research Paper

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Abstract

This paper presents a gradient-based optimization method for interference suppression of linear arrays by controlling the electrical parameters of each array element, including the amplitude-only and phase-only. Gradient-based optimization algorithm (GOA), as an efficient optimization algorithm, is applied to the optimization problem of the anti-interference arrays that is generally solved by the evolutionary algorithms. The goal of this method is to maximize the main beam gain while minimizing the peak sidelobe level (PSLL) together with the null constraint. To control the nulls precisely and synthesize the radiation pattern accurately, the full-wave method of moments is used to consider the mutual coupling among the array elements rigorously. The searching efficiency is improved greatly because the gradient (sensitivity) information is used in the algorithm for solving the optimization problem. The sensitivities of the design objective and the constraint function with respect to the design variables are analytically derived and the optimization problems are solved by using GOA. The results of the GOA can produce the desired null at the specific positions, minimize the PSLL, and greatly shorten the computation time compared with the often-used non-gradient method such as genetic algorithm and cuckoo search algorithm.

Introduction

Pattern synthesis of uniform linear anti-interference arrays (ULAAs) is a traditional and classical topic, which has been extensively studied in the literature [1–3]. Many synthesis methods have been proposed to achieve the radiation pattern performances with different requirements, such as the gain or directivity enhancement, sidelobe minimization, null controlling, and so on [4–6]. Among them, a particularly noteworthy application is to synthesize the radiation pattern with the sidelobe minimization and null controlling. In this type of method, the anti-interference function is realized by placing the nulls in the required directions of the interfering signals, and the radiation performance is improved by minimizing the peak sidelobe level (PSLL) while maintaining the main-beam gain. For the ULAAs design, the target can be achieved by optimizing the electrical parameters of each array element, e.g. the amplitude-only and phase-only. The design optimization of the ULAAs is demonstrated as a convenient way to search for the better array parameter arrangement [6, 7]. In the design optimization, it is important to determine an appropriate optimization algorithm for solving this problem efficiently.

In recent years, many research works are devoted to studying the optimization algorithms for pattern synthesis, which have been successfully used in the optimization design of the ULAAs [8, 9]. The most often used optimization algorithm for solving the problems of the PSLL minimization and the null controlling is the evolutionary algorithm (EA). Based on the EAs' idea, many optimization algorithms are developed through improvement and modification of the classical EAs to overcome some of its limitations or increase the diversity of the searching way by means of the nature-inspired EAs (i.e. a type of optimization algorithms formed by imitating the evolution of the natural biological or social behavior). For example, classical EAs such as particle swarm optimization [1], differential evolution algorithm [10], simulated annealing algorithm [11] and genetic algorithm (GA) [12, 13] have been used to minimize the PSLL and control the nulls at the specific positions. Among them, an improved GA based on the dynamic weight vector avoids the repeated calculation of the fitness function in each generation through a pre-computed discrete cosine transform matrix, which reduces the computational complexity of GA [14]. Besides, most of the research works focused on the nature-inspired EAs, such as ant lion algorithm [15], invasive weed algorithm [16], cat swarm algorithm [17], biogeography-based optimization [18], whale optimization algorithm [19], and so on, have also been developed. The advantage of this kind of algorithm lies in the relatively flexible searching way. However, the optimization process needs to execute a complex random search process and consider a large number of tuning parameters. Although the cuckoo search (CS) algorithm may have fewer parameters and simpler operations than other EAs [20, 21], it still has the problem of slow convergence speed in the later stage of optimization. For the large-scale optimization problems with complex constraints, these kinds of methods require excessive analysis and iteration of optimization. To overcome this limitation, various improved methods were proposed to reduce the computational cost and increase the convergence speed [22, 23].

Gradient information is very important to determine the direction of iterative search in the optimization process. Gradient-based optimization algorithms (GOAs), such as MMA algorithm [24], SQP algorithm [25], etc., are common algorithms that use the gradient information to accelerate the design optimization. The design optimization can be solved by a GOA more efficiently for the case when the sensitivities of the optimization are derived analytically [26, 27]. An optimization method based on the GOA was proposed to achieve the pattern synthesis by element rotation [28]. In this work, the radiation performance of low PSLL is considered and the sensitivity of the design objective with respect to the rotation angle of each array element is derived analytically. Then, the optimization problem is solved by the GOA, which improves the searching efficiency and saves the computation time. To simplify the analysis procedure, the array elements are generally assumed to be a series of identical currents, and the array patterns are characterized by the array factor. In this way, many SOAs are feasible to search for the global optimal solution without increasing the computational burden [17–19, 21]. However, it is generally believed that the mutual coupling affects the depth of nulls, and for some arrays, it also affects the position of nulls. To maintain the position of nulls precisely and calculate the radiation performance accurately, the full-wave method of moments (MoM) analysis is adopted to consider the mutual coupling of the array elements rigorously. Therefore, in the optimization procedure where the full-wave MoM simulations and the calculated sensitivities analytically are involved, the GOA is an effective choice to solve the optimization problem of the PSLL minimization and null controlling.

In this paper, a gradient-based optimization method for the PSLL minimization and null controlling of ULAAs is presented. In section "Optimization formulation and procedure", the optimization formulation is given and the optimization procedure based on the GOA is proposed. The full-wave MoM is used to maintain the nulls and calculate the pattern performances accurately. The sensitivities of the design objective, the constraint function with respect to the different design variables (excitation amplitude or phase) are derived analytically. The optimization problem of the anti-interference array is solved by the GOA. In section "Numerical examples", typical numerical examples are provided to verify the effectiveness of the proposed method. In section "Conclusion", the advantage of the GOA is summarized and the conclusion is given.

Optimization formulation and procedure

The optimization formulation

The optimization problem of ULAAs for the interference suppression is defined as: maximize the main-beam gain and minimize the PSLL along with the constraints of null depth in the desired directions through optimizing the electrical parameters of each array element. The optimization problem can be formulated as:

$$\begin{cases} \text{find: } \chi \\ \text{min: } \Phi_0 = -G_m + \max(G_{\mathrm{SLL}}) \\ \text{s.t.: } ZJ = V(\chi) \\ \max(G_{\mathrm{NULL}}) \leq G_0 \\ \chi_{\min} \leq \chi \leq \chi_{\max} \end{cases} \tag{1}$$

where G_m is the maximum of the gain in the main-beam direction. G_{SLL} and G_{NULL} are the vectors of the gains within the sidelobe region and in the desired null directions, respectively. The design objective Φ_0 is an explicit function that can describe a specific functional requirement, and evaluate the high-quality optimization solution. The constraint is defined to control the null depth by constraining the maximum gain value of the null depth G_{NULL} in all specified directions less than a given value G_0. Φ_1 is defined as a constraint function to characterize the difference between the null depth constraint and G_0 in the optimization process. The constraint of null depth and Φ_1 in the ULAAs optimization problem are expressed as:

$$\max(G_{\mathrm{NULL}}) = \ln\left(\sum_{m=1}^{M_0} e^{-qG_{\mathrm{NULL,m}}}\right)^{1/q} \leq G_0 \tag{2a}$$

$$\Phi_1 = \max(G_{\mathrm{NULL}}) - G_0 \tag{2b}$$

where q is the operator that calculates the approximate maximum value, which is determined to be a sufficiently large positive integer. M_0 is the total number of nulls. $G_{\mathrm{NULL,m}}$ represents the gain value of m-th null.

The optimization procedure

The GOA takes the advantage of the analytical properties of the optimization problem to improve the searching efficiency. In order to carry out the GOA to achieve the optimization design of coupled antenna arrays, it is necessary to establish the relationship among Φ_0 and Φ_1 (collectively referred to as Φ_k, $k = 0$ or 1) and χ. This relationship requires that the change in Φ_k caused by the iteratively changed arrangements of array parameters can be accurately calculated. According to the relationships, the sensitivities of Φ_k and χ can be derived analytically. Then, the sensitivity information is introduced into the optimization procedure to speed up the convergence.

The computational steps for solving the optimization problem by the GOA are summarized as:

(1) The array parameter to be optimized is set as the design variable χ and the upper and lower bounds are limited to the range of $[\chi_{\min}, \chi_{\max}]$. Here, χ represents the vector of the excitation amplitude or phase of each array element. In the design of ULAAs, the $2N$ symmetric geometric elements are assumed to be placed along the x-axis, as shown in Fig. 1. At the beginning of the optimization, a parameterized ULAAs model is generated from χ and an initial arrangement of χ_{ini} is set up.
(2) Define Φ_k of the optimization problem. Φ_k can be calculated through an appropriate analysis method which requires the

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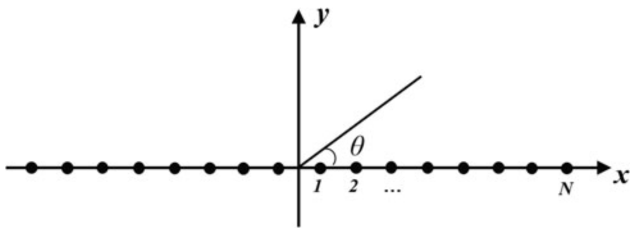

Fig. 1. The geometry of 2*N*-elements symmetric ULAAs.

radiation pattern performances. The full-wave MoM is a common electromagnetic analysis method, and the radiation performance of the ULAAs can be accurately calculated by the full-wave MoM simulation. The following equation is formulated in matrix form as:

$$\begin{bmatrix} Z_{11} & \dots & Z_{1j} & \dots & Z_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{i1} & \dots & Z_{ij} & \dots & Z_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1} & \dots & Z_{Nj} & \dots & Z_{NN} \end{bmatrix} \begin{Bmatrix} J_1 \\ \vdots \\ J_i \\ \vdots \\ J_N \end{Bmatrix} = \begin{Bmatrix} V_1(\chi_1) \\ \vdots \\ V_i(\chi_i) \\ \vdots \\ V_N(\chi_N) \end{Bmatrix} \tag{3}$$

where Z_{ij} represents the mutual impedance matrix between the i-th and j-th array element. J_i and V_i represent the unknown surface current and the incident field at the i-th element, respectively. The unknown currents J can be solved by assembling the Z and V, and the gain of the radiation pattern G can be calculated by the equivalent dipole model [28]. The desired Φ_k can be obtained by the G_m, G_{SLL}, and G_{NULL}, subsequently.

(3) Calculate the sensitivities of Φ_k with respect to the χ. Based on the parametric governing equation, the sensitivity of Φ_k with respect to χ is derived as:

$$\frac{d\Phi_k}{d\chi} = \frac{df(G)}{dG}\frac{dG}{d\chi}. \tag{4}$$

The derivation of $\partial f(G)/\partial G$ depends on the specific Φ_0 or Φ_1. According to the chain rule, the sensitivity of G with respect to χ is derived as:

$$\frac{dG}{d\chi} = \frac{\partial G}{\partial \chi} + \frac{\partial G}{\partial J}\frac{\partial J}{\partial \chi}, \tag{5}$$

where only J is an implicit function of χ, and it is difficult to manifest the function expression. Therefore, a complex adjoint vector λ is determined to eliminate the unknown $\partial J/\partial \chi$ by the adjoint method [26]. The following derivation is given by:

$$\begin{aligned} \frac{dG}{d\chi} &= \frac{\partial G}{\partial \chi} + \frac{\partial G}{\partial J}\frac{\partial J}{\partial \chi} + \lambda^{\dagger}\frac{\partial[ZJ - V(\chi)]}{\partial \chi} \\ &= \frac{\partial G}{\partial \chi} + \left(\frac{\partial G}{\partial J} + \lambda^{\dagger}Z\right)\frac{\partial J}{\partial \chi} + \Re\left(\lambda^{\dagger}\frac{\partial Z}{\partial \chi}J - \lambda^{\dagger}\frac{\partial V(\chi)}{\partial \chi}\right), \end{aligned} \tag{6}$$

where $\Re$ is the real operator and $\dagger$ is the conjugated transpose operator. To complete the sensitivity analysis, the λ should satisfy the following equation:

$$Z^{\dagger}\lambda = -\left(\frac{\partial G}{\partial J}\right)^{\dagger}. \tag{7}$$

Finally, the sensitivity analysis is completed and the formula (6) can be reduced to

$$\frac{dG}{d\chi} = -\Re\left(\lambda^{\dagger}\frac{\partial V(\chi)}{\partial \chi}\right). \tag{8}$$

(4) Update the next generation of the χ. It is noted that $V(\chi)$ should be automatically updated with χ during the iteration process. Then, update Φ_k of the optimization problem and calculate their sensitivities. The optimization procedure needs to be evaluated based on the iteration of Φ_k and χ. If the convergence criterion or the maximum number of iterations is satisfied, the optimization will stop, otherwise, the optimization will go to (2).

(5) Obtain the optimal solution χ_{opt} and synthesize the desired radiation pattern.

The flow chart of the GOA is illustrated in Fig. 2, where ζ is each iteration step in the optimization procedure and M is the maximum number of iterations. *Tol* is the calculation tolerance given based on the convergence criterion. According to the above optimization procedure, the optimization problem of the ULAAs can be solved efficiently.

Numerical examples

In order to verify the effectiveness of the proposed method, typical numerical examples are provided to synthesize the radiation patterns of ULAAs. In Fig. 1, the design model consists of a series of dipoles with 0.04λ in width and 0.46λ in height, where λ is the wavelength of the simulation frequency. The distance between any two adjacent dipoles is 0.5λ. In the optimization design of the ULAAs, the main-beam has a width of $|\theta| \leq 5^{\circ}$ in the direction of $\theta = 0^{\circ}$ and $\varphi = 0^{\circ}$, where θ and φ denote the elevation and azimuth coordinates on the truncation boundary. The q and the G_0 are set to 10 and −45 dB, respectively. The proposed method uses the full-wave MoM to achieve the precise design and correct some existing optimization strategies that the mutual coupling was ignored or approximately considered. The full-wave MoM is utilized to calculate the antenna performances by considering the mutual coupling rigorously. The GOA is used to solve the optimization problem with the PSLL minimization and null controlling. Typical optimization algorithms, including the common GA and CS algorithm, are selected to solve the same optimization model and compared with the GOA in terms of the radiation

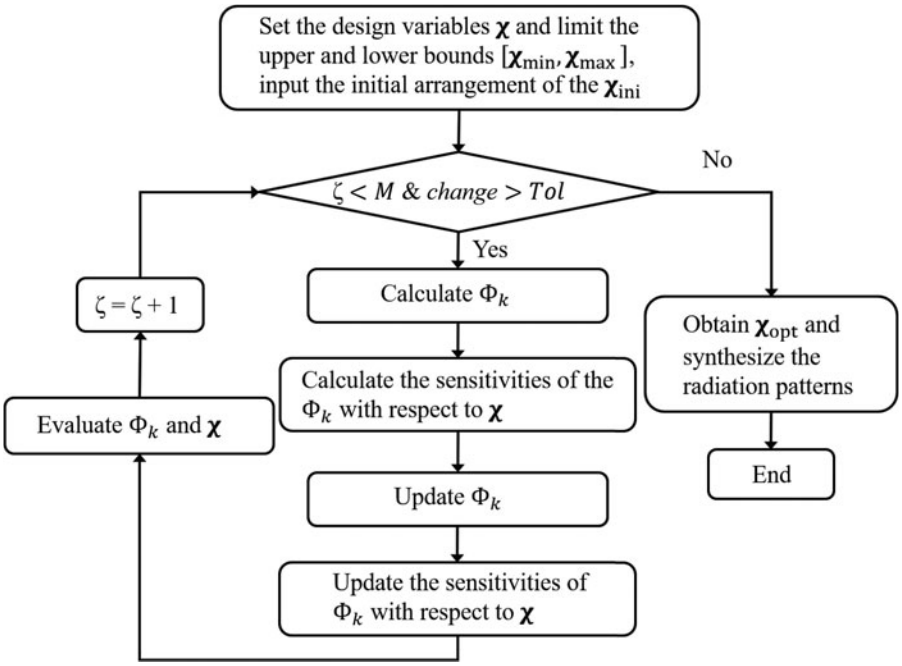

Fig. 2. The optimization flow chart of the GOA.

Table 1. The information and pattern properties of the excitation amplitudes optimization

Algorithms	G_m (dB)	PSLL (dB)	Null positions (°)	Null depth (dB)	The iterations of optimization	CPU time (min)
GA	16.66	−2.79	±30, ±40	−48.01, −45.20	699	210.1
CS	16.74	−3.59	±30, ±40	−46.72, −45.24	762	315.8
GOA	16.74	−3.25	±30, ±40	−46.16, −45.04	392	10.6

Table 2. The design result of the excitation amplitudes optimization

Algorithms	The design results of the excitation amplitudes optimization
GA	0.9647, 0.9886, 0.7409, 1.0000, 0.9656, 0.4852, 0.9387, 0.2594, 0.9756, 0.4565, 0.7954, 0.1251
CS	0.9980, 0.6717, 0.8396, 0.9123, 0.7133, 0.6590, 0.7600, 0.3240, 0.8051, 0.1846, 0.9570, 0.1820
GOA	0.8389, 0.5378, 0.6675, 0.7807, 0.5333, 0.6508, 0.4514, 0.4509, 0.5005, 0.3144, 0.7007, 0.1481

pattern performance and computational time. Both GA and CS algorithm have a population size of 25. The maximum number of iterations in the three algorithms is 1000. The optimization is computed using a 3.60 GHz i7–7700K CPU and 8 GB RAM. In numerical examples, the number of array elements and the nulls are distinguished to illustrate the scalability of the proposed method.

Excitation amplitudes optimization

The first example is to optimize the excitation amplitudes of the 24-element ULAA with the double nulls at $\pm 30^\circ$, $\pm 40^\circ$. Each dipole element is excited by a 1χ V voltage gap generator with 50 Ω characteristic impedance, where χ represents the excitation amplitude of each array element. $\chi_{\min}$ and $\chi_{\max}$ are set to 0 and 1, respectively. The initial arrangement χ_{ini} is set to 0.5. The optimization information and the pattern properties obtained based on the three algorithms are shown in Table 1, which includes the maximum gain in the main-beam direction, the PSLL, the null positions, the iterations of optimization, and the CPU running time. Table 2 describes the design results of the excitation amplitudes optimization. The comparisons of the radiation pattern performances are calculated by the full-wave MoM analysis, which is illustrated in Fig. 3.

It is seen that GA and CS algorithms can provide the G_m of 16.66 and 16.74 dB with a PSLL minimization of −2.79 and −3.59 dB, respectively. The CPU time of optimization is 210.1 and 315.8 min, respectively. The CS algorithm uses the MoM to analyze the radiation performances twice in each iteration for obtaining a good optimization result. The design results by the GOA can provide the G_m of 16.74 dB with a PSLL minimization of −3.25 dB, and the CPU time of optimization is 10.6 min. It is illustrated that the proposed method reduces the CPU time of optimization compared with the GA and CS. The iterations of

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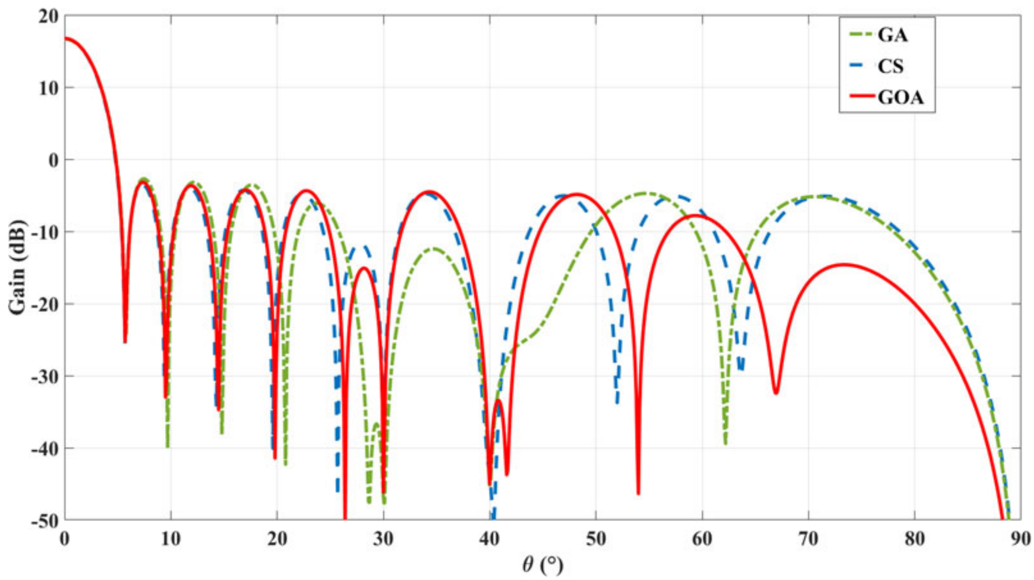

Fig. 3. The comparisons of the radiation pattern performances in the excitation amplitudes optimization.

Table 3. The information and pattern properties of the excitation phases optimization

Algorithms	G_m (dB)	PSLL (dB)	Null positions (°)	Null depth (dB)	The iterations of optimization	CPU time (min)
GA	16.60	1.08	±9	−45.95	421	131.4
CS	16.90	−0.54	±9	−45.54	423	261.1
GOA	16.90	−0.81	±9	−45.10	232	9.9

Table 4. The design result of the excitation phases optimization

Algorithms	The design results of the excitation phases optimization
GA	352.8917, 161.3554, 0.1008, 62.7602, 314.7662, 359.9992, 308.8652, 359.9984, 359.9993, 354.5269, 5.6464, 12.4412, 340.6669, 347.9753, 359.2387, 338.8875
CS	353.4290, 360.0008, 0.0000, 359.7946, 359.8691, 5.6551, 350.1001, 2.6241, 358.9008, 353.0222, 0.3953, 46.5013, 246.8188, 28.3901, 334.5157, 90.3784
GOA	66.4115, 74.6793, 80.2141, 89.6335, 84.0357, 85.8463, 87.0896, 82.4945, 91.3467, 82.9700, 53.3424, 208.1785, 85.2217, 117.5881, 17.6414, 147.4621

optimization based on the three algorithms may increase due to the null-depth constraint of multiple positions. All the design results maintain the nulls at the specific directions ($\pm 30^\circ$, $\pm 40^\circ$) by the full-wave MoM analysis and satisfy the constraint of the nulls. By comparing the optimization information and the pattern properties, the amplitude-only optimization by the GOA can greatly shorten the computational time, and obtain nearly the same radiation pattern performance as the other two algorithms.

Excitation phases optimization

The second example is the 32-element ULAA with the null positions at $\pm 9^\circ$ and each dipole element is excited by a $1e^{j\chi}$ V voltage gap generator, where χ represents the excitation phase of each array element. It is determined to match with a 50 Ω transmission line. $\chi_{\min}$ and $\chi_{\max}$ are set to 0 and 2π, respectively, where χ_{ini} is set to the middle value in the range of $[0, 2\pi]$. Table 3 shows the optimization information and the pattern properties based on the three algorithms, and Table 4 shows the design results of the excitation phases optimization. Figure 4 describes the comparisons of the radiation pattern performances. In the design optimization of the excitation phases, the constraints of null-depth are satisfied less than −45 dB. The design results are calculated by the full-wave MoM analysis for controlling the nulls at the specific directions ($\pm 9^\circ$) precisely and synthesizing the radiation patterns accurately. Compared with GA and CS algorithm, the results of the GOA provide the lowest PSLL, and the G_m is higher than GA. The iterations and the computation time of optimization by the GOA are 232 and 9.9 min, respectively. This validates that solving the optimization problem of the ULAAs through the GOA can enhance the searching efficiency of the optimization efficiently and improve the radiation performances.

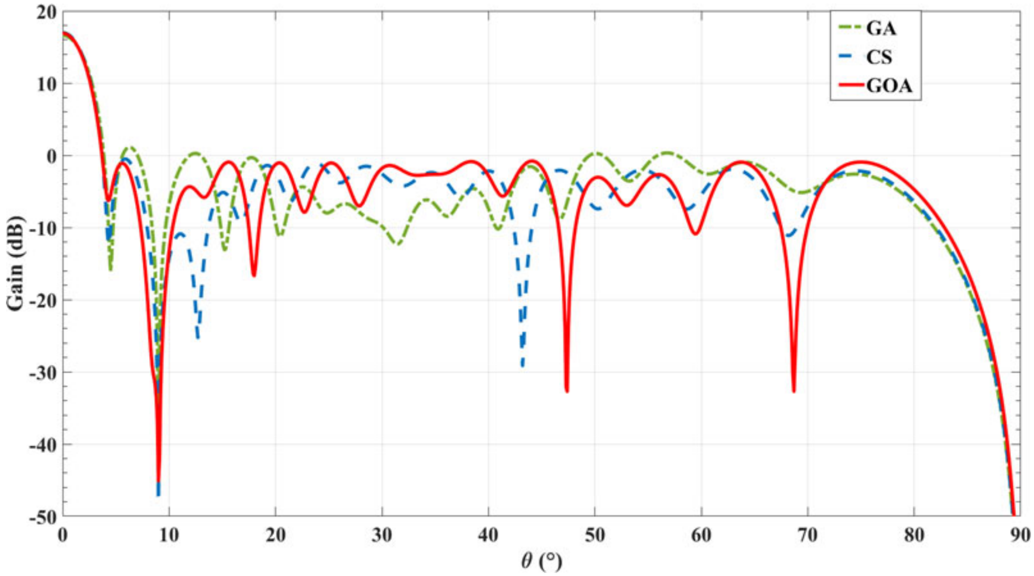

Fig. 4. The comparisons of the radiation pattern performances in the excitation phases optimization.

Conclusion

This paper presents a gradient-based optimization method for the PSLL minimization and null controlling of linear arrays. The optimization procedure of the GOA is described, the optimization problem is defined and the optimization formulation is given. The sensitivities of the design objective, the constraint function with respect to the design variables are derived analytically. The full-wave MoM is used to consider the mutual coupling among the array elements rigorously. Typical numerical results show that, compared with the non-gradient GA and CS algorithm, the GOA can produce the desired null at the specific positions and minimize the PSLL, and greatly shorten the computation time. The proposed method contributes to synthesize the radiation pattern of coupled arrays efficiently and accurately. The results also validate its potential for solving the PSLL minimization and null controlling optimization problem with different numbers of radiating elements and nulls. In future work, we hope to apply the proposed method to practical examples to take advantage of considering the mutual coupling rigorously and solving the optimization problems efficiently.

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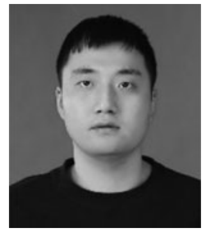

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