J. Plasma Physics (2002), *vol.* 67, *part* 5, *pp.* 329–337.
DOI: 10.1017/S0022377802001666 Printed in the United Kingdom

The effect of quantum oscillation in plasmas

B. SHOKRI

Physics Department and Laser Center of Shahid Beheshti University, Tehran, and Institute for Studies in Theoretical Physics and Mathematics, PO Box 19395-1795, Tehran, Iran
(shokri@theory.ipm.ac.ir)

(Received 2 July 2001 and in revised form 8 November 2001)

Abstract. Making use of the dielectric permitivitty of a solid state plasma obtained from linearizing a quantum hydrodynamic equation, volume and surface waves in cold semibounded plasma-like media and thin layers of solid state plasmas are investigated in the presence and absence of an external magnetic field. It is shown that quantum oscillation of free charged particles and its spatial dispersion even in cold plasmas lead to new spectra of collective oscillations. Furthermore, a new volume ion-acoustic-type wave is obtained with a quadratic dependence on the wavenumber in the long-wavelength limit. Moreover, it is shown that quantum oscillation affects the surface wave spectrum and extends it to a wider frequency region.

1. Introduction

The degenerate electron gas has been a subject of great activity [1–3]. A comprehensive treatment of the quantities related to this system, such as inelastic particle–solid and particle–plasma interactions, can be formulated in terms of the dielectric response function, obtained from the electron-gas model. The results have important applications in radiation and solid state physics [4], and in studies of energy deposition by ion beams in plasmas fusion targets [5,6]. Moreover, quantum effects may be very important in nondegenerate and degenerate plasmas [7–12].

On the other hand, it has been shown that, starting from the quantum hydrodynamics of cold plasmas, one can describe the linear kinetics of a collisionless quantum plasma and obtain the dielectric permittivity of a cold quantum plasma-like medium [13]. Moreover, it has been shown that in a cold plasma, taking into account free quantum oscillation, the effect of quantum spatial dispersion may be important [13]. Therefore, it is expected that it can change the spectra of volume and surface waves in degenerate and nondegenerate plasmas.

Furthermore, it is very important to know that, even in a cold plasma, spatial dispersion in the dielectric permittivity of a medium, due to single-particle quantum oscillations, may result in new spectra of collective oscillations, especially for surface and volume waves.

In the present paper, starting from the quantum hydrodynamics of cold plasmas, we first study the validity of this description and then describe the effect of quantum spatial dispersion on volume and surface waves spectra in semibounded plasmas and plasma layers in the short-wavelength limit.

It should be noted here that we expect that quantum oscillation and the diffusion mode of particles play the same role. In other words, even in cold plasma-like media, spatial dispersion, arising from quantum oscillation, plays the same role that the thermal motion of particles plays in warm plasmas.

This paper is presented in six sections. In Sec. 2, we study the validity of the abovementioned description. In Sec. 3, we treat the effect of quantum oscillation on volume waves in the presence and absence of an external magnetic field. In Sec. 4, we consider the abovementioned effect on surface waves and in Sec. 5 its effect on plasma-layer modes in the absence and presence of an external magnetic field. Finally, a summary and conclusions are presented in Sec. 6.

2. Validity conditions

2.1. Unmagnetized plasmas

Making use of the linearized quantum hydrodynamic equation obtained from the Schrödinger equation for free spinless electrons (and holes or light ions) and taking into account the quantum oscillation of free electrons (and holes or light ions), one can obtain the following relation between the dielectric permittivity of a quantum and classical isotropic collisionless electron (multicomponent) plasma [13]:

$$\varepsilon_q(\omega, \mathbf{k}) = \varepsilon_{cl}(\omega, \mathbf{k}) - \frac{\omega_q^2}{\omega_{Le}^2} \frac{[\delta\varepsilon_{cl}(\omega, \mathbf{k})]^2}{1 + (\omega_q^2/\omega_{Le}^2)\delta\varepsilon_{cl}(\omega, \mathbf{k})}, \tag{1}$$

where $\varepsilon_q(\omega, \mathbf{k}) = 1 + \delta\varepsilon_q(\omega, \mathbf{k})$ and $\varepsilon_{cl}(\omega, \mathbf{k}) = 1 + \delta\varepsilon_{cl}(\omega, \mathbf{k})$ are the dielectric permittivity of a quantum and classical collisionless electron (multicomponent) plasma; $\delta\varepsilon_q$ and $\delta\varepsilon_q$ are the charged-particle contributions to the classical and quantum dielectric permittivity, respectively. In addition, $\omega_{qe} = \hbar k^2/2m$ is the frequency of quantum oscillation of a free electron with effective mass m and wave vector $\mathbf{k}$; ω_{Le} is the plasma frequency of electrons. From the latter equation, we can obtain the longitudinal and transverse dielectric permittivities of an isotropic cold collisionless electron plasma [13,14]:

$$\varepsilon_q^l = 1 - \frac{\omega_{Le}^2}{\omega^2 - \omega_{qe}^2}, \qquad \varepsilon_q^{\mathrm{tr}} = 1 - \frac{\omega_{Le}^2}{\omega^2}. \tag{2}$$

As expected, the pole of the longitudinal dielectric permittivity shows the quantum oscillation of electrons. Moreover, the quantum effect is not manifested in the transverse dielectric permittivity. Here the plasma is cold because the dielectric permittivity, obtained from the linearized hydrodynamic equation, may also be obtained from quantum kinetic considerations when thermal motion is negligible [13].

It is important to know the extent to which this description is appropriate and when (2) is valid. To answer these questions, we first consider the longitudinal dielectric permittivity of an isotropic electron plasma, obtained from the Wigner kinetic equation [15].

$$\begin{aligned}\varepsilon^l(\omega, \boldsymbol{k}) = 1 + 3\frac{\omega_{Le}^2}{k^2 v_{Fe}^2} - \frac{3}{8}\frac{\omega_{Le}^2}{\omega_q k^3 v_{Fe}^3}\Bigg\{&[(\omega+\omega_q)^2 - k^2 v_{Fe}^2]\ln\left(\frac{\omega+\omega_q+kv_{Fe}}{\omega+\omega_q-kv_{Fe}}\right)\\ &-[(\omega-\omega_q)^2 - k^2 v_{Fe}^2]\ln\left(\frac{\omega-\omega_q+kv_{Fe}}{\omega-\omega_q-kv_{Fe}}\right)\Bigg\}.\end{aligned} \tag{3}$$

It is well known that the longitudinal modes can be obtained from the dispersion equation $\varepsilon^l(\omega, \mathbf{k}) = 0$ [14]. Therefore, from (3), when the spatial dispersion is weak, we find

$$\omega^2 = \omega_{Le}^2 + \left(\frac{\hbar k^2}{2m}\right)^2 + \frac{3}{5}k^2 v_{Fe}^2. \tag{4}$$

This spectrum without the final term could be obtained from (2). Therefore, if the energy of quantum oscillation of a free electron is greater than its thermal energy (or Fermi energy), we consider the plasma to be cold. On the other hand, the second term in (4) should be a small correction to the plasma frequency (the first term). But, in this case, the gas approximation is not valid. Therefore, (2) cannot describe the screening of the static electric field in this plasma. Screening of the longitudinal field arises when $\hbar^2 k^4/4m^2 \approx \omega_{Le}^2$ and is given by the usual Debye radius $r_{De} = 3v_{Fe}/\omega_{Le}$ where, v_{Fe} is the electron Fermi velocity.

We can study longitudinal surface waves in an isotropic electron plasma to explain the validity condition of the abovementioned description. This will be done in Sec. 4, where, by comparing the quantum correction with the classical spectrum of a surface wave, one can show that, under the condition $\omega \gg kv_{Fe}$, the plasma can be assumed to be cold.

2.2. Magnetized plasmas

We consider a strongly magnetized plasma, i.e. $\boldsymbol{\Omega}_e = eB_0/mc \gg \omega_{Le} \geqslant \omega$, where $\boldsymbol{\Omega}_e$ is the electron Larmor frequency and B_0 is the external magnetic field. The longitudinal dielectric permittivity can be obtained from the quantum hydrodynamic equations [13] and (1):

$$\varepsilon(\omega, \mathbf{k}) = \frac{k_i k_j}{k^2}\varepsilon_{ij}(\omega, \mathbf{k}) = 1 - \frac{\omega_{Le}^2 k_z^2/k^2}{\omega^2 \omega_q^2 k_z^2/k^2}. \tag{5}$$

If we average this equation with respect to the Fermi distribution, we find the quantum longitudinal dielectric permittivity [14]:

$$\begin{aligned}\varepsilon^l(\omega, \mathbf{k}) = 1 + \frac{3}{2}\frac{\omega_{Le}^2}{k^2 v_{Fe}^2} - \frac{3}{8}\frac{\omega_{Le}^2}{\omega_q k^3 v_{Fe}^3}\Bigg\{\left[\left(\omega + \omega_q \frac{k_z}{k}\right)^2 - k_z^2 v_{Fe}^2\right] \\ \times \ln\left(\frac{\omega + \omega_q(k_z/k) + kv_{Fe}}{\omega + \omega_q(k_z/k) - kv_{Fe}}\right) - \left[\left(\omega - \omega_q \frac{k_z}{k}\right)^2 - k_z^2 v_{Fe}^2\right] \\ \times \ln\left(\frac{\omega - \omega_q(k_z/k) + kv_{Fe}}{\omega - \omega_q(k_z/k) - kv_{Fe}}\right)\Bigg\}.\end{aligned} \tag{6}$$

From this equation, under the weak-spatial-dispersion condition, we find the following spectrum:

$$\omega^2 = \omega_{Le}^2 \frac{k_z^2}{k^2} + \omega_q^2 \frac{k_z^2}{k^2} + \frac{3}{5}k_z^2 v_{Fe}^2. \tag{7}$$

As seen, here again when $\hbar\omega_q \gg E_{Fe}$, the quantum correction may be important in the dispersion of longitudinal volume waves. Here E_{Fe} is the electron Fermi energy. Moreover, the screening of the static longitudinal field in magnetized plasmas, as in unmagnetized plasmas, does not depend on the quantum dispersion effect.

3. Volume waves

From the preceeding section, we study the quantum dispersion effect in the volume-wave spectrum in both unmagnetized and magnetized plasma-like media, respectively.

3.1. *Unmagnetized plasmas*

We first consider an unbounded cold plasma-like medium in the absence of an external magnetic field. Therefore, from (2), we find the following spectrum for the volume longitudinal waves in an unbounded quantum isotropic collisionless electron plasma-like medium:

$$\omega^2 = \omega_{Le}^2 + \omega_q^2. \tag{8}$$

This expression is valid when $\hbar\omega > kv_0$, where $v_0 = \sqrt{T_e/m}$ is the thermal velocity for a nondegenerate plasma and $v_0 = v_{Fe}$ is the Fermi velocity for a degenerate plasma. This leads to the condition $\hbar\omega_{Le} > T_e$ or E_{fe}. Here T_e is the thermal energy of electrons.

Compared with the frequency spectra of a classical plasma, this spectrum shows that quantum oscillation of electrons in a cold plasma plays the same role as thermal motion of electrons in a warm plasma. It should be noted that the quantum correction is important when the quantum oscillation has the same order of magnitude as the collective plasma frequency. Starting from (1), we can write it as

$$(1 + \delta\varepsilon_{\mathrm{cl}})(1 + \delta\varepsilon_q) = \frac{\omega_{qe}^2}{\omega_{Le}^2}\varepsilon_{\mathrm{cl}}^2. \tag{9}$$

This relation shows the coupling of classical and quantum modes due to the spatial dispersion arisen from quantum oscillations.

In order to estimate the effect of quantum oscillation, we first consider metals, where the particle density $n \geqslant 10^{21}\,\mathrm{cm}^{-3}$. In this case, the quantum dispersion effect is negligible. However, in semiconductors such as InSb where $n \leqslant 10^{17}\,\mathrm{cm}^{-3}$ and the ratio of effective mass to mass is about $\frac{1}{30}$, we find that when $k \leqslant 10^7\,\mathrm{cm}^{-1}$, $\omega_{qe} \geqslant \omega_{Le}$, and consequently quantum effects become important.

We consider a two-component cold plasma. In this case, we have

$$\varepsilon_q^l = 1 - \frac{\omega_{Le}^2}{\omega^2 - \omega_{qe}^2} - \frac{\omega_{Lh}^2}{\omega^2 - \omega_{qh}^2}, \tag{10}$$

where ω_{qh} and ω_{Lh} are the quantum oscillation and plasma frequencies of holes or ions. In this case, due to the large mass difference between charged-particle species, a new spectrum may arise. This new spectrum is very similar to the ion-acoustic wave in a plasma. But this type of acoustic wave arises due to the difference between the hole or ion quantum oscillation and the electron quantum oscillation. Therefore, in the intermediate region where $\omega \ll \omega_{qe}$, we find a new ion-acoustic type wave with frequency spectrum

$$\omega^2 = \frac{\omega_{Lh}^2\omega_{qe}^2}{\omega_{Le}^2 + \omega_{qe}^2}. \tag{11}$$

Here it is clear that this acoustic-type wave, in contrast to the usual ion-acoustic wave, has a quadratic dependence on k in the long-wavelength limit, i.e. $\omega^2 = (m/M)\omega_{qe}^2 \sim k^4$, where M is the ion or hole mass.

https://doi.org/10.1017/S0022377802001666 Published online by Cambridge University Press

3.2. Magnetized plasmas

We impose an external magnetic field. For a longitudinal wave in an unbounded pure electron plasma, we find the following dispersion equations, assuming the magnetic field to be along the Z axis and the X axis to be across it:

$$\delta\varepsilon^l_{\rm cl} = \frac{\omega^2_{Le}}{\omega^2}\frac{k^2_z}{k^2} - \frac{\omega^2_{Le}}{\omega^2 - \Omega^2_e}\frac{k^2_x}{k^2}, \qquad \delta\varepsilon^q_{\rm cl} = \frac{\omega^2_{qe}}{\omega^2}\frac{k^2_z}{k^2} - \frac{\omega^2_{q}}{\omega^2 - \Omega^2_e}\frac{k^2_x}{k^2}, \tag{12}$$

where k_z and k_x are the longitudinal and transverse wave vectors, respectively.

Making use of (1), (6) and (12), we find the following dispersion equation for the longitudinal quantum volume wave in a cold magnetoactive plasma-like medium:

$$\begin{aligned}\varepsilon^l = 1 &- \frac{\omega^2_{Le}}{\omega^2}\frac{k^2_z}{k^2} - \frac{\omega^2_{Le}}{\omega^2 - \Omega^2_e}\frac{k^2_x}{k^2} \\ &- \frac{\hbar^2\omega^2_{Le}}{4m^2}\frac{\dfrac{k^4_z}{\omega^4} + \dfrac{k^2_x}{(\omega^2-\Omega^2_e)^2} + \dfrac{2k^2_z k^2_x}{\omega^2(\omega^2-\Omega^2_e)}}{1 - \dfrac{\hbar^2 k^2}{4m^2}\left(\dfrac{k^2_z}{\omega^2} + \dfrac{k^2_x}{\omega^2 - \Omega^2_e}\right)} = 0.\end{aligned} \tag{13}$$

From (13), we find the following spectra:

$$\omega^2 = \begin{cases} \dfrac{\hbar^2 k^4_z}{4m^2} + \omega^2_{Le} + \Omega^2_e, \\ \Omega^2_e \dfrac{\dfrac{\hbar^2 k^4_z}{4m^2} + \omega^2_{Le}\dfrac{k^2_z}{k^2}}{\dfrac{\hbar^2 k^4_z}{4m^2} + \omega^2_{Le} + \Omega^2_e}. \end{cases} \tag{14}$$

When the magnetic field strength is infinite, we find

$$\omega^2 = \frac{\hbar^2 k^2_z}{4m^2}k^2 + \omega^2_{Le}\frac{k^2_z}{k^2}. \tag{15}$$

This shows that, in a cold plasma-like medium, the pressure due to the quantum oscillation acts like the kinetic pressure in warm plasmas.

4. Surface waves on semibounded plasma-like media

Here we consider the quantum dispersion effect on the frequency spectrum of surface waves propagating on a semibounded cold plasma-like medium. In the classical case, the spectrum of surface waves has already been obtained [16,17]. It is obvious that the character of surface waves depends essentially on the properties of the plasma surface. As we are interested in solid state plasmas, the structure of surface waves is determined by the crystal lattice surface, where it is assumed that the plasma has a sharp surface [16,17].

4.1. Unmagnetized semibounded plasmas

As with the quantum effect on the frequency spectrum of surface waves, for an isotropic plasma-like medium, assuming the Z axis to be along the surface of the plasma medium and the X axis to be across it, we find the following dispersion

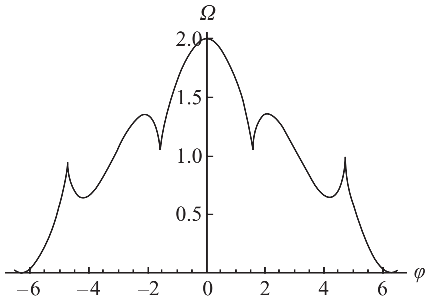

Figure 1. Plot of $\Omega = \omega/\omega_{Le}$ as a function of φ in one period.

equation for surface waves [16,17]:

$$\sqrt{k_z^2 - \frac{\omega^2}{c^2}} + \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{dk_x}{k^2}\left[\frac{k_z^2}{\varepsilon^l(\omega, k)} - \frac{k_x^2\omega^2}{k^2c^2 - \omega^2\varepsilon^{\mathrm{tr}}(\omega, k)}\right] = 0. \tag{16}$$

Substituting (1) into (16), we find

$$\sqrt{k_z^2c^2 - \omega^2}\left(1 - \frac{\omega_{Le}^2}{\omega^2}\right) - \frac{\omega_{Le}^2}{\omega^2}|\cos\varphi|^{1/2}\cos\frac{\varphi}{2}|k_z|c + \sqrt{k_z^2c^2 + \omega_{Le}^2 - \omega^2} = 0, \tag{17}$$

where $\tan^2\varphi = (\omega_{Le}^2 - \omega^2)/(\hbar^2k_z^4/4m^2)$. From (17), we find the following spectra:

$$\omega^2 = \begin{cases} \dfrac{\omega_{Le}^2}{2}\left[1 + |\cos\varphi|^{1/2}\cos\dfrac{\varphi}{2}\right] \ll k_z^2c^2, \\ k_z^2c^2\left[1 - \dfrac{1}{\sqrt{2}}\dfrac{mc^2}{\hbar\omega_{Le}}\dfrac{k_z^2c^2}{\omega^2} - \cos\varphi\cos^2\dfrac{\varphi}{2}\right] \ll \omega_{Le}. \end{cases} \tag{18}$$

As can be seen, for long waves, quantum oscillation plays a very small role, and consequently one can neglect it. But, for short waves, the quantum effect increases the frequency of surface waves from the classical value $\omega_{Le}/\sqrt{2}$, and thus manifests itself. The behavior of the upper equation in (18) is illustrated in Fig. 1.

If we consider only longitudinal surface waves, i.e. go to the limit $c \to \infty$, from the dispersion equation (17) and under the condition $\omega_{Le}^2 \gg \hbar^2k_z^4/4m^2$ (near to the classical limit), we obtain

$$\omega^2 = \frac{\omega_{Le}^2}{2}\left[1 + \frac{1}{2^{1/4}}\left(\frac{\hbar k_z^2}{2m\omega_{Le}}\right)^{1/2}\right]. \tag{19}$$

It is clear that the quantum oscillation affects the classical surface wave spectrum, i.e. it extends the classical surface wave spectrum to the region $\omega > \omega_{Le}/\sqrt{2}$. Therefore, comparing this quantum correction with the classical spectrum of surface waves, we conclude that, under the condition $\hbar\omega_{qe} \gg E_{Fe}$, cold quantum hydrodynamics can be used. The behavior of (19) is illustrated in Fig. 2.

4.2. Magnetized semibounded plasmas

In this subsection, we consider the previous case in the presence of an external magnetic field. It should be noted that for the classical case, in this circumstances, there is no any surface wave. As with the quantum surface wave, in the presence of an external magnetic field another situation arises. Making use of (16) in the limit $c \to \infty$ and substituting (13) into it, we find the following spectrum for surface

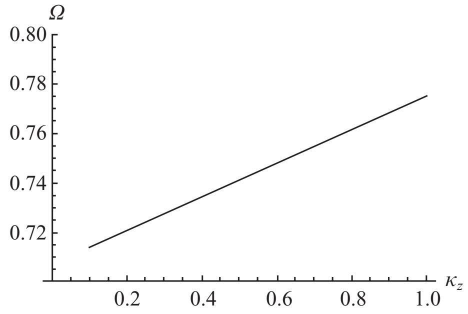

Figure 2. Plot of $\Omega = \omega/\omega_{Le}$ as a function of $\kappa_z = 10^7 k_z$.

waves in the presence of an infinitely strong magnetic field:

$$\omega^2 = 2\omega_{Le}\frac{\hbar}{2m}k_z^2. \tag{20}$$

However, this spectrum is not valid because it contradicts the gas approximation. Therefore, surface waves also do not appear in the presence of the magnetic field.

5. Plasma-like layers

Generalizing earlier work [16,17], we now study the quantum oscillation effect in the frequency spectrum of a thin plasma layer with thickness a. Here we suppose that the geometry of this layer is similar to what was used in the semibounded case in the previous section.

5.1. Longitudinal surface modes

First, we investigate only longitudinal modes. Starting from the dispersion equation for longitudinal surface wave in a plasma-like medium [16,17],

$$1 + \frac{2}{a|k_z|}\sum_{n=0}^{\infty}{}'[1 \pm (-1)^n]\frac{k_z^2}{k^2\varepsilon^l(\omega, k)} = 0, \tag{21}$$

and substituting (1) into it, we find the following dispersion relation for symmetric (with sign +) and antisymmetric (with sign −) modes, respectively:

$$\begin{aligned}1 &+ \frac{2a|k_z|}{\pi^2}\left\{\frac{\omega^2}{\omega'^2}\left[\frac{1}{\eta^2} + \frac{\pi}{2\eta}\left(\coth\pi\eta + \frac{1}{\sinh\pi\eta}\right)\right]\right.\\ &- \frac{\omega_{Le}^2}{2\omega'^2}\left[\frac{1}{\xi_1^2} + \frac{1}{\xi_2^2} + \frac{\pi}{2\xi_1}\left(\coth\pi\xi_1 + \frac{1}{\sinh\pi\xi_1}\right)\right.\\ &+ \left.\left.\frac{\pi}{2\xi_2}\left(\coth\pi\xi_2 + \frac{1}{\sinh\pi\xi_2}\right)\right]\right\} = 0,\end{aligned} \tag{22}$$

$$\begin{aligned}1 + \frac{2a|k_z|}{\pi^2}&\left\{\frac{\pi}{2\eta}\left(\coth\pi\eta - \frac{1}{\sinh\pi\eta}\right) - \frac{\omega_{Le}^2}{2\omega'^2}\left[\frac{\pi}{2\xi_1}\left(\coth\pi\xi_1 - \frac{1}{\sinh\pi\xi_1}\right)\right.\right.\\ &+ \left.\left.\frac{\pi}{2\xi_2}\left(\coth\pi\xi_2 - \frac{1}{\sinh\pi\xi_2}\right)\right]\right\} = 0,\end{aligned} \tag{23}$$

https://doi.org/10.1017/S0022377802001666 Published online by Cambridge University Press

where

$$\eta^2 = \frac{a^2 k_z^2}{\pi^2}, \qquad \xi_{1,2}^2 = \frac{a^2 k_z^2}{\pi^2}\left(1 \pm \sqrt{\frac{\omega'^2}{\beta k_z^4}}\right),$$

$$\beta = \frac{\hbar^2}{4m^2}, \qquad \omega'^2 = \omega^2 - \omega_{Le}^2, \tag{24}$$

and the prime on the summation symbol means that for $n = 0$ it should be divided by 2.

We now study (22) and (23) in different frequency limits to obtain the frequency spectrum of a thin plasma-like layer where $\eta \ll 1$, i.e. where the surface effect is important. At first it should be noted that when $\eta \gg 1$, we return to the frequency spectrum of the semibounded case. Therefore, we assume that $\eta \ll$ and $\xi_{1,2}^2 \ll 1$. In this case, we find the following frequency spectrum for symmetric modes:

$$\omega^2 = \frac{\hbar^2 k_z^4}{4m^2} + \omega_{Le}^2 \frac{a|k_z|}{4}. \tag{25}$$

As with antisymmetric modes, due to the quantum correction terms, no valid frequency spectrum can be obtained. In other words, the quantum effect prevents antisymmetric modes from appearing.

We now go to the other limit where $\xi_{1,2}^2 \gg 1$. In this case, we find the classical result [17]:

$$\omega^2 = \omega_{Le}^2 \frac{a|k_z|}{4} \tag{26}$$

for symmetric longitudinal modes, and again antisymmetric modes cannot appear.

5.2. *Transverse surface modes*

Finally, we study the general case with the dispersion equation [16,17]

$$\sqrt{k_z^2 - \frac{\omega^2}{c^2}} + \frac{2}{a}\sum_{n=0}^{\infty}{}' \frac{[1 \pm (-1)^n]}{k_z^2 + \dfrac{n^2\pi^2}{a^2}} \left[\frac{k_z^2}{\varepsilon^l(\omega, k)} - \frac{\dfrac{n^2\pi^2}{a^2}\omega^2}{k_z^2 c^2 + \dfrac{n^2\pi^2}{a^2}c^2 - \omega^2 \varepsilon^{\mathrm{tr}}(\omega, k)}\right] = 0. \tag{27}$$

Making use of this equation, we can investigate transverse modes as well. Substituting ε^l and $\varepsilon^{\mathrm{tr}}$ from (1) into (27), we find that, due to the presence of the last term, showing transverse modes, only transverse antisymmetric modes $\omega = k_z c$ may appear, apart from longitudinal modes. Therefore, we find that in this case, only longitudinal symmetric modes with spectra (25) and (26) and a transverse antisymmetric mode $\omega = k_z c$ may appear. Moreover, we can conclude that the presence of the quantum effect prevents the formation of transverse symmetric modes. Consequently, any significant relation between the quantum dispersion effect and boundedness of a plasma-like layer has no basis in our linear approximation.

6. Summary and conclusions

Making use of the dielectric permittivity of a solid state plasma obtained from a linearized quantum hydrodynamic equation and taking into account the quantum spatial dispersion of free charged particles, we have obtained a quantum collective effect in the frequency spectrum of volume and surface waves on semibounded plasma-like media and thin plasma-like layers in the presence and absence of an external magnetic field. In addition, we have found that in a cold plasma-like medium the pressure related to the quantum oscillation acts as a kinetic pressure. Furthermore, a new volume ion-acoustic type wave has been obtained with a quadratic dependence on the wavenumber in the long-wavelength limit. Moreover, it has been shown that the quantum oscillation affects the surface waves spectrum and extends it to a wider frequency region. Also, it has been found that quantum oscillation prevents antisymmetric surface modes from appearing. Finally, it has been shown that there is no significant relation between the quantum dispersion effect and the boundedness of a plasma-like medium in the linear approximation.

References

[1] Bohm, D. and Pines, D., *Phys. Rev.* **82**, 625 (1951).
[2] Bohm, D. and Pines, D., *Phys. Rev.* **85**, 338 (1952).
[3] Ritchie, R. H., *Phys. Rev.* **114**, 644 (1959).
[4] Tung, C. J. and Ritchie, R. H., *Phys. Rev.* **B16**, 256 (1977).
[5] Arista, N. R. and Brandt, W., *Phys. Rev.* **A23**, 1898 (1981).
[6] Mehlhorn, T. A., *J. Appl. Phys.* **52**, 6522 (1981).
[7] Shokri, B. and Rukhadye, A. A., *Phys. Plasmas* **6**, 4467 (1999).
[8] Shokri, B. and Rukhadze, A. A., *Phys. Plasmas* **6**, 3450 (1999).
[9] Klimantovich, Yu. L. and Silin, V. P., *Sov. Phys. Dokl.* **85**, 49 (1952).
[10] Lindhard, J., *Kgl. Danske Videnskab. Selskab. Mat. Fys. Medd.* **28**, 8 (1954).
[11] Pines, D. and Nozieres, F., *Theory of Quantum Liquids*. New York: Benjamin (1966).
[12] Bonitz, M., *Quantum Kinetic Theory*. Stuttgart: Teubner, 1998.
[13] Kuzelev, M. V. and Rukhadze, A. A., *Sov. Phys. Usp.* **6**, 42 (1999).
[14] Alexandrov, A. F., Bogdenkovich, L. S. and Rukhadze, A. A., *Principles of Plasma Electrodynamics*. Heidelberg: Springer-Verlag, 1984.
[15] Liboff, R. L., *Kinetic Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1990.
[16] Rukhazde, A. A. and Shokri, B., *Physica Scripta* **57**, 127 (1998).
[17] Shokri, B., *Phys. Plasmas* **7**, 3867 (2000).