



# Relativistic self-focusing in the interaction of laser beam and plasma with periodical density ripple

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## Research Article

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### Abstract

In the paper, relativistic self-focusing in the interaction of laser beam and plasma with periodical density ripple has been studied by the applied WKB approximation and higher-order paraxial theory. The result shows that under the influence of relativistic nonlinear effect, the dielectric function shows the fierce oscillational variation with similar periodicity, which then leads to the intense relativistic beam self-focusing along the propagation distance, such self-focusing also presents similar periodic variation. Besides, in the plasma with periodical density ripple, the initial density and the density ripple amplitude have obvious influence on self-focusing. When the two factors increase, then there will be more strength self-focusing. Choosing the appropriate initial density and the periodic density parameter is benefit to the formation of the more stable self-focusing.

### Introduction

Self-focusing generally refers to a nonlinear phenomenon which is caused by the incident high-intensity laser under the action of an electric field, as the intensity of the light increases, and the refractive index of the plasma increases at the same time, thus causing the outer profile of the laser beam to deflect toward the center. Previous researches have shown many effects in self-focusing (Gao and Shim, 2019; Kovalev and Bychenkov, 2019; Shao *et al.*, 2019) such as the coupling efficiency (Kodama *et al.*, 2001), fluid instability (Srinivasan *et al.*, 2019), and other nonlinear processes. Self-focusing (Simmons and Godwin, 1983) is considered as one of the most important issues in the total designing, engineering advancement such as direct-drive laser fusion (Watkins and Kingham, 2018; Gopalaswamy *et al.*, 2019), the research of which is of vital significance both in theory and practical applications.

To date, there have been four types of formation mechanisms (Sharma *et al.*, 2003) of self-focusing from discovery and which have been confirmed: ponderomotive self-focusing (Brandi *et al.*, 1993; Aggarwal *et al.*, 2015b; Patil *et al.*, 2018; Rawat and Purohit, 2019), thermal self-focusing (Craxton and McCrory, 1984), resonance self-focusing (Joshi *et al.*, 1982; Gill and Saini, 2007; Zare *et al.*, 2015), and relativistic self-focusing (Sprangle *et al.*, 1987; Hora *et al.*, 2000). Different formation mechanisms correspond to different laser–plasma interaction (LPI) systems, and a large number of experimental studies have shown that the mechanism of self-focusing often appears two or more and does not appear alone (Lu *et al.*, 2006). The different approaches to analyze the contributions of self-focusing have been reported such as paraxial-ray theory (Lam *et al.*, 1977), variational approach (Wilson *et al.*, 2019), and source-dependent expansion method (Malekshahi *et al.*, 2014). Alexopoulos and Uslenghi (1981) obtained the second-order Wentzel–Kramers–Brillouin (WKB) solutions for the TE and TM waves, the method of which then has been put forward in the research developments from Bud’ko and Liberman (1992). Based on the expansion of the eikonal and nonlinear constant, the ray theory could expand the distance from the axis to the beam up to square term. Then, Gill *et al.* (2010) developed the ray theory to the expansion of the distance up to the higher-order term.

The formation plasma of intense laser and matters interaction, the density of plasmas mainly exhibits several forms such as the homogeneous manners (Fuchs *et al.*, 2000; Sen *et al.*, 2010; Kant *et al.*, 2012), inhomogeneous manners (Gahn *et al.*, 1999), and even exponential manners (Kemp *et al.*, 2008). Applied paraxial-ray theory, Wang *et al.* (2019) studied the nonlinear propagation characteristics of a Gaussian beam in collisionless plasma by the high-order paraxial-ray theory. Pathak *et al.* (2018) applied the paraxial axis approximation to study the laser dynamics in a laterally inhomogeneous plasma and its correlation with the wakefield acceleration and it was found that the nonuniformity of the plasma can lead to a stronger self-focusing. Kaur *et al.* (2017) studied the self-focusing and defocusing of the Hamiltonian hyperbolic cosine-type Gaussian laser beam (HChG) in an inhomogeneity corrugated density plasma. Tikhonchuk *et al.* (1997) studied the effect of laser light self-focusing in speckles on stimulated Brillouin scattering (SBS) in an inhomogeneous plasma.

Bornatici and Maj (2003) put forward the various methods for the description of paraxial wave beams propagating in weakly inhomogeneous media. Varshney *et al.* (2006) investigated the relativistic self-focusing of intense laser radiation in an axially inhomogeneous plasma. In the above studies, as the significant nonlinear effect, self-focusing must be taken into consideration.

According to the research results by Zhang *et al.* (2017) and Trtica and Gaković (2003), they have found that there are such plasmas similar to the plasma with periodical density ripple. Ripples could be produced by different mechanisms, the following references could be declared, Liu and Tripathi (1995) proposed a mechanism for short-wavelength electromagnetic wave generation in a periodic dielectric material. For a moderately energetic electron beam, when passing through a periodic dielectric, then electromagnetic wave which will get amplified along the propagation, then the periodic density ripple would be formed. Another mechanism has been noted by Liu and Tripathi (2008) which noted that impinging on a gas jet target, the intense machining laser beam causes space periodic ionization of the gas and heats the electrons, the inhomogeneous plasma pressure leads to the redistribution of atomic density, when the intense laser pulse propagates after the certain time delay, there could produce the plasma density ripple. Thakur and Kant (2018) have presented stronger self-focusing of chirped pulse laser with exponential plasma density ramp profile in cold quantum magnetoplasma, and they observed the low value of the beam width parameter in the focal region. What is more, the significant contribution of the cold quantum magnetoplasma with the optimized value of magnetic field lead to enhanced self-focusing. Also, the intensity of the laser beam imparts a chief role in achieving the stronger self-focusing. Thakur *et al.* (2019) have revealed an exploration of self-focusing of Hermite-cosine-Gaussian laser beam in a collisionless plasma under relativistic nonlinearity. Wani and Kant (2016) have studied nonlinear propagation of Gaussian laser beam in an inhomogeneous plasma under plasma density ramp and derived the differential equation for beam width parameter by the parabolic wave equation approach under paraxial approximation. Aggarwal *et al.* (2015a) have investigated the propagation of circularly polarized quadruple Gaussian laser beam in underdense magnetoplasma. Their results revealed that the propagation of the quadruple laser beam can be studied in three different regimes, that is, steady divergence, oscillatory divergence, and self-focusing regime depending on the initial point. For an initial point not lying on the critical curve, the beam width parameter will either increase or decrease. The beam is more focused at lower intensity in both cases, namely extraordinary and ordinary mode.

In this paper, we have used high-order paraxial-ray theory and WKB approximation to study the relativistic laser self-focusing in plasmas with periodic density ripple, revealing some properties of self-focusing.

**The theory of laser beam propagation in plasma**

In the process of the interaction between laser and plasma, the wave equation that determines the laser propagation in the plasma is:

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2} \epsilon \vec{E} = 0 \tag{1}$$

where  $\vec{E}$  is the electric field of incident laser which may be the function of position and time,  $c$  refers to the light speed,  $\epsilon$  is the effective

dielectric constant of the plasma,  $\omega$  is the relativistic electron plasma frequency in the absence of electromagnetic beam and wave frequency.

For Gaussian laser beam, when the laser propagates along the axis in the  $z$ -direction, according to the wave equation, at the initial position  $z = 0$ , the laser intensity distribution can be expressed as  $EE^* = E_0^2 \exp(-r^2/r_0^2)$ , where  $r_0$  is the width of the initial laser beam,  $r$  is the radial component in the cylindrical coordinate system,  $E_0$  is the axial amplitude of the beam.

When the laser intensity reaches to  $10^{19}$  W/cm<sup>2</sup>, the relativistic nonlinear effect becomes very obvious at this time. When the value is lower than  $10^{19}$  W/cm<sup>2</sup>, there also exists the nonlinear effect, such as the electron mass begins to change significantly (Umstadter, 2003), changes in electronic mass will form the difference between and the rest mass of electron, and which further leads to the form of relativistic nonlinear effect. Under the effect of relativistic effect, the dielectric constant of the plasma (Walia *et al.*, 2017) can be expressed as follows:

$$\epsilon = \epsilon_0 \varphi(EE^*) \tag{2}$$

where  $\epsilon_0$  is the linear part and  $\varphi(EE^*)$  is the nonlinear part, the expression of  $\epsilon_0$  is  $\epsilon_0 = 1 - (\omega_{p0}^2/\omega^2) \exp(-\beta E_{00}^2/f^2)$ , and  $\varphi(EE^*) = [1 - N_e/(N_{0e}\gamma)](\omega_{p0}^2/\omega^2)$ ,  $\omega_{p0}$  and  $\omega$  are the relativistic electron plasma frequency in the absence of electromagnetic beam and wave frequency. Here,  $\omega_{p0}^2 = 4\pi N_{0e}e^2/(m_0\gamma)$  and  $\beta = e^2/(8m\omega^2 k_B T)$ , where  $e$  is the charge of an electron,  $m_0$  is the rest mass,  $N_{0e}$  is the initial entity of plasma electrons, and  $k_B$  is the Boltzmann constant, respectively.

Under the effect of intense laser, the relationship between the relativistic factor and light intensity has the following relationship:

$$\gamma = \left[ 1 + \frac{e^2}{c^2 m_0^2 \omega^2} EE^* \right]^{1/2} \tag{3}$$

According to the previous research results (Xia and Lin, 2012), under the influence of relativistic ponderomotive force, there is such a relationship between the plasma density and the plasma frequency along the axial direction in the process of laser and plasma interaction:  $N_e/N_{e0} = (\omega_p/\omega_{p0})^2$ , where  $N_e$  is the density of electron plasma under the influence of intense laser,  $\omega_p = \sqrt{N_e e^2 / (\epsilon_0 m_e)}$  is the plasma frequency in the presence of electromagnetic beam, which is related to the initial dielectric constant  $\epsilon_0$  and the electron mass  $m_e$  in it, under the influence of the intense laser, the  $\omega_p$  is relativistic.  $\omega_{p0}$  is the plasma frequency at  $z = 0$ .

According to the discovery by Zhang *et al.* (2017) and Trtica and Gaković (2003) above, the plasmas with periodical density ripple have been found. So, in this paper, based on their results, the plasma density with periodical distribution is assumed as sinusoidal and cosine profile:

$$N_e/N_{e0} = (\omega_p/\omega_{p0})^2 = C_1 + D_1 \sin(F_1 \zeta) \tag{4}$$

and

$$N_e/N_{e0} = (\omega_p/\omega_{p0})^2 = C_2 + D_2 \cos(F_2 \xi) \tag{5}$$

where  $C_1$  and  $C_2$  refer to the initial density without the influence of periodical density ripple,  $D_1$  and  $D_2$  refer to 2, and  $F_1$  and  $F_2$  refer to the frequency of periodic variation in density.

According to the wave equation [see Eq. (1)] in the plasma, the electric field change along the direction is assumed as:

$$\vec{E} = \vec{A}(x, y, z) \exp [i(\omega t - k_0 z)] \tag{6}$$

Substituting the value of  $\vec{E}$  to Eq. (1), one can get the result:

$$-k_0^2 - 2ik_0 A + \left( \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial z} \right) A = \frac{\omega^2}{c^2} \epsilon A \tag{7}$$

where  $A$  is a complex function of space and  $k_0 = \omega\sqrt{\epsilon_0}/c$  is the wave number which is referring to the electromagnetic beam in the plasma. Following Nanda *et al.* (2018),  $A$  can be written as follows:

$$A = A_0(r, z) \exp [ - ik_0 S_0(r, z) ] \tag{8}$$

where  $A_0$  and  $S_0$  are the real function of space,  $S_0 = S_2 r^2/r_0^2 + S_4 r^4/r_0^4$  and  $S_2 = r^2 f'(z)/(2f)$ ,  $S_2$  and  $S_4$  are the components of the eikonal contributions.

Applied higher-order paraxial-ray theory and WKB approximation, during the calculation, it used the dimensionless propagation parameter  $\xi = cz/(\omega r_0^2)$ , one can obtain the laser beam intensity, equations for controlling beam width parameters:

$$A_0^2 = \frac{E_{00}^2}{f^2} \left( 1 + \frac{a_2 r^2}{r_0^2 f^2} + \frac{a_4 r^4}{r_0^4 f^4} \right) \exp \left( - \frac{r^2}{r_0^2 f^2} \right) \tag{9}$$

where  $A_0^2$  refers to the square of the amplitude in optical electric field, which reflects the beam intensity as well,  $f$  is the dimensionless beam width parameter of laser beam, and  $\alpha = \alpha_0 A_0^2$ . In Eq. (10), the first term in right-hand side is the discrete of the differential equation, which is the linear part related to the initial beam width  $f$  along the laser propagation. While the second term is the convergence term which shows the nonlinearity, reflecting the characters of the periodical variations of the plasma, and governed by the term in right hand of Eqs (4) and (5).

Following earlier investigators (Kaur *et al.*, 2018), using WKB approximation and higher-order paraxial theory, and then separating the real and imaginary parts for Eq. (7), the eikonal contributions and the equation for the coefficient  $a_2$  and  $a_4$  have been

obtained:

$$\frac{da_2}{d\xi} = - \frac{16S_4 f^2}{r_0^2} \tag{12}$$

$$\frac{da_4}{d\xi} = \frac{8S_4 f^2}{r_0^2} - \frac{24a_2 S_4 f^2}{r_0^2} \tag{13}$$

where  $a_2$  and  $a_4$  are indicative of the departure of the beam from the Gaussian nature.

### Numerical results and analysis

In this paper, applied the fourth order of Runge–Kutta method to solve Eqs (6) and (9)–(13). Some parameters are given as follows: beam intensity  $I \approx 10^{19}$  W/cm<sup>2</sup>, frequency  $\omega \approx 10^{15}$  rad/s, wavelength  $\lambda = 0.5$   $\mu$ m, and the initial electron density  $N_{e0} \approx 10^{21}$  cm<sup>-3</sup>. The parameters above could further be used to obtain the functions of the spot size which the function  $I = I(r, z) = I_0(\omega_0^2/\omega^2(z)) \exp [ - 2r^2/\omega^2(z) ]$  is to be involved, where  $r$  is the radial distance from the center axis of the beam,  $\omega_0$  is the waist size, and  $\omega(z)$  is the spot size of the beam. Besides, beam diameter is defined as the distance across the center of the beam for which the  $I = I_{\max}/e^2$ . The spot size of the beam is the radial distance from the center of  $I = I_{\max}$  to the  $I = I_{\max}/e^2$  points.

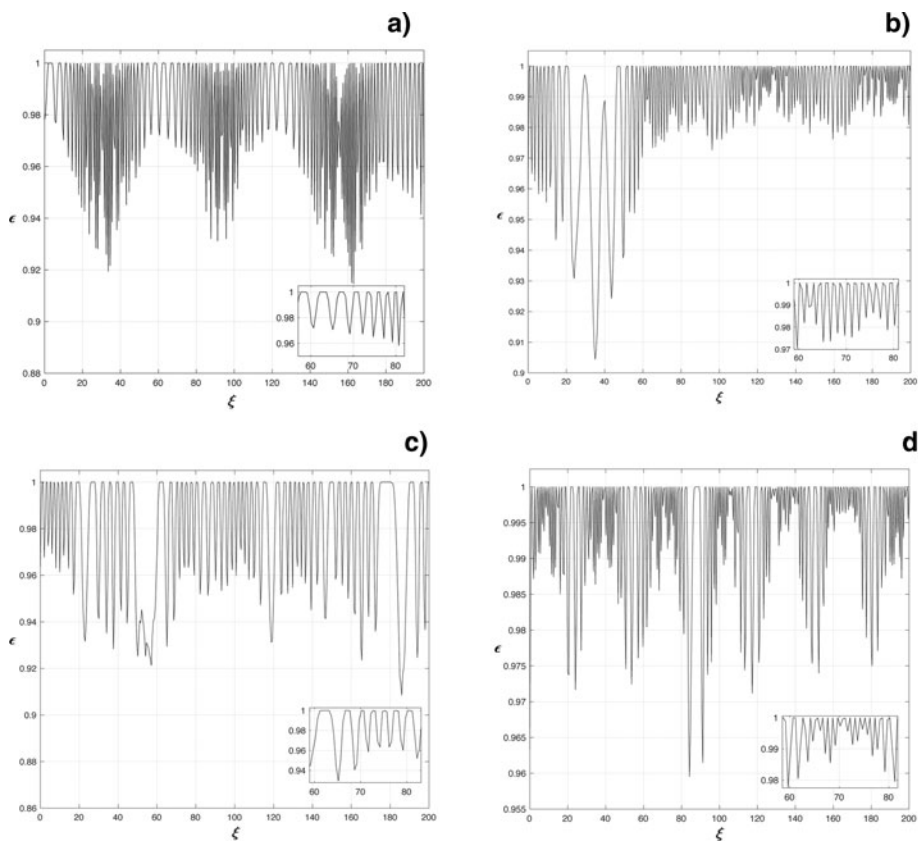
The effective dielectric constant in plasmas with cosine density and sinusoidal density ripple in different variations have been shown in Figure 1 that due to the relativistic nonlinearity, varia-

$$\frac{d^2 f}{d\xi^2} = \frac{(1 + 8a_4 - 3a_2^2 - 2a_2)c^2}{\omega_0^2 \epsilon_0 r_0^4 f^3} + \frac{\omega_p^2 \alpha}{\omega_0^2 \epsilon_0} \left[ \frac{a_2 - 1}{2\gamma^3 r_0^2 f^3} - \frac{4c^2}{\omega_p^2 r_0^2 f^3} \left( \frac{2a_4 - 2a_2 + 1}{\gamma^2 r_0^2 f^2} - \frac{\alpha a_2^2 - 2\alpha a_2 + \alpha}{\gamma^4 r_0^4 f^4} \right) \right] \tag{10}$$

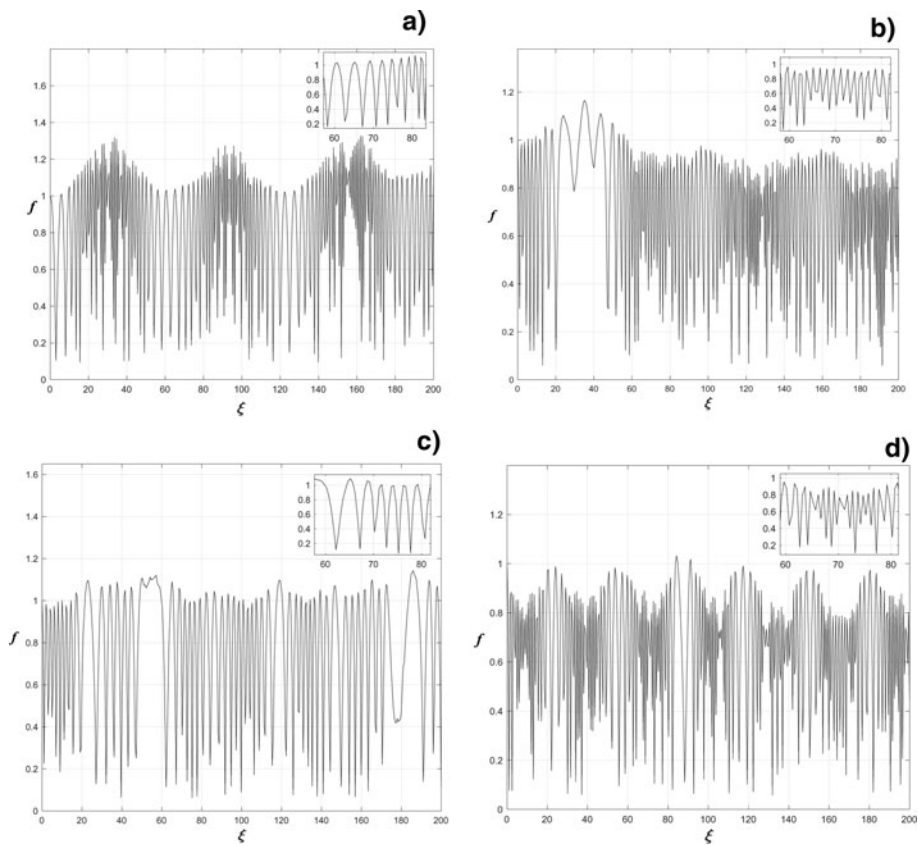
tions in plasma frequency and electron energy result in redistribution of electron density and affects the dielectric constant. On the one hand, under the influence of relativistic nonlinear effect, dielectric function has shown the sharp oscillating change with the similar periodical variation. On the other hand, with the increase of initial density (C) and the density ripple amplitude (D), dielectric function presents the fiercer oscillating variation.

It can be seen from the enlargement of the details that the dielectric function shows the faster variation within the same distance. In addition, when the plasma density shows cosine distribution (see Fig. 1a and 1b), with the increase of initial density, the envelope of dielectric function oscillation presents the poor periodicity. And for the sinusoidal distribution (see Fig. 1c and 1d), with the increase of the density ripple amplitude, the

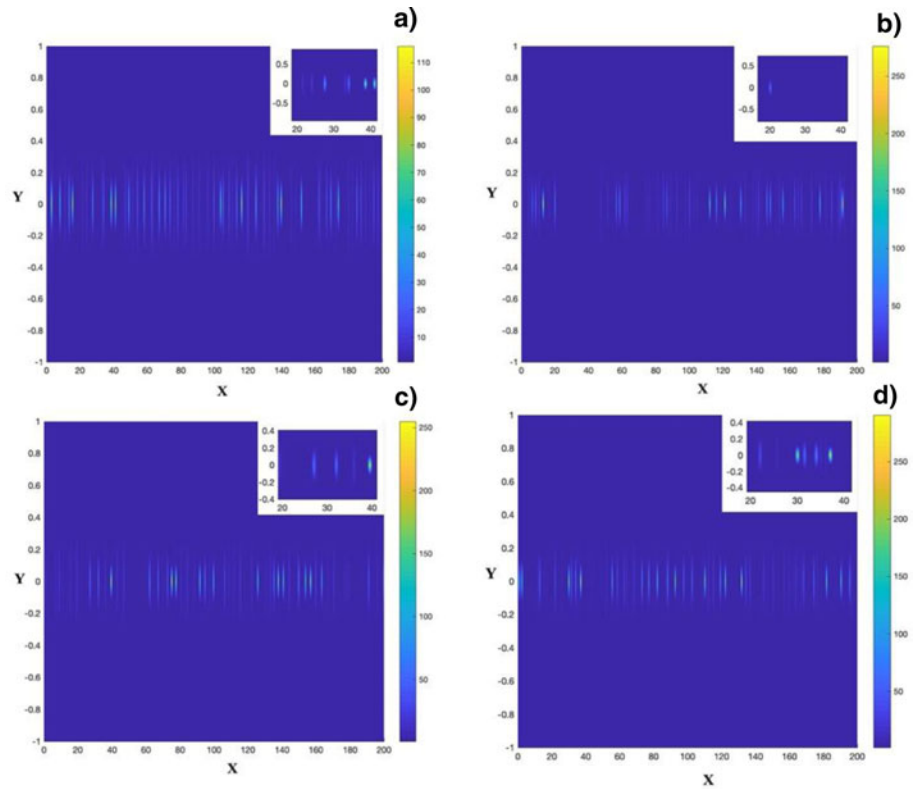
$$\frac{dS_4}{d\xi} = \frac{(a_2^3 - a_2^2 - 7a_2 a_4)c^2}{\omega_0^2 \epsilon_0 r_0^2 f^6} - \frac{\omega_p^2 r_0^4}{4\omega_0^2 \epsilon} \left\{ \frac{3\alpha^2(a_2 - 1)^2}{4\gamma^5 r_0^4 f^8} - \frac{\alpha(2a_4 - 1)}{2\gamma^3 r_0^2 f^6} + \frac{2\alpha c^2}{\omega_p^2 r_0^2 f^4} \left[ \frac{(9a_2 - 18a_4 - 3)}{\gamma^2 r_0^4 f^4} + \frac{\alpha(a_2 - 1)(17a_2 - 18a_4 - 8)}{\gamma^4 r_0^4 f^6} + \frac{6\alpha^2(a_2 - 1)^3}{\gamma^6 r_0^4 f^8} \right] \right\} \tag{11}$$



**Fig. 1.** Variation of axial dielectric function  $\epsilon$  with dimensionless propagation distance  $\xi$  in different plasmas. (a)  $N_e/N_{0e} = 1 + 0.2 \cos(0.1\xi)$ , (b)  $N_e/N_{0e} = 2 + 0.2 \cos(0.1\xi)$ , (c)  $N_e/N_{0e} = 2 + 0.1 \sin(0.2\xi)$ , and (d)  $N_e/N_{0e} = 2 + 0.3 \sin(0.2\xi)$ .



**Fig. 2.** Variation of beam width parameter  $f$  with dimensionless propagation distance  $\xi$  in different plasmas. (a)  $N_e/N_{0e} = 1 + 0.2 \cos(0.1\xi)$ , (b)  $N_e/N_{0e} = 2 + 0.2 \cos(0.1\xi)$ , (c)  $N_e/N_{0e} = 2 + 0.1 \sin(0.2\xi)$ , and (d)  $N_e/N_{0e} = 2 + 0.3 \sin(0.2\xi)$ .



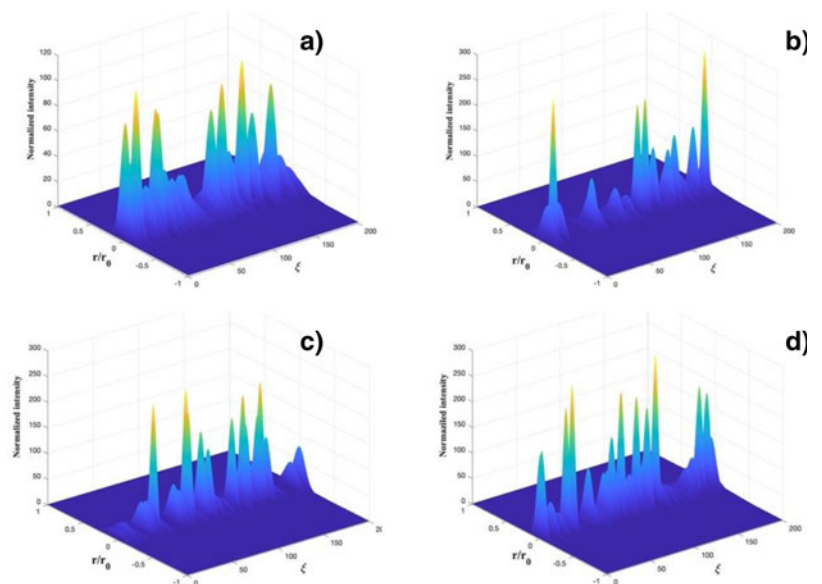
**Fig. 3.** The spatial plots of the normalized pulse intensity, the  $X$ - and  $Y$ -axes present the transverse pulse width and radial width, respectively, the bar shows the variation in the normalized intensity. (a)  $N_e/N_{0e} = 1 + 0.2 \cos(0.1\xi)$ , (b)  $N_e/N_{0e} = 2 + 0.2 \cos(0.1\xi)$ , (c)  $N_e/N_{0e} = 2 + 0.1 \sin(0.2\xi)$ , and (d)  $N_e/N_{0e} = 2 + 0.3 \sin(0.2\xi)$ .

envelope of dielectric function oscillation presents the better periodicity. And what is more, in such conditions, the frequency increases as well, the variations in **Figure 1d** is the same as **Figure 1a** and **1b** in comparison.

**Figure 2** shows the variation of the beam width along the dimensionless propagation distance. Under the influence of the relativistic nonlinear effect, the beam width shows the obvious oscillating variation with fiercer self-focusing, the envelope of which also presents the similar periodicity. With the increase of initial density and the density ripple amplitude, the variation of beam width becomes fiercer, and the variation frequency increases as well. From the enlargement of the

details, the variation of beam width becomes faster, which implies beam self-focusing also becomes faster. Moreover, for the plasma density showing cosine distribution (see **Fig. 2a** and **2b**), when the initial density decreases, the periodical variation of beam self-focusing becomes better. However, for the sinusoidal distribution (see **Fig. 2c** and **2d**), with the density ripple increasing, the envelope of beam width parameter presents the better periodicity, which means the more stable self-focusing.

**Figure 3** depicts the spatial plots of the normalized pulse intensity profile in two kinds of plasmas. For the plasma density with cosine and sinusoidal distribution, with the increase of the initial



**Fig. 4.** Variation of normalized beam intensity with dimensionless propagation distance  $\xi$  and radial distance  $r$ . (a)  $N_e/N_{0e} = 1 + 0.2 \cos(0.1\xi)$ , (b)  $N_e/N_{0e} = 2 + 0.2 \cos(0.1\xi)$ , (c)  $N_e/N_{0e} = 2 + 0.1 \sin(0.2\xi)$ , and (d)  $N_e/N_{0e} = 2 + 0.3 \sin(0.2\xi)$ .

density and the density ripple, some of the top values of laser intense increase, which shows the further compression of beam and stronger self-focusing. For the cosine distribution, when the initial density increase, the number of light spot increases in the same propagation distance. What is more, the number of light spot also increases for the plasma density with the sinusoidal distribution.

Figure 4 shows the variation of the dimensionless axial beam intensity for different plasmas assumed above. Along with the propagation distance, the beam presents obvious self-focusing and filamentation due to the relativistic effect. The main differences are the amplitude of beam intensity and numbers of beam filamentation. Comparing with Figure 4a and 4b, with the decrease of the initial density, the intensity of the light is greatly increased, which implies the enhanced self-focusing, and the number of filaments is also slightly decreased. Obviously, the self-focusing effect is enhanced over a longer distance. In Figure 4c and 4d, with the increasing density ripple amplitude, the number of filaments of the beam is increased, and the stability of self-focusing is enhanced.

## Conclusion

In the paper, the self-focusing in LPI system has been studied, where the plasma with periodical density ripple and laser beam with relativistic characters have been chosen. Applied WKB approximation and higher-order paraxial theory, dielectric function and relativistic laser wave equations have been attained. Under the influence of relativistic nonlinear effect, the laser beam presents the intense relativistic self-focusing with a similar periodic variation. During the formation of the relativistic self-focusing, it is shown that the plasma density profile has the important effect on self-focusing. In the plasma with periodic density ripple, both the increase of the initial density and the amplitude of density ripple could lead to the fiercer variation of the dielectric function oscillation, the beam width and its variation frequency as well, which further leads to the strength of self-focusing. The former is more likely to destroy the current self-focusing, the latter is more likely to form the stable self-focusing. In theory and practical applications, if one needs to apply the self-focusing effect, then plasmas with property initial density and the density ripple amplitude value would be selected, so that the better periodicity and more stable self-focusing could be attained.

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